M-theory of all self-dual ones: AdS/CFT, stringy and celestial holography

From Good Cuts to Celestial Holography, Oxford

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FREEDOM TO RESEARCH

Main Messages/Plan

- **Higher spin gravity** (HiSGRA) is one of ideas to construct (solvable) models of quantum gravity, which is based on ∞-dim symmetry, **Higher spin symmetry** (stronger than SUSY)
- **Chiral HiSGRA** is the "M-theory" of all self-dual ones: it contains all of them and has many more interactions that conspire to give a UV-finite theory that exists for any $\Lambda <, =, > 0$
- AdS/CFT relates it to $3d$ CFT's — Chern-Simons vector models, which can explain the $3d$ bosonization duality therein and implies the existence of new self-dual CFTs, which works up to ABJ(M) and, hence, applies to tensionless strings on $AdS_4 \times \mathbb{CP}_3$
- Being self-dual, it has lots of connections to Twistor space
- It can find its place within celestial holography

Why higher spins?

How high is higher spin?

Why higher spins? Epistemology

Usual assumptions: to "solve all problems" with particles/fields. Massless fields have helicity $\lambda = 0, \pm\frac{1}{2}, \pm 1, \pm\frac{3}{2}, \pm\mathbf{2}, \ldots$. Higher $s > 2$ seems needed, like in strings. Masslessness $\rightarrow$ (gauge) symmetry.

To probe the QG problem one can take massless higher spins

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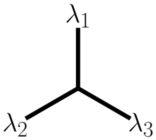
For any triplet of helicities λ_i, $\lambda_1+\lambda_2+\lambda_3>0$ there is a unique interaction vertex (Brink, Bengtsson^2, Linden, 1983-7):

$$V_3 \sim \boldsymbol{C_{\lambda_1,\lambda_2,\lambda_3}}[12]^{\lambda_1+\lambda_2-\lambda_3}[23]^{\lambda_2+\lambda_3-\lambda_1}[31]^{\lambda_3+\lambda_1-\lambda_2} \sim$$



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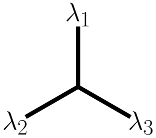

Questions: Classification of (classical) theories? Quantum gravity? Right math? Physics? Physics drastically depends on spin!

HiSGRA = the smallest extension of gravity by massless, i.e. gauge, higher spin fields. Vast gauge symmetry should render it finite.

Quantizing Gravity via HiSGRA $\sim$ Classical HiSGRA?

Wish list

- Some sensible examples of HiSGRA
- Cancellation of UV divergences
- Applications to physics (opt.)
- Beautiful math to explain all of that (opt.)
- Holographic applications (opt.)

Higher spin symmetry: S-matrix summary

Asymptotic higher spin symmetries (HSS)

$$\delta\Phi_{\mu_1\ldots\mu_s}(x) = \partial_{(\mu_1}\xi_{\mu_2\ldots\mu_s)}$$

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$$\delta\Phi_{\mu_1\ldots\mu_s}(x) = \nabla_{(\mu_1}\xi_{\mu_2\ldots\mu_s)} \quad \xleftrightarrow{AdS/CFT} \quad \partial^m J_{ma_2\ldots a_s} = 0$$

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seem to completely fix (holographic) S-matrix to be

$$S_{\text{HiSGRA}} = \begin{cases} 1^{****}, & \text{flat space, (Weinberg)} \\ \text{free CFT, e.g. } \Box\phi = 0, & \text{asymptotic AdS, unbroken HSS}^{\dagger} \\ \textbf{Chern-Simons Matter}, & \textbf{asymptotic AdS}_4\textbf{, SB HSS}^{\diamond} \end{cases}$$

Most interesting applications are to vector models, (Klebanov, Polyakov; Sezgin, Sundell; Maldacena, Zhiboedov;†,◇ Giombi, Yin, ...;◇ ...)

