Deep Hedging: from Theory to Practice
From Greeks to Hedging under Market Frictions

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Deep Hedging

Summary

– **Greek Hedging** is a legacy approach once justified by lack of data and computational power

– **Statistical Hedging** brings data-driven risk management but still relies on classic models for pricing

– **Deep Hedging** defines a new data-driven “AI” reinforcement learning risk and pricing concept for derivatives.

Its challenges are

- Realistic and robust simulation of markets
- Efficient modern Reinforcement Learning techniques for rapid evolution
Framework

Portfolio

– We are given a portfolio $Z$ of securities and derivatives, all of which are assumed to expire before some terminal maturity $T$. Negative values represent losses.

  ▪ We assume interest rates are deterministic, hence we may consider discounted variables.
  ▪ We assume that FX transactions are cost-free, hence we may assume w.l.o.g. that all assets are denominated in the same currency.
  ▪ The portfolio and all subsequent instruments are considered as "total return" assets. The total return of $Z$, i.e. the sum of all cash flows of $Z$ at $T$ is denoted by $Z_T$.

– We set $T=t_m > 0$ and denote by $0=t_0 < ... < t_{m-1} < T$ possible intermediate hedging days.
Mark-to-Model

– At each $t$ we observe relevant market data such as spots, implied vols, rates, etc. We denote this set of market data by $X=(X_1^1,...,X_K^K)$.

– We assume that for each element of our portfolio we have a way to compute a “mark-to-model” value from $X$.

  ▪ Volume-weighted mid-prices for equity, FX
  ▪ Classic derivative pricing models such as Stochastic-Local Vol for derivatives.

– Combined, this yields a (mark-to-) model value $Z_t$ for our portfolio.

  ▪ This is not a tradable quantity.
  ▪ This meta model will yield a range of classic Greeks in the form of first or higher order derivatives.

Quant Finance as an Interpolation Problem

– Classic derivative models are neither equivalent to the statistical measure $Q$, nor have they been designed to behave realistically. Their primary objective is interpolation between observable market data in $X$.

A “good” model is measured by:

  ▪ Quality of fit to reference market data in $X$, e.g. implied volatilities.
  ▪ Speed of calibration and execution
  ▪ Stylized dynamics such as stochastic volatility or stochastic interest rates.
Hedging

- We are given a range of liquid hedging instruments $H=H^1,\ldots,H^n$ such as options, swaps, futures, ETFs, stocks, FX etc.
- The mid-price at time $t$ is denoted by $H_t$, which is a model value computed from $X_t$, for example the volume-weighted mid-price for an equity. It is not a tradable quantity.
- The actual price for trading $a=(a^1,\ldots,a^n)$ is given by
  \[
  H_t(a) := a H_t + c_t(a)
  \]
  in terms of a non-negative and normalized cost function $c_t$. We usually assume $c_t$ is convex, but there are valid examples it is not, e.g. fixed fees per trade.
  - Cost can depend on past trading activity to model impact.
    Research topic: consistent impact model for option prices.
- The formal mark-to-model P&L over the period $dt$ due to trading $a$ in $t$ is given as
  \[
  a \, dH_t - c_t(a).
  \]
  We note that this does not take into account unwind cost.
Liquidity

— Not all instruments are tradable at all times:
  
  ▪ An exchange traded option \((S_r - k S_t)^+\) for \(r > t\) may only be traded at times \(u \in [t, r)\), i.e. when \(S_i\) is known and therefore the strike is fixed.

— We denote by \(A_t\) the convex, non-empty set of **admissible actions** at time \(t\).
  
  ▪ The set \(A_t\) may depend all observable market data and our historic trading decisions. For example,
    
    – Short-sell restrictions
    – Available liquidity as a function of past trading activity
    – Risk limits for our overall position (e.g. maximum Vega exposure)
  
  ▪ We call \(\pi = (a_0, \ldots, a_{m-1})\) with \(a_t \in A_t\) a **trading policy**; we will usually omit the “\(\in A_t\)” unless necessarily.
  
