Examiners' Report: Preliminary Examination in Mathematics Trinity Term 2024

November 12, 2024

Part I

A. STATISTICS

180 candidates in Mathematics and Mathematics & Statistics were awarded an overall year outcome. Candidates on both degrees submit the same assessments and no distinction is made between the two groups in this document.

Table 1: Numbers in each outcome class

	Numbers				Percentages			
	2024	(2023)	(2022)	(2021)	2024	(2023)	(2022)	(2021)
Distinction	57	(52)	(53)	(60)	31.67	(29.21)	(29.78)	(30.61)
Pass	109	(113)	(116)	(124)	60.56	(63.48)	(65.17)	(63.27)
Partial Pass	10	(11)	(6)	(7)	5.56	(6.18)	(3.37)	(3.57)
Incomplete	0	(0)	(0)	(2)	0	(0.00)	(0.00)	(1.02)
Fail	4	(2)	(3)	(3)	2.22	(1.12)	(1.69)	(1.53)
Total	180	(178)	(178)	(196)	-	-	-	-

B. NEW EXAMINING METHODS AND PROCEDURES

None.

C. CHANGES IN EXAMINING METHODS AND PROCEDURES CURRENTLY UNDER DISCUSSION OR CONTEMPLATED FOR THE FUTURE

None.

D. NOTICE OF EXAMINATION CONVENTIONS FOR CANDIDATES

The Notice to Candidates, containing details of the examinations and assessments, was issued to all candidates at the beginning of Trinity term. The Examination Conventions in full were made available at

https://www.maths.ox.ac.uk/members/students/undergraduate-courses/examinations-assessments/examination-conventions.

Part II

A. GENERAL COMMENTS ON THE EXAMINATION

Acknowledgements

First, the Moderators should like to thank the Undergraduate Studies Administration Team.

We should also like to thank Charlotte Turner-Smith for her invaluable experience with the Mitigating Circumstances Panel, and Matthew Brechin and Waldemar Schlackow for maintaining and running the examination database and their assistance during the final examination board meeting.

We should like to thank the lecturers for their feedback on proposed exam questions; the assessors for their extraordinary assistance with marking; Patrick Farrell for coordinating the Computational Mathematics assessments, and the team of graduate checkers for their rapid work checking the marks on the papers.

Timetable

The examinations began on Monday 17th June and ended on Friday 21st June.

Setting and checking of papers

The Moderators set and checked the questions, model solutions, and mark schemes. Every question was carefully considered by at least two moderators, and feedback was sought from lecturers. In a small number of cases feedback from lecturers was not available, and those were discussed in more detail until the Board of Moderators was satisfied that all questions were appropriate.

The questions were then combined into papers which were considered by the Board of Moderators and small changes were made to satisfy the Board that the papers were appropriate. After this a final proof-reading of the papers was completed before the Camera Ready Copies (CRCs) were produced. The whole Board of Moderators signed off the CRCs which were submitted to Examination Schools.

Marking and marks processing

The Moderators and Assessors marked the scripts according to the mark schemes and entered the marks. Small adjustments to some mark schemes were made at this stage, and care was take to ensure these were consistently applied to all candidates.

A team of graduate checkers, supervised by Academic Admin Team sorted all the scripts for each paper and carefully cross checked these against the mark scheme to spot any unmarked parts of questions, addition errors, or wrongly recorded marks. A number of errors were corrected, with each change checked and signed off by a Moderator, at least one of whom was present throughout the process.

Determination of University Standardised Marks

Marks for each individual assessment are reported as a University Standard Mark (USM) which is an integer between 0 and 100 inclusive. The Moderators used their academic judgment to map the raw marks on individual assessments to USMs using a process similar to previous years. In coming to this judgement the board followed the advice from the Mathematics Teaching Committee that the percentages awarded for each USM range of the examination should be in line with recent years. This alignment can be seen in Table 1; in more detail, for Papers I–V, a piecewise linear map was constructed as follows:

1. Candidates' raw marks for a given paper were ranked in descending order.
2. The default percentages p_1 of Distinctions and p_2 of Nominal Upper Seconds were selected.
3. The candidate at the p_1 percentile from the top of the ranked list was identified and assigned a USM of 70, and the corresponding raw mark denoted R_1.
4. The candidate at the $(p_1 + p_2)$ percentile from the top of the list was assigned a USM of 60 and the corresponding raw mark denoted R_2.
5. The line segment between $(R_1, 70)$ and $(R_2, 60)$ was extended linearly to USMs of 72 and 57 respectively, and the corresponding raw marks denoted C_1 and C_2 respectively.

6. A line segment through $(C_2, 57)$ was extended towards the vertical axis, as if it were to join the axis at $(0, 10)$, but the line segment was terminated at a USM of 37 and the raw mark at the termination point was denoted C_3.

With these data a piecewise linear map was constructed with vertices at $\{(0,0),\ (C_3,37),\ (C_2,57),\ (C_1,72),\ (100,100)\}$.

Reports from the Assessors describing the apparent relative difficulty and the general standard of solutions for each question were then considered, and the Board decided that the values of $p_1 = 31\%$ and $p_2 = 48\%$ were suitable for all papers.

In line with previous years, for the Computational Mathematics assessment the linear map with gradient 2.5 was used to map from raw marks to USMs.

The vertices of the final maps used in each assessment are listed in Table 2.

