Examiners' Report: Preliminary Examination in Mathematics Trinity Term 2019

November 21, 2019

Part I

A. STATISTICS

- **Numbers and percentages in each class.**
  See Table 1. Overall 185 candidates were classified.

Table 1: Numbers in each class (Preliminary Examination)

	Numbers					Percentages %				
	2019	(2018)	(2017)	(2016)	(2015)	2019	(2018)	(2017)	(2016)	(2015)
Distinction	54	(58)	(62)	(59)	(55)	29.19	(29.44)	(30.85)	(30.89)	(30.73)
Pass	120	(126)	(124)	(119)	(105)	64.86	(63.96)	(61.69)	(62.3)	(58.66)
Partial Pass	8	(10)	(13)	(7)	(13)	4.32	(5.08)	(6.47)	(3.66)	(7.26)
Incomplete	1	(0)	(0)	(0)	(1)	0.54	(0)	(0)	(0)	(0.56)
Fail	2	(3)	(2)	(6)	(6)	1.08	(1.52)	(0.99)	(3.14)	(3.35)
Total	185	(197)	(201)	(191)	(180)	100	(100)	(100)	(100)	(100)

- **Numbers of vivas and effects of vivas on classes of result.**
  As in previous years there were no vivas conducted for the Preliminary Examination in Mathematics.

- **Marking of scripts.**
  As in previous years, no scripts were multiply marked by the Examiners; however all marking was conducted according to a detailed marking scheme, strictly adhered to. For details of the extensive checking process, see Part II, Section A.

B. NEW EXAMINING METHODS AND PROCEDURES

No new examining methods and procedures were used for 2018/19.

C. CHANGES IN EXAMINING METHODS AND PROCEDURES CURRENTLY UNDER DISCUSSION OR CONTEMPLATED FOR THE FUTURE

No changes are under discussion for 2019/20.

D. NOTICE OF EXAMINATION CONVENTIONS FOR CANDIDATES

The Notice to Candidates, containing details of the examinations and assessment, including the Examination Conventions, was issued to all candidates at the beginning of Trinity term. All notices and the Examination Conventions in full are available at

https://www.maths.ox.ac.uk/members/students/undergraduate-courses/examinations-assessments/examination-conventions.

Part II

A. GENERAL COMMENTS ON THE EXAMINATION

Acknowledgements

The Examiners express their sincere gratitude to the Academic Administration team, and in particular to Nia Roderick, for all their work in running the examinations system and supporting the Examiners at every turn whilst being careful always to facilitate but never to influence academic decisions made by the Examiners.

We also thank Waldemar Schlackow for maintaining and running the examination database and in particular for his assistance during the final examination board meeting.

We express our sincere thanks to Prof Vidit Nanda and Dr Alberto Paganini for administering the Computational Mathematics projects. We would like to thank the Setters Prof Capdeboscq and Dr Wilkins for setting some of the questions, and Assessors Dr Maria Christodoulou, Dr Radu Cimpeanu, Dr Adam Gal, Dr Marcelo Goncalves De Martino, Dr David Hume, Dr Chris Lester and Dr Andrew Mellor for their assistance with marking. We would also like to thank the team of graduate script checkers for their work checking and sorting the scripts.

Timetable

The examinations began on Monday 24th June at 2.30pm and ended on Friday 28th June at 11.30am.

Mitigating Circumstances Notices to Examiners

A subset of the Examiners (the 'Mitigating Circumstances Panel') attended a pre-board meeting to band the seriousness of the individual notices to examiners. The outcome of this meeting was relayed to the Examiners at the final exam board. The Examiners gave careful regard to each case, scrutinised the relevant candidates' marks and agreed actions as appropriate.

See Section E for further detail.

Setting and checking of papers and marks processing

The Examiners first set questions, a checker then checked the draft papers and, following any revisions, the Examiners met in Hilary term to consider the questions on each paper. They met a second time to consider the papers at the end of Hilary term making further changes as necessary before finalising the questions. A meeting was held in early Trinity term for a final proof read. The Camera Ready Copy (CRC) was prepared and each Moderator signed off the papers. The CRC was submitted to Examination Schools in week 4 of Trinity term.

The examination scripts were collected from Ewert House and delivered to the Mathematical Institute.

Once the scripts had been marked and the marks entered, a team of graduate checkers, under the supervision of Charlotte Turner-Smith and Nia Roderick, sorted all the scripts for each paper of the examination. They carefully cross checked against the mark scheme to spot any unmarked questions or parts of questions, addition errors or wrongly recorded marks. A number of errors were corrected, with each change checked and signed by an Examiner, at least one of whom was present throughout the process. A check-sum was also carried out to ensure that marks entered into the database were correctly read and transposed from the marks sheets.

Determination of University Standardised Marks

The candidates under consideration are Mathematics and Mathematics & Statistics candidates, 185 in total. We do not distinguish between them as they all take the same papers.

Marks for each individual paper are reported in university standardised form (USM) requiring at least 70 for a Distinction, 40–69 for a Pass, and below 40 for a Fail.

As last year the Mathematics Teaching Committee issued each examination board with broad guidelines on the proportion of candidates that might be expected in each class. This was based on the average proportion in each class over the past five years.

The raw marks were recalibrated to arrive at the USMs reported to candidates, adopting the procedures outlined below. These procedures are similar to the ones used in previous years.

To ensure equal weightings across all subjects, papers were first standardised to have broadly similar proportions of candidates attaining each class. A piecewise linear mapping was adopted to produce a USM from a raw mark. The default algorithm for each paper works as follows.

1. Candidates' raw marks for a given paper are ranked in descending order. Here the population data used is the set of marks for all candidates in Mathematics or Mathematics & Statistics.

2. The default percentages p_1 of Distinctions and p_2 of nominal upper seconds (USM 60-69) are selected, these percentages being similar to those adopted in previous years.

