Examiners' Report: Preliminary Examination in Mathematics Trinity Term 2022

November 30, 2022

Part I

A. STATISTICS

178 candidates in Mathematics and Mathematics & Statistics were awarded an overall year outcome. Candidates on both degrees submit the same assessments and no distinction is made between the two groups in this document.

Table 1: Numbers in each outcome class

	Numbers					Percentages				
	2022	(2021)	(2019)	(2018)	(2017)	2022	(2021)	(2019)	(2018)	(2017)
Distinction	53	(60)	(54)	(58)	(62)	29.78	(30.61)	(29.19)	(29.44)	(30.85)
Pass	116	(124)	(120)	(126)	(124)	65.17	(63.27)	(64.86)	(63.96)	(61.69)
Partial Pass	6	(7)	(8)	(10)	(13)	3.37	(3.57)	(4.32)	(5.08)	(6.47)
Incomplete	0	(2)	(1)	(0)	(0)	0.00	(1.02)	(0.54)	(0.00)	(0.00)
Fail	3	(3)	(2)	(3)	(2)	1.69	(1.53)	(1.08)	(1.52)	(0.99)
Total	178	(196)	(185)	(197)	(201)	-	-	-	-	-

B. NEW EXAMINING METHODS AND PROCEDURES

The methods and procedures reverted to the examining methods used prior to the COVID-19 pandemic.

C. CHANGES IN EXAMINING METHODS AND PROCEDURES CURRENTLY UNDER DISCUSSION OR CONTEMPLATED FOR THE FUTURE

None.

D. NOTICE OF EXAMINATION CONVENTIONS FOR CANDIDATES

The Notice to Candidates, containing details of the examinations and assessments, was issued to all candidates at the beginning of Trinity term. The Examination Conventions in full were made available at

https://www.maths.ox.ac.uk/members/students/undergraduate-courses/examinations-assessments/examination-conventions.

Part II

A. GENERAL COMMENTS ON THE EXAMINATION

Acknowledgements

First, the Moderators should like to thank the Undergraduate Studies Administration Team and their coordinating officer, Clare Sheppard.

We should also like to thank Charlotte Turner-Smith for her invaluable experience with the Mitigating Circumstances Panel, and Matthew Brechin and Waldemar Schlackow for maintaining and running the examination database and their assistance during the final examination board meeting.

We should like to thank the lecturers for their feedback on proposed exam questions; the assessors for their extraordinary assistance with marking; Nick Trefethen for coordinating the Computational Mathematics assessments, and Pablo Brubeck, Julia Krol, and Fabian Laakmann for marking them; and the team of graduate checkers for their rapid work checking the marks on the papers.

Finally the Moderators are extremely grateful to Haleigh Bellamy for her fantastic work running the examinations process this year – thank you!

Timetable

The examinations began on Monday 20th June and ended on Friday 24th June.

Setting and checking of papers

The Moderators set and checked the questions, model solutions, and mark schemes. Every question was carefully considered by at least two moderators, and feedback was sought from lecturers. In a small number of cases feedback from lecturers was not available, and those were discussed in more detail until the Board of Moderators was satisfied that all questions were appropriate.

The questions were then combined into papers which were considered by the Board of Moderators and small changes were made to satisfy the Board that the papers were appropriate. After this a final proof-reading of the papers was completed before the Camera Ready Copies (CRCs) were produced. The whole Board of Moderators signed off the CRCs which were submitted to Examination Schools.

Marking and marks processing

The Moderators and Assessors marked the scripts according to the mark schemes and entered the marks. Small adjustments to some mark schemes were made at this stage, and care was take to ensure these were consistently applied to all candidates.

A team of graduate checkers, supervised by Haleigh Bellamy and Clare Sheppard, sorted all the scripts for each paper and carefully cross checked these against the mark scheme to spot any unmarked parts of questions, addition errors, or wrongly recorded marks. A number of errors were corrected, with each change checked and signed off by a Moderator, at least one of whom was present throughout the process.

Determination of University Standardised Marks

Marks for each individual assessment are reported as a University Standard Mark (USM) which is an integer between 0 and 100 inclusive. The Moderators used their academic judgment to map the raw marks on individual assessments to USMs using a process similar to previous years. In coming to this judgement the board followed the advice from the Mathematics Teaching Committee that the percentages awarded for each USM range of the examination should be in line with recent years. This alignment can be seen in Table 1; in more detail, for Papers I–V, a piecewise linear map was constructed as follows:

1. Candidates' raw marks for a given paper were ranked in descending order.
2. The default percentages p_1 of Distinctions and p_2 of Nominal Upper Seconds were selected.
3. The candidate at the p_1 percentile from the top of the ranked list was identified and assigned a USM of 70, and the corresponding raw mark denoted R_1.
4. The candidate at the $(p_1 + p_2)$ percentile from the top of the list was assigned a USM of 60 and the corresponding raw mark denoted R_2.
5. The line segment between $(R_1, 70)$ and $(R_2, 60)$ was extended linearly

to USMs of 72 and 57 respectively, and the corresponding raw marks denoted C_1 and C_2 respectively.

