# Examiners' Report: Final Honour School of Mathematics
# Part C Trinity Term 2025

October 15, 2025

## Part I

### A. STATISTICS

- **Numbers and percentages in each class**

  See Table 1, page .

- **Numbers of vivas and effects of vivas on classes of result.**
  As in previous years there were no vivas conducted for the FHS of Mathematics Part C.

- **Marking of scripts.**
  The dissertations and mini-projects were double marked. The remaining scripts were all single marked according to a pre-agreed marking scheme which was very closely adhered to. For details of the extensive checking process, see Part II, Section A1.

- **Numbers taking each paper.**
  See Table 7 on page .

Table 1: Numbers in each class (post-2021 classification)

| | Number | | | | | Percentages % | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| * | 2025 | (2024) | (2023) | (2022) | (2021) | 2025 | (2024) | (2023) | (2022) | (2021) |
| Distinction | 57 | (71) | (61) | (46) | (60) | 60.64 | (66.36) | (64.89) | (58.23) | (60) |
| Merit | 21 | (26) | (21) | (19) | (20) | 22.34 | (24.3) | (22.34) | (24.05) | (20) |
| Pass | 13 | (10) | (12) | (14) | (18) | 13.83 | (9.35) | (12.77) | (17.72) | (18) |
| Fail | 3 | (0) | (0) | (0) | (2) | 3.19 | (0) | (0) | (0) | (20) |
| Total | 94 | (107) | (94) | (79) | (100) | 100 | (100) | (100) | (100) | (100) |

Table 2: Numbers in each class (pre-2021 classification)

| | Number | | | | | Percentages % | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 2020 | (2019) | (2018) | (2017) | (2016) | 2020 | (2019) | (2018) | (2017) | (2016) |
| I | 63 | (58) | (53) | (48) | (44) | 67.74 | (57.43) | (56.99) | (57.14) | (50.57) |
| II.1 | 30 | (40) | (26) | (23) | (31) | 32.26 | (39.6) | (27.96) | (27.38) | (35.63) |
| II.2 | 0 | (2) | (13) | (12) | (9) | 0 | (1.98) | (13.98) | (14.29) | (10.34) |
| III | 0 | (1) | (1) | (1) | (3) | 0 | (0.99) | (1.08) | (1.19) | (3.45) |
| F | 0 | (0) | (0) | (0) | (0) | 0 | (0) | (0) | (0) | (0) |
| Total | 93 | (101) | (93) | (84) | (87) | 100 | (100) | (100) | (100) | (100) |

### B. Changes in examining methods and procedures currently under discussion or contemplated for the future

The examiner's are not aware of any planned changes to the examinations currently under consideration.

### C. Notice of examination conventions for candidates

The first notice to candidates was issued on 17th March 2025 and the second notice on 14th May 2025. These contain details of the examinations and assessments.

All notices and the examination conventions for 2025 examinations are on-line at https://www.maths.ox.ac.uk/members/students/undergraduate-courses/teaching-and-learning/part-c-students/examinations-and-assessments/part-c-and-omms.

## Part II

### A1. General Comments on the Examination

The examiners would like to convey their grateful thanks for their help and cooperation to all those who assisted with this year's examination, either as assessors or in an administrative capacity. The chair would like to thank Anwen Amos and the rest of the academic administration team, and Waldemar Schlackow and Matt Brechin for their support of the Part C and OMMS examinations.

In addition the internal examiners would like to express their gratitude to Prof Rosemary Dyson and Prof Roger Moser for carrying out their duties as external examiners in a constructive and supportive way during the year, and for their valuable input at the final examiners' meetings.

#### Timetable

The examinations began on Monday 2nd June and finished on Friday 13th June.

### Mitigating Circumstances Notice to Examiners and other special circumstances

A subset of the examiners (the 'Mitigating Circumstances Panel') attended a pre-board meeting to band the seriousness of the individual notices to examiners. The outcome of this meeting was relayed to the Examiners at the final exam board, who gave careful regard to each case, scrutinised the relevant candidates' marks and agreed actions as appropriate. See Section E for further details.

### Setting and checking of papers and marks processing

Following established practice, the questions for each paper were initially set by the course lecturer, with the lecturer of a related course involved as checker before the first draft of the questions was presented to the examiners. The course lecturers also acted as assessors, marking the questions on their course(s).

The internal examiners met in early January to consider the questions on Michaelmas Term courses, and changes and corrections were agreed with the lecturers where necessary. The revised questions were then sent to the external examiners. Feedback from external examiners was given to examiners, and to the relevant assessor for each paper for a response. The internal examiners met a second time late in Hilary Term to consider the external examiners' comments and assessor responses. The cycle was repeated for the Hilary Term courses, with the examiners' meetings in the Easter Vacation. Before questions were submitted to the Examination Schools, setters were required to sign off a camera-ready copy of their questions.

Exams were held in-person in the Exams Schools. Papers were collected by the Academic Administration team and made available to assessors approximately half a day following the examination. Assessors were made aware of the marking deadlines ahead of time and all scripts and completed mark sheets were returned, if not by the agreed due dates, then at least in time for the script-checking process.

A team of graduate checkers, under the supervision of Anwen Amos, reviewed the mark sheets for each paper of this examination, carefully cross checking against the mark scheme to spot any unmarked questions or parts of questions, addition errors or wrongly recorded marks. Sub-totals for each part were also checked against the mark scheme, noting correct addition. In this way a number of errors were corrected, with each change approved by one of the examiners who were present throughout the process.

### Determination of University Standardised Marks

The scaling process is outlined in full on the Mathematical Institute's website: https://www.maths.ox.ac.uk/mem courses/examinations-assessments/examiners-reports/scaling-algorithm.

Table 3: Vertex positions for the piecewise linear scaling maps, $P_1$, $P_2, P_3$ with the raw marks rescaled to be out of 50. The difficulty score for each paper is also given.

| Paper | $P_1$ | $P_2$ | $P_3$ | Additional corners | Difficulty score |
|---|---|---|---|---|---|
| C1.1 | 10;37 | 20.19;64.5 | 34.59;71 | 42.3;80 ; 50;100 | 0.41 |
| C1.2 | 13.29;37 | 26.83;64.5 | 41.23;71 | 45.62;80 ; 50;100 | -6.23 |
| C1.3 | 10.67;37 | 25;64.5 | 35.93;71 | 42.97;80 ; 50;100 | -0.93 |
| C1.4 | 11.36;37 | 25;64.5 | 37.34;71 | 43.67;80 ; 50;100 | -2.34 |
| C2.2 | 10.35;37 | 27;64.5 | 35.29;71 | 42.65;80 ; 50;100 | -0.29 |
| C2.4 | 10.49;37 | 21.17;64.5 | 35.57;71 | 42.79;80 ; 50;100 | -0.57 |
| C2.5 | 8.62;37 | 17.39;64.5 | 31.79;71 | 40.9;80 ; 50;100 | 3.21 |
| C2.7 | 12.1;37 | 30;64.5 | 38.82;71 | 44.41;80 ; 50;100 | -3.82 |
| C2.3 OS | 11.94;37 | 24.1;64.5 | 38.5;71 | 44.25;80 ; 50;100 | -3.5 |
| C2.6 OS | 14.75;37 | 29.77;64.5 | 44.17;71 | 47.09;80 ; 50;100 | -9.17 |
| C3.1 | 7.29;37 | 23;64.5 | 29.11;71 | 39.56;80 ; 50;100 | 5.89 |
| C3.2 | 8.58;37 | 24;64.5 | 31;71 | 40.86;80 ; 50;100 | 3.28 |
| C3.3 | 11.74;37 | 23.7;64.5 | 38.1;71 | 44.05;80 ; 50;100 | -3.1 |
| C3.4 | 8.49;37 | 22;64.5 | 31.53;71 | 40.77;80 ; 50;100 | 3.47 |
| C3.5 | 9.38;37 | 18.94;64.5 | 30;71 | 41.67;80 ; 50;100 | 1.66 |
| C3.6 | 13.68;37 | 27.61;64.5 | 42.01;71 | 46.01;80 ; 50;100 | -7.01 |
| C3.7 | 10.21;37 | 27;64.5 | 35.01;71 | 42.51;80 ; 50;100 | -0.01 |
| C3.8 | 7.37;37 | 25;64.5 | 29.27;71 | 39.64;80 ; 50;100 | 5.73 |
| C3.10 | 7.44;37 | 22;64.5 | 30;71 | 39.71;80 ; 50;100 | 5.59 |
| C3.11 | 12.23;37 | 24.69;64.5 | 39.09;71 | 44.55;80 ; 50;100 | -4.09 |
| C3.12 | 7.66;37 | 23;64.5 | 29.87;71 | 39.94;80 ; 50;100 | 5.13 |
| C4.1 | 7.97;37 | 27;64.5 | 30.48;71 | 40.24;80 ; 50;100 | 4.52 |
| C4.3 | 9.84;37 | 30;64.5 | 34.27;71 | 42.14;80 ; 50;100 | 0.73 |
| C4.6 | 10.83;37 | 32;64.5 | 36;71 | 43.14;80 ; 50;100 | -1.27 |
| C3.8 OS | 9.13;37 | 18.43;64.5 | 32.83;71 | 41.42;80 ; 50;100 | 2.17 |
| C4.9 | 10.33;37 | 20.85;64.5 | 35.25;71 | 42.63;80 ; 50;100 | -0.25 |
| C5.1 | 9.97;37 | 30;64.5 | 34.52;71 | 42.26;80 ; 50;100 | 0.48 |
| C5.2 | 11.26;37 | 29;64.5 | 37.12;71 | 43.56;80 ; 50;100 | -2.12 |
| C5.5 | 9.5;37 | 25;64.5 | 33.58;71 | 41.79;80 ; 50;100 | 1.42 |
| C5.6 | 12.18;37 | 24.59;64.5 | 38.99;71 | 44.5;80 ; 50;100 | -3.99 |
| C5.7 | 9.53;37 | 25;64.5 | 33.63;71 | 41.82;80 ; 50;100 | 1.37 |
| C5.11 | 9.58;37 | 30;64.5 | 33.73;71 | 41.87;80 ; 50;100 | 1.27 |
| C5.12 | 9.77;37 | 28;64.5 | 31;71 | 42.06;80 ; 50;100 | 0.88 |
| C6.1 | 9.61;37 | 25;64.5 | 33.8;71 | 41.9;80 ; 50;100 | 1.2 |
| C6.2 | 11;37 | 30;64.5 | 36.6;71 | 43.3;80 ; 50;100 | -1.6 |
| C6.4 | 13.03;37 | 26.3;64.5 | 40.7;71 | 45.35;80 ; 50;100 | -5.7 |
| C7.4 | 11.48;37 | 23.18;64.5 | 37.58;71 | 43.79;80 ; 50;100 | -2.58 |
| C7.5 | 11.41;37 | 23.04;64.5 | 37.44;71 | 43.72;80 ; 50;100 | -2.44 |
| C7.6 | 7.56;37 | 15.27;64.5 | 29.67;71 | 39.84;80 ; 50;100 | 5.33 |
| C7.7 | 12.15;37 | 31;64.5 | 38.92;71 | 44.46;80 ; 50;100 | -3.92 |
| C8.1 | 8.58;37 | 23;64.5 | 31.71;71 | 40.86;80 ; 50;100 | 3.29 |
| C8.2 | 9.58;37 | 28;64.5 | 33.73;71 | 41.87;80 ; 50;100 | 1.27 |
| C8.3 | 12.74;37 | 31;64.5 | 38;71 | 45.06;80 ; 50;100 | -5.11 |
| C8.4 | 8.97;37 | 23;64.5 | 32.5;71 | 41.25;80 ; 50;100 | 2.5 |
| C8.7 | 11.49;37 | 23.19;64.5 | 30;71 | 43.8;80 ; 50;100 | -2.59 |

| Paper | $P_1$ | $P_2$ | $P_3$ | Additional corners | Difficulty score |
|---|---|---|---|---|---|
| SC1 | 11.95;37 | 27;64.5 | 38.53;71 | 44.27;80 ; 50;100 | -3.53 |
| SC2 | 12.99;37 | 30;64.5 | 40.63;71 | 45.32;80 ; 50;100 | -5.63 |
| SC4 | 10.69;37 | 21.58;64.5 | 35.31;71 | 42.66;80 ; 50;100 | 0.09 |
| SC5 | 10.44;37 | 32;64.5 | 35.47;71 | 42.74;80 ; 50;100 | -0.47 |
| SC6 | 10.84;37 | 32;64.5 | 36.28;71 | 43.14;80 ; 50;100 | -1.28 |
| SC7 | 9.78;37 | 30;64.5 | 34.15;71 | 42.08;80 ; 50;100 | 0.85 |
| SC9 | 7.75;37 | 23;64.5 | 30.05;71 | 40.03;80 ; 50;100 | 4.95 |
| SC10 | 8.44;37 | 20;64.5 | 27;71 | 40.72;80 ; 50;100 | 3.57 |
| SC11 | 8.33;37 | 23;64.5 | 29;71 | 40.61;80 ; 50;100 | 3.79 |