Table 2: Vertices of final piecewise linear model

Paper	Vertices				
I	0;0	24.4;37	42.5;57	80;72	100;100
II	0;0	22.1;37	38.5;57	72.5;72	100;100
III	0;0	29.6;37	51.6;57	97.1;72	120;100
IV	0;0	27;37	41;57	69;72	100;100
V	0;0	20.2;37	35.1;57	66.1;72	80;100
CM	(0.0,0)	(40,100)			(0.0,0)

With the USMs, a provisional outcome class for each candidate was produced as follows: Write MI, MII, $MIII$, MIV and MV for the USMs on Papers I–V respectively, and CM for the USM on the Computational Mathematics assessment. Write Av_1 and Av_2 for the quantities

$$\frac{MI + MII + \frac{6}{5}MIII + MIV + \frac{4}{5}MV + \frac{1}{3}CM}{5\frac{1}{3}}$$

and

$$\frac{MI + MII + \frac{6}{5}MIII + MIV + \frac{4}{5}MV}{5}$$

respectively, symmetrically rounded. With these auxiliary statistics the provisional outcome class was determined by the definitions:

Distinction: both $Av_1 \geq 70$ and $Av_2 \geq 70$ and a USM of at least 40 on each paper and for the Computational Mathematics assessment;

Pass: not a Distinction and a USM of at least 40 on each paper and for the Computational Mathematics assessment;

Partial Pass: not a Pass or Distinction and a USM of at least 40 on three or more of Papers I–V;

Fail: not a Partial Pass, Pass, or Distinction, and a USM of less than 40 on three or more of Papers I–V.

The scripts of those candidates at the boundaries between outcome classes were scrutinised carefully to determine which attained the relevant qualitative descriptors and changes were made to move those into the correct class.

Mitigating Circumstances were then considered using the banding produced by the Mitigating Circumstances Panel, and appropriate actions were taken and recorded.

Table 3 gives the rank list ordered by the average of Av_1 and Av_2 (as defined above), showing the number and percentage of candidates with USM greater than or equal to each value.

Table 3: Rank list of average USM scores

		Candidates with USM $\geq x$	
USM (x)	Rank	Number	Percentage
88.29	1	1	0.56
87.56	2	2	1.11
86.96	3	3	1.67
86.2	4	4	2.22
85.69	5	5	2.78
85.1	6	6	3.33
84.1	7	7	3.89
82.06	8	8	4.44
81.04	9	9	5
80.49	10	10	5.56
80.27	11	11	6.11
80.24	12	12	6.67
79.96	13	13	7.22
79.52	14	14	7.78
79.2	15	15	8.33
78.92	16	16	8.89
78.89	17	17	9.44
78.76	18	18	10
78.44	19	19	10.56
78.36	20	20	11.11
77.62	21	21	11.67
76.64	22	22	12.22
76.4	23	23	12.78

Table 3: Rank list of average USM scores (continued)

USM (x)	Rank	Candidates with average USM $\geq x$ Number	Percentage
76.08	24	24	13.33
75.84	25	25	13.89
75.68	26	26	14.44
75.2	27	27	15
75.12	28	28	15.56
74.36	29	29	16.11
74.08	30	30	16.67
74	31	31	17.22
73.96	32	32	17.78
73.84	33	33	18.33
73.44	34	34	18.89
73.03	35	35	19.44
73	36	36	20
72.8	37	37	20.56
72.76	38	39	21.67
72.76	38	39	21.67
72.08	40	40	22.22
71.95	41	41	22.78
71.88	42	42	23.33
71.56	43	43	23.89
71.45	44	44	24.44
71.2	45	45	25
71.16	46	46	25.56
71.05	47	47	26.11
71.04	48	48	26.67
70.8	49	49	27.22
70.75	50	50	27.78
70.64	51	51	28.33
70.44	52	52	28.89
70.04	53	53	29.44
70	54	54	30
69.76	55	55	30.56
69.64	56	56	31.11
69.4	57	57	31.67
69.16	58	58	32.22
69.12	59	59	32.78
69	60	60	33.33
68.6	61	61	33.89
68.4	62	62	34.44
68.32	63	63	35

Table 3: Rank list of average USM scores (continued)

		Candidates with average USM $\geq x$	
USM (x)	Rank	Number	Percentage
68.29	64	64	35.56
68.24	65	65	36.11
67.92	66	66	36.67
67.72	67	68	37.78
67.72	67	68	37.78
67.68	69	69	38.33
67.64	70	70	38.89
67.61	71	71	39.44
67.6	72	72	40
67.52	73	73	40.56
67.4	74	74	41.11
67.28	75	75	41.67
67.21	76	76	42.22
67.14	77	77	42.78
67.1	78	78	43.33
66.92	79	79	43.89
66.89	80	80	44.44
66.8	81	81	45
66.48	82	82	45.56
66.4	83	85	47.22
66.36	86	86	47.78
66.28	87	87	48.33
66.23	88	88	48.89
66.08	89	89	49.44
65.95	90	90	50
65.88	91	91	50.56
65.84	92	93	51.67
65.8	94	94	52.22
65.62	95	95	52.78
65.6	96	96	53.33
65.32	97	97	53.89
65.26	98	98	54.44
65	99	99	55
64.65	100	100	55.56
64.48	101	101	56.11
64.39	102	102	56.67
64.2	103	103	57.22
64.08	104	104	57.78
63.88	105	105	58.33
63.8	106	106	58.89

Table 3: Rank list of average USM scores (continued)