3. The candidate at the p_1 percentile from the top of the ranked list is identified and assigned a USM of 70. Let the corresponding raw mark be denoted by R_1.
4. Similarly, the candidate at the $(p_1 + p_2)$ percentile from the top of the list is assigned a USM of 60 and the corresponding raw mark is denoted by R_2.
5. The line segment between $(R_1, 70)$ and $(R_2, 60)$ is extended linearly to USMs of 72 and 57 respectively. Denote the raw marks corresponding to USMs of 72 and 57 by C_1 and C_2 respectively. For a graph of the mapping between raw marks and USMs, a line segment is drawn, connecting $(C_1, 72)$ to $(100, 100)$ with a further line segment between $(C_2, 57)$ and $(C_1, 72)$.
6. A line segment through $(C_2, 57)$ is extended down towards the vertical axis, as if it were to join the axis at $(0, 10)$, but the line segment is terminated at a USM of 37. The associated raw mark at the termination point is denoted C_3.
7. Finally a line segment between $(C_3, 37)$ and $(0, 0)$ completes the graph of the piecewise linear mapping between the raw marks and the USM.

Thereby a piecewise linear map is constructed whose vertices, at $\{(0,0), (C_3, 37), (C_2, 57), (C_1, 72), (100, 100)\}$, are located away from any class boundaries.

A first run of the outlined scaling algorithm was performed. It was confirmed that the procedure resulted in a reasonable proportion of candidates in each class. The Examiners then used their academic judgement to make adjustments where necessary as described below. The Examiners were not constrained by the default scaling map and were able, for example, to insert more vertices if necessary.

To obtain the final classification, a report from each Assessor was considered, describing the apparent relative difficulty and the general standard of solutions for each question on each paper. This information was used to guide the setting of class borderlines on each paper.

The scripts of those candidates in the lowest part of each ranked list were scrutinised carefully to determine which attained the qualitative class descriptor for a pass on each paper. The gradient of the lower section of the scaling map was adjusted to place the pass/fail borderline accordingly.

Careful consideration was then given to the scripts of candidates at the Distinction/Pass boundary.

Adjustments were made to the scaling maps where necessary to ensure that the candidates' performances matched the published qualitative class descriptors.

The Computational Mathematics assessment was considered separately. In consultation with the relevant Assessor it was agreed that no recalibration was required, so the raw marks (out of 40) were simply multiplied by 2.5 to produce a USM.

Finally, the class list for the cohort was calculated using the individual paper USMs obtained as described above and the following rules:

Distinction: both $Av_1 \geq 70$ and $Av_2 \geq 70$ and a mark of at least 40 on each paper and for the practical assessment;

Pass: not meriting a Distinction and a mark of at least 40 on each paper and for the practical assessment;

Partial Pass: awarded to candidates who obtained a standardised mark of at least 40 on three or more of Papers I-V but did not meet the criteria for a pass or distinction;

Fail: a mark of less than 40 on three or more papers.

Here Av_2 is the average over the five written papers, weighted by length, and Av_1 is the weighted average over these papers together with Computational Mathematics (counted as one third of a paper). The Examiners verified that the overall numbers in each class were in line with previous years, as shown in Table 1.

The vertices of the final linear model used in each paper are listed in Table 2, where the x-coordinate is the raw mark and the y-coordinate the USM.

Table 2: Vertices of final piecewise linear model

Paper	Positions of vertices				
I	(0,0)		(50.9,57)	(76.4,72)	(100,100)
II	(0,0)		(42.9,57)	(68.4,72)	(100,100)
III	(0,0)	(39,40)	(59,57)	(89,72)	(120,100)
IV	(0,0)	(37,37)	(54.9,57)	(80.4,72)	(100,100)
V	(0,0)	(28,37)	(42.8,57)	(63.8,72)	(80,100)
CM	(0,0)				(40,100)

Table 3 gives the rank list of average USM scores, showing the number and percentage of candidates with USM greater than or equal to each value.

Table 3: Rank list of average USM scores

		Candidates with USM $\geq x$	
USM (x)	Rank	Number	%
90	1	1	0.54
87	2	2	1.08
85	3	3	1.62
83	4	4	2.16
82	5	7	3.78
80	8	8	4.32
79	9	11	5.95
78	12	13	7.03
77	14	16	8.65
76	17	20	10.81
75	21	25	13.51
74	26	29	15.68
73	30	34	18.38
72	35	36	19.46
71	37	47	25.41
70	48	54	29.19
69	55	62	33.51
68	63	74	40
67	75	83	44.86
66	84	89	48.11
65	90	99	53.51
64	100	112	60.54
63	113	123	66.49
62	124	132	71.35
61	133	139	75.14
60	140	147	79.46
60	140	147	79.46
59	148	153	82.7
58	154	158	85.41
57	159	163	88.11
56	164	170	91.89
56	164	170	91.89
55	171	172	92.97
55	171	172	92.97
54	173	174	94.05
53	175	175	94.59
52	176	177	95.68
52	176	177	95.68
51	178	178	96.22

Table 3: Rank list of average USM scores (continued)

USM (x)	Rank	Candidates with USM $\geq x$ Number	%
49	179	179	96.76
48	180	180	97.3
47	181	181	97.84
43	182	182	98.38
42	183	183	98.92
36	184	184	99.46
18	185	185	100

Recommendations for Next Year's Examiners and Teaching Committee

The Examiners noted the gender split and are aware of ongoing work within the Department to monitor this.

B. EQUAL OPPORTUNITIES ISSUES AND BREAKDOWN OF THE RESULTS BY GENDER

Table 4 shows the performances of candidates broken down by gender.

Table 4: Breakdown of results by gender

Class	Number								
	2019			2018			2017		
	Female	Male	Total	Female	Male	Total	Female	Male	Total
Distinction	8	46	54	7	51	58	12	50	62
Pass	49	71	120	57	69	126	36	88	124
Partial Pass	4	4	8	6	4	10	4	9	13
Incomplete	0	1	1	0	0	0	0	0	0
Fail	1	1	2	2	1	3	0	2	2
Total	62	123	185	72	125	197	52	149	201
Class	Percentage								
	2019			2018			2017		
	Female	Male	Total	Female	Male	Total	Female	Male	Total
Distinction	12.9	37.4	29.19	9.72	40.8	29.44	23.08	33.56	30.85
Pass	79.03	57.72	64.86	79.17	55.2	63.96	69.23	59.06	61.69
Partial Pass	6.45	3.25	4.32	8.33	3.2	5.08	7.69	6.04	6.47
Incomplete	0	0.81	0.54	0	0	0	0	0	0
Fail	1.61	0.81	1.08	2.78	0.8	1.52	0	1.34	1
Total	100	100	100	100	100	100	100	100	100

C. STATISTICS ON CANDIDATES' PERFORMANCE IN EACH PART OF THE EXAMINATION

The number of candidates taking each paper is shown in Table 5. The performance statistics for each individual assessment are given in the tables below: Paper I in Table 6, Paper II in Table 7, Paper III in Table 8, Paper IV in Table 9, Paper V in Table 10 and Computational Mathematics in Table 11. The number of candidates who received a failing USM of less than 40 on each paper is given in Table 5.