  ▪ \(\delta_t := a_0 + \ldots + a_{t-1}\) is our current position in \(H\).
  
  ▪ At time \(t\) our mark-to-market is \(M_t := Z_t + \delta_t H_t\).
The End of the Greek Era
The End of the Greek Era

From Greeks to Statistical Hedging

– Before we focus on Deep Hedging, we discuss alternative approaches.
– Each of these will hedge only over a given period $dt$. 
The Greek Era

Risk-Neutral Hedging

– We denote by $\delta^S := (X_1 \cdot \partial X_1, ..., X_K \cdot \partial X_K)$ the first order “Cash Delta” derivative operator to relevant observable market parameters. We denote by $\Gamma^S$ the respective second order “cash Gamma” derivatives and by $\Theta$ “Theta”.

– The risk-neutral paradigm stipulates to reduce all Delta exposure to zero, i.e. to minimize regardless of cost in each $t$

$$
\sup_{a_t} - \|\delta^S M_t + a_t \delta^S H_t\| \quad (M_t := Z_t + \Delta_t H_t)
$$

NB: for the $L^2$ norm this can be solved using the Pseudo-inverse of $\delta^S H_t$.

– This formulation does not take into account trading cost.
Ad-hoc heuristics

$$
\sup_{a_t} - \lambda \|\delta^S M_t + a_t \delta^S H_t\|_2 - c_t(a_t)
$$

Pros

– Fast
– Needs only today’s market data.

Cons

– Inconsistent:
  - No sense of carry
  - Difficult to add cost to this approach as the “Cash Deltas” have only nominally a $ interpretation: how do we know whether spending 100k$ to hedge a nominal 1m$ vega position is worth it?

– Unrealistic: does not account for “Skew Delta”.
Parameter Hedging

- We assume that we are estimating a simple normal model

\[
\frac{dX_t}{X_t} = \mu_t(X_t)dt + \Sigma_t(X_t)dW_t
\]

for our market parameters, e.g. simply by replaying historic data. This model gives rise to the operator

\[
d(\cdot) = \Delta^s\Sigma_t dW_t + \left\{ \Delta^s\mu_t + \frac{1}{2} \Sigma_t \Gamma^s\Sigma_t + \Theta \right\} dt
\]

- Applying Markoviz’ mean-variance approach to \( M_t := Z_t + \Delta_t H_t \) yields the intuitive “carry” expression

\[
\sup_{a_t} \left\{ a_t \Delta^s\mu_t + \frac{1}{2} (a_t \Sigma_t)'\Gamma^s(a_t \Sigma_t) + \Theta \right\} dt - \frac{\lambda}{2} (a_t \Delta^s\Sigma_t)'(a_t \Delta^s\Sigma_t)dt - c_t(a_t)
\]

Carry term combined both of Gamma-Theta carry and drift
Risk term driven by the covariance matrix of \( X \)
Implementation cost
The Greek Era

Parameter Hedging

\[
\sup_{a_t}\left\{ a_t \delta^\mu_t + \frac{1}{2} (a_t \Sigma_t)' \Gamma^\delta (a_t \Sigma_t) + \Theta \right\} dt - \frac{\lambda}{2} (a_t \delta^\Sigma_t)' (a_t \delta^\Sigma_t) dt - c_t(a_t)
\]

Pros

- Data-driven combination of carry and risk
- Captures well-known effects such as “skew delta”
- Fast

Cons

- Normal Approximation:
  - Does not capture strong non-linearities such as short-term barriers
  - I.e. can lead to strictly worse “hedged” portfolios
- Hedges are only locally optimal / what is the optimal horizon \(dt\) vs. the absolute cost term
- Still requires mark-to-model values for Greeks and pricing
Beyond Greeks

Statistical Hedging I – quadratic case

– In our previous example, we effectively approximated the return of a derivative over $dt$ as a normal.

– In *Statistical Hedging* [2013] we proposed replacing this approximation with genuine historic returns of “the same” derivatives.
  