6. A line segment through $(C_2, 57)$ was extended towards the vertical axis, as if it were to join the axis at $(0, 10)$, but the line segment was terminated at a USM of 37 and the raw mark at the termination point was denoted C_3.

With these data a piecewise linear map was constructed with vertices at $\{(0,0), (C_3, 37), (C_2, 57), (C_1, 72), (100, 100)\}$.

Reports from the Assessors describing the apparent relative difficulty and the general standard of solutions for each question were then considered, and the Board decided that the values of $p_1 = 31\%$ and $p_2 = 48\%$ were suitable for all papers.

In line with previous years, for the Computational Mathematics assessment the linear map with gradient 2.5 was used to map from raw marks to USMs.

The vertices of the final maps used in each assessment are listed in Table 2.

Table 2: Vertices of final piecewise linear model

Paper	Vertices				
I	(0.0,0)	(28.1,37)	(48.9,57)	(74.4,72)	(100.0,100)
II	(0.0,0)	(28.8,37)	(50.1,57)	(84.6,72)	(100.0,100)
III	(0.0,0)	(29.4,37)	(51.1,57)	(100.6,72)	(120.0,100)
IV	(0.0,0)	(20.9,37)	(36.4,57)	(69.4,72)	(100.0,100)
V	(0.0,0)	(17.1,37)	(29.7,57)	(61.2,72)	(80.0,100)
CM	(0.0,0)				(40.0,100)

With the USMs, a provisional outcome class for each candidate was produced as follows: Write MI, MII, $MIII$, MIV and MV for the USMs on Papers I–V respectively, and CM for the USM on the Computational Mathematics assessment. Write Av_1 and Av_2 for the quantities

$$\frac{MI + MII + \frac{6}{5}MIII + MIV + \frac{4}{5}MV + \frac{1}{3}CM}{5\frac{1}{3}}$$

and

$$\frac{MI + MII + \frac{6}{5}MIII + MIV + \frac{4}{5}MV}{5}$$

respectively, symmetrically rounded. With these auxiliary statistics the provisional outcome class was determined by the definitions:

Distinction: both $Av_1 \geq 70$ and $Av_2 \geq 70$ and a USM of at least 40 on each paper and for the Computational Mathematics assessment;

Pass: not a Distinction and a USM of at least 40 on each paper and for the Computational Mathematics assessment;

Partial Pass: not a Pass or Distinction and a USM of at least 40 on three or more of Papers I–V;

Fail: not a Partial Pass, Pass, or Distinction, and a USM of less than 40 on three or more of Papers I–V.

The scripts of those candidates at the boundaries between outcome classes were scrutinised carefully to determine which attained the relevant qualitative descriptors and changes were made to move those into the correct class.

Mitigating Circumstances were then considered using the banding produced by the Mitigating Circumstances Panel, and appropriate actions were taken and recorded.

Table 3 gives the rank list ordered by the average of Av_1 and Av_2 (as defined above), showing the number and percentage of candidates with USM greater than or equal to each value.

Table 3: Rank list of average USM scores

		Candidates with USM $\geq x$	
USM (x)	Rank	Number	Percentage
92.32	1	1	0.56
88.12	2	2	1.12
87.68	3	3	1.69
87.58	4	4	2.25
85.88	5	5	2.81
85.86	6	6	3.37
85.28	7	7	3.93
85.08	8	8	4.49
84.52	9	9	5.06
82.89	10	10	5.62
82.52	11	11	6.18
82.40	12	12	6.74
82.24	13	13	7.30
81.88	14	14	7.87
81.80	15	15	8.43
81.64	16	16	8.99
81.60	17	17	9.55
81.04	18	18	10.11
80.40	19	19	10.67
80.12	20	20	11.24
79.48	21	21	11.80

Table 3: Rank list of average USM scores (continued)

USM (x)	Rank	Candidates with average USM $\geq x$ Number	Percentage
78.76	22	22	12.36
78.64	23	23	12.92
78.24	24	24	13.48
77.40	25	25	14.04
76.76	26	26	14.61
76.36	27	27	15.17
76.28	28	28	15.73
76.06	29	29	16.29
75.56	30	30	16.85
75.48	31	31	17.42
75.36	32	32	17.98
74.44	33	33	18.54
73.70	34	34	19.10
73.48	35	35	19.66
73.24	36	37	20.79
73.24	36	37	20.79
73.16	38	38	21.35
73.04	39	39	21.91
72.92	40	40	22.47
72.84	41	42	23.60
72.84	41	42	23.60
72.56	43	43	24.16
72.12	44	44	24.72
71.76	45	46	25.84
71.76	45	46	25.84
71.68	47	47	26.40
70.48	48	48	26.97
70.44	49	49	27.53
70.28	50	50	28.09
70.00	51	51	28.65
69.80	52	52	29.21
69.69	53	53	29.78
69.44	54	55	30.90
69.44	54	55	30.90
69.24	56	58	32.58
69.24	56	58	32.58
69.24	56	58	32.58
69.12	59	59	33.15
69.04	60	60	33.71
68.96	61	61	34.27