Note that Paper I, II and IV are marked out of 100 (being 2.5 hours in duration), Paper III is marked out of 120 (being 3 hours in duration) and Paper V is marked out of 80 (being 2 hours in duration).

Table 5: Numbers taking each paper

Paper	Number of Candidates	Avg RAW	StDev RAW	Avg USM	StDev USM	Number failing	% failing
I	184	64.49	12.98	65.13	10.22	4	2.17
II	184	56.33	13.49	64.32	10.88	4	2.17
III	185	76.05	16.04	65.88	10.32	4	2.16
IV	185	69.3	12.56	66.17	10.15	3	1.62
V	185	54.45	9.94	65.67	9.6	2	1.08
CM	184	30.8	4.64	77.28	11.6	1	0.54

Table 6: Statistics for Paper I

Question	Average Mark		Std	No. of Attempts	
Number	All	Used	Dev	Used	Unused
Q1	10.80	10.80	2.73	158	0
Q2	14.75	14.75	2.59	142	0
Q3	13.91	13.91	4.00	80	0
Q4	12.81	12.81	4.07	172	0
Q5	11.52	11.63	4.20	86	1
Q6	13.70	13.76	3.88	144	1
Q7	13.00	13.00	3.22	136	0

Table 7: Statistics for Paper II

Question Number	Average Mark All	Average Mark Used	Std Dev	No. of Attempts Used	No. of Attempts Unused
Q1	11.36	11.40	3.16	169	1
Q2	14.31	14.31	3.57	51	0
Q3	11.70	11.77	3.15	147	1
Q4	10.95	10.95	3.64	131	0
Q5	14.02	14.02	5.04	92	0
Q6	11.68	11.68	4.82	145	0
Q7	8.48	8.48	3.77	184	0

Table 8: Statistics for Paper III

Question Number	Average Mark All	Average Mark Used	Std Dev	No. of Attempts Used	No. of Attempts Unused
Q1	16.61	16.61	3.11	152	0
Q2	13.83	13.83	3.87	82	0
Q3	15.57	15.57	4.41	136	0
Q4	8.48	8.48	5.33	54	0
Q5	11.24	11.24	4.07	161	0
Q6	12.05	12.05	3.85	153	0
Q7	12.48	12.48	4.97	120	0
Q8	8.97	9.03	4.24	114	1
Q9	12.55	12.55	3.52	132	0

Table 9: Statistics for Paper IV

Question Number	Average Mark All	Average Mark Used	Std Dev	No. of Attempts Used	No. of Attempts Unused
Q1	14.28	14.28	2.19	169	0
Q2	12.80	12.80	3.03	140	0
Q3	15.08	15.08	3.68	61	0
Q4	8.32	8.32	4.56	68	0
Q5	13.88	13.88	4.54	171	0
Q6	11.91	11.96	5.45	127	1
Q7	17.49	17.49	1.83	185	0

Table 10: Statistics for Paper V

Question	Average Mark		Std	No. of Attempts	
Number	All	Used	Dev	Used	Unused
Q1	13.50	13.50	2.36	175	0
Q2	13.81	13.81	4.45	52	0
Q3	11.42	11.42	3.92	142	0
Q4	15.27	15.27	3.44	181	0
Q5	13.64	13.64	4.31	114	0
Q6	14.26	14.40	4.58	73	1

Table 11: Statistics for Computational Mathematics

Question	Average Mark		Std	No. of Attempts	
Number	All	Used	Dev	Used	Unused
Q1	15.08	15.08	2.89	182	0
Q2	16.90	16.90	2.60	10	0
Q3	15.65	15.65	2.66	176	0

D. COMMENTS ON PAPERS AND ON INDIVIDUAL QUESTIONS

Paper I

Question 1.

A popular question, though not particularly well done with a low average and very few attempts gaining high or full marks. In (a)(ii) many candidates often wrote long answers gaining little credit by not addressing the particular question; it is clear that it may be assumed that a matrix may be put into RRE form using EROs, so the main points to address were that EROs are invertible and that a matrix is invertible iff its RRE form is the identity. Also most answers to (a)(iv) were incorrect. If N is an $n \times m$ inverse of M then it can be argued that

$$\begin{aligned} n &= \text{rank} I_n = \text{rank}(NM) \leqslant \text{rank} M \leqslant m; \\ m &= \text{rank} I_m = \text{rank}(MN) \leqslant \text{rank} N \leqslant n. \end{aligned}$$

Most successful attempts were implicitly along these lines, either making use of the rank-nullity theorem or discussing the RRE form of a non-square matrix. One clever answer used $\text{trace}(MN) = \text{trace}(NM)$.

Part (b) was often successfully done, however many scripts did not connect this part with the matrix M from (a)(iii) and instead proved afresh that the linear system is invertible. For (c) the expected argument was that if $A = P(-B)P^{-1}$, so that $AP + PB = 0$, then the linear map $X \mapsto AX + XB$ has a non-zero kernel and so cannot be onto. A few scripts made this argument; a few other scripts inventively completed this part by focussing on the trace of certain matrices.

Question 2.

A popular question, largely well done. In (a)(ii) a surprising number struggled to give a basis for the solution space of $f'' + f = 0$. Some suggesting e^x, e^{-x}, others $1, x$ and the more subtly wrong e^{ix}, e^{-ix} which, whilst solving the ODE, are not in the given subspace. For part (c)(ii) note that a finite-dimensional rational vector space is isomorphic to $\mathbb{Q}^n$, for some n, and so countable, whereas $\mathbb{C}$ is uncountable. Many argued that $\mathbb{Q}[\zeta]$ is infinite-dimensional which is not generally the case; it is though if ζ is transcendental and so some partial credit was awarded.

Question 3.

This was not such a popular question, but candidates who chose it often did well.

In 3(a)(ii), candidates were expected to state the conditions of the Rank-Nullity Theorem as well as the conclusion, but often for example did not mention that the domain should be finite-dimensional.

3(b): Candidates who were familiar with the proof seemed comfortable producing it, sometimes getting confused between m and n, and sometimes writing things like $T(v_i) = \sum_{j=1}^{m} a_{ij} w_j$ where it should be a_{ji}. Some candidates skipped this part, but were nonetheless able to make good progress on other parts of the question.