Both Minkowski/(A)dS cases reveal certain non-localities since HSS mixes ∞ spins and derivatives, invalidating the local QFT approach

Chiral HiSGRA:
M-theory of all self-dual ones

Self-dual Yang-Mills (SDYM)

It is easy to get SDYM as "truncation" of YM ($\boldsymbol{A}_\mu \to \boldsymbol{\Phi}^\pm$)

$$\mathcal{L}_{\mathsf{YM}} = \mathrm{tr}\, \boldsymbol{F}_{\mu\nu} \boldsymbol{F}^{\mu\nu}$$

$$\wr\wr$$

$$\mathcal{L}_{\mathsf{YM}/\mathsf{SDYM}} = \boldsymbol{\Phi}^- \Box \boldsymbol{\Phi}^+ + \boldsymbol{V}^{++-} + \boldsymbol{V}^{--+} + \boldsymbol{V}^{++--}$$

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SDYM can also be described covariantly with the help of

$$F\wedge F = F^2_{AB} - F^2_{A'B'} \qquad\qquad F^2_{\mu\nu} = F^2_{AB} + F^2_{A'B'}$$

where F_{AB}, $F_{A'B'}$ are the (anti)self-dual components in the $sl(2,\mathbb{C})$-language. Next, a couple of tricks due to (Chalmers, Siegel)

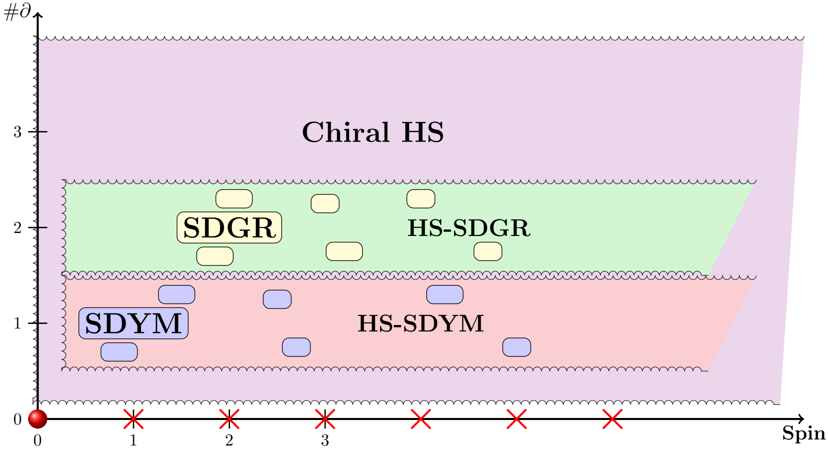
$$S_{YM} = \int F^2_{\mu\nu} \sim \int F^2_{AB} \sim \int \boldsymbol{\Psi^{AB}F_{AB}} - \boldsymbol{\frac{g'}{2}\Psi^2_{AB}}\,,$$

- actions are not real in Minkowski space
- actions are simpler than the complete theories
- twistors, integrability, instantons (Penrose; Wald; ADHM; ...)
- **SD theories are consistent truncations**, so anything we can compute will be a legitimate observable in the full theory; **Unitary physics from nonunitary theories!**

In general: amplitudes (MHV, BCFW, double-copy, ...), strings, QFT, Twistors, ... encourage to go outside Minkowski

In higher spins: little explored (Adamo, Hähnel, McLoughlin; E.S., Ponomarev; Ponomarev; Tran; Adamo, Tran; Krasnov, Herfray, E.S.; Mason, Sharma), can be the only reasonably local theories

M-theory of self-dual theories



(Ponomarev; Krasnov, E.S., Tran; Monteiro; Serrani) similar to

GR+more $\longrightarrow$ SUGRAs $\longrightarrow$ maximal (gauged) SUGRA

Chiral Higher Spin Gravity

(light-cone gauge, UV-cancellations)

Chiral HiSGRA: light-cone approach

Chiral HiSGRA (Metsaev; Ponomarev, E.S.) is a 'higher spin extension' of SDYM/SDGR. It has fields of all spins $s = 0, 1, 2, 3, ...$:

$$\boldsymbol{\mathcal{L}} = \sum_{\boldsymbol{\lambda}} \boldsymbol{\Phi}^{-\boldsymbol{\lambda}} \Box \boldsymbol{\Phi}^{+\boldsymbol{\lambda}} + \sum_{\boldsymbol{\lambda}_i} \frac{\boldsymbol{\kappa}\, \boldsymbol{l}_{\mathsf{Pl}}^{\boldsymbol{\lambda}_1+\boldsymbol{\lambda}_2+\boldsymbol{\lambda}_3-1}}{\boldsymbol{\Gamma}(\boldsymbol{\lambda}_1+\boldsymbol{\lambda}_2+\boldsymbol{\lambda}_3)} \boldsymbol{V}^{\boldsymbol{\lambda}_1,\boldsymbol{\lambda}_2,\boldsymbol{\lambda}_3} + \mathcal{O}(\boldsymbol{\Lambda})$$

where the vertices correspond to the textbook amplitudes

$$\boldsymbol{V}^{\boldsymbol{\lambda}_1,\boldsymbol{\lambda}_2,\boldsymbol{\lambda}_3} \sim [\mathbf{12}]^{\boldsymbol{\lambda}_1+\boldsymbol{\lambda}_2-\boldsymbol{\lambda}_3}[\mathbf{23}]^{\boldsymbol{\lambda}_2+\boldsymbol{\lambda}_3-\boldsymbol{\lambda}_1}[\mathbf{13}]^{\boldsymbol{\lambda}_1+\boldsymbol{\lambda}_3-\boldsymbol{\lambda}_2}$$

