Examiners' Report: Preliminary Examination in Mathematics and Philosophy Trinity Term 2018

November 14, 2018

Part I

A. STATISTICS

(1) Numbers and percentages in each class

See Table 1. Overall, 14 candidates were classified.

Table 1: Numbers in each class (Preliminary Examination)

	Numbers					Percentages %				
	2018	(2017)	(2016)	(2015)	(2014)	2018	(2017)	(2016)	(2015)	(2014)
Distinction	6	4	7	6	4	42.86	23.53	50	42.86	30.77
Pass	7	13	4	7	8	50	76.47	28.57	50	61.54
Partial Pass	1	0	3	1	1	7.14	0	21.43	7.14	7.69
Fail	0	0	0	0	0	0	0	0	0	0
Total	14	17	14	14	13	100	100	100	100	100

(2) Vivas

No vivas were given.

(3) Marking of Scripts

In Mathematics, all scripts were single marked according to a pre-agreed marking scheme which was strictly adhered to. There is an extensive checking process. In Philosophy, all scripts were single marked except for failing scripts, which were double-marked.

B. New examining methods and procedures

There were no new examining methods or procedures this year. This was the sixth year of the new examining structure following the change in 2013 from Honour Moderations to Preliminary Examination.

C. Changes in examining methods and procedures currently under discussion or contemplated for the future

There are no changes under discussion.

D. Notice of examination conventions for candidates

The Notice to Candidates, containing details of the examinations and assessment, including the Examination Conventions, was issued to all candidates at the beginning of Trinity term. All notices and examination conventions in full are on-line at https://www.maths.ox.ac.uk/members/students/undergraduate-courses/examinations-assessments/examination-conventions.

Part II

A. GENERAL COMMENTS ON THE EXAMINATION

Timetable

The examinations began on Monday 18th June at 2.30pm and ended on Friday 22nd June at 12:30pm.

B. EQUAL OPPORTUNITIES ISSUES AND BREAKDOWN OF THE RESULTS BY GENDER

The breakdown of the final classification by gender is as follows:-

Table 2: Breakdown of results by gender

Class	Number								
	2018			2017			2016		
	Female	Male	Total	Female	Male	Total	Female	Male	Total
Distinction	1	5	6	2	2	4	1	6	7
Pass	4	3	7	5	8	13	1	3	4
Partial Pass	1	0	1	0	0	0	1	2	3
Fail	0	0	0	0	0	0	0	0	0
Total	6	8	14	7	10	17	3	11	14
Class	Percentage								
	2018			2017			2016		
	Female	Male	Total	Female	Male	Total	Female	Male	Total
Distinction	16.67	62.5	42.86	28.57	20	25.53	33.33	54.55	50
Pass	66.67	37.5	50	71.43	80	76.47	33.33	27.27	28.57
Partial Pass	16.67	0	7.14	0	0	0	33.33	18.18	21.43
Fail	0	0	0	0	0	0	0	0	0
Total	100	100	100	100	100	100	100	100	100

C. DETAILED NUMBERS ON CANDIDATES' PERFORMANCE IN EACH PART OF THE EXAMINATION

Mathematics I

	Maths and Philosophy		Single School	
Question	Mean	Std Dev	Mean	Std Dev
Q1	12.42	4.14	12.84	3.77
Q2	10.15	4.12	12.20	4.41
Q3	11.83	4.71	14.40	4.02
Q4	10.55	3.78	12.23	4.02
Q5	10.00	3.08	10.57	3.48
Q6	10.63	3.38	11.45	4.66
Q7	16.36	2.38	15.75	4.21

Mathematics II

	Maths and Philosophy		Single School	
Question	Mean	Std Dev	Mean	Std Dev
Q1	14.23	1.54	13.66	2.23
Q2	11.58	3.60	12.58	3.83
Q3	15.33	3.79	11.43	4.71
Q4	10.79	3.29	11.43	4.54
Q5	8.83	5.56	9.77	5.74
Q6	6.75	3.15	7.34	4.12
Q7	13.00	6.08	12.84	5.64

Mathematics III(P)

	Maths and Philosophy		Single School	
Question	Mean	Std Dev	Mean	Std Dev
Q1	8.85	4.95	12.16	5.14
Q2	16.25	4.35	13.97	5.10
Q3	11.18	3.89	14.79	3.74
Q4	13.88	5.34	15.31	4.18
Q5	13.13	3.18	15.50	3.11
Q6	10.92	4.21	11.95	4.78

Elements of Deductive Logic

AvgUSM	StdDevUSM
68.14	12.01

Introduction to Philosophy

AvgUSM	StdDevUSM
61.36	9.64

D. COMMENTS ON INDIVIDUAL PAPERS

See the Mathematics report for reports on the following papers:

Mathematics I

Mathematics II

Mathematics III(P)

Report on Elements of Deductive Logic

This report on the EDL paper covers students in Computer Science & Philosophy, Maths & Philosophy, and Physics & Philosophy.