## B. Equality and Diversity issues and breakdown of the results by gender

Table 5: Breakdown of results by gender

| Class | Number | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 2025 | | | 2024 | | | 2023 | | | 2022 | | | 2021 | | |
| | Female | Male | Total | Female | Male | Total | Female | Male | Total | Female | Male | Total | Female | Male | Total |
| Distinction | 12 | 45 | 57 | 11 | 60 | 71 | 7 | 54 | 61 | 10 | 36 | 46 | 15 | 45 | 60 |
| Merit | 6 | 15 | 21 | 9 | 17 | 26 | 8 | 13 | 21 | 9 | 10 | 19 | 8 | 12 | 20 |
| Pass | 3 | 10 | 13 | 3 | 7 | 10 | 3 | 9 | 12 | 5 | 9 | 14 | 5 | 13 | 18 |
| Fail | 0 | 3 | 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 2 |
| Total | 21 | 73 | 94 | 23 | 84 | 107 | 18 | 76 | 94 | 24 | 55 | 79 | 29 | 71 | 100 |
| Class | Percentage | | | | | | | | | | | | | | |
| | 2025 | | | 2024 | | | 2023 | | | 2022 | | | 2021 | | |
| | Female | Male | Total | Female | Male | Total | Female | Male | Total | Female | Male | Total | Female | Male | Total |
| Distinction | 57.14 | 61.64 | 60.64 | 47.83 | 71.43 | 66.36 | 38.89 | 71.05 | 64.89 | 40 | 65.45 | 58.23 | 51.72 | 63.38 | 60 |
| Merit | 28.57 | 20.55 | 22.34 | 39.13 | 20.24 | 24.3 | 44.44 | 17.11 | 22.34 | 36 | 18.18 | 24.05 | 27.59 | 16.9 | 20 |
| Pass | 14.29 | 13.7 | 13.83 | 13.04 | 8.33 | 9.35 | 16.67 | 11.84 | 12.77 | 20 | 16.36 | 23.73 | 17.24 | 18.31 | 18 |
| Fail | 0 | 4.11 | 3.19 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 3.45 | 1.41 | 2 |
| Total | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |

Table 6: Breakdown of results by gender (pre-2021 classification)

| Class | Number | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 2020 | | | 2019 | | | 2018 | | |
| | Female | Male | Total | Female | Male | Total | Female | Male | Total |
| I | 16 | 47 | 63 | 8 | 50 | 58 | 6 | 47 | 53 |
| II.1 | 4 | 26 | 30 | 9 | 31 | 40 | 7 | 19 | 26 |
| II.2 | 0 | 0 | 0 | 0 | 2 | 2 | 3 | 10 | 13 |
| III | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 1 |
| F | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Total | 20 | 73 | 93 | 17 | 84 | 101 | 17 | 76 | 93 |

| Class | Percentage | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 2020 | | | 2019 | | | 2018 | | |
| | Female | Male | Total | Female | Male | Total | Female | Male | Total |
| I | 80 | 64.38 | 72.19 | 47.06 | 59.52 | 57.43 | 35.29 | 61.84 | 56.99 |
| II.1 | 20 | 35.62 | 27.81 | 52.94 | 36.9 | 39.6 | 41.18 | 25 | 27.96 |
| II.2 | 0 | 0 | 0 | 0 | 2.38 | 1.98 | 17.65 | 13.16 | 13.98 |
| III | 0 | 0 | 0 | 0 | 1.19 | 0.99 | 5.88 | 0 | 1.08 |
| F | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Total | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |

## C. Detailed numbers on candidates' performance in each part of the exam

Data for papers with fewer than six candidates are not included in the public versions of the examiners' report.

Table 7: Numbers taking each paper

| Paper | Number of Candidates | Avg RAW | StDev RAW | Avg USM | StDev USM |
|---|---|---|---|---|---|
| C1.1 | 6 | 37.67 | 8.57 | 78 | 11.37 |
| C1.2 | 6 | 37.83 | 10.82 | 75 | 14.55 |
| C1.3 | 16 | 34.81 | 8.17 | 71.75 | 9.15 |
| C1.4 | 11 | 37.91 | 9.59 | 74.55 | 12.31 |
| C2.2 | 17 | 31 | 7 | 63.82 | 18.51 |
| C2.4 | 8 | 28.5 | 9.43 | 66.12 | 10.4 |
| C2.5 | - | - | - | - | - |
| C2.7 | 20 | 33.15 | 7.51 | 67.3 | 10.87 |
| C2.6 OS | - | - | - | - | |
| C3.1 | 17 | 26.12 | 10.42 | 66.29 | 14 |
| C3.2 | 10 | 30.2 | 7.69 | 70.6 | 8.9 |
| C3.3 | 14 | 34.14 | 11.35 | 70.07 | 16.29 |
| C3.4 | 8 | 27 | 11.66 | 67.12 | 15.93 |
| C3.5 | 9 | 29.78 | 6.08 | 71.33 | 4.12 |
| C3.6 | - | - | - | - | - |
| C3.7 | 13 | 29.54 | 7.42 | 65.23 | 9.06 |
| C3.8 | 12 | 28.25 | 12.2 | 67.5 | 16.93 |
| C3.10 | 9 | 26.22 | 11.81 | 64.33 | 16.54 |
| C3.11 | - | - | - | - | - |
| C3.12 | - | - | - | - | - |
| C4.1 | 14 | 29.5 | 8.72 | 68.07 | 12.42 |
| C4.3 | 8 | 36.5 | 6.76 | 73.62 | 9.35 |
| C4.6 | 6 | 35.17 | 2.14 | 69.5 | 3.15 |
| C4.9 | | - | - | - | - |
| C5.1 | 8 | 29.62 | 5.55 | 64.12 | 7.66 |
| C5.2 | 14 | 35.86 | 6.43 | 71.57 | 8.67 |
| C5.5 | 22 | 31.43 | 5.36 | 65.91 | 15.74 |
| C5.6 | 17 | 38.24 | 7.72 | 74.59 | 9.66 |
| C5.7 | 12 | 31.75 | 7.48 | 69.92 | 8.08 |
| C5.11 | 19 | 31.79 | 7.03 | 67.53 | 10.49 |
| C5.12 | 18 | 30.83 | 6.79 | 68.22 | 9.25 |
| C6.1 | 19 | 32.16 | 8.32 | 70.58 | 10.82 |
| C6.2 | 10 | 31.3 | 9.93 | 64.5 | 13.03 |
| C6.4 | 6 | 38.67 | 9.42 | 74.83 | 13.35 |
| C7.4 | - | - | - | - | - |
| C7.5 | - | - | - | - | - |
| C7.6 | - | - | | - | - |
| C7.7 | 7 | 36.86 | 7.15 | 71.43 | 11.86 |

| Paper | Number of Candidates | Avg RAW | StDev RAW | Avg USM | StDev USM |
|---|---|---|---|---|---|
| C8.1 | 18 | 29.83 | 7.29 | 70.39 | 8.29 |
| C8.2 | 8 | 33 | 8.38 | 70.75 | 10.87 |
| C8.3 | 26 | 37.85 | 7.53 | 73 | 11.78 |
| C8.4 | 19 | 30 | 6.82 | 69.11 | 7.38 |
| C8.7 | - | - | - | - | - |
| SC1 | 12 | 35.67 | 5.99 | 69.92 | 6.08 |
| SC2 | 9 | 38.89 | 8.64 | 72 | 11.1 |
| SC4 | 6 | 32.5 | 10.78 | 72.33 | 12.14 |
| SC5 | - | - | - | - | - |
| SC6 | - | - | - | - | - |
| SC7 | - | - | - | - | - |
| SC9 | 7 | 26.57 | 5.41 | 67 | 6.32 |
| SC10 | - | - | - | - | - |

Table 9: *

Table 8: Statistics by paper (USM only assessments)

| Paper | Number of Candidates | AvgUSM | StdevUSM |
|---|---|---|---|
| C2.3 | - | - | - |
| C2.6 | - | - | - |
| C3.9 | 8 | 69.38 | 8.8 |
| C5.4 | 17 | 72.06 | 6.41 |
| C6.5 | 13 | 70.69 | 7.33 |
| CCD | 88 | 72.93 | 14.87 |
| COD | 6 | 77.5 | 10.48 |
| SC8 | 6 | 79.17 | 3.37 |
| CS3 | - | - | - |
| CS7 | - | - | - |

The tables that follow give the question statistics for each paper for Mathematics candidates. Data for papers with fewer than six candidates are not included in the public version of the report.

**Paper C1.1: Model Theory**

| Question | Mean Mark | | Std Dev | Number of attempts | |
|---|---|---|---|---|---|
| | All | Used | | Used | Unused |
| Q1 | 14 | 14 | 6.32 | 4 | 0 |
| Q2 | 20.5 | 20.5 | 2.43 | 6 | 0 |
| Q3 | 23.5 | 23.5 | 2.12 | 2 | 0 |

**Paper C1.2: Gödel's Incompleteness Theorems**

| Question | Mean Mark | | Std Dev | Number of attempts | |
|---|---|---|---|---|---|
| | All | Used | | Used | Unused |
| Q1 | 12 | 12 | 0 | 2 | 0 |
| Q2 | 20.4 | 20.4 | 6.19 | 5 | 0 |
| Q3 | 18.17 | 20.2 | 6.18 | 5 | 1 |

**Paper C1.3: Analytic Topology**

| Question | Mean Mark | | Std Dev | Number of attempts | |
|---|---|---|---|---|---|
| | All | Used | | Used | Unused |
| Q1 | 14.38 | 14.38 | 5.15 | 8 | 0 |
| Q2 | 15.33 | 15.33 | 5.15 | 9 | 0 |
| Q3 | 20.27 | 20.27 | 4.03 | 15 | 0 |

**Paper C1.4: Axiomatic Set Theory**

| Question | Mean Mark | | Std Dev | Number of attempts | |
|---|---|---|---|---|---|
| | All | Used | | Used | Unused |
| Q1 | 18.1 | 18.1 | 6.26 | 10 | 0 |
| Q2 | 18.5 | 18.5 | 6.09 | 8 | 0 |
| Q3 | 18.33 | 22 | 7.45 | 4 | 2 |

**Paper C2.2: Homological Algebra**

| Question | Mean Mark | | Std Dev | Number of attempts | |
|---|---|---|---|---|---|
| | All | Used | | Used | Unused |
| Q1 | 18.06 | 18.06 | 3.36 | 16 | 0 |
| Q2 | 12.94 | 12.94 | 5.76 | 16 | 0 |
| Q3 | 3 | | | 0 | 1 |

**Paper C2.4: Infinite Groups**

| Question | Mean Mark | | Std Dev | Number of attempts | |
|---|---|---|---|---|---|
| | All | Used | | Used | Unused |
| Q1 | 14.62 | 14.62 | 5.4 | 8 | 0 |
| Q2 | 19.67 | 19.67 | 3.21 | 3 | 0 |
| Q3 | 8.71 | 10.4 | 5.79 | 5 | 2 |

**Paper C2.7: Category Theory**

| Question | Mean Mark | | Std Dev | Number of attempts | |
|---|---|---|---|---|---|
| | All | Used | | Used | Unused |
| Q1 | 16.45 | 16.45 | 3.87 | 20 | 0 |
| Q2 | 17.41 | 17.88 | 4.53 | 16 | 1 |
| Q3 | 16 | 16 | 2 | 3 | 0 |

**Paper C3.1: Algebraic Topology**

| Question | Mean Mark | | Std Dev | Number of attempts | |
|---|---|---|---|---|---|
| | All | Used | | Used | Unused |
| Q1 | 12.3 | 15.25 | 7.93 | 8 | 2 |
| Q2 | 10.92 | 10.92 | 7.68 | 13 | 0 |
| Q3 | 13.85 | 13.85 | 5.94 | 13 | 0 |

**Paper C3.2: Geometric Group Theory**

| Question | Mean Mark | | Std Dev | Number of attempts | |
|---|---|---|---|---|---|
| | All | Used | | Used | Unused |
| Q1 | 18.6 | 18.6 | 3.1 | 10 | 0 |
| Q2 | 9.86 | 10.17 | 4.71 | 6 | 1 |
| Q3 | 13.75 | 13.75 | 6.9 | 4 | 0 |

**Paper C3.3: Differentiable Manifolds**

| Question | Mean Mark | | Std Dev | Number of attempts | |
|---|---|---|---|---|---|
| | All | Used | | Used | Unused |
| Q1 | 19.14 | 19.14 | 6.21 | 14 | 0 |
| Q2 | 12.9 | 12.9 | 5.51 | 10 | 0 |
| Q3 | 20.25 | 20.25 | 5.19 | 4 | 0 |

**Paper C3.4: Algebraic Geometry**

| Question | Mean Mark | | Std Dev | Number of attempts | |
|---|---|---|---|---|---|
| | All | Used | | Used | Unused |
| Q1 | 10.75 | 10.75 | 10.44 | 4 | 0 |
| Q2 | 14.71 | 16.83 | 7.57 | 6 | 1 |
| Q3 | 12 | 12 | 6.48 | 6 | 0 |

**Paper C3.5: Lie Groups**

| Question | Mean Mark | | Std Dev | Number of attempts | |
|---|---|---|---|---|---|
| | All | Used | | Used | Unused |
| Q1 | 14 | 13.67 | 4.24 | 6 | 1 |
| Q2 | 15.88 | 15.88 | 3.6 | 8 | 0 |
| Q3 | 14.75 | 14.75 | 2.22 | 4 | 0 |

**Paper C3.7: Elliptic Curves**

| Question | Mean Mark | | Std Dev | Number of attempts | |
|---|---|---|---|---|---|
| | All | Used | | Used | Unused |
| Q1 | 14.25 | 14.25 | 2.6 | 8 | 0 |
| Q2 | 13.12 | 13.12 | 4.79 | 8 | 0 |
| Q3 | 16.5 | 16.5 | 4.45 | 10 | 0 |