USM (x)	Rank	Candidates with average USM $\geq x$ Number	Percentage
63.76	107	107	59.44
63.68	108	108	60
63.64	109	111	61.67
63.52	112	112	62.22
63.39	113	113	62.78
63.36	114	116	64.44
63.28	117	117	65
63.12	118	118	65.56
63.02	119	119	66.11
62.92	120	120	66.67
62.8	121	121	67.22
62.7	122	122	67.78
62.68	123	123	68.33
62.62	124	124	68.89
62.53	125	125	69.44
62.4	126	126	70
62.36	127	127	70.56
62.28	128	128	71.11
62.2	129	130	72.22
62.2	129	130	72.22
61.92	131	131	72.78
61.88	132	132	73.33
61.8	133	133	73.89
61.72	134	134	74.44
61.61	135	135	75
61.56	136	136	75.56
61.4	137	137	76.11
61.36	138	138	76.67
61.24	139	139	77.22
61	140	142	78.89
60.96	143	143	79.44
60.88	144	144	80
60.6	145	145	80.56
60.44	146	146	81.11
60.28	147	147	81.67
60.24	148	148	82.22
59.76	149	149	82.78
59.4	150	150	83.33
58.56	151	152	84.44
57.96	153	153	85

Table 3: Rank list of average USM scores (continued)

USM (x)	Rank	Candidates with average USM $\geq x$: Number	Percentage
57.8	154	154	85.56
57.12	155	155	86.11
56.84	156	156	86.67
56.72	157	157	87.22
56.32	158	158	87.78
56.24	159	159	88.33
56.16	160	160	88.89
55.66	161	161	89.44
54.88	162	162	90
54.69	163	163	90.56
53.64	164	164	91.11
53.04	165	165	91.67
53	166	166	92.22
52.88	167	167	92.78
52.8	168	168	93.33
52.2	169	169	93.89
51.96	170	170	94.44
51	171	171	95
50.84	172	172	95.56
48.76	173	173	96.11
48.19	174	174	96.67
47.88	175	175	97.22
43	176	176	97.78
35	177	177	98.33
29.6	178	178	98.89
16.6	179	179	99.44
0	180	180	100

B. EQUAL OPPORTUNITY ISSUES AND BREAKDOWN OF THE RESULTS BY GENDER

Table 4 shows the performances of candidates by gender. Here gender is the gender as recorded on eVision.

Table 4: Breakdown of results by gender

Outcome	Number								
	2024			2023			2022		
	Female	Male	Total	Female	Male	Total	Female	Male	Total
Distinction	4	53	57	9	43	52	8	45	53
Pass	35	74	109	38	75	113	43	73	116
Partial Pass	2	8	10	4	7	11	2	4	6
Incomplete	0	0	0	0	0	0	0	2	2
Fail	1	3	4	1	1	2	0	3	3
Total	41	138	180	52	126	178	53	125	178

Outcome	Percentage								
	2024			2023			2022		
	Female	Male	Total	Female	Male	Total	Female	Male	Total
Distinction	9.52	38.41	31.67	17.23	34.15	29.21	15.09	36.00	29.78
Pass	83.33	53.62	60.56	73.08	59.52	63.53	81.13	58.40	65.17
Partial Pass	4.76	5.8	5.56	7.69	5.56	6.17	3.77	3.20	3.37
Incomplete	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Fail	2.38	2.17	2.22	1.92	0.79	1.12	0.00	2.40	1.69

C. STATISTICS ON CANDIDATES' PERFORMANCE IN EACH PART OF THE EXAMINATION

Table 5: Numbers taking each paper

Paper	Number of Candidates	Average raw mark	Std dev of raw marks	Average USM	Std dev of USMs	Number failing
I	178	65.94	14.3	67.24	8.24	1
II	179	58.36	16.85	66.2	11.17	5
III	178	78.85	18.47	66.74	9.25	1
IV	177	56.63	15.01	65.11	11.35	3
V	177	54.05	13.45	66.77	11.78	4
CM	177	29.51	4.95	73.46	13.38	8

Tables 6–11 give the performance statistics for each individual assessment, showing for each question the average mark, first over all attempts, and then over the attempts used; the standard deviation over all attempts; and finally the total number of attempts, first those that were used, and then those that were unused.

Table 6: Statistics for Paper I

Question Number	Average mark All	Used	Std dev	No. of Attempts Used	Unused
Q1	16.66	16.66	3	176	0
Q2	13.66	13.66	4.5	104	0
Q3	13.39	13.39	3.53	169	0
Q4	11.76	11.89	5.55	82	2
Q5	15.63	15.63	3.76	127	0
Q6	10.31	10.4	3.75	98	2
Q7	8.92	8.92	3.16	128	0

Table 7: Statistics for Paper II

Question Number	Average mark All	Used	Std dev	No. of Attempts Used	Unused
Q1	14.67	14.75	3.08	167	2
Q2	9.17	9.24	3.16	95	4
Q3	16.14	16.14	3.56	92	0
Q4	12.55	12.55	4.25	163	0
Q5	11.55	11.59	4.61	167	1
Q6	13.41	13.92	5.87	26	1
Q7	7.2	7.2	4.47	177	0

Table 8: Statistics for Paper III

Question Number	Average mark All	Used	Std dev	No. of Attempts Used	Unused
Q1	15.08	15.12	4.13	153	1
Q2	15.92	16.01	3.86	120	1
Q3	14.48	14.7	3.27	82	2
Q4	11.13	11.17	3.77	121	2
Q5	13.24	13.34	3.73	120	2
Q6	14.58	14.58	3.76	115	0
Q7	11.01	11.01	4.3	151	0
Q8	11.57	11.57	5.62	136	0
Q9	11.98	11.98	4.3	61	0