Comments on single questions

Question 1 (17 answers, mean 16.6, std dev 3.6) This was a reasonably popular question, and many of the answers offered were strong. Part (b) was intended as a gentle introduction to the key concepts, and most candidates managed to show that claim (i) is true and claims (ii) and (iii) false. Part (c) proved a bit trickier. Quite a few students slipped up on either the inductive step in (ci), and one or two on the deduction in (cii). The key to the former was to show that, if A_{i+1} falsifies some finite subset of Γ, then it is the case, not just that there is a finite subset Δ_1 of Γ that is falsified by the extension of A_i that agrees with A_i on all sentence letters other than p_{i+1} and assigns 0 to $p_i + 1$, i.e. A_{i+1} itself, but also that there is a finite subset Δ_2 of Γ that is falsified by the extension of A_i that agrees with A_i on all sentence letters other than p_{i+1} and assigns 1 to p_{i+1}. A contradiction with the inductive hypothesis that A_i doesn't falsify any finite subset of Γ can then be shown to follow. The deduction in (cii) only involves a few moves. Suppose Γ is not satisfiable. Then the union, A, of the A_is will be an $\mathcal{L}_1$-structure that assigns 0 to some formula ϕ in Γ. So let p_k be the first sentence in the enumeration p_1, p_2, ... such that, if p_i occurs in ϕ then $k \geq i$. Then A_k falsifies the finite subset $\{\phi\}$ of A. Using the result in (ci), it then follows that Γ is not finitely satisfiable.

Question 2 (20 answers, mean 16.0, std dev 4.4) This was another reasonably popular question, but while most answers to it were strong, one or two students fared quite badly on it. This was a bit surprising, as the material was covered in *The Logic Manual*, and is fairly straightforward. Some slipped up on part (a), mainly by failing to get the right answer in (aii) or by getting the right answer but failing to provide an adequate justification for it. It was enough to simply enumerate the different equivalence relations (there are five) and to explain why these and no other relations had the relevant feature. “Yes” was the answer to each of the remaining questions, except (cii) and (ciii), for both of which $D = \{a\}$ and $R = \{\langle a, b\rangle\}$ was a counterexample, and (di) and (dii), for both of which $D = \{a\}$ and $R = \{\langle b, a\rangle\}$ was a counterexample. Full marks for these required clear justifications for the answers given; merely providing an appropriate counterexample wasn’t enough.

Question 3 (27 answers, mean 15.7, std dev 4.4) This was the second most popular question, but the quality of the answers given was mixed. To get all three marks for part (a), students needed to provide a full recursive definition of the sentences of $\mathcal{L}_1'$. For part (b), some students offered a general characterisation of the notion of the dual of a connective, rather than definitions of each of the connectives of $\mathcal{L}_1'$; these only got partial credit. Part (c) also required a full recursive definition; again, some students offered a general characterisation of the notion of the dual of a sentence, and only got partial credit. Part (d) called for a proof by induction on the complexity of a sentence. Not all students attempted such a proof, and some of those who did didn’t cover all the relevant cases in the inductive step. Quite a few students got the right answer for part (ei): the sentence letters are the only negation free sentences of $\mathcal{L}_1'$ that are their own duals. But some tripped up on part (eii), mistakenly thinking that a sentence is logically equivalent to its dual iff it is *identical* to it, and inferring from the answer to (ei) that the answer is “no”. In fact, the answer is “yes”, as illustrated by the case of $P \wedge P$. On the whole, attempts at part (f) were to a good standard.