Table 3: Rank list of average USM scores (continued)

		Candidates with average USM $\geq x$	
USM (x)	Rank	Number	Percentage
68.88	62	62	34.83
68.76	63	63	35.39
68.60	64	64	35.96
68.44	65	65	36.52
68.40	66	66	37.08
68.32	67	67	37.64
67.92	68	68	38.20
67.84	69	69	38.76
67.68	70	70	39.33
67.52	71	71	39.89
67.41	72	72	40.45
67.40	73	74	41.57
67.40	73	74	41.57
67.32	75	75	42.13
67.00	76	76	42.70
66.96	77	77	43.26
66.76	78	78	43.82
66.56	79	80	44.94
66.56	79	80	44.94
66.40	81	81	45.51
66.35	82	82	46.07
66.04	83	83	46.63
65.88	84	84	47.19
65.80	85	85	47.75
65.68	86	86	48.31
65.64	87	89	50.00
65.64	87	89	50.00
65.64	87	89	50.00
65.48	90	90	50.56
65.08	91	91	51.12
65.06	92	92	51.69
64.88	93	93	52.25
64.80	94	94	52.81
64.72	95	95	53.37
64.64	96	96	53.93
64.58	97	97	54.49
64.56	98	98	55.06
64.55	99	99	55.62
64.28	100	100	56.18
64.24	101	101	56.74

Table 3: Rank list of average USM scores (continued)

		Candidates with average USM $\geq x$	
USM (x)	Rank	Number	Percentage
64.20	102	102	57.30
64.16	103	103	57.87
63.92	104	104	58.43
63.60	105	105	58.99
63.48	106	107	60.11
63.48	106	107	60.11
63.44	108	108	60.67
63.36	109	109	61.24
63.00	110	110	61.80
62.80	111	111	62.36
62.76	112	112	62.92
62.72	113	113	63.48
62.64	114	114	64.04
62.60	115	115	64.61
62.52	116	116	65.17
62.38	117	117	65.73
62.32	118	118	66.29
62.24	119	120	67.42
62.24	119	120	67.42
62.12	121	121	67.98
62.00	122	122	68.54
61.80	123	124	69.66
61.80	123	124	69.66
61.72	125	125	70.22
61.70	126	126	70.79
61.60	127	127	71.35
61.56	128	129	72.47
61.56	128	129	72.47
61.40	130	131	73.60
61.40	130	131	73.60
61.36	132	132	74.16
61.32	133	133	74.72
61.28	134	134	75.28
61.20	135	135	75.84
61.16	136	136	76.40
61.12	137	137	76.97
61.08	138	138	77.53
60.96	139	139	78.09
60.80	140	140	78.65
60.76	141	141	79.21

Table 3: Rank list of average USM scores (continued)

		Candidates with average USM $\geq x$	
USM (x)	Rank	Number	Percentage
60.68	142	142	79.78
60.64	143	143	80.34
60.56	144	144	80.90
60.08	145	145	81.46
59.92	146	146	82.02
59.52	147	147	82.58
59.48	148	148	83.15
59.24	149	149	83.71
58.96	150	150	84.27
58.20	151	151	84.83
58.00	152	152	85.39
57.96	153	153	85.96
57.84	154	154	86.52
57.72	155	155	87.08
57.68	156	156	87.64
57.64	157	157	88.20
57.40	158	158	88.76
56.84	159	159	89.33
56.56	160	160	89.89
55.52	161	162	91.01
55.52	161	162	91.01
55.28	163	163	91.57
53.88	164	164	92.13
53.80	165	165	92.70
52.96	166	166	93.26
51.84	167	167	93.82
51.12	168	168	94.38
50.84	169	170	95.51
50.84	169	170	95.51
47.91	171	171	96.07
46.80	172	172	96.63
45.75	173	173	97.19
42.72	174	174	97.75
39.68	175	175	98.31
38.60	176	176	98.88
38.04	177	177	99.44
0.00	178	178	100.00

Recommendations for next year's Moderators and Teaching Committee

1. It is recommended that markers completing assessor reports be told that a detailed mapping between raw marks and USMs will be arrived at by the Board of Moderators later and so their report does not need to include this.
2. It is recommended that assessor reports be produced for the Computational Mathematics assessments.
3. The Board noted that there are definitions of gender that do not partition populations into those who are male and those who are female and asks Teaching Committee for guidance on whether the reporting in §B could usefully be different or expanded in future years to capture this or other equal opportunity issues.

B. EQUAL OPPORTUNITY ISSUES AND BREAKDOWN OF THE RESULTS BY GENDER

Table 4 shows the performances of candidates by gender. Here gender is the gender as recorded on eVision.