**Paper C3.8: Analytic Number Theory**

| Question | Mean Mark | | Std Dev | Number of attempts | |
|---|---|---|---|---|---|
| | All | Used | | Used | Unused |
| Q1 | 13.22 | 14 | 6.96 | 8 | 1 |
| Q2 | 14.75 | 14.75 | 6.18 | 8 | 0 |
| Q3 | 13.62 | 13.62 | 6.48 | 8 | 0 |

**Paper C3.10: Additive and Combinatorial Number Theory**

| Question | Mean Mark | | Std Dev | Number of attempts | |
|---|---|---|---|---|---|
| | All | Used | | Used | Unused |
| Q1 | 15 | 16.43 | 8.45 | 7 | 1 |
| Q2 | 11 | 11 | 5.6 | 4 | 0 |
| Q3 | 10.62 | 11 | 5.21 | 7 | 1 |

**Paper C4.1: Further Functional Analysis**

| Question | Mean Mark | | Std Dev | Number of attempts | |
|---|---|---|---|---|---|
| | All | Used | | Used | Unused |
| Q1 | 15.62 | 15.62 | 3.28 | 13 | 0 |
| Q2 | 14.71 | 14.71 | 4.83 | 14 | 0 |
| Q3 | 4 | 4 | | 1 | 0 |

**Paper C4.3: Functional Analytical Methods for PDEs**

| Question | Mean Mark | | Std Dev | Number of attempts | |
|---|---|---|---|---|---|
| | All | Used | | Used | Unused |
| Q1 | 17.86 | 17.86 | 3.98 | 7 | 0 |
| Q2 | 17.5 | 17.5 | 3.7 | 4 | 0 |
| Q3 | 19.4 | 19.4 | 3.05 | 5 | 0 |

**Paper C4.6: Fixed Point Methods for Nonlinear PDEs**

| Question | Mean Mark | | Std Dev | Number of attempts | |
|---|---|---|---|---|---|
| | All | Used | | Used | Unused |
| Q1 | 17 | 17 | 2.68 | 6 | 0 |
| Q2 | 18 | 18 | 2.65 | 3 | 0 |
| Q3 | 18.33 | 18.33 | 1.15 | 3 | 0 |

**Paper C5.1: Solid Mechanics**

| Question | Mean Mark | | Std Dev | Number of attempts | |
|---|---|---|---|---|---|
| | All | Used | | Used | Unused |
| Q1 | 13.62 | 13.62 | 5.9 | 8 | 0 |
| Q2 | 14.33 | 14.33 | 6.35 | 3 | 0 |
| Q3 | 17 | 17 | 1 | 5 | 0 |

**Paper C5.2: Elasticity and Plasticity**

| Question | Mean Mark | | Std Dev | Number of attempts | |
|---|---|---|---|---|---|
| | All | Used | | Used | Unused |
| Q1 | 18.91 | 18.91 | 3.39 | 11 | 0 |
| Q2 | 14.25 | 14.25 | 3.2 | 8 | 0 |
| Q3 | 20 | 20 | 2.45 | 9 | 0 |

**Paper C5.5: Perturbation Methods**

| Question | Mean Mark | | Std Dev | Number of attempts | |
|---|---|---|---|---|---|
| | All | Used | | Used | Unused |
| Q1 | 10.09 | 13.83 | 5.99 | 6 | 5 |
| Q2 | 16.2 | 16.2 | 2.46 | 20 | 0 |
| Q3 | 15.81 | 15.81 | 4.28 | 16 | 0 |

**Paper C5.6: Applied Complex Variables**

| Question | Mean Mark | | Std Dev | Number of attempts | |
|---|---|---|---|---|---|
| | All | Used | | Used | Unused |
| Q1 | 17.79 | 17.79 | 4.23 | 14 | 0 |
| Q2 | 16.6 | 16.6 | 3.97 | 5 | 0 |
| Q3 | 21.2 | 21.2 | 3.43 | 15 | 0 |

**Paper C5.7: Topics in Fluid Mechanics**

| Question | Mean Mark | | Std Dev | Number of attempts | |
|---|---|---|---|---|---|
| | All | Used | | Used | Unused |
| Q1 | 11.08 | 12.8 | 5.96 | 10 | 2 |
| Q2 | 19.67 | 19.67 | 4.21 | 12 | 0 |
| Q3 | 8.5 | 8.5 | 2.12 | 2 | 0 |

**Paper C5.11: Mathematical Geoscience**

| Question | Mean Mark | | Std Dev | Number of attempts | |
|---|---|---|---|---|---|
| | All | Used | | Used | Unused |
| Q1 | 15.23 | 15.23 | 4.97 | 13 | 0 |
| Q2 | 15.69 | 15.69 | 3.53 | 16 | 0 |
| Q3 | 17.22 | 17.22 | 3.27 | 9 | 0 |

**Paper C5.12: Mathematical Physiology**

| Question | Mean Mark | | Std Dev | Number of attempts | |
|---|---|---|---|---|---|
| | All | Used | | Used | Unused |
| Q1 | 17.5 | 17.5 | 3.71 | 18 | 0 |
| Q2 | 10.25 | 10.25 | 4.27 | 4 | 0 |
| Q3 | 14 | 14.21 | 4.17 | 14 | 1 |

**Paper C6.1: Numerical Linear Algebra**

| Question | Mean Mark | | Std Dev | Number of attempts | |
|---|---|---|---|---|---|
| | All | Used | | Used | Unused |
| Q1 | 15.14 | 16 | 5.33 | 13 | 1 |
| Q2 | 15.69 | 15.69 | 4.97 | 13 | 0 |
| Q3 | 16.58 | 16.58 | 4.52 | 12 | 0 |

**Paper C6.2: Continuous Optimization**

| Question | Mean Mark | | Std Dev | Number of attempts | |
|---|---|---|---|---|---|
| | All | Used | | Used | Unused |
| Q1 | 14.88 | 14.88 | 6.42 | 8 | 0 |
| Q2 | 13.78 | 14.88 | 6.51 | 8 | 1 |
| Q3 | 15.8 | 18.75 | 6.87 | 4 | 1 |

**Paper C6.4: Finite Element Methods for Partial Differential Equations**

| Question | Mean Mark | | Std Dev | Number of attempts | |
|---|---|---|---|---|---|
| | All | Used | | Used | Unused |
| Q1 | 17.33 | 17.33 | 7.23 | 6 | 0 |
| Q2 | 21.33 | 21.33 | 2.5 | 6 | 0 |
| Q3 | - | - | - | - | - |

**Paper C7.7: Random Matrix Theory**

| Question | Mean Mark | | Std Dev | Number of attempts | |
|---|---|---|---|---|---|
| | All | Used | | Used | Unused |
| Q1 | 18.57 | 18.57 | 3.99 | 7 | 0 |
| Q2 | 19 | 19 | 5.79 | 5 | 0 |
| Q3 | 11.33 | 16.5 | 9.29 | 2 | 1 |

**Paper C8.1: Stochastic Differential Equations**

| Question | Mean Mark | | Std Dev | Number of attempts | |
|---|---|---|---|---|---|
| | All | Used | | Used | Unused |
| Q1 | 13.75 | 17 | 7.15 | 6 | 2 |
| Q2 | 11.6 | 12.43 | 4.47 | 14 | 1 |
| Q3 | 16 | 16.31 | 4.69 | 16 | 1 |

**Paper C8.2: Stochastic Analysis and PDEs**

| Question | Mean Mark | | Std Dev | Number of attempts | |
|---|---|---|---|---|---|
| | All | Used | | Used | Unused |
| Q1 | - | - | - | - | - |
| Q2 | 16.88 | 16.88 | 5 | 8 | 0 |
| Q3 | 16.12 | 16.12 | 5.19 | 8 | 0 |

**Paper C8.3: Combinatorics**

| Question | Mean Mark | | Std Dev | Number of attempts | |
|---|---|---|---|---|---|
| | All | Used | | Used | Unused |
| Q1 | 17.72 | 17.72 | 3.73 | 25 | 0 |
| Q2 | 21.14 | 21.14 | 3.88 | 21 | 0 |
| Q3 | 14.88 | 16.17 | 5.62 | 6 | 2 |

**Paper C8.4: Probabilistic Combinatorics**

| Question | Mean Mark | | Std Dev | Number of attempts | |
|---|---|---|---|---|---|
| | All | Used | | Used | Unused |
| Q1 | 15.94 | 15.94 | 5.75 | 18 | 0 |
| Q2 | 11 | 11 | 4.69 | 4 | 0 |
| Q3 | 14.94 | 14.94 | 3.26 | 16 | 0 |

**Paper SC1: Stochastic Models in Mathematical Genetics**

| Question | Mean Mark | | Std Dev | Number of attempts | |
|---|---|---|---|---|---|
| | All | Used | | Used | Unused |
| Q1 | 14.43 | 16.67 | 6.5 | 6 | 1 |
| Q2 | 15 | 15 | 5.55 | 6 | 0 |
| Q3 | 19.83 | 19.83 | 2.59 | 12 | 0 |

**Paper SC2: Probability and Statistics for Network Analysis**

| Question | Mean Mark | | Std Dev | Number of attempts | |
|---|---|---|---|---|---|
| | All | Used | | Used | Unused |
| Q1 | 18.89 | 18.89 | 4.91 | 9 | 0 |
| Q2 | 21.25 | 21.25 | 5.6 | 8 | 0 |
| Q3 | 8.5 | 10 | 2.12 | 1 | 1 |

**Paper SC4: Statistical Data Mining and Machine Learning**

| Question | Mean Mark | | Std Dev | Number of attempts | |
|---|---|---|---|---|---|
| | All | Used | | Used | Unused |
| Q1 | 17.25 | 17.25 | 6.7 | 4 | 0 |
| Q2 | 15 | 15 | 6.57 | 6 | 0 |
| Q3 | 18 | 18 | 2.83 | 2 | 0 |

**Paper SC9: Interacting Particle Systems**

| Question | Mean Mark | | Std Dev | Number of attempts | |
|---|---|---|---|---|---|
| | All | Used | | Used | Unused |
| Q1 | 14.57 | 14.57 | 2.64 | 7 | 0 |
| Q2 | 6 | 6 | | 1 | 0 |
| Q3 | 13 | 13 | 2.37 | 6 | 0 |

## D. Recommendations for Next Year's Examiners and Teaching Committee

The exam board recommends that the department introduces strong repercussions for markers who consistently do not meet the standards of marking required, or return their marking late. The examiners felt that there was a high volume of errors in markings. It was noted that the department offers thorough guidelines on marking, as well as a marking seminar. The examiners would encourage a review of all guidance to ensure consistency across markers.

Additionally, the examiners note their concern at the volume of mistakes within the exam papers and solutions this year. The exam board recommends that the guidance for paper setters should emphasise the importance in double checking formulas and mathematical aspects of questions to avoid mistakes.

The exam board would be in strong support of the removal or easing of the University regulation that prevents candidates from achieving a merit or distinction where one paper has been failed. For example, the board would ask that Teaching Committee consider whether this regulation could be reserved for cases where more than one paper has been failed, or alternatively whether results should be capped at a merit instead of a pass. Additionally, the exam board would welcome further guidance on when this regulation should be applied and when it can be waived by the examiners.

The board notes that the database should be updated to prevent candidates with excusals from being included in the difficulty rating of those papers.

Finally, the board would recommend that Teaching Committee review whether it is possible to introduce automated emails on Inspera to notify students when they have successfully submitted work. It is felt that this would reduce the amount of technical fails overall.

## E. Comments on papers and on individual questions

The following comments were submitted by the assessors.

### C1.1: Model Theory

**Question 1** 13 out of 16 candidates attempted this question. It is the question with the lowest average performance. Parts (a) and (b) were close to bookwork. Part (c)(ii) was somewhat challenging. And for (c)(iv), many bizarre suggestions came up, often invoking Gödel's First Incompleteness Theorem — the easiest example of a complete $L$-theory which is not $\forall\exists$-axiomatizable is the theory of $\langle \mathbb{Z}; +; 0\rangle$.

**Question 2** Everyone did this question. Parts (a) and (b) were mostly bookwork or similar. Part (c) was more demanding: in (c)(ii), one had to realize that in an $L_n$-reduct $\mathcal{A}_n$ of models $\mathcal{A}$ of $T_\infty$ any boolean combination of $P_1, \ldots, P_n$ must be realisable infinitely often to see that $T_n$ is $\aleph_0$-categorical. In part (c)(iii) it was often claimed that $T_\infty$ is also $\aleph_0$-categorical, but this is not the case. One rather has to see first that the $T_n$ are complete to conclude the same for $T_\infty$. In part (c)(iv) it was often argued that any model of $T_\infty$ realizes $2^{\aleph_0}$ many types, but this is just wrong: it is true that there are $2^{\aleph_0}$ many complete types of $T_\infty$, but, as $T_\infty$ has countable models, they are obviously not always all realized.

**Question 3** Only 3 candidates chose this question, possibly because it covered material which only came up towards the end of the course. Part (b) was just bookwork, whereas part (c) required a solid understanding: the most difficult part was to show that $\langle \mathbb{R}; +; 0 \rangle$ is $\aleph_1$-saturated (and hence also $\aleph_0$-saturated): the underlying theory is that of non-zero divisible torsion-free abelian groups which can be reinterpreted as the theory of non-zero $\mathbb{Q}$-vector spaces.

### C1.2: Gödel's Incompleteness Theorems

On the whole this paper was done well.

Question 1. was not very popular. Notwithstanding this, there were some good answers, some quite ingenious.

For question 2., part (a) was done well. Part (b) could largely done by using the Second Incompleteness Theorem and Löb's Theorem. Forgetting about Löb's Theorem was a common mistake.

In question 3., there were some common errors in part (a).