Question 4 (14 answers, mean 12.4, std dev 4.5) This was among the least popular questions, and most who attempted it struggled. While it only demanded knowledge of material covered in *The Logic Manual*, it was a much more demanding question than would be found in an *Introduction to Logic* paper. The formalization in part (a) should have been straightforward: the obvious strategy was to take $\forall x \exists y(Rxy \wedge Py)$ as the sole premise, $\exists x Rxx \vee \exists x \exists y(Px \wedge Py \wedge \neg x = y)$ as the conclusion, and offer the obvious dictionary, noting that one was treating ‘x is self-causing’ as ‘x has x as a cause’ and not attempting to formalize ‘x is a thing’, but instead assuming that all objects in the domain are things. But the subsequent proof in Natural Deduction was challenging. One strategy was to first prove an instance of the Law of Excluded Middle, $\exists x Rxx \vee \neg\exists x Rxx$, and then, using this, prove the conclusion via disjunction elimination. One of the subproofs, discharging $\neg\exists x Rxx$, would then employ existential elimination twice. The translation task in part (b) was also quite tough, with only (bi) being translatable into anything particularly elegant: ’The sum of one thing and a second is the same as the sum of the second and the first’ or, more briefly, ’Addition is commutative’. Quite a few students were tripped up by the fact that (biii) is false in its intended interpretation; it is, however, consistent, as is shown by the fact that it is true in a structure with $\{0, 1\}$ as its domain and in which the interpretation of R is $\{\langle 1, 0, 1\rangle\}$. Note that, as was announced in the exam, there is an unfortunate typo in (biii): the formula should have been $\neg\forall x \forall y(Ryxy \to \forall z(Rzxy \leftrightarrow Rzyx))$.

Question 5 (26 answers, mean 18, std dev 3.2) This was the third most popular question. Most answers to it were very strong. Two different answers to (ai) often cropped up: that Γ is inconsistent IFF, for any $\mathcal{L}_1$-sentence ϕ, $\Gamma \vdash \phi$, and that Γ is inconsistent IFF, for some particular contradiction ϕ, e.g. $P \wedge \neg P$, $\Gamma \vdash \phi$. Both were accepted, though the first is the officially correct answer. Some students attempted to answer part (b) by appealing to the fact that Natural Deduction underwrites redundancy (i.e., if $\Gamma \cup \{\phi\} \vdash \neg\phi$ then $\Gamma \vdash \neg\phi$) and double negation elimination (i.e., if $\Gamma \vdash \neg\neg\phi$ then $\Gamma \vdash \phi$) rather than by showing how the relevant proofs can be constructed directly. This was fine, so long as the relevant fact was stated clearly, and the proof was otherwise complete. Parts (c), (d), and (e) posed few problems, but some students misunderstood (f), reading the phrase 'for every maximally consistent Γ^+ such that $\Gamma \subseteq \Gamma+$' as taking wide scope over the biconditional. Read correctly, the toughest bit of the question was establishing the R-to-L direction of the claim, i.e. that if, for every maximally consistent Γ^+ such that $\Gamma \subseteq \Gamma^+$, $\phi \in \Gamma^+$, then $\Gamma \vdash \phi$. The key was to use the result in (b), and show that, on the assumption that the consequent of this conditional fails, $\Gamma \cup \{\neg\phi\}$ is consistent. There is therefore a maximally consistent set Γ^* such that $\Gamma \cup \{\neg\phi\} \subseteq \Gamma^*$, and for which it holds both that $\Gamma \subset \Gamma^*$ and that $\neg\phi \in \Gamma^*$. Since Γ^* is consistent, that means that $\phi \notin \Gamma^*$. So it's not the case that, for every maximally consistent Γ^+ such that $\Gamma \subseteq \Gamma^+$, $\phi \in \Gamma^+$. Read incorrectly, the R-to-L direction of the claim is in fact false. Partial credit was given to students who misunderstood the question in this way, so long as they explained how they were understanding the question, and tried to show the claim so understood is false.