  ▪ For each historic day, use a derivative with the *then*-same moneyness and time-to-maturity, and compute that derivative’s return over *then*-dt.
    - Do not use today’s fixed derivative terms.
    - Compute returns of fixed instruments.
    - For path-dependent options, keep past states consistent (e.g. past barrier breaches).
  
  ▪ This yields genuine historic returns of both $Z$ and $H$ from $t$ to $t+dt$.

– In practise, we will want to adjust the drift term to take into account views on relevant carry: e.g. just because the S&P went up for the last years does not mean we wish to capture this with our model → topic of model uncertainty.
Beyond Greeks

Statistical Hedging I – quadratic case

Given returns \(dM_t\) and \(dH_t\) we may now solve the Markoviz problem

\[
\sup_{a_t} \mathbb{E}[dM_t + a_t dH_t] dt - \frac{\lambda}{2} \text{Var}[dM_t + a_t dH_t] - c_t(a_t)
\]

- We note that if all variables are well approximated by our previous normal representation, then the two approaches coincide.
Beyond Greeks

What is wrong with Markoviz for Derivatives?

Mean-Dispersion measures are not monotone

\[ U(A) := E[A] - \frac{\lambda}{2} \|A - E[A]\|_p \]

... it makes very little sense to optimize over a non-monotonic objective.

Graph shows the fair risk-adjusted price for taking on a short position in a digital call.
The lower the strike, the higher the required compensation.

NB: Mean-Variance, Mean-Volatility, Mean-Square, ... all have the same issue of not being monotone.

Risk aversion levels manually calibrated to roughly fit between CVaR and Mean-Dispersion measure: Mean-MAD 1, Mean-Vol 1, CVaR 1.5, Quadratic VaR 0.5.
What is wrong with Markoviz for Derivatives?

Using entropy vs. mean-variance to hedge a net long butterfly position maturing tomorrow. Data generated with BS model, 50% vol. The same vanillas which compose the butterfly are available to trade for free.

Increased Risk Aversion

Mean-variance sells a net long position for positive cost to reduce perceived “variance” risk.
Beyond Greeks

Statistical Hedging II – Convex Risk Measures

– Mean-Variance is not monotone; we therefore move to a systematic approach to measure carry/risk: we call $U$ a **risk-adjusted return** if
  - $U$ is normalized to $U(0) = 0$.
  - $U$ is monotone, i.e. $A \geq B$ then $U(A) \geq U(B)$.
  - $U$ is concave.
  - $U$ is cash-invariant, $U(A+c) = U(A) + c$ for all constants $c$.

– We note $-U$ is a **convex risk measure**.

– A classic example is the **entropy** risk-adjusted return $U(A) := \frac{-1}{\lambda} \log E[\exp(-\lambda A)]$. Another important example is (the negative of) CVaR.

– Boundary cases for most reasonable $U$'s: $E[A] \geq U(A) \geq -\inf(A)$

– We now solve the local problem

$$\sup_{a_t} U(dM_t + a_t dH_t) - c_t(a_t)$$

which we may also write in terms of the associated dispersion risk measure $\rho(A) := -U(A - E[A])$ as

$$\sup_{a_t} E[dM_t + a_t dH_t] - \rho(dM_t + a_t dH_t) - c_t(a_t)$$

This resembles the previous carry-risk-cost representation.
Beyond Greeks

Cost vs. Risk Aversion – Quadratic CVaR Example

Hedge only spot

Increased Cost

Super-Hedging under extreme risk aversion

Here we see less hedging vs. cash injection: trading is more expensive than injecting more cash

\[ U(A) := -\inf_w (w + (1 + \lambda) \| (A + w)^- \|_2) \]
Beyond Greeks

Statistical Hedging

\[ \sup_{a_t} U(dM_t + a_t dH_t) - c_t(a_t) \]

Pros

- Data-driven combination of carry and risk which captures non-linearities
- Captures well-known effects such as “skew delta”
- Monotone, convex hedged portfolios.