Table 9: Statistics for Paper IV

Question Number	Average mark All	Used	Std dev	No. of Attempts Used	Unused
Q1	10.22	10.24	4.36	100	1
Q2	13.19	13.19	4.98	108	0
Q3	11.64	11.67	3.83	146	1
Q4	8.7	8.78	5.28	113	1
Q5	11.55	11.55	2.53	172	0
Q6	12.56	12.56	6.17	59	0
Q7	12.23	12.23	3.85	176	0

Table 10: Statistics for Paper V

Question Number	Average mark All	Used	Std dev	No. of Attempts Used	Unused
Q1	14.38	14.4	4.16	127	2
Q2	11.79	11.81	3.81	112	1
Q3	11.6	11.76	3.97	109	2
Q4	14.42	14.42	4.63	170	0
Q5	16.17	16.17	4.17	142	0
Q6	10.21	10.21	4.48	38	0

Table 11: Statistics for Computational Mathematics

Question Number	Average mark All	Used	Std dev	No. of Attempts Used	Unused
Project A	15.02	15.02	2.38	172	0
Project B	16.33	16.33	2.94	6	0
Project C	14.61	14.61	2.75	173	0

D. COMMENTS ON PAPERS AND ON INDIVIDUAL QUESTIONS

Paper I

Question 1. Almost all candidates solved part (a)(i) correctly. Most attempts on part (a)(ii) and (a)(iii) were correct, but some students made computational mistakes, especially those who had forgotten the formula for the inverse of a 2×2 matrix. Almost all students stated the rank-nullity theorem in part (b)(i) correctly. Part (b)(ii) was the most challenging: while many candidates noted that the kernel is two-dimensional, most struggled to present a correct proof of the desired result.

Question 2. Most students stated the change-of-basis formula in part (a)(i) correctly, but only few gave full details. Many gave the correct definition of the trace in part (a)(ii), but not all managed to prove its invariance in complete detail. A common mistake was to assume that the trace was multiplicative. Almost all students presented a correct definition of inner products in part (b)(i) and showed that part (b)(ii) provides an example of such an inner product on matrices. Only relatively few candidates managed to construct an orthonormal basis in part (b)(ii), and many did not verify that it indeed has the asserted properties.

Question 3. Candidates generally correctly answered part (a): defined eigenvalues and eigenvectors (sometimes forgetting eigenvectors should be non-zero), and showing that eigenvalues are roots of the characteristic polynomials. A common mistake in (a)(i) was to only show one implication (an eigenvalue is a root of the characteristic polynomial but not vice versa). Part (b) was also mostly solved correctly, although only proper proofs by induction received full points in (b)(i). Part (c) was found to be the most difficult by the students. A lot of students realized that PTP^{-1} has to be strictly upper-triangular and immediately concluded that $(PTP^{-1})^n = 0$; the crux of the problem was to prove this! Some students used the heuristic argument *every time you multiply by the matrix PTP^{-1}, the non-zero diagonal moves up by one*, without giving a formal proof. The correct approach was to proceed by induction, which some of the candidates executed correctly. Part (d) was done correctly by almost all the students.

Question 4. This question was attempted by less of the candidates, but those who attempted it usually succeeded in reproducing the proof of the Spectral Theorem in part (a) correctly. Part (b)(i) also received many correct attempts; the students were at least able to diagonalize B, use the positive-definiteness to deduce its eigenvalues are positive, and take their square roots to show B is similar to the identity matrix. The final step was to further diagonalize the matrix resulting from A, and much fewer candidates succeeded in that. Not all the students attempted part (b)(ii); those who did often got the first part right, but struggled to properly deduce the final assertion.

Question 5. Question 5, on normal subgroups and the isomorphism theorem, was generally well done. In the final section, some failed to show that the inverse of an element of the subgroup B of upper triangulars was itself in B. Many candidates got the right idea in the final part of finding a homomorphism from B onto the diagonal matrices with kernel U.

Question 6. Question 6 proved more difficult. The standard material in parts (a) and (b) was done well, but only a few candidates managed a complete answer to (c), the main omission being a proper proof that the order 2 element acted by inversion on the normal subgroup generated by an order p element. The final part (d) proved difficult, with only a few candidates giving a proper proof.

Question 7. Question 7 was also found difficult. Many candidates had difficulty with the argument that the centre of a p-group was nontrivial, although there were also many good answers. In part (c), most candidates were fairly happy with the idea of the coset space action, but many did not check it was well defined and there was some confusion about stabilisers. The last part was found very challenging and only a few people managed this.

Paper II

Question 1. This question tested knowledge of convergence of real sequences. It was on the easy side. Candidates only found (b)(ii) challenging. Part (a)(i) was simple bookwork, though several candidates failed to formulate clearly when a sequence is convergent. Most solutions for (a)(ii) were correct. The majority of candidates did not show why $1/\sqrt{n} \to 0$ in (a)(iii) and why $\sqrt{1+1/n} \to 1$ in (b)(i); it was not sufficient to simply mention the Algebra of Limits. Most candidates tried to show that $\sqrt{2+\sqrt{s_n}} > s_n$ using a wide variety of flawed arguments involving inequalities, instead of simply substituting the recursive formula for s_n on the right-hand side and using induction. Also, most solutions claimed that $s_2 = \sqrt{2+\sqrt{2}}$, whereas actually $s_2 = \sqrt{2+\sqrt[4]{2}}$.