Question 6 (28 answers, mean 16.1, std dev 4.2) This was the most popular question. It was generally well done. For full marks in part (b), it was not enough to merely state the Interpolation Theorem for both $\mathcal{L}_1$ and $\mathcal{L}_1^+$; students also needed to explain the difference between the two, i.e. that in the case of $\mathcal{L}_1^+$, there is an interpolant for $\phi \vDash \psi$ even if ϕ and ψ do not share any sentence letters in common (in which case, either ϕ is a contradiction, and $\bot$ is an interpolant, or ψ is a tautology, and $\top$ is an interpolant.) There were various possible answers to part (c). The crucial thing was to offer a genuine method for finding interpolants, which could be applied in practice, as opposed to an abstract definition of one. In demonstrating how the method worked for the entailment in question, it was enough to arrive at an interpolant; students did not need to simplify their answers. Part (d) was generally well done. For part (diii), some students showed that $\iota_1 \vDash \iota_2$ but neglected to show that ι_1 and ι_2 have any sentence letters in common. Some—perhaps pressed for time—tripped up in the final part, and either failed to offer an example at all, or offered examples that didn't succeed. (For one that does, let ϕ be $P \wedge Q \wedge R \wedge P_1$, ι and ι_1 be $P \wedge Q \wedge R$, ι_2 be P, and ψ be $P \vee Q \vee R$.) Part (e) posed few difficulties, with almost everyone showing that it *does* follow that ι is an interpolant for $\psi \vDash \phi$. (Since ι is an interpolant for $\phi \vDash \psi$, we have both that $\phi \vDash \iota$ and that $\iota \vDash \psi$. From the former, together with the fact that $\psi \vDash \phi$, it follows that $\psi \vDash \iota$. From the latter, again together with the fact that $\psi \vDash \phi$, it follows that $\iota \vDash \phi$.) A small number of students, however, seemed quite confused, and attempted proofs by induction in this part.

Question 7 (13 answers, mean 15.6, std dev 6.3) This was the least popular question, but attracted some strong answers—as well as one or two quite weak ones. The material was entirely drawn from *The Logic Manual*, but was not straightforward. M&P students generally fared better than others on this and Question 4, the other Natural Deduction question. Of the six claims in part (a), four were true and two false. The false claims were (iii) and (iv). Any structure in which $\|P\|$ is the empty set constituted a counter-example to (iii). For a counter-example to (iv), a structure with an infinite domain was required, on which $\|R\|$ was transitive and reflexive (to secure the truth of the premises) and also serial (to secure the falsity of the conclusion). The proof in (ii) required a couple of applications of disjunction elimination. The proof in (iv) required applying $\forall$-intro after an application of disjunction elimination, otherwise one would be left with an unwanted constant in an undischarged assumption. The proof in (vi) clearly required an application of existential elimination, as the main operator of the premise was an existential quantifier. In any such case, it is wise to follow the rule of thumb that existential elimination be used as the last step. Surprisingly, there were few, if any, completely satisfactory answers to part (b), which called for examples of unwanted "proofs" that would be allowed were the various restrictions to be lifted.

Question 8 (14 answers, mean 17.6, std dev 3.6) This was among the least popular questions, but attracted some excellent answers. The substance of the question was part (c). For full marks in (ci) students needed, not just to identify which connectives expressed monotonic functions and which didn't, but also to justify their choices; the monotonic ones are $\wedge$ and $\vee$. In (cii), by contrast, once one had determined that a connective was truth-functional, it was enough to simply correctly report whether or not it was monotonic. The first connective, which expresses the function that yields F for $\langle T, T\rangle$ and T otherwise, is not monotonic; the second, which yields T if A is T and F otherwise, is; the third is not truth-functional, yielding T for$\langle T, F\rangle$ in some cases but not others. Part (ciii) was a bit fiddly. What was needed was a proof by induction on the complexity of $\mathcal{L}_1$-sentences containing only $\wedge$ and $\vee$. The tricky bit was the inductive step. Let ϕ be an $\wedge$ and $\vee$-sentence containing $n+1$ occurrences of $\wedge$ and $\vee$ (and no occurrences of any other connective), and let f_ϕ be the function expressed by ϕ. Then ϕ is of the form $\psi \circ \chi$, where $\circ$ is either $\wedge$ and $\vee$, and ψ and χ are $\mathcal{L}_1$-sentences containing n or fewer occurrences of occurrences of $\wedge$ and $\vee$ (and no occurrences of any other connective). By the inductive hypothesis, it follows that the functions expressed by ψ and χ are both monotonic. Using this and the fact that the function expressed by $\circ$ is also monotonic, it is then fairly easy to show that f_ϕ must be too. The deduction in part (civ) turns on the observation that not all truth-functions are monotonic—something that should have been established in part (ci). It then follows from part (ciii) that some truth-functions cannot be expressed by $\mathcal{L}_1$-sentences containing only $\wedge$ and $\vee$.