HS-SDYM and HS-SDGR: $\sum_i \lambda_i = 1$ or 2.
All theories are smooth in the cosmological constant.

This is the smallest higher spin theory and it is unique.
Graviton and scalar field belong to the same multiplet

No UV Divergences! One, two, ... -loop finiteness

Tree amplitudes vanish! The interactions are "non-renormalizable":

$$V^{\lambda_1,\lambda_2,\lambda_3} \sim \partial^{|\lambda_1+\lambda_2+\lambda_3|}\Phi^3$$

there are **no UV divergences!** (E.S., Tsulaia, Tran). Some loop momenta factor out, just as in $\mathcal{N}=4$ SYM, but ∞-many times.

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At one loop we find three factors: (1) SDYM or all-plus 1-loop QCD; (2) higher spin dressing to account for λ_i; (3) total number of d.o.f.:

$$\boldsymbol{A^{\text{1-loop}} = A^{++\ldots+}_{\text{QCD,1-loop}} \times D_{\lambda_1,\ldots,\lambda_n}} \times \sum_\lambda 1 \rightarrow 0$$

d.o.f.= $\sum_\lambda 1 = 1+2\zeta(0) = 0$ to comply with no-go's, (Beccaria, Tseytlin) and agrees with many results in AdS, where $\neq 0$

Two-loop cut = one-loop $\times$ tree, but tree $=0$, so no $\log$-div

Divergent sums

The number of effective degrees of freedom "wants" to be 0. The same and similar sums appear in string/M-theory (Giombi, Tseytlin, ...)

$$\sum_\lambda 1 = 0 \qquad \longleftrightarrow \qquad F^1 = \log\det \frac{\Box_0}{1}\frac{\Box_1}{\Box_0}\frac{\Box_2}{\Box_1}...\frac{\Box_s}{\Box_{s-1}}...$$

The theory wants to be "topological"; analogs of SUSY "spin sums"

$$\sum_s (-)^{2s} d(s) s^p = 0 \qquad \text{vs.} \qquad \frac{1}{360} + \sum_s \left(\frac{1}{180} - \frac{s^2}{24} + \frac{5s^4}{24}\right) = 0$$

In AdS_d with the help of ζ-function one can get 0 and

$$F^1_{AdS} \sim \sum_s \log\det[\Box + m_s^2] = 0 \qquad \text{or} \qquad \tfrac{1}{16}(2\log 2 - \frac{3\zeta(3)}{\pi^2})$$

Zeta-function seems the right regularization!

(Beccaria, Bekaert, Giombi, Joung, Klebanov, Pufu, Tseytlin, E.S, Tran, ...)

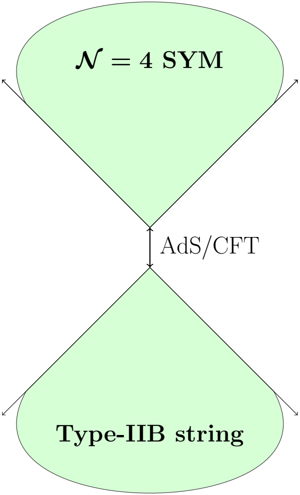
Chiral HSGRA in Minkowski

- **stringy 1**: the spectrum is infinite $s = 0, (1), 2, (3), 4, ...$
- **stringy 2**: admit Chan-Paton factors, $U(N)$, $O(N)$ and $USp(N)$
- **stringy 3**: we have to deal with spin sums $\sum_s$ (worldsheet takes care of this in string theory) and ζ-function helps
- **stringy 4**: the action contains parts of YM and Gravity
- **stringy 5**: higher spin fields soften amplitudes
- consistent with Weinberg etc. $S = 1^{***}$ (in Minkowski)
- gives all-plus QCD or SDYM amplitudes from a gravity