In part (i), some people stated the rule of Necessitation as: from $\phi$ deduce $\Box\phi$, which is incorrect. Others stated Modus Ponens as: if $\phi$ and $(\phi \to \psi)$ are provable, then so is $\psi$, which is also incorrect.

In part (ii), there were many correct statements of Solovay's Completeness Theorem, but also many incorrect ones. It has to do with certain maps $\phi \mapsto \phi^*$ from the language of Gödel-Löb logic to the language of arithmetic. Many people said, incorrectly, that a formula $\phi$ of GL logic was a theorem if and only if it was verified by one map $\phi \mapsto \phi^*$, rather than by all of them.

In part (b), it was again a question of exploiting the Second Incompleteness Theorem and Löb's Theorem, but also the third provability rule.

### C1.3: Analytic Topology

**Question 1** Part (a) was generally done to a high standard. In part (b) a number of candidates implicitly assumed that the cover consisted of basic open sets instead of remarking that this is what they were doing and why this does not lose generality. A lot of candidates claimed incorrectly claimed that if $\{A_j : j \in J\}$ refines $\{U_i : i \in I\}$ and $\{B_k : k \in K\}$ refines $\{V_i : i \in I\}$ then $\{A_j \times B_k : j \in J, k \in K\}$ refines $\{U_i \times V_i : i \in I\}$, thereby short-cutting the question substantially. Part (c) stumped most candidates with only very few credible attempts.

**Question 2** Again, part (a) was generally well done. In part (b)(i) a number of candidates claimed incorrectly that $f^\star(X \setminus f^{-1}(x))$ and $f^\star(X \setminus f^{-1}(y))$ would be disjoint (in $f(X)$) for distinct $x, y$. Other candidates obtained disjoint open sets $U, V$ containing the fibres of $x$ and $y$ respectively but then did not verify that $f^\star(U), f^\star(V)$ were disjoint or contained $x$ and $y$ respectively. In part (b)(ii) $(\alpha) \Rightarrow (\beta)$ a number of candidates tried to unsuccessfully use that the maps witnessing the order between compactifications of $X$ map remainders to remainders despite $Z$ not being a compactification of $X$. In $(\delta) \Rightarrow (\alpha)$ many candidates

claimed that a closed subset of $X$ would be compact instead of considering the closure in $\beta X$.

**Question 3** Overall question 3 was much better done than the others, however, in part (a) a number of candidates did not provide full details, e.g. why it is enough to show that subbasic open sets containing $x$ being in a filter implies that this filter converges to $x$. Part (b)(i) and (ii) was done very well, although often not in the most efficient way with unnecessary but correct claims made. Part (iii) was considerably harder and again candidates failed to be explicit about some of their implied arguments, e.g. using the second part of (i) to shrink elements of the open cover to basic open sets which also must be in the cover provided by the hint. It was positive that a number of candidates remarked how a minor variations of (ii) together with Zorn's Lemma proved the hint.

### C1.4: Axiomatic Set Theory

Question 1. gained quite a spread of marks.

In (a)(iv), the point is that in the presence of Foundation, it is enough to characterise ordinals as being transitive and totally ordered by $\in$, which is $\Sigma_0$. In part (vi), it is necessary to use the construction of the $V_\alpha$ hierarchy by iterating $\wp$. A few people claimed that in $L$, $\wp L_\alpha \subseteq L_{\alpha+1}$ which, even assuming $V = L$, is true only if $\alpha$ is finite.

There were many good solutions to question 2.

In part (b)(ii), it is not absolutely essential, but it is extremely useful, to prove a lemma saying that there is a monotonically increasing unbounded function from $\operatorname{cof}\alpha$ to $\alpha$. The absence of this lemma led some people into error.

There were various correct solutions to (b)(iv), none of which (I think) used the line of reasoning that I expected: that if $\langle \kappa_\alpha : \alpha < \mu \rangle$ is a monotonically increasing cofinal sequence in $\kappa$, then any subset $A$ of $\kappa$ can be recovered from the sequence $\langle A \cap \kappa_\alpha : \alpha < \mu \rangle$.

Question 3. frightened off some people, but there were good answers submitted.

Part (a) received many good answers.

More people got the essential idea of the recursion needed for (b)(i) than were able to carry it out in detail.

People who attempted part (a) got it right, on the whole.

### C2.2: Homological Algebra

Q1a was bookwork and most candidates did well on it. Q1bi was done well by most candidates. Q1bii was diffcult and most candidates did not manage to show that adjoints exist. For Q1biii one had to use a similar reasoning to the one used in proving enough injectives in categories of modules. Some candidates did that but many did not. In Q1c many candidates showed that the full subactegory is additive or pre-abelian but not everyone checked that the canonical map from the coimage to the image is an isomorphism.

Q2ai was standard bookwork. Q2aii was also bookwork or very similar to bookwork (dimension shifting) but not all candidates remembered how to prove it. Not many candidates

gave a good argument for Q2bi. In Q2bii the point was to check for n=0 and then use Q2aii. This was done by some candidates. Given Q2bii Q2c was quite straightforward and many candidates gave quite a good solution.

Only one candidate attempted Q3 and only the first part of Q3a.

### C2.3: Representation Theory of Semisimple Lie Algebras (Old Syllabus)

This is an exam written under old regs for a single student. I cannot comment on the difficulty of the exam or any common trends given that there was one candidate.

### C2.4: Infinite Groups

**Question 1.** This question was attempted by all candidates. Even though a large portion of it was book work, the responses only covered part of it. Surprisingly, some candidates thought that from the two questions of whether the torsion of a polycyclic group is a group or is finite, only one is false while the other is true, and the counter-example they provided for one actually worked for both. Some confusion between solvable and polycyclic was displayed in the answers to the final part of the question.

**Question 2.** This question was the least popular, probably because it uses material from the end of the course. The few candidates who attempted it produced very good answers. A few candidates wrote the isomorphism between the matrix group and the wreath product, but did not prove that it was one. The rest of the question was well attempted and answered, including the application of Milnor's theorem.

**Question 3.** This question was attempted by many candidates. In part (b), despite the fact that similar exercises have been seen in classes, many candidates first argued that the normal subgroup that is direct sum is residually finite, but after that, instead of attempting to extend the construction to a finite index subgroup of the whole group, argued that the normal subgroup can appear as a quotient, obviously wrong.

In (c), while the very first part required a bit of thought, the second and the third part were consequences easy to derive, and should therefore have been better attempted.

Most candidates saw how to derive (d) from (c) and explained the argument well.

### C2.5: Non-Commutative Rings

It is difficult to analize the results due to the very low number of candidates taking this exam.

Question 1 had several attempts but none of the candidates made a serious progress at part (c) despite the generous hint.

Question 2 had one almost complete solution, it was attempted by just 2 candidates.

Question 3 had the best results, two candidates spotted that the solution to the last part was a direct consequence of the Bernstein inequality.

### C2.6 Introduction to Schemes

### C2.7 Category Theory

**Question 1** Most candidates produced relatively strong solutions, but a few recurring issues stood out. In part (a), almost everyone stated and proved the Yoneda Lemma correctly, but some did not describe how the Yoneda functor acts on morphisms. In part (b), some candidates forgot to explain what makes a cone universal, and a noticeable number conflated the answers for limits and colimits in (b)(ii), or forgot to provide a proof and a counterexample, respectively. Part (c)(i) was generally answered well, but many candidates misidentified the ring in (c)(ii) as either the zero ring or $\mathbb{Z}$. Most attempts on part (c)(iii) were correct, but some responses described only the cardinality of the image of $*$ and not the relevant $G$-action

**Question 2** The bookwork parts (a)(i) and (a)(ii) were answered well by almost all candidates, although some had difficulties checking the associativity axiom for monads arising from adjunctions. Most candidates also realised that part (a)(iii) was asking about non-monadic adjunctions. The true–false questions in part (b) were answered with varying degrees of accuracy across the different items. Most candidates gave correct solutions to (b)(i) and (b)(ii), which were bookwork, and answered (b)(iii) by either providing a counterexample or citing the Barr–Beck theorem. Most candidates struggled with the final part (b)(iv), which contained unseen material.

**Question 3** Only very few candidates attempted Question 3. Answers to part (a) were generally of high quality, while for part (b), the performance was more varied. Many candidates struggled with the bookwork, either misstating the definition of filtered colimits in part (b)(i) or omitting key steps in the proof required for part (b)(ii). The final parts (b)(iii) and (b)(iv) were handled well overall, although most answers would have benefited from a more detailed justification of the counterexample required in part (b)(iv).

### C3.1: Algebraic Topology

1.(a) was reasonably easy, and yet there was a surprising number of students that didn't manage to answer it. 1.(b) required familiarity with the universal coefficient theorem, and was a good test of the students understanding. Again, it was reasonably easy for those who understood that part of the course material. The fist part of 1.(c) was unexpectedly easy, while the second part of 1.(c) was maybe a bit too hard.

2.(a) I was rather disappointed by the student's performance in that question, which was supposed to be really easy: many students completely failed the understand the notion of homotopy equivalence between spaces (confusing it with the notion of homotopy between maps). 2.(b) relied of geometric intuition, and might have disadvantaged those students whose visualisation capacity is not as strong. 2.(c) was as hard as I had intended it to be, meaning hard but yet doable by the strong students.

3.(a) was fun to mark, and worked really well for most. 3.(b) was generally ok, though sometimes led to messy/confused answers. 3(c) required detailed understanding of how to justify the ring structure on cohomology. Most students fallen short of what I would have liked to see, so I ended up being lenient in my way of marking that part of the question. A small number of students were able to provide perfect answers, which made me happy.

### C3.2 Geometric Group Theory

**Question 1.** Almost all candidates attempted this question. A large majority forgot to mention, in the enumeration of the homomorphisms required in (a), (ii), that one should proceed by order of word length of the images of the generators.

In answer to question (c), (ii), many candidates attempted to prove first that the group satisfying the given assumption should be residually finite. This is not necessarily true, since, for instance, simple groups satisfy the given assumption as well.

**Question 2.** This question was attempted by about half of the candidates. Part (a) was well answered. In part (b), where the arguments required the use of the geometry of actions on simplicial trees, most answers were rather approximate and not well justified. A clear picture would have sufficed for (b), but even that was hard to find. Part (c) followed from a combination of (a) and (b), but only few candidates saw this.

**Question 3.**

Parts (a) and (c), (i) and (ii), were well answered and well explained. There were few attempts to answer part (b), even if it was a consequence of (a), (iii), and similar arguments have been seen both in lectures and in classes. The candidates attempting (c), (iii), noted, correctly, the obstruction to a quasi-isometry between the Euclidean plane and a hyperbolic space, but proceeded with a use of the Morse Lemma, instead of using the statement proved in (b), as suggested by the hint.

### C3.3: Differentiable Manifolds

**Question 1.** This was the easiest question and attempted by all students, with many good answers. In 1(b), the 'standard result' the question intended was the Künneth Theorem. My preference was for people to give the full statement, not only the isomorphism $\bigoplus_{i+j=k} H^i(X) \otimes H^j(Y) \cong H^k(X \times Y)$, but also the map $\alpha \otimes \beta \mapsto \Pi_X^*(\alpha) \cup \Pi_Y^*(\beta)$, which many people omitted. If people did not state this, but used it in (c) without explanation, I deducted a mark.

**Question 2.** Students found the first part of (b), 'Prove that ...', difficult, and noone answered this correctly. This was worth 6 marks, and I usually gave 3/6 for an attempt that showed some understanding. The rest of the question was easier. For example, in (d) you combine (b),(c) to get $\mathrm{vol}(\mathcal{S}^{n+1}) = \frac{2\pi}{n}\mathrm{vol}(\mathcal{S}^{n-1})$ and then give inductive formulae for $\mathrm{vol}(\mathcal{S}^n)$ for $n$ even or odd, which is 5 marks in the bag. Students should have been able to abandon the first part of 2(b), go on to answer the rest of the question, and score a high mark. But a significant proportion of students, after hitting a problem in 2(b), then went to pieces, and missed out on easy later marks. This happened more often for students on the MSc courses, who I guess are much less used to Oxford exam questions.

For the second part of 2(b), 'Deduce ...', a point often missed was that the deletions in the domain and target of $\Phi$ have measure zero, so do not change the integrals. Most people did not notice the deletions.

For 2(c), the idea is to apply Stokes' Theorem to the given form $\alpha$, and note that $\mathrm{d}\alpha$ is the volume form for $\mathcal{S}^1 \times B^n$, and the restriction of $\alpha$ to the boundary $\mathcal{S}^1 \times \mathcal{S}^{n-1}$ is $\frac{1}{n}$ times

the volume form of $\mathcal{S}^1 \times \mathcal{S}^{n-1}$. I had thought that having to state Stokes' Theorem earlier would be enough of a hint for this. However, many students instead tried to use the map $\Phi$ somehow, and got stuck.

**Question 3.** This was the least popular question. There was unfortunately a mistake in 3(b)(iii): the students were asked to prove that $\frac{\mathrm{d}^2}{\mathrm{d}t^2}(r^2 x_i(t)) = 0$, but in fact $\frac{\mathrm{d}^2}{\mathrm{d}t^2}(r^{-2} x_i(t)) = 0$. This also made it more difficult to answer 3(c) sensibly. This was dealt with in the marking by redistributing some of the marks for 3(b)(iii),(c) to earlier parts of the question, and giving the remaining marks as credit for a good attempt (for example, if the students assumed that $\frac{\mathrm{d}^2}{\mathrm{d}t^2}(r^2 x_i(t)) = 0$ and then correctly wrote down the general solution this implies in the second part of 3(b)(iii), I gave marks for this). Only 4 students answered 3(b)(ii) correctly, and so were in a position to attempt 3(b)(iii). None of them found the mistake.