Question 2. This question tested determining when a series is convergent. Candidates generally found it difficult, even though none of the questions were particularly challenging on their own. This partly seems to be because after proving the ratio test, the majority thought that is the only technique required for the rest of the question, even though the hint at the end explicitly states they could use the comparison test without proof. Part (a) was bookwork and solutions were typically correct, with occasional minor gaps in the proof or in the statement of the ratio test. Most solutions for part (a)(i) were correct, with sometimes details missing when showing the ratio converges to e^{-1}. While part (a)(ii) was straightforward with comparison with a geometric series together with the hint, very few candidates pursued that route and instead chose the ratio test. The ratio test could also be made to work once one showed that $\sqrt[n]{n}$ is monotonically decreasing for n large enough, but was more involved and the majority of solutions were incomplete. There were many partially correct solutions for (b)(iii), but relatively few complete solutions, and a worrying number of candidates wrote inequalities between complex numbers. But many conjectured that the series converges when $|z| > 1$ and diverges when $|z| < 1$, with few dealing with the $|z| = 1$ case correctly. There were few correct solutions for part (c), even though a simple inequality between the arithmetic and geometric mean of two numbers and the comparison test suffices, or comparison of $\sqrt{a_n}/n$ with $\max(a_n, 1/n^2)$ depending on whether $\sqrt{a_n} > 1/n$ or not. Instead, most candidates tried to use the ratio test, but in reverse, which of course does not hold: a_{n+1}/a_n does not need to be convergent just because $\sum a_n$ is convergent. Also, $\sqrt{a_n} > a_n$ when $a_n < 1$; another common mistake was writing $\sqrt{a_n} < a_n$.

Question 3. The majority of the students correctly got the main ideas of part a, but what really set them apart was how the complex numbers were handled. By far, the most common mistake was to write something on the lines of $|a| < b$ where both a, b are complex numbers. The technical subtlety of part a.ii was to use absolute convergence to be allowed to talk about order, and then go back to normal convergence, and almost half the candidates lost points over using complex numbers as if they were real. The part a.iii was generally fine.

Part b was slightly harder, but was still solved by the majority of the candidates. In part b.i, what set candidate apart was the care in talking about the convergence rate, and in particular to remember that it has to be positive. Parts b.ii and b.iii were mostly solved using the differentiation method and comparing the derivative of the series and of the closed formula, but there were other strategies as well. Some candidates computed teh series values directly by cleverly rearranging the terms, for instance. By far, computational mistakes and carelessness with the indices of the summations were the biggest source of lost points.

Question 4. Essentially all students did it correctly. (b) (i) The vast majority did not notice the function converges pointwise to $f(x) = 1$ if $x = 1$ and 0, otherwise. Many also were not able to show it does not converge uniformly.
(b) (ii) This one most students got it right that is converges uniformly to 0 but they did not give full details that the $sup_{x \epsilon \mathbb{R}}\{|\frac{x}{n+x^2}|\}$ tends to 0 as n tends to infinity. Namely, that the max $f_n(x)$ is attained at an extreme point. (c) (i) The first part was generally fine using the alternate sign test. Some problems appeared when showing it is a lipschitz continous function... (c) (ii) This was good. (c) (iii) Most students were able to come up with an example. But examples like $f_n(x) = nx$ I did not count as valid as the limit function is not defined.

Question 5. Almost all students attempted this question. The bookwork (part (a)) was generally fine, although some care needs to be taken as to whether the points found are in the open or the closed interval. In part (b), many students tried to argue that $f'(0) = -1$ was impossible using continuity of $f'(x)$, which is not always satisfied. Others tried to argue that f is strictly increasing directly from the definition of derivative, claiming that if $(f(x) - f(x_0))/(x - x_0)$ tends to a positive limit as $x \to x_0$ then it is positive for all choices of x. There are several valid solutions to (c)(i), although almost all students who successfully answered the question followed the hint. However students often then just claimed, without proof, that monotonicity of f contradicted the given limits. Part (c)(ii) seemed to cause the most trouble. One common and incorrect approach was to try and apply (c)(i) to f', with students often claiming that $f(x) \to 0$ as $x \to \infty$ implies $f'(x) \to 0$ as well, which is of course not true in general.

Question 6. Very few students attempted this question, although it was usually done well by those that did. This is a pity, as I am sure many students would have done better on this question than on questions 4 and 5. Quite a few students failed to differentiate $1 + f(\alpha x)$ correctly, and so got into a mess in parts (b)(i) and (ii). However (b)(iii) can still be completed without the previous parts. Other specific mistakes were rare. Some students failed to note that the bound in (b)(i) is supposed to be independent of n. One or two students just assumed infinite series versions of Taylor's Theorem just work if the series converges, which is false. Some missed the fact that in (b)(iii) we can't use the previous parts of the question as these assume that this f exists.

Question 7. For part (a), most of the students managed to give the definition of integrability, but many omitted important details, for example, many did not define the value $I(\phi)$ for a step function ϕ. Part (b), despite being bookwork, caused difficulties for many students. Many gave incorrect proofs based on the assumption that the function f must change sign a finite number of times. Part (c)(i) was done correctly by very few students. A very common misconception was that a function of the form $x^\alpha cos(1\backslash x^\beta)$ is differentiable at 0 if and only if its derivative has a limit as $x \to 0$. There was a reasonable number of good attempts for (c)(ii), and a good number of complete solutions for (c)(iii), even among students who struggled with earlier parts of this question. Other frequent mistakes throughout the question involved implicit continuity/smoothness-related assumptions (e.g. assuming that supremum/infimum is taken, or incorrect applications of the fundamental theorem of calculus) and incorrect quantifications (very often $\forall$, $\exists$ were in incorrect order). Overall, it appears that many students found the concepts in this question rather difficult, and had problems even with the bookwork.