Table 4: Breakdown of results by gender

Outcome	Number								
	2022			2021			2019		
	Female	Male	Total	Female	Male	Total	Female	Male	Total
Distinction	8	45	53	7	53	60	8	46	54
Pass	43	73	116	50	74	124	49	71	120
Partial Pass	2	4	6	2	5	7	4	4	8
Incomplete	0	0	0	0	2	2	0	1	1
Fail	0	3	3	3	0	3	1	1	2
Total	53	125	178	62	134	196	62	123	185
Outcome	Percentage								
	2022			2021			2019		
	Female	Male	Total	Female	Male	Total	Female	Male	Total
Distinction	15.09	36.00	29.78	11.29	39.55	30.61	12.90	37.40	29.19
Pass	81.13	58.40	65.17	80.65	55.22	63.27	79.03	57.72	64.86
Partial Pass	3.77	3.20	3.37	3.23	3.73	3.57	6.45	3.25	4.32
Incomplete	0.00	0.00	0.00	0.00	1.02	0.74	0.00	0.81	0.54
Fail	0.00	2.40	1.69	4.84	0.00	1.53	1.61	0.81	1.08

C. STATISTICS ON CANDIDATES' PERFORMANCE IN EACH PART OF THE EXAMINATION

Table 5: Numbers taking each paper

Paper	Number of Candidates	Average raw mark	Std dev of raw marks	Average USM	Std dev of USMs	Number failing
I	177	63.66	13.33	66.05	10.05	3
II	177	69.93	16.41	66.91	11.52	2
III	177	79.46	21.69	66.38	11.03	4
IV	176	56.02	16.79	66.47	10.36	4
V	177	48.77	13.81	67.05	10.09	2
CM	177	32.29	5.02	80.99	12.54	3

Tables 6–11 give the performance statistics for each individual assessment, showing for each question the average mark, first over all attempts, and then over the attempts used; the standard deviation over all attempts; and finally the total number of attempts, first those that were used, and then those that were unused.

Table 6: Statistics for Paper I

Question Number	Average mark All	Average mark Used	Std dev	No. of Attempts Used	No. of Attempts Unused
Q1	15.77	15.77	3.03	177	0
Q2	16.52	16.52	3.97	173	0
Q3	13.37	13.37	4.56	59	0
Q4	11.31	11.31	3.49	121	0
Q5	8.97	9.02	3.40	115	1
Q6	8.46	8.54	3.94	89	2
Q7	11.54	11.54	3.45	144	0

Table 7: Statistics for Paper II

Question Number	Average mark All	Average mark Used	Std dev	No. of Attempts Used	No. of Attempts Unused
Q1	16.35	16.35	2.81	163	0
Q2	11.76	11.76	4.60	111	0
Q3	13.16	13.16	4.34	80	0
Q4	15.86	15.86	3.71	166	0
Q5	13.00	13.00	4.43	27	0
Q6	15.17	15.17	3.85	161	0
Q7	11.14	11.14	5.31	173	0

Table 8: Statistics for Paper III

Question Number	Average mark All	Average mark Used	Std dev	No. of Attempts Used	No. of Attempts Unused
Q1	14.22	14.32	4.80	164	2
Q2	14.87	14.87	3.71	69	0
Q3	14.92	14.92	3.74	121	0
Q4	14.16	14.16	4.23	141	0
Q5	12.39	12.39	4.26	108	0
Q6	14.28	14.28	4.26	105	0
Q7	9.38	9.39	6.35	128	2
Q8	12.58	12.58	4.35	142	0
Q9	13.81	13.99	4.25	76	1

Table 9: Statistics for Paper IV

Question Number	Average mark All	Average mark Used	Std dev	No. of Attempts Used	No. of Attempts Unused
Q1	15.41	15.41	4.49	152	0
Q2	12.65	12.87	6.25	87	2
Q3	13.67	13.67	3.77	111	0
Q4	7.94	7.94	3.20	170	0
Q5	8.27	8.27	4.30	169	0
Q6	10.22	11.25	8.06	8	1
Q7	11.74	11.74	4.93	174	0

Table 10: Statistics for Paper V

Question Number	Average mark All	Average mark Used	Std dev	No. of Attempts Used	No. of Attempts Unused
Q1	15.53	15.53	3.66	153	0
Q2	13.34	13.34	4.19	50	0
Q3	11.28	11.28	4.81	151	0
Q4	13.78	13.78	4.97	176	0
Q5	5.10	5.28	3.79	58	2
Q6	10.07	10.11	4.76	114	1

Table 11: Statistics for Computational Mathematics

Question Number	Average mark All	Average mark Used	Std dev	No. of Attempts Used	No. of Attempts Unused
Project A	16.06	16.06	2.40	124	0
Project B	16.63	16.63	2.52	128	0
Project C	15.80	15.80	3.25	101	0

D. COMMENTS ON PAPERS AND ON INDIVIDUAL QUESTIONS

Paper I

Question 1. This was attempted by every candidate. The bookwork from part (a) did not give trouble beyond candidates overlooking the last line asking for the definition of dimension. A fair number of candidates had difficulties in part (b). A common issue here was treating the spanning and the linear independence separately and working directly from the definition of linear independence, which leads to long-winded answers with plenty of opportunity for missteps. Successful approaches usually worked with the matrix for the basis transformation or observed that linear dependence comes down to co-linearity. Candidates did well on Part (c)(i), with a few exceptions. The next part (ii) was hard for the majority of candidates, with many interpreting the word "deduce" to indicate that they should work with the basis from part (i), rather than just the dimension found there. The final part (iii) was almost universally successfully answered, though perhaps the question should have asked for a non-diagonal B to make it more interesting.