3(c): (i) and (ii) were done well. Some candidates simply wrote down bases for the kernel and the image, but the question says "Find..." (not "Write down...", so candidates were expected to give some brief justification.

Many candidates tackled (iii) successfully. Some did not use the Change of Basis Theorem, and so received no credit (even if the matrix they found was correct). Some candidates worked with the transpose of the matrices P and Q that they should have been using.

There were lots of good answers given to (iv). The most streamlined noted that the matrix in the question has rank 1, whereas we know from (c)(i) that T has rank 2.

Question 4.

This was a popular question, tackled by almost all candidates.

(a)(i) Quite a few candidates forgot to specify that an eigenvector must be nonzero, or got confused and said that the eigenvalue must be nonzero instead. It is important to require that v is nonzero, otherwise $Tv = \lambda v$ would be satisfied for all λ, by choosing $v = 0$.

(a)(iii) There were lots of good attempts at this, either by using contradiction and a minimal counterexample, or using induction. Some had a good overall strategy, but were not quite careful enough about the subtleties. One common issue was that candidates applied the 'linear map' $T - \lambda_i$ — when they meant to apply $T - \lambda_i I$ (writing I for the identity linear map).

(b)(i) A very large number of candidates gave an eigenvector for each eigenvalue, but did not describe *all* eigenvectors. The question says "Find the eigenvalues and eigenvectors...", which is meant to imply "Find the eigenvalues, and all the eigenvectors for each". Candidates who described the whole eigenspace (for example as the span of a particular set) were awarded full marks. Some candidates found the eigenspace and then wrote something like "And so the eigenvector is ..." — and so received partial credit. Some candidates made an algebraic or arithmetic slip when computing the eigenvalues, and so ended up with an incorrect eigenvalue. This should become apparent when there seem to be no corresponding eigenvectors. Several candidates discovered there were no eigenvectors, but did not apparently recognise that this meant there *must* be an issue with the calculation of the eigenvalues.

(b)(ii) Candidates who argued that the matrix is not diagonalisable because "there are only 2 eigenvectors" or "there are only 2 distinct eigenvectors" received partial credit. In fact there are infinitely many eigenvectors (simply by taking scalar multiples). Candidates who used a phrase such as "there are only 2 linearly independent eigenvectors" received full marks.

(c) Lots of candidates made a good attempt at this, and had some idea of a strategy. Many of the difficulties arose in identifying and managing the different cases. The most successful candidates clearly articulated what cases needed considering, and structured their arguments around this. One fairly frequent misconception was that if $\lambda = 0$ then the matrix can't be diagonalisable because it is not invertible — perhaps this is an underlying misconception that diagonal matrices must be invertible. A few candidates, but not many, noted that if $\mu = 0$ then the matrix already is diagonal and so is certainly diagonalisable. Some considered the case $\mu = 0$, but did a significant chunk of calculation to ascertain that the matrix is diagonalisable, rather than recognising that if a matrix is diagonal then it is certainly diagonalisable.

Question 5.

At various points in this question, some candidates seemed to assume that a cyclic group must be finite, which is not the case. Another common issue was asserting something like $\langle g \rangle = \{e, g, g^2, \dots\}$, apparently not considering the negative powers of g. (Note, for example, that $(\mathbb{Z}, +)$ is generated by 1, but is not the set $\{0, 1, 2, \dots\}$.)

(b)(v) and (vi) were clearly less familiar for students (as was the intention). Candidates who were reasonably confident with quotient groups did a nice job of showing that if we take a quotient of a cyclic group by a subgroup (which must be normal because a cyclic group is Abelian, and every subgroup of an Abelian group is normal) then we must get a cyclic group, just from the definitions. If G is generated by g, and H is a subgroup, then candidates argued that gH generates G/H. To get full marks they needed to show some understanding of the group operation in the quotient group, for example noting that $(gH)^k = g^k H$.

For (vi), the most common counterexample was S_3 (which, as the smallest non-Abelian group, is perhaps the most natural to try). The candidates who suggested S_3 but incorrectly listed its subgroups did not get the marks. Sometimes candidates proposed groups that were not in fact counterexamples (perhaps the group was a bit larger, and they overlooked some subgroups), sometimes candidates misread the last word of the question as "cyclic" rather than "Abelian".

Question 6.

(a)(i) Some candidates did not specify the group operations in the definition of a homomorphism, but it is important to be careful about this since there are two group operations involved.

(b)(ii) For the direction that starts by supposing that H is the kernel of a homomorphism, quite a few candidates forgot that in order to show that H is a normal subgroup of G, they needed to check that it is a subgroup at all, as well as checking the condition for normality. For the other direction (taking a normal subgroup and showing that it is the kernel of a homomorphism with domain G), several candidates wrote answers that somehow referred to ϕ without having ever introduced ϕ. The standard argument defines the quotient map $\phi : G \to G/H$ by $\phi(g) = gH$, and argues that this is a homomorphism with kernel H. Amongst candidates adopting this strategy, several did not check that the map really is a homomorphism.

(b)(iii) Candidates were expected to state the conditions of the theorem carefully, not just to write down the conclusion.

(c)(i) Some candidates did a nice job of drawing together relevant parts of the theory to describe the normal subgroups, for example using: a normal subgroup is a union of conjugacy classes, conjugacy classes in S_n are determined by cycle type, a subgroup must contain the identity, and Lagrange's theorem which tells us that for a finite group the order of a subgroup must divide the order of the group.

(c)(ii) Candidates seemed to find this relatively challenging — or perhaps lacked time to complete it.

Question 7.

(a)(iii) Again, candidates were supposed to state all the conditions (that we have a *finite* group *acting* on a set) as well as the conclusion.

Some candidates did a good job of proving Orbit-Stabiliser. Others showed that they had a partial understanding. Some tried to show that there was an isomorphism between $G/\mathrm{Stab}(s)$ and $\mathrm{Orb}(s)$, but this is not a good plan because in general $\mathrm{Orb}(s)$ is a subset of S, a set that need not have a group structure. We cannot have a structure-preserving map to a set that does not have structure.

(b)(i) was done very well, on the whole. Some candidates did a great job on (b)(ii), others seemed uncertain about how to proceed, or became confused about what sorts of objects should be involved in their answer (for example, $\mathrm{Orb}(H)$ should consist of elements of S, that is, subgroups of G, not of elements of G).