To get $S \neq 1$ we can either take backgrounds (e.g. shock-waves) in Minkowski space or go to $(A)dS_4$, where the holographic S-matrix turns out to be nontrivial ... and related to **Chern-Simons matter theories**

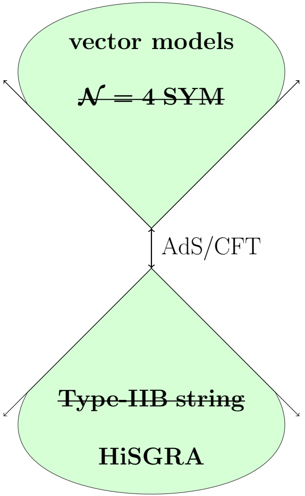
AdS/CFT with Chiral HiSGRA

(vector models & 3d bosonisation duality)



Let's look for AdS/CFT simpler than Strings/SYM

HiSGRA vs. Vector models



great attempts to construct one: Vasiliev; Jevicki, Mello Koch et al

Let's look for AdS/CFT simpler than Strings/SYM: HiSGRA/vector models (Klebanov, Polyakov; Sezgin, Sundell; ...)

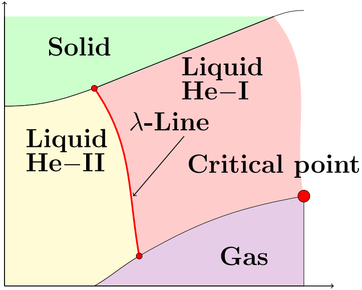
Not every CFT has a nice dual!

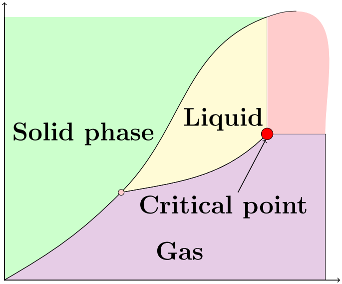
1. quasi-classical $\sim N \to \infty$
2. finitely-many fields (?) $\sim$ SUGRA
3. local? **Deadly even with 1, 2 ok**

We can have only quasi-classical $G \sim N^{-1}$, but still very nonlocal (Bekaert et al; Maldacena et al; Sleight, Taronna; Ponomarev)

The HiSGRA is not local/known

Chern-Simons Matter Theories and bosonization duality





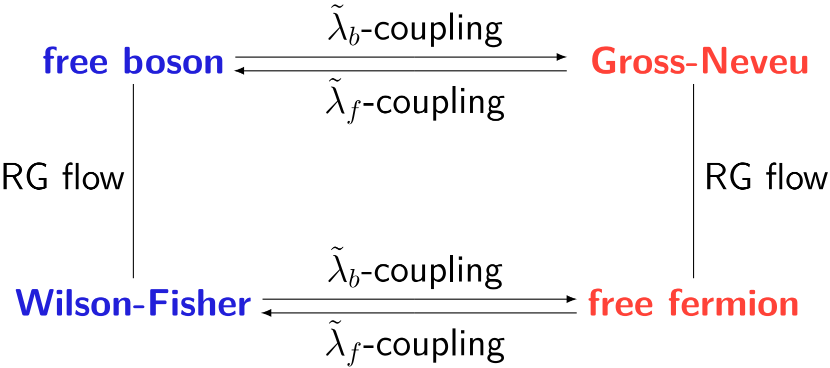
Chern-Simons Matter theories and dualities

CFT$_3$: Chern-Simons Matter theories, which span CFTs from vector models to BLG/ABJ(M). Let's consider the simplest 4 vector models

$$\frac{k}{4\pi}S_{CS}(A)+\text{Matter}\begin{cases}(D\phi^i)^2 & \textbf{free boson}\\ (D\phi^i)^2+g(\phi^i\phi^i)^2 & \textbf{Wilson-Fisher (Ising)}\\ \bar{\psi}^i \not{D}\psi_i & \textbf{free fermion}\\ \bar{\psi}^i \not{D}\psi_i+g(\bar{\psi}^i\psi_i)^2 & \textbf{Gross-Neveu}\end{cases}$$

- describe some physics (Ising, quantum Hall, ...)
- break parity in general (due to Chern-Simons)
- two parameters $\lambda = N/k$, $1/N$ (λ continuous for N large)
- exhibit remarkable dualities, e.g. ***3d* bosonization duality** (Aharony, Alday, Bissi, Giombi, Jain, Karch, Maldacena, Minwalla, Prakash, Seiberg, Tong, Witten, Yacobi, Yin, Zhiboedov, ...)