Question 2. This was attempted by almost every candidate. The only stumbling block in (a)(i) was the linear independence part of the proof of the Rank-Nullity theorem, with candidates falsely claiming that the relevant vector did not lie in the kernel of T. Part (ii) exposed a lack of familiarity with the notion of 1-1 for some candidates, with some interpreting it to mean bijective. Otherwise most candidates gave correct answers. In part (b) many candidates lost a mark for either not showing that T was linear or not showing it mapped to polynomials of degree at most n. Candidates used a plethora of methods to show that T is onto when $\lambda \neq 0$, with some using a matrix for T; many solving directly for coefficients; many using the defining property of the exponential function; and some resorting to differential equation techniques. A small majority of candidates completed this part successfully. The final part of this question where λ was assumed to be zero was clearly easy for almost all candidates.

Question 3. This question was not very popular with candidates, perhaps because of its abstract nature. Part (a) was found to be easy, and many candidates got full marks for this question. Part (a)(i) was an 'if and only if' question but some candidates only answered the 'only if' direction. (b) was about checking definitions of vector spaces and it was done well. Part (c) was harder, most of the candidates left it blank, but for those who tried to answer this question, their attempts were in general reasonable and mostly correct.

Question 4. Most of the candidates did part (a) correctly. For part (b), it was generally done well but some candidates could not prove that the matrix is invertible after they showed the matrix has full rank. Most of the candidates left parts (c)(ii) and (iii) blank. For (c)(ii) many candidates mistakenly applied the result in (b) directly as they did not notice all the eigenvectors are in $\mathbb{R}^n$ instead of $\mathbb{Q}^n$.

Question 5. The early parts of the question were completed well. For (a)(v) it was expected that candidates would adapt the proof of non-triviality of the centre in groups of order p^r from the lecture notes. This seems to have prevented a number of attempts from proceeding further and resulted with a lot of scripts receiving 8/20. It might have helped to switch the order of (a)(v) and (b)(i). Those who continued to part (b)(i) in many cases did well showing by induction that Abelian groups of order p^r are generated by at most r elements. The general idea of (b)(ii) was appreciated by many of those who got that far, with all the ingredients for the argument found in earlier parts of the question.

Question 6. (a)(ii) was from the problem sheets but proved difficult with a lot of attempts focussing on manipulating Lagrange's Theorem rather than the expected path of Bezout's Lemma. For (b) some candidates incorrectly thought that Q_8 is Abelian. (b)(i) was generally well done, and (b)(ii) too by those who saw that (a)(ii) applied. A number of attempts at (b)(iii) gave a subgroup in the centre of $Q_8 \times C_4$, but a number of others appreciated that subgroups of the form $\langle (i, 1) \rangle$ ought to be interesting.

Question 7. This was more popular than the other two group theory questions. Part (a) was mostly done well, while (b) was frequently omitted, perhaps because it seemed unapproachably abstract. Part (c) was grasped by many, but there were lots of cases to check and it was easy to miss some of them.

Paper II

Question 1. Candidates generally knew their bookwork and scored very well on part (a). Parts (b)(i) and (b)(ii) were generally done very well by most candidates. Part (b)(iii) was seen as being much more difficult, with many candidates leaving this part blank. Common mistakes were claiming that the sequence ($\sqrt[n]{b_n}$) converges or that $\sqrt[n]{b_N 2^{-N}}$ is decreasing (it is not if $b_N 2^{-N} < 1$).

Question 2. Candidates were very comfortable with the bookwork (a)(i) and most recognised that (a)(ii) can be shown by completing the square. For part (a)(ii) many candidates gave their argument in the form of an induction even though it was not strictly necessary. Part (a)(iii) appeared slightly more difficult. The most common omission was not showing that the sequences a_n and b_n converge in the first place.

This part was perceived as being extremely difficult across the board, with only a handful of candidates being able to compute all three limits and many returning blanks for this part. For (b)(i) the most common mistake was an incorrect use of AOL to get that the limit is 0, which it is not. For (b)(ii) some candidates found correct alternatives to the ratio test, either by grouping parts of $n!$ cleverly or by using Gauss' trick to pair the terms $(n-k)$ and k in the factorial and noting that $k(n-k) \leq n^2/4$. Some solved (b)(ii) by quoting Stirling's approximation. Part (b)(iii) was mostly solved by bounding the number of prime factors $\alpha(n)$ either by $\log_2 n$ or by $2\sqrt{n}$ (by symmetric counting of the divisors of n). Here many candidates were able to see that the limit must be 0 but not give a complete proof – some even bounded $\alpha(n)$ by the number of all primes up to n and invoked the Prime Number Theorem (a nuking of a mosquito if ever there was one).