As was intended, (c) was more challenging. Many students tried to use the hint, and some recognised that if G contains a non-identity element g, then under the action in the hint the element g has orbit $G \setminus \{e\}$. Orbit-Stabiliser then tells us that the order of this must divide $|G|$. Many candidates who reached this point then asserted that this means that $|G| = 2$, only some gave a justification for this (perhaps noting that if $n-1$ divides n then, since it also divides $n-1$, it must divide 1). Some candidates were a little confused about the action, deducing that the orbit of a non-identity element must be the whole group (whereas in fact the only element conjugate to the identity is the identity).

Paper II

Question 1.

This was quite a popular question since almost half of the allocated points were bookwork. This, typically awarded the student forty percent of the total marks. The other sixty percent was a bit more challenging. Students that used the continuity of basic functions claiming it was part of the "Algebra of Limits" in order to conclude results about convergence of sequences got only a fraction of the points. The previous to the last item was quite manageable, but a common mistake was that students did not separate the average into natural numbers n with respect to a certain bound, say N, after which one could write $|a_n - l| < \epsilon$ if $n \geq N$ and treat the finite exceptions $|a_n - l|$ with $n < N$ as a constant. The last item was challenging, and those who realised that, similarly to the previous item, there is a bound N for which $n \geq N$ one could write $(l-\epsilon)^{n-N} < a_n/a_N < (l+\epsilon)^{n-N}$ and then use a previous item, managed to find the required solution.

Question 2.

This question was less popular than Question 1, probably because there were not too many bookwork points available. However, this question was well streamlined, and, in particular in the second half of the question, each item used a previous one to conclude its results. Typically, those students that attempted this questions managed to bear fruit. The students were allowed to use the hint at will in the first half, and in the second, the proofs typically required to be done by simple induction arguments.

Question 3.

Part (a) was successfully done by most students. Part (b) turned out to be challenging and many failed to justify the change of the order of summation which is the main point of the problem. In part (c), again, many failed to justify the change of the order of summation. Also, many students argued that the series is bounded by a divergent series and, hence, divergent.

Question 4.

The bookwork part (a) was not challenging for most students. About half of the students gave a proof using bisection and the other half used suprema. If done correctly, both approaches were given the full mark. Part (b) was rather straightforward, but some wrote an overly complicated solution. Part (c) was done by very few students and many of them failed to justify some of the steps.

Question 5

Item a (bookwork) was answered satisfactorily by the vast majority of students. There was one common mistake where the student wrote f_n instead of $\sum_k = 1^n f_k$ in the definition.

Item b was solved by many students, but proved harder in general. The common mistakes were: b.i) Claiming that the general term of the series tends to 0 implies convergence and uniform convergence. Using the M-test with a non-constant bound.

b.ii) Confusing uniform convergence and uniform continuity. Claiming erroneously (without an attempt at proof) that uniform convergence on every bounded interval implies uniform convergence everywhere.

Question 6.

Quite a few students gave the statement of the MVT instead of the Cauchy MVT and tried to use it in the proof of L'Hôpital's rule. In both statements, many forgot some essential conditions or added assumptions that make it a very simple, but useless statement. In particular, many assumed that both functions are continuously differentiable everywhere. Many proofs contained mistakes showing that the proof was memorised without understanding. Part (b)(i) is a standard bookwork that was done by almost everyone. Part (b)(ii) is a standard but a bit lengthy computation. Too many students made computational mistakes evaluating derivatives. Many tried to write the function in terms of $\sin x^{1/x^2}$ which is not well defined.

Question 7

In part (a)(i) many students missed details in the definition of Riemann integrability; (a)(ii) was very well-handled except by students who stated that a bounded monotone function is continuous, or is continuous except at finitely many points.

Students struggled with (b)(i): many incorrectly applied Taylor's theorem which yields expressions involving $f'(a)$ and/or $f'(b)$, neither of which necessarily exist.

Students who attempted (b)(ii) generally answered it very well - including many who wrote nothing for (b)(i). The two most common approaches which did not work were to attempt to break the expression down into a Riemann sum, or to attempt a proof by induction on n.

Paper III

Question 1.

This was a very popular question with a high standard of solutions. Most people solved a) well but a fair number did not indicate why the positive and not the negative root brach was taken. The most common issue with part b) was either an incorrect or a very complicated guess for the particular solution. Part c) was mostly done well, the common mistakes was failure to indicate how (i) was being applied to (ii) and confusion with negative and positive powers.

Question 2.

Part a) was a straightforward chain rule application and there were very few issues with it. Part b) caused the most confusion with a fair proportion of students computing either an entirely wrong object, multiplying the gradient by the point and not the tangent vector at the point, or forgetting to normalise the tangent vector. Part c) involved a fair amount of chain rule and some solutions got confused. Some had small mistakes but which meant the PDE did not simplify as requested. Many in this group were nevertheless able to find the general form of u assuming the given PDE holds.

Question 3.

Part a) was mostly done well. Its first two components (i) and (ii) were bookwork and simple transformation and did not cause any difficulties. Some solutions however failed to use (ii) to support their classification of critical points in (iii). Part b) was mostly solved fully and correctly, some scripts failed to compute the critical points correctly which made it impossible to solve this part. Part c) was again mostly solved well but with a significant number of scripts attracting lower marks. Some students simply made silly errors when plugging in function values but some either computed a wrong object, used $r(t)$ in place of $r'(t)$, or tried to use a different path.

Question 4.

This question was least popular, and many who did it seemed to find it difficult. There was no consensus about which part was hardest. In particular, there were about equal numbers who did part (a) well and had nothing for part (c), as vice versa, and few had both completely.

Part (a) was an expectation calculation for continuous random variables that required them to take account of the fact that a given formula for a density must be normalised for all choices of parameter. Unfortunately, many did not recognise this, and indeed quite a few attempted to calculate the integrals directly, despite the question's explicit instruction that this was not necessary.