Chern-Simons Matter theories and dualities



$3d$ **bosonization: these 4 families/theories are just 2 theories** (Aharony et al; Giombi et al; Maldacena, Zhiboedov; Alday et al; Jain et al; ...) many checks, but no proof ... (Sharapov, E.S.) for large-N

Slightly-broken higher spin symmetry

The simplest gauge-invariant operators are **higher spin currents**:

$$\boldsymbol{J_s = \phi D ... D\phi} \qquad \text{and} \qquad \boldsymbol{J_s = \bar{\psi}\gamma D ... D\psi}$$

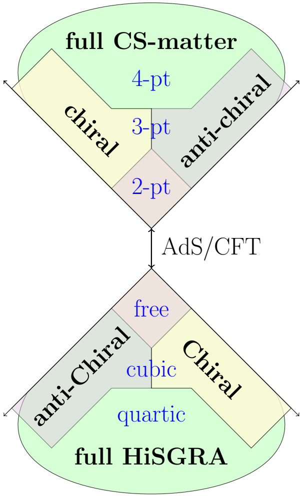
which are conserved to the leading order in $1/N$ $\rightarrow$ higher symmetry

HS-currents are responsible for their own non-conservation:

$$\partial \cdot J_s = \sum_{s_1,s_2} C_{s,s_1,s_2}(\lambda) \frac{1}{N} [J_{s_1} J_{s_2}]$$

which is a useful (!) exact equation (Maldacena, Zhiboedov).
This is called **slightly-broken higher-spin symmetry**.
But what it is? Sign of an ∞-dimensional symmetry ...

What is the right math? $\boldsymbol{L_\infty}$ (Lie algebroid). Can be used to prove $3d$-bosonization at least in the large-N (Sharapov, E.S.)



The existence of Chiral HiSGRA implies: there are two closed subsectors of Chern-Simons vector models (Sharapov, E.S.)

One can define them holographically, but it would be interesting to identify them on the CFT side (Aharony, Kalloor, Kukolj; Jain, Dhruva, E.S.); This corresponds to $k \to \pm i\infty$

SDYM/SDGR capture subsectors;

All 3pt-functions computed (E.S.);

$3d$ bosonization up to 4pt (Yin, E.S.);

There are two new **self-dual CFTs!**

Chiral Higher Spin Gravity

(beautiful math: covariant formulation, twistors)

Twistors treat positive and negative helicities differently:

$$\nabla_{B}{}^{A'}\,\Psi^{BA(2s-1)} = 0 \qquad \text{(Penrose, 1965)}$$

$$\nabla^{A}{}_{B'}\,\Phi^{A(2s-1),B'} = 0 \qquad \delta\Phi^{A(2s-1),B'} = \nabla^{AB'}\xi^{A(2s-2)}$$

(Hitchin, 1980) entertains a possibility to introduce a connection

$$\omega^{A(2s-2)} \ni e_{BB'}\Phi^{A(2s-2)B,B'} \qquad \delta\omega^{A(2s-2)} = \nabla\xi^{A(2s-2)}$$

where $e_{AA'}$ is the vierbein and with $H^{AB} \equiv e^{A}{}_{C'} \wedge e^{BC'}$ we can write

$$\boldsymbol{S = \int \Psi^{A(2s)} \wedge H_{AA} \wedge \nabla\omega_{A(2s-2)}}$$

which is also invariant under $\delta\omega^{A(2s-2)} = e^{A}{}_{C'}\,\eta^{A(2s-3),C'}$ to get rid of the extra component. **The simplest action for HS!**

N.B: for $s=1$ we have Ψ^{AB} and $A^{CC'}$, for $s=2$ Ψ^{ABCD} and ω^{AB}

HS-SDYM

Let's add more indices to the Charlmers-Siegel action

$$\mathbf{tr} \int \boldsymbol{\Psi}^{AB} \boldsymbol{H}_{AB} \wedge \boldsymbol{F} \qquad \rightarrow \qquad \sum_{s} \mathbf{tr} \int \boldsymbol{\Psi}^{A(2s)} \boldsymbol{H}_{AA} \wedge \boldsymbol{F}_{A(2s-2)}$$

where all A's are symmetrized inside F

$$F = d\omega + \tfrac{1}{2}[\omega, \omega] \qquad\qquad \omega = \sum_{s} (\omega^{A(2s)})^{i}{}_{j}\; y_A ... y_A$$

Feature: describes gauge, one-derivative, interactions of higher spin fields. The higher-spin symmetry is loop algebra $\mathfrak{g} \times C[y^A]$.