Question 3. Part (a) was mostly fine although (ii) was not straightforward. In (b)(i), many candidates forgot to check the endpoints of the convergence interval. Part (c) was the most difficult part of the question.

Question 4. This was very well done. Most candidates successfully used the IVT to do the last part. Some were too sketchy in part (b).

Question 5. This did not prove popular and there were few really good solutions.

Question 6. This was reasonably well done. Some candidates failed to see that the MVT was needed in part (b). Many did not manage the examples in (d).

Question 7. This was mostly fine, though a little on the easy side. Some candidates chose to prove in (a)(ii) that the uniform limit of continuous functions is continuous, hence integrable (rather than prove integrability with minorants/majorants). This was a valid approach. Part (b) was generally solved well with some mistakes in the choice of majorant. Part (c) turned out not to be too difficult: the main point was that x^n does not converge uniformly on the whole interval $[0, 1]$, so one instead could work with intervals $[0, \epsilon]$ and $[\epsilon, 1]$, but there is also a simpler solution, just by bounding $f(x)$ and integrating x^n using the FTC.

Paper III

Question 1. Part (a) was answered well by most candidates, although a significant minority made algebraic slips when using the given substitution. In Part (b) a variety of successful substitutions were used, although some candidates were unable to find a suitable substitution. Some candidates struggled with the chain rule in part (c) and so did not derive the correct transformed differential equation.

Question 2. Some candidates failed to spot the link between parts (a)(i) and (a)(ii) and as a consequence they did more work than was needed, and sometimes this introduced errors. Not many candidates were able to deal correctly with the given condition in part (a)(iii). Part (b) was answered well although some answers to (b)(ii) were unnecessarily long, again the link with (b)(i) was not seen.

Question 3. Whilst a lot of candidates did successfully locate the stationary points in part (a), this was not always done rigorously and some potential stationary points were not discussed/discounted. The change of variables in part (b) did cause quite a few problems, particularly with the new limits of integration. Most candidates did very well in part (c), although some methods were unnecessarily lengthy as shortcuts with Euler's identity were not spotted.

Question 4. This question was generally done very well. In (a)(i), marks were lost for not justifying an application of the linearity of expectation, or for unclear argumentation. Quite a few candidates slightly misremembered the variance formula in (a)(ii). Most candidates found (b)(i)–(iv) straightforward, although some struggled to show that the covariance is negative in (iv). In (b)(v), quite a few candidates could not correctly remember Chebyshev's inequality (some mixed it up with Markov's inequality). Marks were also lost for failing to recall that multiplying a random variable by a constant c gives a factor of c^2 in the variance. There were quite a few incorrect limits taken, or correct limits insufficiently well justified, but most candidates who attempted this part got at least substantial partial marks.

Question 5. This seems to have been found harder than the other two probability questions. In the definition of the probability generating function in (a)(i), many candidates failed to make any reference to the fact that the series might not converge for all reals. The vast majority did (a)(ii) correctly. Many answers to (b)(i) could barely be described as explanations, and were often quite hard to understand. Many candidates lost marks for not explaining why the variables on the right-hand side of the expression are mutually independent. The most common error in the bookwork in (b)(ii) was to use the partition theorem to condition on $\{N = n\}$, but to not then justify the removal of the conditioning by reference to the independence between $Z_1, Z_2, \ldots$ and N. Quite a few candidates erroneously factorised $\mathbb{E}[s^{Z_1+\cdots+Z_N}]$ without first conditioning on the value of N, though. The manipulations in (b)(iii) were generally done well but surprisingly many candidates simply assumed that the correct solution would be the '+' one without looking for a justification. On the other hand, some particularly outstanding solutions referred to the need for the solution to be an increasing function, or to the impossibility of the two solutions intersecting in the interval $[0, 1]$.

Question 6. This question was generally done very well. Almost all candidates gave a correct answer to (a)(i), although concerningly a few thought the probability density function was $\mathbb{P}(X = x)$. Answers to (a)(ii) often lost marks for omitting some of the necessary conditions, but answers to (a)(iii) were mostly convincing. In (a)(iv), many candidates forgot to specify that the cumulative distribution function obtained is only valid for positive values of its argument, and also that the name of the distribution should also include the value of its parameter. Part (b) seems to have been straightforward for most candidates. Some lost marks in (b)(i) for not justifying a use of independence, or for incorrectly applying the hint. In (b)(ii), the most common errors were to not spot that one could apply the binomial theorem, or inaccurate manipulations of signs in the integral.

Question 7. This question got the lowest marks, mainly due to sloppiness in the answers. Although almost all candidates were confident in how to find the MLE in (a)(i), most failed to check that it was indeed a maximum. Some candidates did not realise that X is binomially distributed. A common mistake was to try to work out the likelihood for multiple mutations rather than a single genetic mutation in multiple individuals. In the derivation of the confidence interval in (a)(iii), many candidates failed to check all the conditions for the Central Limit Theorem to hold, and many failed to use the hint. There were surprisingly many errors in the calculation of the confidence intervals in (b)(i). The interpretation and choice of which interval should be preferred in (b)(ii) presented some difficulties because candidates focussed on the fact that the first one was empty in the particular example considered in (b)(i): they did not think in general about the performance of the confidence intervals at different values of p.