Cons

- Hedges are only locally optimal / what is the optimal horizon \( dt \) vs. the absolute cost term
- Still requires mark-to-model values for return computation and pricing
- Compute intensive
The End of the Greek Era

Summary

- **Classic Greek Hedging** is unsuitable for data-driven risk management
- **Parametric Hedging** works for very smooth portfolios
- **Statistical Hedging** expands this to strong non-linear features

Challenges

- Future hedging cost for the now-optimal portfolio are not taken into account: tomorrow’s mark-to-model value is assumed to be realizable.
- Pricing relies on classic model prices, possibly with some ad-hoc adjustment due to immediate hedging cost.

Deep Hedging

- Simulate the market to maturity and then solve the generic problem

\[
\sup_{a_0 \ldots a_{T-1}} U \left( Z_T + \sum_{t=0}^{T-1} \delta_t dH_t - \sum_{t=0}^{T-1} c_t(a_t) \right)
\]
Deep Hedging
Deep Hedging

Simulate the market to maturity and then solve the generic problem

$$\sup_{a_0 \ldots a_{T-1}} U \left( Z_T + \sum_{t=0}^{T-1} \delta_t dH_t - \sum_{t=0}^{T-1} c_t(a_t) \right)$$

Classic “hedging under market frictions” problem.

Challenges

- Theory
- Numerical implementation
- Market dynamics
Deep Hedging

\[ v^*(Z) := \sup_{\pi=a_0...a_{T-1}} U \left( Z_T + \sum_{t=0}^{T-1} \delta_t dH_t - \sum_{t=0}^{T-1} c_t(a_t) \right) \]

Statistical Arbitrage

– We say the market has statistical arbitrage if \( v^*(0) > 0 \).

   • This does not imply presence of strict arbitrage. Example: Black & Scholes model with positive drift.

   • Strictly speaking, presence of strict arbitrage also does not imply presence of statistical arbitrage. Example: market has 100 scenarios, in 6 of which the asset returns 0%. In all other scenarios the asset returns 1%. Under CVaR@95%, \( v^*(0) = 0 \).

– Statistical Arbitrage is real – it means there are opportunities in the market, depending on one’s risk aversion.

   • Example: realized-implied vol carry; rates curve carry etc.

– However, it can pollute the question of risk management: just as in classic cash portfolio optimization the estimation of “alpha” is much more involved.

   • We may therefore separate the estimation of carry from the estimation of the higher moments of the distribution of our instruments.

– We call a trade a static arbitrage opportunity if the return is non-negative under any market dynamics (e.g. violation of butterfly or calendar arb).
Deep Hedging

\[ v^*(Z) := \sup_{\pi = a_0 \ldots a_{T-1}} : U\left( Z_T + \sum_{t=0}^{T-1} \delta_t dH_t - \sum_{t=0}^{T-1} c_t(a_t) \right) \]

Pricing

Consider a current position of \( Z \).

The price of selling a derivative \( Y \) to a customer is given by the marginal cost

\[ p(Y) := v^*(Z - Y) - v^*(Z) \]

- Of course, \( v^*(Z - Y + p(Y)) = 0 \).
- Reflects naturally a bid/ask spread.
- The model-price is no longer used.
Deep Hedging

\[ v^*(Z) := \sup_{\pi = a_0 \ldots a_{T-1}} : U \left( Z_T + \sum_{t=0}^{T-1} \delta_t dH_t - \sum_{t=0}^{T-1} c_t(a_t) \right) \]

Hamilton-Jacobi-Bellman

- One of the biggest shortcomings of the approach presented here is that \( Z \) is fixed and not a part of the “state” of the market.
- The Bellman form of the problem can be written as follows:
  - Denote by \( M^t \) all future cash flows of our portfolio on and after \( t \), and by \( m_t \) the cashflow arising from holding \( M^t \) at \( t \). This gives the classic HJB reward form:
    \[ V^*(M^t | S_t) = \sup_{a_t} : U \left( V^*(M^{t+1} + a_t H^{t+1}) | S_t \right) - c(a_t | S_t) + m_t \]
  - Research topics:
    - Find a representation for a portfolio of derivatives which is efficient for this to be applicable.
    - Under what conditions does this equation have a fixed point for all combined states \( (M^t, S_t) \).
Deep Hedging

$$v^*(Z) := \sup_{\pi=a_0 \ldots a_{T-1}} \mathbb{E} \left( Z_T + \sum_{t=0}^{T-1} \delta_t dH_t - \sum_{t=0}^{T-1} c_t(a_t) \right)$$