Paper III

Question 1. Q1: This question was generally popular with students and was done well on the whole. In part (b) some candidates struggled to find an ansatz for the particular solution — they did not try a high enough power of x early on. In part (c), successful candidates either spotted that $m = -1/2$ simplified the ODE significantly (the intended solution) or did a little more work with $m = -1$ and the additional substitution $V = v(x)^2$.

Question 2. This question was very well done. Most students were able to do parts (a) and (b) relatively easily. Part (c) was well done in general, though some candidates failed to realize that a triangle had to be subtracted from $\int_0^{\pi/3} r(\theta)^2 \, \mathrm{d}\theta$ to determine the area to the right of the line $x = a$ correctly.

Question 3. The bookwork parts of this question were done well on the whole, though a number of candidates did not *explain* how a critical point is classified using insight from (a)(ii) instead quoting standard results by rote. Part (b) was also well done, though many candidates suffered from not being sufficiently careful with their algebra, making simple algebraic errors or introducing spurious critical points. Part (c) was not well done — very few candidates seemed to spot that the fourth-order Taylor expansion about the critical point $(0, 1)$ is exact and demonstrates that this critical point is a minimum.

Question 4. For this question, there were a number of very good answers, but also many who mixed up various formulas and ended up with wrong results or did not manage to solve the problem. Some few answers were somewhat disappointing, in that they did not seem to know where to start. There were also quite a number of copies where it seems the student ran out of time - after a good start, the answers stop.

Part a) was generally done very well, except for subpart iv), on which many did only achieve partial marks. The subpart a)i) was generally done well. Some people lost a mark here for not justifying the use of the infinite geometric series. a)ii) was equally not a problem for most people, though some did not justify the crucial step where one drops the intersection with the event that Z is larger than 2. a)iii) was the question that almost everyone managed to solve and got full marks on. For question iv), most people found the mean after some calculation and using the hint, but struggled to show that the variance diverges.

For part b), many people were able to derive at least some of the distributions, but equally many used the wrong formula or forgot to take into account part of the given information, deriving results that did not make sense. Subpart ii) depended on subpart i), hence those who struggled with i) generally could not answer ii).

Part c) was often answered well, with people recognizing the correct link to part a).

Question 5.

This was a popular question achieving a good spread of marks. (a) was mostly done well, with the occasional mark lost for a complete lack of logical structure and terminology. (b) was also done well by many, but a significant number either assumed $e_0 = n^2$ or in some incorrect way deduced two unknowns from a single linear equation instead of realising that $e_0 = 1 + e_1$, which can serve as a boundary condition. Some students also completed (c) in one of several different ways, and many who did not complete (c) (or made mistakes) scored some good partial marks.

Question 6.

This was a popular question and generally done well with a good spread. Some students lost one or both marks by only stating a finite special case in (a)(i) or by forgetting the disjointness assumption. Many students did not remember an accurate definition of what we call a continuous random variable. Some only proved special cases of (a)(ii) and (a)(iv) rather than explicitly using (a)(i). (b)(i) and (b)(ii) were usually done well, with some marks lost for not specifying the cdf for x outside [-1,1]. Answers to (b)(iii) were more variable, with many scoring well on the first transformation, which is bijective, but gradually less well on the second and third transformations, which are not bijective.

Question 7.

While this was the most popular of the Statistics questions, there was a wide spread of marks with a considerable number of very weak or very incomplete answers, as well as a good number of very good answers. Specifically, quite a few students lost one or both marks on (a) for incomplete or incorrect statements of the CLT. Others considered a binomial sample in (b)(i) and some then fudged their answer to conclude. (b)(ii) was fine. (b)(iii) is not the same as (b)(i) and only those who wrote down the likelihood (without binomial coefficient!), as well as the MLE, got both marks. Students who wrote as much detail as in (b)(i) wasted time. Many made some progress on (b)(iv) and some found the MLEs, except that nobody noticed this is not meaningful when $N_0 + N_1 = 0$. (c)(i) and/or (c)(ii) were often done well even if earlier parts were not. Some marks were lost, however, by those who forgot where the random variable is in the confidence interval calculation, as well as by those who did not plug in actual values for the MLE and the variance estimate in their final answer of (c)(i).

Question 8.

This was the second most popular Statistics question, also with a wide spread of marks ranging from very weak to very good answers. Most students started off correctly for (a)(i) whereby some assumed wrongly that the errors were normally distributed. Quite a few students did not realise that one was able to simplify the least-squares equations by the special structure of the explanatory variables. (a)(ii) was answered correctly by most students, some forgot to mention the constant variance. Quite a lot of students struggled with (a)(iii) and gave an example where the explanatory variable was not discrete or did not allow for negative integers. (b) was answered correctly by most students. (c)(i) seemed to be difficult for a lot of students, especially for those who already struggled with (a)(i). c(ii) was mostly fine, a few did not understand correctly the second plot and thought that the line is the actual fitted regression curve. c(iii) was answered correctly by most students with the exception that most students thought that the predicted value would be a good prediction. However, they didn't lose a mark on that.

Question 9.