Question 8. Part (a) was done very well. The majority of candidates knew the bookwork well and were able reproduce it. Some candidates lost marks for incomplete calculations. In (b)(i), the examples given were often not continuous variables, or were not plausibly normal (very often counts were used, which are characteristically not normally distributed). In some cases, for (b)(ii), the likelihood was given rather than the log-likelihood. For (b)(iii), which was the hardest part of the question, most candidates were able to relate the equations (A) and (C) to the equations (B) and (D), and could see that they should all be equal to 0, but they were not able to deduce what this would mean for the MLEs, or what this would mean for the models under consideration.

Question 9. This question was very well done overall. The bookwork in (a) was consistently done very well. In part (b), most candidates separated the first and second principal components, but they failed to relate their interpretation to the set-up of the question. Part (b)(iv) was the main place that marks were lost, often because candidates failed to write down the expression for the approximation. Some scripts demonstrated a confusion between clustering and PCA. Other candidates thought they needed to use the 3rd and 4th principal components because the first two explained too much of the variation!

Paper IV

Question 1. In part (a)(iii) the most natural answer consisted in noting that the 3 points are co-linear if and only if $\mathbf{u} - \mathbf{v}$ and $\mathbf{u} - \mathbf{w}$ are linearly dependent, which is itself equivalent to $(\mathbf{u} - \mathbf{v}) \wedge (\mathbf{u} - \mathbf{w}) = \mathbf{0}$. Developing the cross products provides the required equality directly. Note that, while the equivalence is obvious here, many candidates decomposed their answer in two steps ($\Rightarrow$ and $\Leftarrow$), which is also an acceptable, albeit cumbersome, answer. Many candidates though failed at proving $\Leftarrow$. A more serious type of problem found in a very significant proportion of the papers, consisted in misinterpreting the question as reading "show that vectors $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ are parallel if and only if $\mathbf{u} \wedge \mathbf{w} + \mathbf{v} \wedge \mathbf{u} + \mathbf{w} \wedge \mathbf{v} = \mathbf{0}$". While $\Rightarrow$ is obvious, the equivalence is false in general. To see this, consider $\mathbf{u} = \mathbf{i}$, $\mathbf{v} = \mathbf{i}$, and $\mathbf{w} = \mathbf{j}$. Here $\mathbf{u} \wedge \mathbf{w} + \mathbf{v} \wedge \mathbf{u} + \mathbf{w} \wedge \mathbf{v} = \mathbf{k} - \mathbf{k} = \mathbf{0}$, but the vectors are not parallel.

Most of the candidates who attempted (a)(iv) provided satisfactory answers. However, as in the previous part, some expressed the normal to the plane as $\mathbf{u} \wedge \mathbf{v}$, which as explained above, is incorrect and does not lead to the required expression. Generally candidates are encouraged to read all the questions very carefully to avoid misinterpretation (as the questions were here clearly not independent).

There were not major difficulties with (b)(i). Many candidates proved the desired result by calculation, however the property could be simply proven by noting that AE is perpendicular to the plane BCD, hence to every vector that is contained in it.

Question 2. No major problem was found with (a)(i). However, some candidates provided the canonical equation for a hyperbola in terms of a and b. Here the parameters given were a and e, thus candidates were expected to express b as a function of a and e.

For (b)(i) most candidates managed to establish the implicit equation for the hyperboloid. Some proposed a parametric formulation in terms of the cylindrical coordinates, which in the spirit of the problem, is not an acceptable answer (since the next question, consisting of computing the intersection of the two surfaces, clearly requires implicit relations). Candidates, in many cases, had difficulty sketching the surfaces correctly and visualising them in 3D. A common inaccuracy found in many scripts, consisted of drawing the asymptotes first, but then sketching a hyperboloid that does not converge to the asymptotes. Although the main point of the question was to identify the general type of hyperboloid (a two sheet hyperboloid here) and the positions of the sheets, this minor inaccuracy shows inadequate understanding of the geometric properties of hyperbolas and hyperboloids. Other candidates started by drawing the two hyperbolas, but then had difficulty representing the rotation in 3D.

Finally, most candidates who attempted (c)(i) managed to answer it correctly (except for a few calculation errors). Other candidates who answered the previous part by providing polar parameterisation failed to compute the intersection. (Note that the question contained a subtlety that none of the candidates noticed: it was possible to show algebraically that the solution satisfied the equation of an ellipse, though, a priori, this does not allow one to conclude that the solution is a full ellipse. From a sketch, one could however easily see that this was actually the case. The omission of this last step of the demonstration was not penalised in the marking.)