Report on Introduction to Philosophy

Many answers managed to critically engage with the literature and surrounding issues, explaining examples well and often offering reconstructions of central arguments. Rather fewer pursued the arguments in depth, including consideration of anticipated objections. The two largest problems faced by the candidates (and problems which appear to recur every year) were: (i) a tendency not to answer the question, with some candidates taking the questions as prompts to launch into a general discussion on a similar theme; and (ii) a tendency to offer pat remarks instead of precise claims, justified through argument. Some candidates seemed not to think of their essays as unified and structured answers, but rather as a sequence of disjointed observations. Highest marks were awarded to answers which managed to stay relevant to the question, demonstrated knowledge of the relevant literature, and offered a sustained argument for a definite conclusion, all in clear and precise language.

General Philosophy Questions

Question 1a (Brains in vats). Candidates seemed to miss the conditional form of the quote, and the relevance of epistemic closure. A couple of answers made attempts to reach reasoned conclusions about the more general topic of scepticism, but all failed to bring their discussion back to the question.

Question 1b (Knowledge and justification). Attempts varied from fairly weak to excellent. The excellent answers stayed relevant to the question and managed to distinguish being in a position to provide a justification from having a justification/being justified. All candidates provided Gettier cases, some apparently original and successful, but there was some variation in quality in the explanation of these examples and their implications for the question being asked.

Question 2a (Induction and laws of nature). Some answers offered accurate reportage, but little by way of argument. Others made attempts at arriving at a reasoned conclusion, but Hume's arguments tended to be explained rather poorly.

Question 2b (Hume on induction). Two attempts.

Question 3a (Gods omnipotence). No attempts.

Question 3b (Problem of evil). Joint-most popular question. The answers to this question tended to be rather good, a couple being excellent. The better answers contained detailed arguments, including a critical exposition of the literature and consideration of objections and replies. Poorer answers offered more superficial argumentation, or contained muddle. All answers managed to stay relevant to the question.

Question 4a (Parfit on survival). Good attempts offering a relevant discussion of Parfit's and Lewis's accounts of personal identity.

Question 4b (Personhood). All attempts managed to stay relevant. Locke's example tended to be explained well, though weaker answers offered rather pat remarks in place of reasoned argument. No answer pushed very hard on the assumption that the animal is preserved under a brain transplant.

Question 5a (The Knowledge Argument). Joint-most popular question in Section One. The excellent answers gave critical discussions of Jackson (and Lewis, mostly), and some expressed convincing reasons to be sceptical of the category of 'physical‘ facts.

Question 5b (Descartes on mind-body dualism). One attempt.

Question 6a (Moral responsibility). Joint-most popular question in Section One. The answers tended to be rather weak, a couple being seriously incomplete. Only a couple of answers addressed the Principle of Alternative Possibilities, despite the contrast set up in the question. One answer made a convincing case for the relevance of intent to moral responsibility, and succeeded in linking this to modality.

Question 6b (Hume on moral blame). No attempts.

Frege Questions

Question 7 (Cardinal numbers as properties). This was a very popular question, answered by 79% of candidates, with a mean of 63. Most answers concentrated on Mill's account of numbers, focussing on the problem of zero and counting nonphysical objects. There was some confusion over Frege's criticism of Mill regarding the formation of agglomerations and the problem of units. Better answers tended to address the problem of inductive definitions. Weaker answers offered little critical engagement, muddled exposition of the literature, or failed to keep focus.

Question 8 (Analyticity and enlargement of knowledge). Two attempts.

Question 9 (Concept vs. object). One attempt.

Question 10 (The Julius Caesar problem). All answers managed to stay largely relevant to the question and to offer an attempt at a reasoned argument. Interestingly Benecerraf's selection problem was raised, but this was not well integrated to the discussion. Another answer articulated well a position according to which numerical identity claims are truth-"gappy", but failed to defend it from anticipated objections.

Question 11 (Frege's definition of the natural numbers). No attempts.

Question 12 (Existence of numbers given logically?). A couple of answers were incomplete. No candidate seemed to address the question whether second-order logic is logic worthy of the name (and the relevance of that to this question), but a couple of answers offered rigorous discussions of the logical status of Hume's Principle.

E. RESERVED BUSINESS

Removed from public version of the report.

F. NAMES OF MODERATORS

- Dr Adam Caulton
- Dr Richard Earl
- Prof. Volker Halbach (Chair for Preliminary Examinations)
- Prof. Oliver Riordan