Part (b) was a series of fairly standard questions about jointly continuous random variables. Many answers simply did not show a knowledge of what a joint density is, which made the following expectation calculation impossible. A number of answers, instead of calculating

$$\mathbb{E}[U \max\{U, V\}] = \iint u \cdot \max\{u, v\} \, \mathrm{d}\, u \, \mathrm{d}\, v,$$

wrote

$$\mathbb{E}[U \max\{U, V\}] = \mathbb{E}[U \max\{U, V\} \,|\, U > V]\mathbb{P}\{U > V\} + \mathbb{E}[U \max\{U, V\} \,|\, U \leq V]\mathbb{P}\{U \leq V\},$$

which is correct, but then either wrote $\mathbb{E}[U \max\{U, V\} \,|\, U > V] = \mathbb{E}[U^2]$ (ignoring the conditioning), which produces a completely wrong answer, or

$$\mathbb{E}[U \max\{U, V\} \,|\, U > V] = \int_0^1 \int_0^u u^2 \, \mathrm{d}\, v \, \mathrm{d}\, u,$$

which confuses the conditional expectation with $\mathbb{E}[U \max\{U, V\} 1\{U > V\}]$. This ends up doing the correct calculation, but with a spurious factor of $\frac{1}{2}$. A few answers confused densities with cdfs, and more wrote formulas for densities without indicating where they are valid, and where the density is 0.

The third part was an exercise in combinatorial probability (ci) followed by an application of the method of indicators. Among those who made serious attempts at this the only common error was to forget to consider that two beads could be connected going either way around the ring. A number of answers ignored (ci) in doing part ii, computing directly the distribution of the number of black beads. This turns out to be only slightly more complicated than the approach using indicators, and most attempts along these lines produced correct solutions.

Question 5. This was a popular question from the probability segment of the exam.

From Part *a*, section *i* challenged the majority of candidates with only few detecting that the rolls were not independent. Many answers appeared to confuse the formal notion of mathematical independence (which is the topic covered in this course) with a looser concept of causal independence. Correct answers obtained through the definition of independence or via more intuitive reasoning based on the correct concept were awarded full marks. In section *ii* most candidates performed well by stating Bayes' Theorem and the Law of Total Probability and then applying them correctly. Minor arithmetic errors occurred but if the candidates demonstrated correct application of the rules they were not penalised. Failure to present the rules being applied resulted in marks being deducted.

Candidates overall performed well in Part *b*, section *i*, setting up the relevant equation correctly and in most cases solving it well. In few cases the quadratic formula was misremembered and factorisation failed leading to marks being deducted. Some candidates did not realise that one of the roots for α had to be rejected which meant they could not successfully complete this sub-section. In section *ii*, candidates encountered some difficulties, although by setting up the equation and stating the partition theorem for expectations they were awarded partial marks even if they did not succeed in the proof. Most solved the following quadratic correctly although many failed to notice that the root with the minus sign had to be selected, which caused some difficulties for section *iii*. In that section, candidates were evenly split between using implicit differentiation and calculating the derivative. In the latter case, mistakes carried over from the solutions of section *ii* were not further penalised. Almost all candidates who used implicit differentiation ended up with the right answer.

Question 6.

Part a asked for statement and proof of Markov's and Chebyshev's inequalities. Many students wrote down expressions for inequalities with no context: something like $\mathbb{P}\{X \geq t\} \leq \mathbb{E}[X]/t$ without saying what X and t are. Many wrote strict inequalities where $\geq$ is conventional, and some switched vaguely between strict and non-strict inequalities. The version of the inequalities stated in the lecture notes includes the assumption that the relevant expectations are finite. This assumption not strictly required for the statements to be correct, though some of the manipulations in the proof become problematic without it. Very few answers included these statements. Similarly, the proof given in the lecture notes depends on the Partition Theorem, which is given in a form that requires the partition sets to have positive probability. This is dismissed in the lecture notes with the lapidary remark "We may assume that $\mathbb{P}(A) \in (0, 1)$, since otherwise the result is trivially true." A very small number of students repeated this formula, but most ignored the problem. (Another small number gave a superior proof that does not involve dividing by $\mathbb{P}(A)$.)

Part b asked for the application of linearity of expectations to show that the sum of random variables with mean 0 also has mean 0 (few had any difficulty with this) and then to derive the variance-sum formula. A fair number of answers did not relate the variance and covariance to the expectation — despite the question's explicit instruction that "standard facts about expectations" may be used — but simply asserted that $\mathrm{Var}(X + Y) = \mathrm{Var}(X) + \mathrm{Var}(Y) + \mathrm{Cov}(X, Y)$ and assumed bilinearity, leaving very little to do. Partial credit was granted for stating both of these facts clearly, and correctly using them to derive the final result. Others asserted that

$$\mathrm{Var}(\sum X_i) = \sum_i \mathrm{Var}(X_i) + \sum_{i \neq j} \mathrm{Cov}(X_i, X_j),$$

and substituted $\mathrm{Var}(X_i) = \sigma_i^2$ in the first term and rewrote the second as $2\sum_{i=1^{n-1}} \sum_{j=i+1}^{n}$, which received no credit.

Part c applied the earlier parts to the distribution of the sum of a sample without replacement from a collection of numbers. The mean and variance of a single draw (ci) were found by most, though the calculations and/or explanations were often not presented in a way that made them readily interpretable. Not a few seemed perplexed by the joint distribution of X and Y, two successive draws without replacement. In many cases nothing was said about the equal distribution, and quite a few asserted that "same distribution" is equivalent to "same mean and variance". cii asked about the joint distribution of two separate choices. Most explained the equivalence of the distributions either with an intuitive argument or with a correct calculation (sometimes more elaborate than had been anticipated). A not insignificant number of answers calculated the mean and variance of Y, and then asserted (incorrectly) that random variables have the same distribution if their means and variances are equal. The covariance was generally computed correctly, with a variety of approaches, sometimes with minor errors. A few students simply asserted that X and Y were independent, hence had covariance 0.

The covariance was challenging, though a significant fraction did get the right answer. Of those who did not, most did not write down any expression like $\mathbb{E}[XY] = \sum xy\mathbb{P}(X = x, Y = y)$; those who did were generally able to extract the answer from the available information, modulo occasional calculation errors. Some confused "same distribution" with "same random variable", and concluded that $\mathrm{Cov}(X,Y) = \mathrm{Cov}(X,X) = \mathrm{Var}(X) = 1$. Part ciii asked for the mean and variance of the sum Z of 500 draws, combining the results of cii and b, and asked for a probability bound, for which Chebyshev's inequality was required. Full credit was given if the wrong covariance was used correctly. The most common error at this stage was to claim $\mathbb{P}\{Z \geq 50\} = \frac{1}{2}\mathbb{P}\{|Z| \geq 50\}$, with or without an assertion that Z is symmetric. This is not true, in general, and even approximately cannot be inferred from the information given. Full credit was given to those who had miscalculated the covariance, but then correctly applied the incorrect covariance.