Pricing

Consider a current position of $Z$.
The price of selling a derivative $Y$ to a customer is given by the marginal cost

$$p(Y) := v^*(Z - Y) - v^*(Z)$$

- Of course, $v^*(Z - Y + p(Y)) = 0$.
- Reflects naturally a bid/ask spread.
- The model-price is no longer used.
Numerical Implementation

Deep Hedging

\[ \sup_{\pi=a_0...a_{T-1}} \mathbb{U} \left( Z_T + \sum_{t=0}^{T-1} \delta_t dH_t - \sum_{t=0}^{T-1} c_t(a_t) \right) \]

Structure of the Problem

- The problem above is convex in $\pi$.
- We will solve this over a fixed set of paths, and therefore fixed set of terminal payoffs.
- Path-dependency of the problem enters due to transaction cost and from path-dependent restrictions on liquidity.

Reinforcement Learning

- Parameterize $a_t$ as a neural network, with the result of the previous step feeding into the next step.
- This is called “model-based policy search” in the ML literature.
- Theoretical result [DH’18]: neural networks approximate any policy arbitrarily well with increasing depth and width.
- Practical choices:
  - Each $a_t$ has its own network $\rightarrow$ rather deep network
  - Share network across $t$ $\rightarrow$ LSTM to capture path
- Example code in Karas is just about a page of code https://people.math.ethz.ch/~jteichma/deep_portfolio_optimization_keras.html
- Efficient, scalable, model-independent implementation.
Numerical Implementation

Deep Hedging

– We show the efficiency and impact of the algorithm for classic derivative model dynamics.
– Used to validate convergence of the numerical scheme against (the few) analytically available results.
– Even here, we are able to compute previously inaccessible problems for entire portfolios of derivatives
Deep Hedging

Comparison with theory: vanilla option with Heston dynamics

- Hedge an ATM 30-day call with spot and var swap
- No costs, no limits, 50% CVaR value function

- Spot hedge
- PnL distribution
Deep Hedging

Comparison with theory: Heston dynamics with cost

- We have asymptotic results for small transaction cost for classic one-factor models such as Black Scholes.
- There are no analytic results for higher order models. We show that the same asymptotics hold using our numerical scheme.
Deep Hedging

Vanilla option: impact of transaction costs

- 30-day ATM call option
- Plot shows spot hedge ("Delta")
- No limits
- Entropic value
- Risk aversion 10
- Black-Scholes simulator

**Delta at day 29**

Transaction Costs
- 0.01
- 0.001
- 0.0001
- 1e-10

**Spots**

-0.2 0.0 0.2 0.4 0.6 0.8 1.0 1.2 1.3
Deep Hedging

Forward starting option: impact of transaction costs
- 30-day ATM call starting in 15 days
- Heston simulator, no limits, entropic value, risk aversion 50
- Hedge with spot only

![Incremental delta graph]
Deep Hedging

Forward starting option: impact of transaction costs

- 30-day ATM call starting in 15 days
- Heston simulator, no limits, entropic value, risk aversion 50
- Hedge with spot only
Deep Hedging

Popular retail payoff:
- Client is short a down-and-in put paid at maturity
- Upper knockout barrier
- Fixed coupons until KO

Market
- 0.1% transaction costs
- No limits
- Risk aversion 20
- Entropic value
- Monthly hedging
- Local volatility simulator

Depending on the state (barrier breached) future delta changes
Deep Hedging

Deep Hedging with Classic Derivatives Generators

- Useful for assessing liquidity and cost impact at scale
  - Straightforward with TensorFlow
  - Inherently parallelizable since risk-adjusted returns tend to be expectation based.
  - Speed independent of number of instruments in portfolio
  - Asset-class agnostic: discounting, FX, own callability etc all no problem
  - Client-callability: master thesis under way @ ETH