This was the least popular question in the Statistics section. For this question, marks were not spread as widely as for Question 8. (a)(i) was mostly fine, sometimes the definition of the mean-centred matrix was not accurate. Quite a lot of student struggled with giving a correct and rigorous proof for (a)(ii) and (a)(iii). (a)(iv) was answered correctly by almost every student. (b)(i) and (b)(ii) was not a problem, for (b)(ii) the students sometimes identified 3 instead of 4 components to explain the desired variability. The majority of students gave a correct definition of the agglomerative clustering in (c)(i), struggling mostly with the definition of complete linkage. Most students who answered c(i) correctly gave also the right answers in c(ii). (c)(iii) was mostly fine with a few incorrect answers. Since many students did not give a justification of their answer, it is hard to identify the aspects they were struggling with.

Paper IV

Question 1. Q1: This question was slightly less popular than the others. The bookwork in (a) was generally well done, though some candidates struggled to provide a full derivation and simply stated the final conditions. Parts (b) was made of 3 sub-questions. The first two ones were done relatively well by the candidates, but the final one was more challenging and few students found the solution. Part (c) was the most challenging and several students did have a good first intuition but struggled to find the analytical expression for the solution.

Question 2. Q2: This question was overall easier than the other two. For part (a), most of the students gave a full solution, but sometimes lacking rigour. In part (b), the hint made the problem relatively easy, yet several students struggled and tried to solve the problem by using matrices, leading to unnecessary complications. Part (c) required long calculations but many students obtained the right expression for the matrix and the angle of rotation. Finding the axis of rotation was more difficult, perhaps due to a lack of time.

Question 3. Q3: This question was the most popular with candidates. Part (a) was bookwork and the vast majority of the candidates gave a good solution. For part (b), many students arrived close to the final solution, sometimes finding a solution line to the right one. Parts (c) and (d) were overall more challenging. Several students started relatively well, but got lost in their calculations.

Question 4. Most candidates were able to complete 4(a), write the equation of motion based on Newton's second law, and solve for the equilibrium position. It is worth noting a minor issue: some candidates' force analysis is not very standardised, leading to minor errors in the sign of the reaction force. For many candidates, question 4(b) represented a challenge, especially 4(b)(ii). For 4(b)(i), while most candidates could derive the general solution from the equation of motion, only a few could use the equation for $z_b(t)$ derived in 4(a) as a direct starting point. Additionally, there were many errors in specifying all required initial conditions and solving eigenvalues. Only a small number of candidates could successfully derive the condition for z_0 and the jump time t_{jum}, because of earlier errors in $z_b(t)$. Furthermore, the foot remains in contact with the floor, equivalent to $N > 0$, was not clearly understood by some candidates. For 4(b)(iv), most candidates could correctly identify the centre of mass but failed to complete the equation of motion.

Question 5. Almost all candidates provided correct answers for 5(a), demonstrating a solid understanding of the central force and Newton's second law in polar coordinates. For 5(b)(i), many candidates could obtain the equilibrium, although some only verified that r_{eqm} given by the problem satisfied the equilibrium equation, rather than solving for the equilibrium position directly. It is recommended to solve for the equilibrium from the equation. Regarding the stability problem, only a few candidates showed all the necessary steps to reach the final conclusion. Calculation mistakes were common, but there are few conceptual problems. A better approach is to add a small perturbation to the equilibrium solution, formulate the associated linearised equation, and then check the eigenvalue. Many candidates finally failed to derive a multiple of $(n-3)$ for the coefficients of the equation. Additionally, some candidates checked the stability of $u = 1/r$ while a few considered r. For 5(b)(ii), most candidates could only get the correct form of $g'(x)$ and failed to go further. Many directly attempted to find the eigenvalue to write down the general solution indicating that most have not yet learned sufficient techniques for solving ordinary differential equations. A critical step is to establish a well-posed system for the unknown, including the equations and boundary/initial conditions. Following this, an exact differential equation can be deduced through proper manipulations. The constant of integration can then be determined from the initial conditions, allowing a solution to be obtained from the given hint.

Question 6. The majority of students who attempted all parts of this question did well. Question (a): all students performed, generally, well. A common error was not considering that the moment of inertia was about

P, not the centre of mass (or necessarily, the origin). Many students complicated things by calculating the full intertial tensor. (b) Several minor calculation errors. Many students showed unclear working by continuing to use the variable M, which is defined as the mass of the complete disc (instead of working with the density). This lead to some confusion which resulted in a correct general approach, but incorrect final answer. The majority of students worked with the full centre of mass formula, projected down to one-dimension, which was very clear. In (c)ii, several students made a small sign error (making the equilibrium point, if they were to calculate it, unstable). A small subset of students tried to copy from memory the solution to the rod pendulum. Another small subset carried on working with M, treating it both as the mass of the full disc, and the mass of the perforated disc.

Question 7. Most candidates were able to convey an idea of Euclid's algorithm for (a), but quite a few answers lacked precision in describing the output of the division algorithm, or did not explain how to iterate. Successful answers to (b) used either a careful application of Bézout's Lemma or described the process of reversing Euclid's algorithm, with a pitfall in (b)(ii) being to claim that $\mathrm{hcf}(a, b)$ divides $\mathrm{hcf}(a, b, c)$. In part (c), apart from computation errors, the most common mistakes were to neglect to check that $g_i([a, b]) \subseteq [a, b]$, or to draw conclusions about the behaviour of g_i on an interval after only checking at the endpoints. Candidates who answered (b)(ii) meaningfully generally had the correct answer and explanation, although some tried to compare γ's arising from earlier calculations which was insufficient.