### C3.4: Algebraic Geometry

Q2 and Q3 were the most popular questions. Q1 was mostly solved correctly by the very few candidates who tackled it. Very few candidates managed to do Q2 (d). The point here is that the complement of $U$ is the image of the set $S := \{(v_1, v_2) \in V \times V \mid f(v_1) = g(v_2)\} \cap \Delta_V$ in $W$. The set $S$ is closed by (b), and its image in $W$ is closed because $V \times V$ is complete. In Q3, many candidates were confused by (d) (i). The point here is that one can start with the graph of a representative of the rational map $\rho$; this gives a subset of $V \times V$, and in order to obtain the variety $V'$, one should take the Zariski closure of this graph. Most candidates did not think of considering this Zariski closure.

### C3.5: Lie Groups

**Question 1.** This was quite a popular question. Part (a) was standard bookwork and well done, with very few errors. Part (b) was usually done well, with only minor omissions in the argument on occasion. Part (c)(i) proved to be surprisingly challenging. Not all students spotted that it was a variation on the bookwork regarding covering maps, but those that did usually did well. Part (c)(ii), on the other hand, was done well by almost all students. Part (d), as expected, proved to be difficult for students. Several students spotted the links to covering maps as suggested by the question, but not all. Quite a few students spotted the idea for part (i), but failed to show the required condition on the differential. More students understood what was going on in part (ii), but often struggled to give a complete argument. The most common issue was not showing that the map needs to be surjective. Part (iii) proved to be the most difficult, with most students giving no or wrong answers, with only few seeing the correct answer. A common issue was that students did not know the centre of SO(4) and that they again did not consider the condition on the differential.

**Question 2.** This was the most popular question, with almost all students attempting it. Part (a) was standard bookwork and was usually done well, but marks for lost for not saying whether objects were left-invariant vector fields or tangent vectors. Part (b)(i) provoked a mixed response. Many students did it well or had the idea, but there were a surprising number which, in particular, did not show that the resulting vector field was left-invariant (i.e. they did not use the hint) or further errors (such as that $f$ takes the identity to the

identity). Part (b)(ii) was usually done well using (b)(i). Part (c)(i) was standard bookwork and usually done well, but there were a number of errors in the definition. Part (c)(ii) was typically done well using (b)(ii), with the common error being that students did not say that conjugation was a Lie group homomorphism. Students took a variety of approaches to (d)(i). Those who chose to take the approach suggested by the hint usually did well, only failing to argue sufficiently why it is enough to restrict to the Jordan normal forms. Other students tried to use the lack of square root argument with mixed success, with the typical issue being errors in computation or incorrect choices of matrices. Students who attempted part (d)(ii) often did well, with the only error being not to argue sufficiently why one can use the exponential map on $G$ by using (b)(ii) and the projection map.

**Question 3.** This was the least popular question, with relatively few attempts. Part (a) was standard bookwork but there were several common errors, including remembering the non-abelian condition and that the subgroups have to be connected in the definition of simple Lie group. As expected, part (b) provoked a mixed response. Most students had a fair attempt, but common errors included not saying that $\mathrm{dexp}_0 = \mathrm{id}$, and arguing why if $\sigma(X)$ preserves $W$ then $\exp(\sigma(X))$ preserves $W$. Part (c)(i) was usually done well, with the only errors being not to say that $\mathrm{dAd}_e = \mathrm{ad}$. Part (c)(ii) was done less well, with the common error not being to consider the issue of abelian Lie groups and Lie algebra. Determining the roots in (d)(i) was usually done well. For the second half of (d)(i), the usual issue was again not considering the non-abelian condition or correctly arguing why there are no non-trivial proper ideals in $\mathfrak{su}(2)$. Part (d)(ii) was typically not answered or not answered correctly. Several students spotted the link to quotients of products, but could not give a complete answer.

### C3.6: Modular Forms

Almost all parts of the questions were done very well by some candidate, and the paper should not have thrown up any great surprises for well-prepared ones.

Question 1: This was attempted by all but two candidates and done well by most. It is similar in content to the first question on other recent papers, and those candidates who prepared suitably will have scored highly.

Question 2: This was attempted by all but one candidate. It had the least amount of original material of any of the questions, and those with a solid understanding across the later parts of the course will have done well.

Question 3: Only taken by three candidates; involves some slightly tricky calculations, and a little insight was needed for the final subpart.

### C3.7: Elliptic Curves

**Question 1** (a) The first part of the question, finding a birational transformation to short Weierstrass form, attracted good answers. Many candidates then computed that the reduction of the curve mod 5 had 5 points, and deduced that the order of the rational torsion subgroup divided 5. Only a few candidates explained clearly why a non-identity rational point must then be a generator of the torsion subgroup (we are told the rank of the curve is zero, so any rational point is a torsion point).

(b) The bookwork part of the question on doubling points was answered well. Moving on to points of order 4, only a few candidates noted that if a rational point $P$ has order 4, then $2P = (0, 0)$ (since the only rational point of order 2 in this case is $(0, 0)$). Having made this observation, most of these candidates reformulated the problem in terms of looking for points whose tangent lines pass through $(0, 0)$, but finding the correct condition on  to ensure the existence of such points was found challenging.

**Question 2** (a) This bookwork part was answered well.

(b) Despite the hint suggesting how to set up the induction, very few candidates successfully carried out the induction step, which required adding one more monomial to cancel the first 'error term' in $h(g_{a,r}(X)) - g_{a,r}(h(X))$. Some tried (unsuccessfully) to come up with the required correction term by inspection, rather than solving for it. In the final part, very few candidates made the key observation that $h(X)$ defining a homomorphism implies that $[p](X)$ satisfies the property characterising $g_p(X)$ (as does $h(X)$).

**Question 3** (a) The bookwork related to 2-descent was answered well.

(b) Finding the rational torsion was found tricky overall, with some computational errors computing points on reductions mod primes leading to diffculties. Several candidates tried to apply the Lutz–Nagell theorem, which usually led to partial credit but not a complete solution — care was needed to correctly compute the discriminant of the curve (which is divisible by $3^2$ ), and some didn't consider the possibility that $|y| = 1$. A correct application of Lutz–Nagell leads to finding some rational points which are in fact nontorsion, so it then needs to be systematically verified that these points have infinite order (e.g. by doubling until you get a non-integral point).

Carrying out the 2-descent was done well by many candidates, as was producing an example of a point of infinite order. To justify that the point had infinite order, some observed that their point was not an element of the torsion subgroup (found in the previous subpart), whilst others observed that they had found $P$ with $2P$ non-integral, and hence of infinite order.

### C3.8 Analytic Number Theory

In general the exam seemed to work quite reasonably, with a good spread of marks.

**Q1.** Part (a) of this question was designed to be quite straightforward. They were done fairly well by the majority of candidates, though many lost 1 mark for not even stating (less still proving) that $D_{f*g} = D_f D_g$. Parts (b) and (c) were done moderately well by a reasonable number of candidates. There were only a few correct solutions to (d).

**Q2.** This question was done fairly well by a good proportion of candidates. There was one small oversight in the setting of (d), where the candidates were not supposed to address the question of whether $c$ is uniform in $a$. Only one candidate got slightly distracted by this.

**Q3.** This question was done reasonably well by a good proportion of candidates, though quite a few proved that the number of zeros in $|T - \gamma| \leq 1$ is $O(\log T)$ and thought they had proven that there are at least $\gg \log T$ in this interval, thus (falsely) trivialising part (f).

### C3.10: Additive and Combinatorial Number Theory

Question 1: This was a question on some of the most basic objects in the course. Most students picked up some marks on it and a few did very well, but there were also a surprisingly large number of students with fairly fundamental misconceptions, or unable to repeat the most basic estimates from the course.

Question 2: This was a question about entropic notions, which is a new part of the course. Students seem to have found it very di cult with some students unable to repeat even the most basic de nitions from this part of the course, which spanned 5-6 lectures. One or two students did fairly well on the question but there were no complete solutions.

Question 3: The first part of this question (worth 8 points) was really a gift to students, being simply a piece of bookwork. Whilst some students were then able to continue further into the question, applying the result of part (a), the extent to which this was so on average was rather disappointing.

### C3.11: Riemannian Geometry

**Question 1.** This question was about the Levi-Civita connection,and was the most popular,with almost all (13) candidates attempting it. Most candidates performed well and there were 8 marks in the 18-25 range. The bookwork was done well, and most candidates were able to see how parallel transport interacted with the covariant constant endomorphism.

The final part also had many successful attempts.

**Question 2.** This question,on totally geodesic submanifolds received 8 attempts of which 2 were in the 18-25 range.

Showing that the fixed point locus of an involution was totally geodesic proved more tricky than expected, but there were some very good solutions. The example of an involution on complex projective space was generally well done. Some candidates mistakenly thought that a totally geodesic submanifold of an Einstein manifold was Einstein.

**Question 3.** This question was the most sophisticated, dealing with Myers's theorem and applications. It received 7 attempts of which 2 were in the 18-25 range.

The bookwork, proving Myers's Theorem, was well done. Applying the theorem proved more challenging. Some candidates got confused about the behaviour of Ricci curvature under products and thought $S^1S^2$ admitted a metric of positive Ricci because $S^2$ did and $S^1$ was flat. However other candidates answered this correctly using Myers and the fact that the fundamental group was infinite.

The final part, using the idea of taking a product with a small sphere to get positive scalar curvature,proved quite difficult, but a couple of candidates did this very well, showing a good understanding of scalar curvature and fundamental group.

### C3.12: Low-Dimensional Topology and Knot Theory

*This section will be published in the Examiners' Report. Please include comments on each of the questions, summarising the standard of answers and noting any common difficulties*

*encountered by the candidates.*

Question 1 (8 attempts): This question tested knowledge of immersions, smooth manifolds, and Sard's theorem. The general level of solutions was mixed. Part (a) was generally fine. There was no complete solution for (b)(i) (even though describing an immersion of $T^2 \setminus \{x\}$ into $\mathbb{R}^2$ is not difficult) and only a few for (b)(ii). There were several essentially correct solutions for (b)(iii) and (b)(iv).

Question 2 (7 attempts): This question tested knowledge of knot theory, the knot group, the Wirtinger presentation, and primeness of knots. The general level of solutions was mixed. There were no serious issues with part (a). There were a few correct solutions for (b)(i), and some candidates got the key ideas right for (b)(ii) (studying the intersection set between the torus and the hypothetical connected sum sphere).

Question 3 (7 attempt): This question tested knowledge of 4-manifolds, Kirby diagrams, and the intersection form. The general level of solutions was mixed. Solutions for (a)(i) and (a)(ii) were typically correct. In (a)(iii), only a few candidates managed to show that the two forms are inequivalent, even though this was a Section A problem on Problem Sheet 4. There were several correct solutions for (b)(i), while a few for (b)(ii) and (b)(iii).

### C4.1: Further Functional Analysis

Question 1. Overall, bookwook was answered very well. In showing completeness of the quotient space in part (a), several students tried to take a Cauchy sequence in the quotient space $X/Z$ directly, but this does not yield a Cauchy sequence in $X$: take any divergent sequence (in $X$) with elements from $Z$. Virtually all students had a good understanding of the canonical quotient operator. All students correctly identified the annihilator (in $X$); virtually all students got the easy direction (kernel belongs to the annihilator); most students correctly applied the Hahn-Banach theorem to obtain a contradiction for the more difficult direction, though several students left it blank despite the *use the Hahn-Banach extension theorem* suggestion provided. In part (b), all students correctly identified the annihilator (now in $X^*$), though several confused where this annihilator lives (when identifying $\{0\}^\circ$, for example: it is the dual $X^*$, not $X$ itself!) and whether it is a set or otherwise (several wrote $X^\circ = 0$). Most students showed that $Z^\circ$ is closed, but several skipped showing that it is a subspace, even briefly. Virtually all students used continuity and linearity with density to conclude $Z^\circ = \{0\}$; the reverse direction is most easily shown using Hahn-Banach again, though some students used that the annihilator of the annihilator is the closure of the set, which was also acceptable. Students struggled with the last part of (b), which is likely due to how brief the section is on complemented subspaces in the course notes. Some students argued using the closed range theorem, which was acceptable, but all details (e.g., the hypotheses required to apply it, like mentioning that $W$ is closed, and the conclusion used). In part (c), virtually all students correctly stated Goldstine's theorem, though several wrote "dense" rather than "weak* dense". Otherwise, most students struggled with the second part. Many students correctly identified the correct answer to be *yes*, but most did not provide justification. Several students correctly identified lower semi-continuity as a key element, even if not written using this language, and were able to reduce the proof for $S_X$ to the original statement for $B_X$.