Question 3. The first part of this question was generally done well, though in part (a)(iii) a significant minority of candidates did not use part (i) correctly, deriving instead that the arc length is given by $\int r(\theta)\mathrm{d}\theta$. In part (b), many candidates calculated the distance between two points in $\mathbb{R}^3$, rather than in the cone; this made it difficult to determine the value of n for which the map is an isometry.

Question 4. Most candidates did (a)(i)–(ii) fairly well, with the omission of the discussion of the different cases on the sign of e occurring frequently. Quite a few candidates did not remember to do the transformation $u = 1/r$, which is bookwork. Similarly, part (iii) was done by significantly fewer candidates, even though it was straightforward or even bookwork. Very few candidates attempted the new part (b) or even finished it correctly, though there where some successful attempts.

Question 5. This was done somewhat better than Q4, but still not as well as expected, given that it was very similar to an example in the lecture notes; in fact, as the lecturer remarked upon reviewing the paper earlier, it is identical to an example in the notes if $K = 0$ (at least for parts (a) and (b)). Most candidates did (a) well, but quite a few had difficulties with (b), though many still did get part of the answer. The new part, (c), was only attempted by a few candidates and only a small number got it right.

Question 6. This was perhaps the best opportunity to get most of the marks, but the topic seems unpopular with candidates and hence the question was attempted only by very few. It is therefore hard to make any general statement. Some candidates got to the end, others clearly had difficulties with the basic theory.

Question 7. In (a) some candidates struggled with applying Bezout's identity. Another approach not shown in the solutions is using the unique prime decomposition. In that solution, clear notation was the key to success.

In (b) some candidates only wrote $g' < 1$ and not $|g'| < 1$. Many candidates forgot to guarantee that g maps an interval to the same interval.

In (c) many candidates failed to notice that Newton's method was applied to $\nabla F = 0$, not $F = 0$. There were many computational mistakes in computing the second iteration. Also many candidates forgot to check whether $\nabla F(x_2, y_2) = 0$, and only said that it is not the easily computed (close) critical point.

Paper V

Question 1. Part (a) was found more difficult than expected by a number of candidates. There was particular confusion over the limits of integration, which led to a variety of answers (including many that were dimensionally inconsistent). Parts (b) and (c) were well done, though again some algebraic errors in part (c) would have been spotted had candidates been more aware of the units expected for the quantity calculated in (c)(ii).

Question 2. Parts (a) and (b) of this question were generally done well, though in part (b) a significant proportion of candidates opted to calculate the gravitational field $\nabla\phi$ by solving Poisson's equation, rather than use Gauss' Flux Theorem (as had been envisaged). This strategy of course worked, provided that the candidates could recall the formula for $\nabla^2\phi$ in spherical coordinates correctly. This issue became particularly acute in part (c) where a majority of candidates attempted to use Poisson's equation to calculate the density via $\rho = -\nabla^2\phi/(4\pi G)$: many solutions were based on the (erroneous) belief that $\nabla^2\phi = -g'(r)$ and hence found that the density must vanish between $R_E/2$ and R_E.

Question 3. There were many mistakes in (a)(ii) in the calculation of the integrals that appear on both sides of Stokes' Theorem. Part (b) was bookwork and not difficult, and part (c) was more challenging.

Marks for the subparts of part (b) were slightly redistributed, but the total remained the same.

Question 4. In (a) some candidates forgot to assume the exchange of integral and sum. Some others forgot the $1/\pi$.

In (b) the computation of a_0 was almost always okay. There are many approaches to finding a_n and b_n, some of them very long. Most of them leading to mistakes, especially in b_n. A very few candidates forgot to clearly write the complete expansion.

In (c) many candidates forgot to explain that they are using the FCT, and some did not complete the second subpart.

Question 5. This was less popular than Q6 and on the whole was poorly done. For (a)(i) many candidates struggled to consider the energy of the wave equation and failed to find the correct integral to use to show uniqueness. (a)(ii) was mixed, some answered this with ease and others were flummoxed despite it being very similar to a question on this year's problem sheets and rather straightforward. (b)(i) had a twist on the classic Fourier series for the wave equation – the temporal domain was simply $t > s$ in lieu of $t > 0$. This seemingly slight change this was enough to derail most candidates' attempts. This shows a fundamental lack of understanding of the basics of this course. (b)(ii) was challenging and there we only a handful of attempts. That said there were a couple of decent solutions amongst all the attempts.

Question 6. This was more popular than Q5 and, on average, was better done. (a): Most candidates correctly determined the problem for S and problem for $\phi = S_1 - S_2$. Many candidates could not find the correct function to show uniqueness which lost a lot of marks. That said, there were also a lot of solid answers to this part. (b): Most candidates correctly followed the hint and found

$$\frac{\dot{H}}{H} = \frac{F_{xx}}{F} - 1 = \omega.$$

Not relabelling $\omega + 1 = \tilde{\omega}$ led to a significant number of sign errors in the solution for H. Another common issue was using a generic Fourier series coefficient formula integrated over $-\pi$ to π; the question needed this to be derived which only a handful of candidate did. Overall, many candidates did present good solutions to the question.

E. MODERATORS AND ASSESSORS

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