Part c asked various questions about sampling uniformly without replacement from a collection of numbers. ci asked expectation and variance, which most did with little difficulty. cii asked about the joint distribution of two separate choices. Most explained the equivalence of the distributions either with an intuitive argument or with a correct calculation (sometimes more elaborate than had been anticipated). A not insignificant number of answers calculated the mean and variance of Y, and then asserted (incorrectly) that random variables have the same distribution if their means and variances are equal. The covariance was generally computed correctly, with a variety of approaches, sometimes with minor errors. A few students simply asserted that X and Y were independent, hence had covariance 0. civ required the students to combine the covariance with the variance formula of part b, and with Chebyshev's inequality. Full credit was given to those who had miscalculated the covariance, but then correctly applied the incorrect covariance.

Question 7.

Part a of the question was straightforward bookwork about definitions of the singular-value decomposition. A surprising number of students who answered this question skipped this part entirely. In part ai some students wrote that Λ is diagonal $n \times n$ and Q is $p \times n$. This could be an alternative representation of the singular value decomposition, but this requires that the orthogonality relations in (ii) be written differently, depending on which of n or p were larger.

Part b asked about single-variable least-squares regression. The first three sections were standard. (bi) asked for the derivation of the least-squares regression coefficients, while (bii) asked for the conditions under which these will be MLEs. Not a few students mistakenly read into the first part the assumption that the errors were independent normal, which then made it impossible to answer the second section (since those were the missing assumptions that needed to be stated). In those cases the points were deducted in (bii), but full credit was given for (bi); when the student restated the assumption in (bii) one point was deducted for erroneously presuming it in (bi). The other major error in (bi) also related to incompletely reading the assumptions of the question, namely neglecting to use the stated fact that $\sum x_i = 0$. Without this the calculations became significantly more complicated. It was not marked as wrong when the student managed nonetheless to get a correct answer, but an incorrect answer that would have been correct with $\sum x_i = 0$ was still given only partial credit. Almost no student addressed the question of whether the critical point for the sum of squares is a minimum. As this is both quite obvious in one sense (because it is a quadratic with positive coefficients), and somewhat complicated in ways not directly germane to this course (testing the critical points of a multivariate function) no points were deducted for this.

Most of the answers for (biii) were essentially correct, though some were inadequate, for neglecting to make clear how the assumptions $\sum x_i = 0$ and $\mathbb{E}[\epsilon_i]$ were being used.

(biv) Many did not recognise that the residuals will be centred at 0. There was some confusion between *residuals* and *studentised residuals*. Many did not seem to have a clear idea of what heteroskedasticity is. The lectures defined the residual plot as having $\hat{y}_i$ on the abscissa, but the notes also said that in the one-dimensional setting this is equivalent to using x_i, so either one was accepted. (Most students used x_i, suggesting that it might not be made clear in the lectures why $\hat{y}_i$ would be preferred.)

Question 8.

In this question many candidates performed better in Part *a* than in Part *b* primarily because they did not attempt Part *b*. Overall low scores reflect this. Almost all candidates correctly calculated the likelihood estimator but only 5 established whether it was a maximum using the second derivative. Not demonstrating whether it is a maximum was consistently penalised. Candidates demonstrated infinite bias well. The alternative estimator caused some difficulty, especially in correctly applying the binomial theorem - many candidates identified but failed to apply it correctly.

Part *b* was found to be more challenging, leading to few candidates even attempting it. Of those who did, a clear definition of the central limit theorem was awarded some marks even if the calculation of the confidence interval was incorrect. Mistakes in the confidence interval from section *i* that were carried over to section *ii* were not penalised twice: If the candidates applied it correctly in section *ii* they were awarded full marks. Section *iii* was challenging, with few establishing that the initial estimate was too low. Well-reasoned intuitive arguments were awarded full marks. Overall, students performed very well on Part *a* and many found Part *b* difficult.

Question 9.

This was a very popular question. Many did not attempt the sections *v* and *vi*, leading to the suspicion that they did not realise there were more questions on the last page.

For Part *a*, the CLT was stated well, and all correct definitions were accepted. Application of the CLT was done overall well.

Most candidates described agglomerative clustering, although a few demonstrated that they did not read the full question in advance and presented the linkage methods in Section *i*. If that was done well they were awarded the relevant points from Section *ii*. In Section *ii* candidates missed marks by not being careful enough with notation, although overall they performed well. In Section *iii* although many worked out which was single linkage, they failed to give reasons, missing some marks. Very few candidates understood biplots and that is reflected in the performance for Section *v*. In Section *vi* candidates had to also explain why the two points could not be G, H instead, which very few did. Overall, this was a well-balanced question, examining both theoretical and practical aspects.

Paper IV

Question 1.

A popular question, largely well done, with many candidates completing up to (b)(i) or (ii). The invertibility of XX^T is most easily seen by noting its determinant is $|\mathbf{a} \wedge \mathbf{b}|^2$ by (a)(iii). To do (b)(iii) it is sufficient to focus on the effect of P on $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{a} \wedge \mathbf{b}$. This can be done by working out PX^T and $P(\mathbf{a} \wedge \mathbf{b})^T$.

Question 2.

A popular question. In (a)(ii) there was some occasional slackness/lack of rigour in explaining quite why a 2×2 orthogonal matrix had the given form, with some arguments closer to showing A_θ and B_θ are orthogonal, rather than the converse. The purpose of introducing co-ordinates X and Y in (c)(i) was that the reflection in (ii) was given by $(X, Y) \mapsto (X, -Y)$. So to find the image $T(x, y)$, one method is to convert $P = (x, y)$ into XY-co-ordinates, find the image, and then convert back to xy-co-ordinates. For (iii) note that $P \in T(E)$ if and only if $T(P) = T^{-1}(P) \in E$.

Question 3.

This question was not particularly popular, though it was well done by those who attempted it, with many gaining high or full marks. The missing part S of the torus in part (b) consists of the outside equator of the torus ($u = 0$), and of the original generating circle ($v = 0$). For (c) candidates needed to appreciate that the formula from (b)(ii) still works with a being the radius of the generating circle (so $a = 1$) and b being the distance of the circle's centre from the axis of revolution (so $b = 4$).