Question 2. Again, bookwork was generally answered very well. All students correctly identified the dual and the bidual, and understood the difference between the algebraic and topological senses. Most students correctly reconstructed the contradiction argument for separability of a space via separability of its dual. Virtually all students then provided a brief justification that $\ell^1$ is *not* reflexive. For part (b), students had no issue with the dual operator and compactness of an operator. Most students argued correctly (using total boundedness or otherwise) that finite-rank operators are compact by looking at the dimension of the range. Part ii) was generally answered well, though several students used inequality rather than strict inequality, which is essential. Part iii) was also generally answered well, with a large majority of students recreating the diagonal sequence argument. Part (c) was more varied, with virtually all students correctly identifying compactness of the ball iff finite dimensional, but most struggled with the second part. The easy direction was okay, though many students did not write what the operators $S$ and $T$ are after stating that the identity is compact in finite dimensional spaces. Some students used the hint, while others appealed to a Neumann series argument, which was also acceptable. For part (d), most students correctly identified this as Schauder's theorem, and were able to apply the theorem of Arzela-Ascoli as done in the course notes. Some students were confused by the last part of this problem: the point is simply that the dual is always complete, which is where we find our Cauchy sequence (it was not enough to say "we did not need completeness of $X$ or $Y$ in the proof", for example, as this does not indicate an understanding of where completeness was used and not used). For the second part, most students had the correct idea: using the hint, we can string together these isomorphisms. However, the point is to then justify this step, either by density or by constructing an isometric isomorphism. Students struggled with the last question in general. Several students correctly identified an operator as suggested in the hint (choosing something like $\alpha_n = 1/n$ or similar), and most who did were able to show that $T^*$ is compact. Most were unable to prove that $T^*$ was not compact.

Question 3. Virtually every student chose not to attempt question 3 at all. The very few that did correctly identified part (a) as Riesz's lemma, correctly reproducing the proof, and had a reasonable attempt at the second and third parts of (a). Otherwise, (b) and (c) were not attempted. This is likely because question 3 has less obvious "bookwork" steps directly from the course material, though many ideas are embedded in these proofs.

Summary: Overall, students did very well with the bookwork portions. A large majority of students have a good grasp of the core material, especially definitions and applying several versions of the Hahn-Banach theorems. Students were generally able to reproduce proofs from the course material well. Students generally struggled with new material, with students either "getting it" or not, with little in between.

### C4.3: Functional Analytic Methods for PDEs

17 students took the exam of the course.

**Question 1** was about the first part of the course regarding regularization of $L^p$ functions by convolution, some properties of the convolution, Mazur's Lemma and a couple of appli-

cations of the latter.
Part 1(a) was bookwork and was done well by most students. Item 1(a)(iii) of the script had a small typo. The correct statement is either that $\|f \star g\|_{L^1} \leq \|f\|_{L^1}\|g\|_{L^1}$ for all $f, g \in L^1(\mathbb{R}^n)$, or that $\|f \star g\|_{L^1} = \|f\|_{L^1}\|g\|_{L^1}$ for all $f, g \in L^1(\mathbb{R}^n)$ with $f \geq 0, g \geq 0$ almost-everywhere with respect to the $n$-dimensional Lebesgue measure on $\mathbb{R}^n$. Both the correct options have been given full marks, and partially correct answers have been given intermediate marks.
Part 1(b) was about some standard properties of the mollification and was done well by most students. A common error was trying to use dominated convergence theorem in proving 1(b)(ii) without realising that the convolution kernel $\rho_\epsilon$ blows up in $L^\infty$ norm as $\epsilon \to 0$.
Part 1(c) was about Mazur's Lemma (covered with proof in Part B Functional Analysis, statement recalled in C4.3 with some applications) and a couple of applications. 1(c)(i) and (ii) were standard and were done well by most students. 1(c)(iii) aimed to prove that a single point has null capacity in $\mathbb{R}^n$, $n \geq 2$. Some students were able to complete the proof for $n \geq 3$, but no-one was able to complete the proof in the (more challenging) case $n = 2$, which needed a less standard application of Mazur's Lemma.

The question was chosen by 14 students. 4 solutions were at distinction level, 4 at merit level, 2 at an intermediate level, 2 just above sufficiency, 2 below pass (with serious gaps in definitions and basic results).

**Question 2** was about weak derivates, Sobolev spaces, and a guided proof of Hardy's inequality.
Parts 2(a)(i)-(ii) and 2(b)(i) were bookwork and were done perfectly (or almost perfectly) by everyone.
Part 2(a)(iii) was standard and was done well by most students. A common gap was trying to use dominated convergence theorem, without stating clearly the dominating function.
Part 2(b)(ii) was standard but not completely straightforward and was done well by around half of the students.
Parts 2(c)(i) and 2c(ii) had a computational component and were done generally well by those attempting. The fact that the final result was claimed in the statement of the exercise made possible to guess the right choice of parameters, without a thorough discussion. For this reason, from the model solutions, we moved one point from 2(c)(ii) to 2(c)(iii), since the latter was more conceptual.
Part 2c(iii) consisted in extending an operator by density and it required a good level of familiarity with the methods of the course. It has been done perfectly by a couple students, and quite well by a couple more.

The question was chosen by 11 students. 3 solutions were at distinction level (1 of which almost perfect), 1 at merit (upper intermediate) level, 3 at an intermediate level, 2 just above sufficiency, 2 below pass (with serious gaps in definitions and basic results).

**Question 3** was about Gagliardo-Niremberg-Sobolev inequality, and applying the functional analytic tools developed in the course to prove existence of solutions to PDEs. The exercise was more advanced, and overall was attempted by students with a better preparation.
3(a) and 3(b)(i) were bookwork with a small variation (in part (ii)) and were done perfectly (or almost perfectly) by everyone.

3(b)(ii) was based on applying Riesz's representation theorem (after using ellipticity and Friedrich's inequality to prove the bilinear form associated to the PDE was equivalent to the scalar product in $H_0^1$) to show existence and uniqueness of solutions to a linear elliptic PDE. This part was done well by most students.
3(c) was about proving existence and uniqueness of a minimizer to a strictly convex functional and interpreting it as the unique solution to a non-linear elliptic PDE. This was a variation of examples from the course (and problem sheets) and required a good familiarity with the course material. The standard parts were done well by most students. Several solutions tried to apply the statement of Tonelli's theorem, but missed to prove the lower semi-continuity of the functional. Instead of applying the statement of Tonelli's theorem it was actually more efficient to argue along its proof, and simplify some steps thanks to Rellich-Kondrachov compact embedding theorem.

The question was chosen by 9 students. 6 solutions were at distinction level (1 almost perfect) and 3 at an intermediate level.

### C4.6 Fixed Point Methods for Nonlinear PDEs

*This section will be published in the Examiners' Report. Please include comments on each of the questions, summarising the standard of answers and noting any common difficulties encountered by the candidates.*

**Q1.** All candidates attempted this question. Most of part (a) was done relatively well although many candidates weren't careful enough about the satisfaction of the boundary condition for the approximating maps in proving (iii), and a couple of candidates complicated (v) substantially by doing another approximation procedure (instead of appealing to (iii) and (iv)). In part (b), (i) and (ii) were done mostly well with minor exceptions. In part (b)(iii), most candidates saw the relevant ingredients but many didn't manage to find the right map or the right ball to apply Brouwer's fixed point theorem.

**Q2.** About half the candidates attempted this question. The bookwork parts were done relatively well. Part (a)(iii) appears somewhat challenging with many candidates incorrectly claimed that $\mathrm{Id}-F$ maps $\bar{B}$ into $\bar{B}$. In part (b)(ii), the identification of $\bar{r}$ was probematic for a few candidates, but those who got the right exponent managed this leg well. Part (c)(ii) was handled reasonably well with either Schauder's fixed point theorem or Leray-Schauder's theorem. A couple of different strategies were proposed for (c)(iii), but the easiest solution is to consider $-\Delta + 1$ instead of $-\Delta$.

**Q3.** About half the candidates attempted this question. The bookwork parts were done mostly well with minor exception in (a)(ii). Part (b)(i) was handled well by most candidates. The coercivity in (b)(ii) proved challenging for all candidates due tot a failure to use the helpful term $u^{p-1}$ in $A(u)$. All candidates attempted to prove that $A$ maps bounded sets into bounded sets using Holder's inequality, but a few candidates messed up the algebra somewhere. In (c), most candidates had the right idea but struggled to prove weak lower semi-continuity, where one needs to split $A$ into a monotone part and a non-monotone but compact part.

**C4.9: Optimal Transport & Partial Differential Equations**

**Question 1:** Parts (a) and (c) were straightforward and were in general answered quite well. Some candidates found part (b) more challenging. Part (d) was mostly well done too. Most of the students understood how to relate the optimal transport map with the radial transport map, but the full proof seem chanllenging for most of the students that tried it. This probably was the most complicated question for students this year.

**Question 2:** Parts (a) and (b) were reasonably straightforward bookwork. Candidates found part (c) and (d) not too diffcult either but most of the students struggle to argue well in part (e) that was the new rider part.

**Question 3:** Parts (a) and (b) were generally well answered, although some students found diffculties in writing the good definition of weak solutions for the PDE. Part (c) was satisfactorily answered by most of the candidates. Some of them did more work than needed. Actually, instead of using directly the course content, they reproved certain parts making their answers longer than needed.

**Summary:** This year's exam seems to have worked quite well to distinguish between the excellent and the very good students in the course. Overall, I am quite happy with the spread of marks, showing the high level of achievement of part of the cohort. The spread of knowledge and work on the material over the cohort seems to be consistent with previous years. Although the first question seemed to have been harder for the students than I expected that is why my scaling raw marks values have been accordingly adapted.

### C5.2: Elasticity and Plasticity

**Question 1:** Unfortunately there was an error in the examination paper in Question 1 in which the Navier equation should have a factor of $\lambda + \mu$ rather than $\lambda + 2\mu$. While the extraneous factor of two does not change the methodology being examined, it made parts (a) and (b)(i) harder than intended because it affects one of the wavespeeds in part (a) and hence the coefficients in part (b)(i). To mitigate against the impact of the error, minor errors and partial progress in parts (a) and (b)(i) were marked generously using equitable marking schemes agreed in advance with the Chair. The bookwork in part (a) was well done overall, with nearly all of the candidates deriving the intended wavespeeds with only minor calculation errors or minor defects in the argument. Part (b)(i) caused more difficulties for those candidates that did not make effective use of the results of part (a) and the majority of candidates got bogged down in the algebra on the way to the dispersion relation. The tail in part (b)(ii) was unaffected by the error and received only a small number of attempts. The error in the question was corrected for the past paper archive.

**Question 2:** The bookwork in part (a) was reasonably well done overall though there was some confusion about the direction of the reaction force exerted by the obstacle on the string and many candidates did not justify the integration by parts nor exploit correctly the identity in part (a)(iii). Part (b)(i) was extremely well done, though only a few candidates spotted how to use it to write down the answer to part (b)(ii). There were a few good attempts at part (c), but the majority faltered in writing down the correct version of Newton's second law for the obstacle

**Question 3:** The book work in part (a) and the similar work in part (b) were well done by all candidates, with many of them scoring full marks. In part (c) the majority derived correctly the solution in the elastic region, but ran into difficulties applying the associated flow rule in the plastic region in the tail of the question. While the dilatation is spatially uniform in the elastic region, it is independent of time in the plastic region, and hence fixed at the value laid down by the advancing elastic/plastic free boundary. The tail was a bit too hard as set and with hindsight I should have given as a hint that the dilatation should be assumed to be continuous at the elastic/plastic free boundary as in the example in the online notes. This modification to the question was made for the past paper archive.

### C5.5: Perturbation Methods

**Question 1.** Part (a) was well done, whereas most candidates struggled with (b) as higher order terms in the expansion needed to be taken.

**Question 2.** Part (a) was mostly answered well, though some candidates lost marks for not stating the distinguished limits. Part (b) was mostly answered relatively well. In part (c) many candidates lost marks by not spotting the conditions on the solutions at the various orders.

**Question 3.** Most candidates provided good solutions to part (a) and part (b) and many, sensibly used the results from (b) to make progress in the early parts of (c). The majority of marks were lost in computing the conditions on $a$ in part (c).

## C5.6: Applied Complex Variables

### Question 1

This was the most popular question, attempted by 88% of candidates. Most answers were good. In part (a) some points were lost by not justifying the sketches of the regions. In particular a number of candidates did not state that $v = 0$ on $AB$ and $DE$, or that $|w'| = 1$ on $BC$. There was some confusion over the labelling of the points. In part (d) the step some candidates found tricky was using double-angle formulae to convert $\cos\theta \tan\theta/2$ to $\sin\theta - \tan\theta/2$.

### Question 2

This was the least popular question, attempted by 40% of candidates. Parts (a) and (b) were handled very well, with some good answers. Part (c) was also handled well, despite being less familiar. Surprisingly, there were no good answers to part (d). No candidate thought to use the results of part (b) to solve the singular integral equation, despite many casting it in that form using the hint.

### Question 3

This was a popular question, attempted by 76% of candidates, and had the highest average mark. The material was challenging, but the format of the question was familiar, and there were some very good answers. The most common difficulty was candidates failing to realise that

$$\int_{-\infty}^{\infty} \mathrm{e}^{-\mathrm{i}kx} f(k)\,\mathrm{d}k$$

is $2\pi$ times the inverse Fourier transform of $f$, so that taking the Fourier transform gives simply $2\pi f$.

## C5.7: Topics in Fluid Mechanics

Question 1 was very popular, with all candidates starting the question and essentially all making a concerted attempt. It produced a wide spread of marks and, overall, the question was implemented well. While not in the markscheme, the possibility of a singularity near $\theta = 0$ escaped the attention of most candidates, though never to the detriment of progress in the question.

All candidates made a concerted attempt at Question 2 and the question was answered well on the whole. The final stage of the final part of the question was hindered by an incomplete specification of the physics required to simplify the boundary conditions. Consequently, the marking took this into account for those candidates that reached this stage of the question.