.... shouldn’t we use the real market to train our model?
Market Dynamics
Market Dynamics

Challenges

– Sparse data set vs. large number of instruments.
– Non-stationarity and robustness

Idea: solve a robust version of our problem,

$$\inf_Q \sup_{\pi=a_0\ldots a_{T-1}} U_Q[Z_T + \sum_{t=0}^{T-1} \delta_t dH_t - \sum_{t=0}^{T-1} c_t(a_t)]$$

– Avoid static arbitrage

A possible approach is parameterizing the implied vol in “discrete local vol” (see also Wissel) which is an arb-free parameterization.

Model Challenges

– Statistical arbitrage.
  - Robustify the estimator.
  - Find the closest risk-neutral measure without drift.
Generative Adversarial Market Dynamics

- We want to solve

\[ p_Q(Z) := \sup_{\pi = a_0 \ldots a_{T-1}} U_Q \left[ Z_T + \sum_{t=0}^{T-1} \delta_t dH_t - \sum_{t=0}^{T-1} c_t(a_t) \right] \]

- However, we are not sure about \( Q \) since we only observe samples \( \hat{Q} \).

Classic robust approach [Follmer, Schied: Stochastic Finance: An Introduction in Discrete Time, 2011]: define set of reasonable measures “close” to \( \hat{Q} \) and a distance \( d \) to \( \hat{Q} \) and solve for \( \alpha \uparrow \infty \):

\[ \hat{p}_F(Z) := \sup_{\pi} \inf_Q \{ \alpha U_Q[G(\pi_t; Z_T)]^+ + d(Q, \hat{Q}) \} \]

- Machine Learning interpretation [Generative Adversarial Networks, Goodfellow 2014]:
  - **Generator** \( Q \) tries to fit the target distribution and take away money
  - **Adversary** \( \pi \) tries to make money

- We use unconditioned Wasserstein distance \( W_1 \) as our metric using a fast stochastic algorithm c.f, [Stochastic Optimization for Large-scale Optimal Transport, 2016, Genevay / Cuturi / Peyré / Bach]
STOXX50 Dataset

- We obtain historical spot and option prices for last 10 years of data ~ 2000 historical data points
- Option Grid with relative strikes
  \[ K = \{0.8, 0.85, 0.9, 0.95, 1.0, 1.05, 1.1, 1.15, 1.2\} \]
  and maturities of
  \[ T = \{20, 40, 60, 80, 100, 120\} \]
- \( H_t = \) 109-dimensional vector = 1 Spot + 54 Calls + 54 Puts
- At every time step, \( H_t = \) generated prices for the grid of options + spot
- Each simulation time step = 20 days to match the maturities
Setup and Training

- Generator network: 2 layer LSTM of size 64, $f \sim 62K$ parameters
- Hedger network: 2 layer LSTM of size 64, $\pi \sim 112K$ parameters
- Use 2% transaction costs to regularize the hedger network
- Training:
  - Train with batch sizes of 32K to minimize noise, RMS Prop with a learning rate of $1e^{-4}$
  - Dropout of 25% between state-to-state LSTM connections to regularize training
  - 50 K updates with a batch size of 32K takes 10hrs to train on Tesla V100 GPU
Results

Risk Neutral Value

$-E[X] = -9.63 \times 10^{-4}$

Wasserstein Distance

$W = 4.81 \times 10^{-2}$
Results

Histogram of generated spot prices compared to historical data.
Results

Histogram of generated call option prices at $t = 3$ compared to historical data.
Results

Graphs show the time series of option prices from simulated by LSTM based generator.
What Next

Next steps

– Define conditional version of GAMeD
– Use distance metric which reflects use case, i.e. DH itself
– Move to recursive version of DH.
– Formalize theoretical relationship between adversarial learning and robust statistics

Thank you very much for your attention