Paper V

Question 1. (a) The majority of students were able to state Stoke's theorem. Common mistakes include not mentioning the condition that $\mathbf{F}$ be differentiable/smooth, and being unclear on the direction of integration. In (b)(i) and (ii), the vast majority of students did not attempt this question as per the original solution guide. Rather, nearly all students parameterised the surface to solve the integrals directly. In (b)(i), this results in a similar effort to the suggested, original approach. In (b)(ii), this results in a (correct), albeit complicated integral. Marks were allocated against the scheme for which the student would receive the highest number of marks (i.e., if they parameterised directly, but made several errors resulting in a low mark, but then proceeded in a fashion similar to the solutions and received a slightly higher mark, the higher mark was awarded). A very small subset of students approached (b)(ii) by finding a function $\mathbf{G}$ such that $\mathbf{F} = \nabla \times \mathbf{G}$, such that they could apply Stokes theorem and integrate the boundary.

Question 2.

Overall candidates generally answered parts a and b well. Part c was less well done.

For part a, almost all students took the same approach as in the solutions, i.e., *i*) $\iff$ *ii*) and *iii*) $\iff$ *ii*). Some students instead showed *i*) $\iff$ *iii*), which was generally well done. Candidates that did not score highly scored rather low, indicating they were less familiar with the theoretical aspects of implication. Some candidates did not seem to interpret "equivalent" as "if and only if" and only one direction was shown in these cases.

For part b, candidates had little issue parametrising the curve and setting up the integral. Points weer not heavily deducted for computational errors. Some candidates did not reach any conclusion, seemingly missing the main point of the question: *deduce whether or not* $\mathbf{F}$ *is conservative.*

Part c was much more computational than parts a and b. Some students were able to identify the points of intersection, but many could not. Curve sketches occasionally had correct intersection points but were otherwise incorrect. Many students were able to set up the integrals appropriately, but most were unable to compute the length and area enclosed by the curve.

Question 3.

Overall candidates generally answered part a well, though many candidates omit key hypotheses of the theorem (in part i) of a). Points were deducted for major omissions, but not always. For example, omitting "piecewise" in "piecewise smooth" had no deduction, but omitting, e.g., where the region lies, smoothness of the boundary, simply writing the integral relation etc. resulted in point deduction. Despite this, part ii) was generally done well and many candidates had an understanding of the correct approach (reduce from 3 dimensions to 2 dimensions in an appropriate way).

Parts b and c were not so successfully answered; deduction was not significant if the final solution was not exactly correct, though several students did obtain the exact answer. Regions were typically set up okay, but the actual calculation was challenging for some. Given that the integrals, done two ways, should give the same result, a surprising number of students were happy obtaining significantly different solutions. Leniency was given to students who recognized that different solutions indicated something went wrong in at least one of the computations. Students were generally more comfortable using Green's theorem to evaluate the integral versus solving it directly. Time constraints likely played a role in some of these cases.

Question 4.

This question was generally well done. In (a), marks were dropped due to incomplete answers, e.g. regarding the convergence of the series or the different range of values for n for a_n and b_n. (b) was mostly done well, with errors arising mostly from mistakes in the algebra. (c) was conceptually somewhat new, and while a good number of candidates got the answer right, some identified the odd/even splitting correctly, but did not know how to get the correct answer from this. For example, the high power in x^{10} prompted some to consider high decay rates n^{-10} or higher. Others formulated their argument too generally, not recognizing that some information on the function at $\pm\pi$ is needed.

Question 5.

(a) was very straightforward – bookwork. Some imprecisions occured when dealing with the constants when integrating the equations for F and G obtained by using the general solutions in the initial conditions. In (b), which was done well overall by a good number of students, mistakes where made by mixing up the domains or, in (b)(ii), by using the information from (i) inadequately.

Question 6.

In (a), typical omissions were incomplete discussions of the cases for the "separation of variables" constant or the conditions used to exclude positive powers of n. In (b)(i), some algebra mistakes were done, often when, instead of by inspection, the coefficients were obtained by integration. Sometimes the final answer had a positive power of r. In (b)(ii), algebra mistakes led to wrong coefficients or A_1 was omitted altogether. Also, some candidates missed that the boundary condition was for a derivative of T and not for T itself. Also, the implication of the fact that taking the derivative T_r of the series eliminates A_0 was not really appreciated, that is, the non-uniqueness (more precisely, uniqueness up to an additive constant) for $\alpha = 2/\pi$ on the one hand and non-existence of a solution for $\alpha \neq 2/\pi$ on the other hand.

E. COMMENTS ON PERFORMANCE OF IDENTIFIABLE INDIVIDUALS

Prizes

The Departmental Prize was awarded to:

Jiongjie Hua, St. Catherine's College.

F. MODERATORS AND ASSESSORS

Moderators: Prof. Andras Juhasz (Chair), Prof. Andrew Dancer, Prof. Dominic Vella, Prof. Renaud Lambiotte, Prof. Andreas Muench, Prof. Matthias Winkel, Prof. Paul Balister.

Assessors: Dr. Aleksander Horawa, Dr. Antonio Girao, Dr. Richard Wade, Dr. Maria Christodoulou, Dr. Guillem Cazassus, Dr. Francis Aznaran, Dr. Gissell Estrada-Rodriguez, Dr. Josh Bull, Dr. Barnabus Janzer, Dr. Davide Spriano, Dr. Andrea Guidici, Dr. David Brantner, Dr. Kathryn Gillow, Prof. Patrick Farrell, Dr. Matija Tapuskovic, Dr. Yurij Salmaniw, Dr. Felix Foutel Rodier, Dr. Jane Tan, Dr. Yang Liu, Dr. Adrian Fischer, Dr. Francis Aznaran, Dr. Tara Trauthwein.