Question 3 was not popular, and only ever considered when Question 1 was attempted without much progress. This may explain the low marks achieved for this question in the very small number of attempts

## C5.11: Mathematical Geoscience

### Question 1

Question 1 - this question was reasonably popular and was done quite well, especially the physical interpretation. The last part was quite challenging, and a number of candidates drew various speculative curves based on things that had been seen in lectures without carefully working out the bifurcation diagram. But a few candidates gave almost complete answers. It was surprising how many candidates didn't seem to know what a graph of $\tanh(x)$ looks like, or seemed to have to go to great lengths to work this out, and this hampered progress from early on for some.

Question 2 - this was the most popular question, but the construction of the characteristic diagrams proved to be quite challenging. The first part was done fine by most candidates. A common difficulty in part (b) was not being clear about which initial data were being used for the characteristics at which stage - the answers often became a sprawling assortment of characteristics that applied in overlapping regions. This was not helped by the fact that many people drew their characteristic diagram the wrong way round, or kept flipping their axes. Quite a few people made more progress with part (c), realising that the characteristics simply split into those on which $c = 0$ and those on which $c = 1$, but most candidates then had difficulty tracing those characteristics clear due to the incomplete or confused answers for $h$ in part (b).

Question 3 - this question was slightly less popular but was generally done quite well by those that attempted it. The first part was found fairly straightforward. Solving the quasi steady model in part (b) was also generally done well, and it was pleasing to see many candidates utilising the solution to correctly interpret the timing of ice thickness maxima/minima in terms of seasons (Spring/Autumn). Part (c) was also generally done well, with many candidates realising the utility of characteristics in a setting they hadn't previously seen. The last part about the net production was however harder, with many candidates thinking about the time-integral of $h$ itself rather than the growth rate.

## C5.12: Mathematical Physiology

**Question 1** Q1 This question was attempted by all candidates. Essentially all candidates gave sound answers for the bookwork on part (a) and most made progress in the initial stages of part (b). The accumulation of calculational errors and misunderstandings from the earlier parts made the final stages of part (b) difficult for many candidates. Even those who had essentially fully correct answers by this stage had difficulty in assessing the global behaviour in the phase plane. Nonetheless, a small number of candidates made substantial progress in these late stages of the question.

### Question 2

This was the least popular question, attempted by only 24% of candidates. There were no good solutions. No candidate managed to get part (e) right. Very few made it to (e)(ii), but those that did failed to realise they could linearise the solution they found in (b) about the solution they found in (d)(ii).

**Question 3** This was a popular question, attempted by 80% of candidates. There was a wide spread of marks, but on the whole the answers were disappointing. For part (b) many

candidates argued that if $p$ increases $\mathrm{d}f/\mathrm{d}x$ decreases without making the next step that since $\mathrm{d}f/\mathrm{d}x = 0$ in steady state, the derivative is then negative, so that $f$ now decreases.

The delay in the equations caused much confusion even for the steady state in part (c), but especially for the linear stability in part (d). Only a handful of candidates managed to get the characteristic equation in (d), and very few made much progress on (e).

### C6.1: Numerical Linear Algebra

Q1 was quite popoular. Some candidates worked out the least-squares solution via the QR factorisation. This is fine, but some then failed to note that $A = QR$ yields $A = Q$ and $R = I$. (iii) Many used insufficient arguments to justify the $V$ factor in the SVD of $A_1$ and $A_2$ are the same. Q1(b) Stability appears to always be challenging for many. The key point here is to see that a small backward error would immediately imply small error in the singular values, by Weyl's theorem. This by contrast does not hold for eigenvalues of nonsymmetric matrices (b-iv).

Q2 was the most popular question. The final question was essentially bookwork (given in lecture notes and lecture), but unsurprisingly not many got to the end. Part (b-iv) also seemed challenging to most. There was a typo in the definition of the Frobenius norm $\|A\|_F = \sqrt{\sum_{i,j} |A_{i,j}|^2}$ in Q2, where the square root was missing. Almost all candidates used the correct definition in their scripts. Where a few candidates used the incorrect/printed definition $\|A\|_F = \sum_{i,j} |A_{i,j}|^2$, full mark was awarded if the argument was otherwise correct.

Q3 was somewhat less popular, but was still attempted by about half of the candidates, and had a slightly higher average mark. (iii) was done with a good success rate, which was a pleasant surprise. Perhaps (b-iv) was not difficult for a final problem, and the other problems are bookwork or a slight variant.

### C6.2: Continuous Optimisation

### C6.4: Finite Element Methods for Partial Differential Equations

**Q1.** The question was concerned with the finite element approximation of a mixed Dirichlet–Neumann boundary-value problem for a linear second-order elliptic partial differential equation of the form $-\mathrm{div}(A\nabla u) + u = f$ on the domain $\Omega := (0,1)^n \subset \mathbb{R}^n$. All candidates attempted the question; several were confused about the correct choice of function space for weak solutions to the boundary-value problem and lost marks for erroneously choosing $H^1_0(\Omega)$ instead of $H^1_{\Gamma_\mathrm{D}}(\Omega) := \{v \in H^1(\Omega) : v|_{\Gamma_\mathrm{D}} = 0\}$.

**Q2.** This, too, was a popular question: it was concerned with the derivation of the Euler–Lagrange equation of the convex functional

$$v \in W^{1,p}(\Omega) \mapsto J(v) := \frac{1}{p}\|v\|^p_{W^{1,p}(\Omega)} - \int_\Omega f(x)v(x)\,\mathrm{d}x,$$

with $f \in L^\infty(\Omega)$ and $p \in (1,\infty)$, and the finite approximation of this in the case of $p = 2$. All candidates who took the exam attempted the question and there were many good answers.

**Q3.** The question was concerned with the formulation of a mixed finite element method for a second-order elliptic boundary-value problem with a zero normal flux boundary condition. There were only two attempts, and neither candidate got much beyond part (a) of the question.

### C6.5: Theories of Deep Learning

The students wrote excellent reports reviewing various aspects of deep learning with a focus on the theoretical aspects of this topic. The reports typically included wonderful literature reviews as well as small scale original experiments contrasting ideas and techniques. Reports were well generally well composed with a good academic tone, structured well with a good balance of stating results and discussing their merits, and generally a pleasure to read.

### C7.4: Introduction to Quantum Information

**Question 1** was generally well answered by those who chose it, and the mistakes seemed to be very similar amongst most students. Quite a few students struggled with the very last part of part (b), and claimed that the rank was always 2. In part (c), many students only got 2 marks due to only showing one direction of the if and only if statement, and there were quite a lot of circular arguments. Part (d) was generally very well done. Parts (e) and (f) seemed to be the source of some confusion (perhaps the wording could have been improved), with many students not actually writing down what the constituent states $|\psi_1\rangle$ and $|\psi_2\rangle$ were but instead merely noting that the output states were indeed separable. However, quite a few students successfully gave the answer to part (f) in part (e) anyway. Finally, part (g) was surprisingly well answered by many students, with almost all who attempted it gaining at least 3 or 4 points.

**Question 2** contained a typo in the second equation defining decoherence, but it could at most have affected parts (b) and (c). The remainder of the question could still be answered using the expression given in part (c). Most students were able to self-correct the typo, as this part of the question was very similar to material covered in class. (The irony is that this question focuses on error correction.)

Out of the 20 students who attempted Question 2, six were, to varying degrees, affected by the typo, as reflected in their scripts. For these students, parts (b) and (c) were marked based on the method used, which is independent of the typo. This turned out to be a sufficient adjustment, counteracting the effect of the typo and enabling affected candidates to perform similarly on Q2 to those who were unaffected. Their performance on Q2 was also on par with that on the other questions.

Otherwise, almost all students answered part (a) correctly. Parts (d), (f), and (g) were also generally well done. However, many students struggled with part (e), failing even to complete the first step — namely, substituting the given expression for $v$ into the probability of obtaining $(0, 0)$ — and consequently scored almost no marks. A more explicit statement that $t$ represents the interaction time might have helped. On the other hand, those students (around seven) who had a good physical understanding of the context were able to score nearly full marks on these questions.

Statistically, this question does not stand out; the average marks for Q2 are comparable to those of the other two questions.

**Question 3** was of medium difficulty. Most students had no problems with parts (a), (c), (d), and (f). Part (b), which required a geometric visualisation of the three elementary operations on a single qubit, was generally well answered, although describing the Hadamard gate in geometric terms proved surprisingly challenging for some. A significant number of students struggled with part (e). Those who answered correctly often did so in a rather pedestrian way, by exhaustively considering the four possible ways of encrypting and decrypting the state. The final two parts, (g) and (h), proved difficult for all who attempted them. In part (g), most students recognised that the $T$ gate is not a Clifford gate, but only a few were able to explain the origin of the 50% success rate. No one provided a fully satisfactory answer to part (h), which required the observation that in the first round, the $T$ and $T^\dagger$ gates are equally likely to be implemented, and that the $T$ gate can be recovered from the $T^\dagger$ gate by composing it with the $S$ gate (a Clifford gate). The textbook-style parts of this question were, by and large, well done. However, the remaining parts, which required a more creative approach, proved challenging, with only a handful of students offering partial answers.

### C7.5: General Relativity I

#### Question 1

This question was attempted by every student and was the question most similar to previous exam questions.

In general people picked up full marks for parts a(i) and a(ii) being simple definitions. On the whole a(iii) was also very well done, with only a few students not getting maximum points for it. Part a(iv) was mostly well done too however a handful of students did not explain why they could strip off the vector field. Some also confused the transformation of a vector field here.

Part b was not as well done and weaker students lost their marks here generally. For b(i) number of students did not give the correct definition of normal coordinates, either writing only that the connection coefficients vanished or only that the metric looked like flat space. Some did not explain that the normal coordinates were valid at a point making it seem

that they could be valid everywhere. b(ii) generally separated the stronger from the weaker students. If the students managed to correctly explain normal coordinates they typically picked up most marks for this part. However a large number of students did not state that they had worked in normal coordinates but because this is a tensorial equation it is valid in all coordinate systems and therefore holds in general, this lost them a mark. For the weaker students they did not realise that derivatives of the Christoffel symbols were non-zero and so ended up with the left-hand side vanishing. They typically justified this by saying that the right-hand side also vanishes, however neither of these statements is true. Most who attempted part b(iii) got most of the marks on offer. Some students used normal coordinates to erroneously argue that the Riemann tensor terms vanished and hence it was obvious.

**Question 2**

The second most popular question and the one with the highest average mark. Most received all four marks for stating Birkhoff's theorem correctly and explaining that it means that the Schwarzschild solution is the solution outside any spherically symmetric distribution of matter (in a vacuum). Those that lost marks here left out one or more of the adjectives in Birkhoff's theorem.

Part b was almost universally well done. Almost everyone correctly obtained the three conserved quantities for part b(i). For b(ii) those that lost marks did not explain why one could choose the correct initial conditions or just stated $\theta = \pi/2$. Most did well here. The strictly illegal method of differentiating the Lagrangian was the most popular method and was accepted even without comment about issues at $\dot{r} = 0$. Some used this method but did not differentiate both sides with respect to $\tau$ and therefore ended up with $\dot{r}$ terms remaining.

In general most said $\ddot{r} = 0$ for a circular orbit. Surprisingly, having stated this, a large number of students then went back to looking at the Lagrangian, finding an expression for $\dot{r}^2 + V_{\text{eff}} = E^2$ and then finding the minima of $V_{\text{eff}}$. This is of course correct but meant they spent extra time rather than using part b(iii). If the result for $h(r)$ was incorrect before propagated errors because of this were ignored and marks were awarded for correctly finding the roots for their $h(r)$. For c(ii) those that went back to computing $\dot{r}^2$ suffered a bit because it was not obvious how to simplify things as easily, though most did in the end. Students lost marks because they only looked at the stable orbit, neglecting to mention anything about the other orbit. Others instead correctly gave the conditions for when the orbits would be stable and unstable but then did not make contact with the location of the orbits computed in c(i).

This was the hardest part of the question. If c(iii) was done correctly then most did c(iv) correctly too. One common issue was trying to go from the period of the orbit to the proper time for the period by using the constrained Lagrangian, setting $\dot{\phi} = 0$ to get $\mathrm{d}t$ in terms of $\mathrm{d}\tau$. This was wrong since $\mathrm{d}\phi \neq 0$. Others gave a result still in terms of the Energy or Angular momentum when the question asked for it in terms of the root and the mass $M$.

**Question 3**

The least popular question though still a healthy number of attempts. Parts a (i) and a(ii) were almost always done satisfactorily, though a few forgot the definition of an integral curve. a (iii) when attempted was done well. Some just stated that the Lie derivative measures the failure of the flow to commute rather than doing any computations.

Those that persevered through the part a typically did well with part b. b (i) was a free mark and part b (ii) though a bit fiddly was done well. For part b(iii) the use of integration by parts was done poorly. Given that there is a connection rather than a partial derivative a little more care is needed to use integration by parts and this was almost always ignored losing a mark.

### C7.6: General Relativity II

**Question 1** This was only attempted by one student who did very well in the question. Despite seeming intimidating the question is actually quite simple. Part a is bookwork which was given in class. Though part b looks like it gets nasty as you progress and the polarisation in the hint looks intimidating there is only one term that contributes and so it all truncates.

Part C was well done with full marks obtained.

**Question 2** Everyone but 1 student did this question. a (i) was correctly done by everyone, but interpreting the horizons was not as well done as expected. For a (iii) many got the idea behind the change of coordinates but could not do the integral. For the most part the Killing horizon was defined correctly though a few students did not say *null*. For part a (v) some students only showed it was null and did not discuss it being a Killing vector. Others did not use the change of coordinates they had found in a(iii) so they obtained a dual vector which was not Killing and ended up confused.

b (i) was done well by almost everyone. b(ii) was on the whole correct however a small minority set $\mathcal{L} = -1$ and then solved rather than $g_{\mu v}\dot{x}^{\mu}\dot{x}^{v} = 1$. This lead to an additional term in their result. b (iii) had been seen in a similar form in tutorials however it was rather polarising. Some students were able to get full marks, others got stuck in the logic of the question and scored only 1 mark.