Question 4.

Question 4 was found the most challenging of the Dynamics questions, and was attempted by relatively few candidates. Many of those failed to notice that θ was defined relative to the downward vertical and drew pictures of the cardioid on its side. Converting the standard expressions for the different energy contributions to be written in terms of θ and $\dot{\theta}$ proved more difficult than anticipated, but those who managed this generally managed to derive the correct equation of motion in part (a). Part (b) was done well by most who moved on to it. A frequent confusion, which was not penalised, was to refer to the right hand side of the equation as '$F(\theta)$', when it depends on $\dot{\theta}$ too. The linearisation in part (c) was slightly more challenging, and the frequency of oscillations was found by very few candidates.

Question 5.

Question 5 was attempted by almost everyone, and was generally done well. Most candidates got the bookwork in part (a). Most also got the marks for part (b), although some explanations of the value of h were incomplete. The sketches in part (c) were not done so well, the most common mistakes being to get confused between the angle of deflection and the polar angle itself (the two extreme limits were often drawn the wrong way around), or to draw the path being deflected around the atom rather than away from it.

Question 6.

Question 6 was quite popular and was mostly done well. Part (a) was done very elegantly by some candidates, but a lot of people confused themselves by not using clear enough different notation for the position vectors of the two different particles. Part (b) was addressed well by most, who realised that circular motion should occur in this case; there were a few nice diagrams showing the second particle stuck in the hole. Part (c) was generally done quite well; the most common difficult was for those who realised this expression represented an energy and tried to write down the energy from scratch (rather than integrating the equation in (a)), which frequently resulted in neglecting the kinetic energy of the second particle.

Question 7

This question was very well answered with only a handful of poor attempts. Only one candidate did not collect full marks in (a)(i) in finding the highest common factor. Most candidates were also happy with finding the solutions in part (a)(ii), although a common error was doubling the homogeneous solution as well as the particular solution.

Candidates were mostly able to use the bisection method and Newton iteration in part (b), with marks lost mainly due to errors in calculation. Part (b)(iii) troubled a significant number of candidates. Either they did not remember/show that the Newton iteration was quadratically convergent, or they used a stopping criterion involving $\sqrt{2}$, which we are trying to find.

Paper V

Question 1. was attempted by almost all of the candidates. Parts (a),(b), and (c)(i)(ii) did not pose too much difficulty. Part (c)(iii) proved harder, with many errors in setting the correct bounds or the correct spherical coordinates. It was also acceptable to solve this using cylindrical (rather than spherical) coordinates.

Question 2. was the least popular question, but those who attempted it did generally well.

Question 3.: part (b) required a complete proof (e.g., $\mathbf{F}$ is conservative therefore there exists a ϕ... was not an acceptable answer) and this was found difficult by the majority of candidates. In (d)(ii), many candidates identified correctly the example $\mathbf{F}(\mathbf{r}) = \frac{1}{r}\mathbf{e}_\theta$, but did not explain properly why this vector field was not conservative.

Question 4

This question was generally well attempted, with a number of attempts gaining full marks. Every candidate was able to find the Fourier coefficients, however some were missing prefactors which caused trouble later in the question. Most candidates were able to draw the limiting function, however many were sloppy with their sketches and omitted the detail around the discontinuity. Part (b) was done well unless errors had been made previously. Showing the second sum was more difficult with most candidates understanding that they needed to consider $x = \pi/2$, however many guessed at the answer and did satisfactorily show how to get there. Part (c) caused the most trouble and invited a number of wildly different attempts. Candidates who realised these are the even and odd expansions generally did well.

Question 5

Question 5 was relatively straightforward, with candidates needing to demonstrate only a limited level of ingenuity to score highly. Whilst a number of candidates did produce faultless or near-faultless answers, a significant minority of responses (approximately 25%) were somewhat flimsy.

Most candidates scored highly on part (a). Whilst part (b) tested bookwork, a significant number of candidates were unable to respond satisfactorily. A surprising number of candidates were flummoxed by the easier tasks of part (c), with some candidates struggling with basic partial differentiation.

Question 6

The question was a relatively accessible exercise on the Helmholtz equation, with separation of variables and standard ODE solving at the heart of the methodology. Many students made very good progress throughout, and, in light of an approachable (less technical) final part, a non-negligible number of full marks were obtained. There were a reasonable number of attempts in which despite it being clear that the students mastered the mathematical techniques well enough, a lack of algebraic rigour and attention to detail at the desired format of the equations for F and G led to significant complications later on. Quite often in (b) a clear statement for the range of C in order to ensure existence of non-trivial solutions was missing. Sign errors also often led to a mischaracterisation of the condition $k^2 = \pi^2(n^2 + m^2)$ as $k^2 = \pi^2(n^2 - m^2)$ or similar, after which significant progress in part (c) was no longer possible. Interestingly, even with the correct condition, there were multiple instances when solving for positive integer pairs (n, m) such that $n^2+m^2 = 50$ led to either missing or additional incorrect solutions.

Computational Mathematics

The students chose two out of three Matlab projects, and each was marked out of 20. More than half the students scored at least 30 of the 40 points allocated. The assessment was based on published reports and submitted code.

Projects **A** [*Solving nonlinear equations*] and **C** [*Solving an initial value problem*] were the overwhelming favourite choices, with only 7 of the 185 students attempting Project **B** [*Nonlinear boundary value problems*]. The few who did attempt **B**, however, were quite successful:

	A	**B**	**C**
Avg	15.04	18.29	15.4
Stdev	2.96	1.38	2.75

Marks were lost for reasons ranging from the trivial (a complete omission of comments in otherwise functioning code) to the serious (no published report submitted, only incorrect code). Fortunately, the former occurred far more frequently than the latter.

E. COMMENTS ON PERFORMANCE ON IDENTIFIABLE INDIVIDUALS

This section has been redacted from the public report.

F. NAMES OF MEMBERS OF THE BOARD OF EXAMINERS

- **Examiners:** Prof. Dmitry Belyaev (Chair), Prof. Dan Ciubotaru, Dr Richard Earl, Prof. Ian Hewitt, Dr Vicky Neale, Prof. Jan Obloj, Prof. David Steinsaltz.
- **Assessors:** Dr Maria Christodoulou, Dr Radu Cimpeanu, Dr Marcelo De Martino, Dr Adam Gal, Dr David Hume, Dr Chris Lester, Dr Andrew Mellor.