**Question 3** The most popular question. On the whole people picked up the most points from part a. For a (iii) you were guided in how to do it by your computations in a (ii) but some used another method which was fine but took longer.

The metric is very similar to Schwarzschild with $t$ and $r$ interchanged. De- spite this, it caused students quite a few problems. For b (i) a few students fell for making $r \in (0, \infty)$ rather than the correct range $r \in (-\infty, \infty)$. The bookwork part in b(ii) was well done. A few struggled to perform the change of coordinates in b (iii). Rather than defining a new time coordinate they tried to define a new radial coordinate like a tortoise coordinate. They ended up trying to solve an impossible equation. b(iv) was done well by stronger students and middle range students picked up a few marks. The Kruskal like coordinates are the same as the ones from Schwarzschild studied extensively in lectures one just needs to replace $t$ and $r$. There were a large number of different Penrose diagrams produced. A few managed to get the correct answer, some even noted that it was a 90 degree rotation of Schwarzschild. If the students did the steps correctly, or outlined the correct steps, despite not drawing the correct diagram they still picked up 2 or 3 marks. If the diagram was wrong but labelled correctly students got 1 mark for this.

### C7.7: Random Matrix Theory

Question 1 was well done in general and attempted by most students. Part (a) involved mostly bookwork on the GOE ensemble and Wick's theorem. For Part (b) most students got (i) right using the symmetry. For (ii), few saw that Wick's can be applied easily and that the dominant term come from the non-crossing partitions. For (c), many students were too brief in explaining that the matrix is non-negative de nite. For the moments, few students were able to compute the third moments using Wick's and the fact that non-crossing partitions are dominant. Some used the general formula from the Marchenko-Pastur which is acceptable but not as straightforward.

Question 2 was also attempted by most students. It involves the particular case of a 3 x 3 GUE matrix. Part (a) involves bookwork. To derive the orthogonal polynomials, some students relied on the general formula which was not necessary as they can be computed from scratch. This was accept- able. Part (b) was well done but few students mentioned that only the first two correlation functions are non-trivial as there are 3 eigenvalues. For part c, (i) was easy bookwork. Very few students were able to get full marks on part (ii) even though the function was given in Part (ii).

Question 3 was attempted by few students (around 10). It involves Dyson Brownian which was covered in lectures and in revisions. From those who attempted, around half got the SDE right in (b). Only one student under- stood that the convergence in part c is related to the stationary uniform distribution discussed in (a). Only two students got full marks in (d) even though it was a question that appeared in problem sheets

### C8.1: Stochastic Differential Equations

**Q1:** Relatively few candidates addressed Q1. The proper discussion of the domain of the generator was not addressed carefully by most of the candidates. Even those how scored high were a bit sloppy on this point. In part (a) some candidates inferred heuristically from the form of the MP the applicability of Levy's theorem without justification. In part (b) the question of existence and uniqueness of solutions of martingale problems was also treaded shallowly, most of the times assuming that the process already satisfies an SDE without justification. Those who addressed part (c) usually correctly invoke DDS theorem. Some other attempts at this question without DDS were quite heuristic and not correct.

**Q2:** Some of the candidates failed the Gaussian computation in part (a), even trying to use stochastic calculus to solve it. Most managed the rest of this part. In part (b) most candidates tried to use Novikov to check for the martingale property, but were not able to justify correctly the finiteness of the expectation. Sometimes there were error of sign in the exponent and then they tried to invoke previous computations in part (a) to conclude. Nobody noticed that the martingale was pathwise bounded due to its explicit form. The derivation of the form of $B$ given $W$ was managed by few. Somebody tried a general function of the form $B_t = f(t, W_t)$ but of course this would not work. Those who implemented the idea of integrating factor managed to derive the result. In part (c) most applied correctly Girsanov, but failed to identify the law of $B$ under the new measure due to a lack of understanding of the second point of part (b). The final question was addressed correctly by those who managed to answer correctly to the previous two points, some candidates also outlined a strategy but could not verify the final formula.

**Q3:** This was the question which most of the candidates attempted quite successfully. Part (a) was easy and most of the candidates got full marks. Part (b) was composed of four points. Most of the candidates managed to conclude correctly the martingale property of the stopped process. Very few found out the key inequality linking the second moment of the martingale with the expectation of the stopping time. Given this, most of the candidates how attempted the third point managed to identify, at least heuristically, the formula for the hitting probability (which corresponds to the Brownian one). Those who correctly answered the third point also managed to conclude the fourth. Sometime there were some computation errors due to the direct application of the Brownian formula, which miss the exponential dependence. Part (c) required to recognize that the form of the coefficients of the SDE do not matters for the above derivations. Many candidates saw this intuitively, but lacked to write down precisely why all the previous steps worked as they did.

### C8.2: Stochastic Analysis and PDEs

The vast majority of candidates choose to do question 2 and 3. Probably the main reason was that Question 1 was an extension of results from time-homogenous to time-inhomogenuous Markov processes which looked probably intimtating but it was probably the easiest and straightforward one among the three questions.

Question 2: Most candidates that did this question managed part a) and nearly all of these also made good progress on 2b. A common hurdle was to realize that the differential equation involved were very easy to solve explicitly and there was no need to involve general ODE theory. Most candidates that made progress on 2b also managed to progress on 2c(i). For 2c(ii), there was a lot variance in approaches and few candidates managed to give a correct argument together with an explicit formula

Question 3: Most managed 3a; suprisingly many didn't manage to correctly spell out the definition of a uniformly elliptic second order differential operator (the positive semi-definiteness of the Hessian was usually forgotten). 3b was fairly straightforward, however, many candidates lost points by not being careful about stating the type of convergence and the function space. Very few managed to complete 3c but many good attempts.

### C8.3: Combinatorics

**Question 1** Q1 was extremely popular, with almost all candidates choosing to do it. The question was generally well done, although a surprising number of candidates constructed a chain decomposition using Hall's Theorem in (a)(ii), rather than using the decomposition from (a)(i). Part (c) was generally found tricky.

Q2 was also popular. Part (a) was done well, and part (b) more variable; many candidates had trouble bounding the binomial coefficients in part (iii).

Q3 had many fewer attempts. Part (a) was done well; parts (b) and (c) less so, although there were some good answers.

### C8.4: Probabilistic Combinatorics

Question 1 was attempted by almost all candidates. (a) is bookwork and was generally done well. Part (b) was ok though for (iii) candidates were often unclear what is independent of what. In part (c) a number of candidates missed the point in (ii) and tried to use Chebyshev's inequality, whereas the aim is to bound (not use) the variance.

Question 2 was by far the least popular even though it is in some sense the most straightforward. The entire question is just an extension to hypergraphs of material in the lectures. There were some reasonable solutions.

Question 3 was the second most popular. The bookwork part (c) was done well. For (a) (which is Prelims material) a few candidates got this wrong. (b) confused many candidates, though there were some good answers. The point is to try to use (a): consider the number of height $n$ trees rooted at children of the root. $A_{n+1}$ holds if there are at least two of these. So we obtain an interative formula for the probability of $A_n$. In (d) the key point is that $A_2$ and $B$ are upsets, but one needs to specify the relevant ground-set $X$.

### C8.7: Optimal Control

This exam was taken by 14 students, with all students attempting Q1, and approximately half attempting Q2 and Q3 respectively.

Question 1 was made more difficult due to an error in Q1c in the printed version of the exam (which, unfortunately, was not accompanied by the prepared correction sheet). Nevertheless, the overall standard was good. Most students gave appropraite statements of the Hamilton–Jacobi equations (however not always with a clear specification of the boundary conditions, or definition of the various terms.) The derivation of Pontryagin's principle was generally well done, but some answers were unclear in how the second-derivative term can be shown to vanish. Part c was attempted well, but (understandably) caused some algebraic difficulties. Part d was not particularly well done – the key insight is that, near the trajectory from part c, and with $t > 1$ the cost function is monotone in $u$, so the trajectory cannot be optimal. It's then possible to notice that taking $u$ very large will generally give a good control in this setting.

Question 2 was reasonably well done. Like in Question 1, the statement of the Hamilton–Jacobi–Bellman equation was generally good, but the boundary conditions often needed to be more clearly stated. Part b varied considerably, with some answers not using the fact $w$ solves the HJB equation clearly. Part c was not well done, as few answers noticed that, if the ansatz $v = -h(t)w(x)$ should hold, then $w$ must be the terminal value up to a scaling factor. This allows simplification of the terms in the HJB equation, giving a simpler solution. In part d many answers noticed that $g$ depends inversely on $u$, but not that there is a factor of $e^{-(1+x^2)}$ next to it, which shrinks the importance of this term quickly when $|x| \to \infty$.

Question 3 was moderately well done. Few answers gave clear statements of appropriate state variables (which need to keep track of sales over the past two days), or if they did, were not able to convert this into appropriate dynamics needed when stating the Bellman equation in part aiii. Many answers also did not account for the upper bound of 10 sales, while allowing an arbitrary number of customers to be approached, or distinguish between

the random number of sales and its expected value. Part b was reasonably well done, while part c varied, due to the need to specify how randomness plays a role in the algorithm.

**Statistics Units**

Reports on the following courses may be found in the Mathematics and Statistics examiners' report.

SC1 - Stochastic Models in Mathematical Genetics
SC2 - Probability and Statistics for Network Analysis
SC4 - Advanced Topics in Statistical Machine Learning
SC5 - Advanced Simulation Methods
SC6 - Graphical Models
SC7 - Bayes Methods
SC8 - Topics in Computational Biology
SC9 - Interacting Particle Systems
SC10- Algorithmic Foundations of Learning

**Computer Science**

Reports on the Computer Science courses may be found in the Mathematics and Computer Science examiners' report.

## F. Comments on performance of identifiable individuals

### 1. Prizes

The following prizes were awarded:

- Gibbs Prize. Franciszek Knyszewski, St Catherine's College
- Gibbs Prize. Yuchen Fan, St Hilda's College
- Gibbs Dissertation Prize. Cheuk Fung Lau, University College Daniel Kelly, The Queen's College
- Junior Mathematics Prize. Norris Lam, Mansfield College
- IMA Prize. Daniel Kelly, The Queen's College

## G. Names of members of the Board of Examiners

- **Examiners:**
  Prof. Emmanuel Breuillard (Chair)
  Prof. Jose Carrillo de la Plata
  Prof. Massimiliano Gubinelli Prof. Ehud Hrushovski
  Prof. Philip Maini
  Prof. Rosemary Dyson (External)
  Prof. Roger Moser (External)

- **Assessors**
  Dr. Naomi Andrew
  Prof. Konstantin Ardakov
  Prof. Louis-Pierre Arguin
  Prof. Ruth Baker
  Prof. Paul Balister
  Dr. Martin Bays
  Dr. Philip Beeley
  Prof. Dmitry Belayev
  Dr. Matthew Bowen
  Dr. Lukas Brantner
  Prof. Helen Byrne
  Prof. Coralia Cartis
  Prof. Jon Chapman
  Prof. Gui-Qiang Chen
  Prof. Dan Ciubotaru
  Prof. Sam Cohen
  Dr. Chris Couzens
  Prof. Mohit Dalwadi
  Prof. Andrew Dancer

Prof. Chris Douglas
Prof. Cornelia Drutu
Prof. Artur Ekert
Prof. Radek Erban
Dr. Karin Erdmann
Dr. Paz Fink Shustin
Prof. Victor Flynn
Prof. Eamonn Gaffney
Dr. Kathryn Gillow
Prof. Christina Goldschmidt
Prof. Alain Goriely
Prof. Ben Green
Prof. Ian Griffiths
Prof. Peter Grindrod
Prof. Lasse Grimmelt
Dr. Gillian Grindstaff
Prof. Ben Hambly
Prof. Robert Hancock
Prof. Raphael Hauser
Prof. André Henriques
Prof. Ian Hewitt
Dr. Christopher Hollings
Prof. Jenni Ingram
Prof. Dominic Joyce
Prof. Andras Juhasz
Prof. Jon Keating
Prof. Dawid Kielak
Prof. Robin Knight
Prof. Jochen Koenigsmann
Prof. Yakov Kremnitzer
Prof. Renaud Lambiotte
Prof. Alan Lauder
Dr Samuel Lewis
Prof. Jason Lotay
Dr Georg Maierhofer
Prof. James Maynard
Prof. Kevin McGerty
Dr. Jori Merikoski
Prof. Andrea Mondino
Prof. Yuji Nakatsukasa
Prof. Vidit Nanda
Prof. James Newton
Prof. Luc Nguyen
Prof. Nikolay Nikolov
Prof. Harald Oberhauser
Prof. Jan Obloj
Prof. James Oliver
Dr. Charles Parker

Dr. Harry Petyt
Prof. Oliver Riordan
Prof. Alexander Ritter
Prof. Damian Rossler
Dr Malena Sabate
Dr Yurij Salmaniw
Prof. Tom Sanders
Prof. Alexander Scott
Dr. Apurva Seth
Dr Jakub Skrzeckowski
Dr. Davide Spriano
Prof. Rolf Suabedissen
Prof Endre Suli
Prof. Jared Tanner
Prof. Dominic Vella
Dr Richard Wade
Dr Ning Wang
Prof. Anne Watson
Prof. Stuart White
Dr. Jinhe Ye
Dr. Zhenghao Zhong