Twistor formulation is available (Tran; Herfray, Krasnov, E.S.) and the HS-extension of the Ward correspondence

A beautiful improvement is in (Mason, Sharma): Chern-Simons on $\mathbb{S}^7$

HS-SDGR

The action is an extension of (Krasnov) and (Krasnov, E.S.)

$$\int \Psi^{ABCD}\, F_{AB} \wedge F_{CD} \quad \to \quad \sum_{m,n} \int \Psi^{A(n+m)}\, F_{A(n)} \wedge F_{A(m)}$$

where $F_{A(2s-s)}$ depends on whether $\lambda = 0$ or $\lambda \neq 0$:

$$\begin{aligned} \lambda = 0 : & \qquad F_{A(n)} = d\omega_{A(n)} \\ \lambda \neq 0 : & \qquad F = d\omega + \tfrac{1}{2}\lambda\,\{\omega, \omega\} \end{aligned}$$

where we define Poisson bracket on $\mathbb{R}^2$ of $f(y)$, same as $w_{1+\infty}$:

$$\{f, g\} = \epsilon^{AB}\, \partial_A f(y)\, \partial_B g(y)$$

Twistor formulation (Herfray, Krasnov, E.S.) and the HS-extension of nonlinear graviton theorem; better twistor formulation (Mason, Sharma)

A bit on higher-spin symmetries

As a space we always have $\omega(y) = \sum_s \omega^{A(2s-2)}\, y_A ... y_A$

HS-SDYM : $\mathfrak{g} \times C[y^A]$

HS-SDGR : $2d$ Poisson $\sim w_{1+\infty}$

Chiral HS : Moyal-Weyl $*_{\boldsymbol{\lambda}} \otimes \mathrm{Mat}_N$

$$I^{\alpha\beta} = \begin{pmatrix} \boldsymbol{\lambda}\epsilon^{AB} & 0 \\ 0 & \epsilon^{A'B'} \end{pmatrix}$$

The finite-dim subalgebra is not Poincare, **but it acts as Poincare**

$$[L_{AA}, L_{BB}] = \boldsymbol{\lambda}\, \epsilon_{..} L_{..} \qquad [\bar{L}_{A'B'}, \bar{L}_{C'D'}] = \epsilon_{..} \bar{L}_{..}$$

$$[L_{AA}, P_{BB'}] = \boldsymbol{\lambda}\, \epsilon_{AB} P_{AB'} \qquad [\bar{L}_{A'A'}, P_{BB'}] = \epsilon_{..} P_{..}$$

$$[P_{AA'}, P_{BB'}] = \epsilon_{A'B'} L_{AB} + \boldsymbol{\lambda}\, \epsilon_{AB} \bar{L}_{A'B'}$$

Maxwell algebra: L_{AB} **is central at** $\boldsymbol{\lambda} = 0$, see (Ponomarev)

One can double-copy all of them (Ponomarev). Higher-d (Basile)

Chiral Higher Spin Gravity

(even more beautiful math we didn't want to know)

Covariant equations of motion are a those of **Poisson Sigma Model**, but in $4d$, (Sharapov, E.S., van Dongen), NC-Poisson (Kontsevich et al)

$$dC^i = \pi^{ij}(C)\, A_j\,, \qquad dA_k = \tfrac{1}{2}\partial_k \pi^{ij}(C)\, A_i \wedge A_j\,.$$

$A_i = \omega + \omega^{AB} y_A y_B + ...$ and $C^i = \phi + F^{AB} y_A y_B + ...$

If we lived in $2d$ the action would be just (topological open string) PSM

Type-A: $$S_{PSM} = \int_\Sigma C^i\, dA_i + \tfrac{1}{2}\pi^{ij}(C)\, A_i \wedge A_j$$

$\pi^{ij} = 0 + \pi^{ij}_k C^k + ...$ and π^{ij}_k is the Moyal-Weyl commutator. The next vertex comes from Shoikhet-Tsygan-Kontsevich formality and we have ∞-many orders on top. A_∞-relations are proved via Stokes theorem. **There must be some topological string theory behind!**

Chiral HiSGRA: Poisson sigma-model

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$$dC^i = \pi^{ij}(C)\, A_j\,, \qquad dA_k = \tfrac{1}{2}\partial_k \pi^{ij}(C)\, A_i \wedge A_j\,.$$

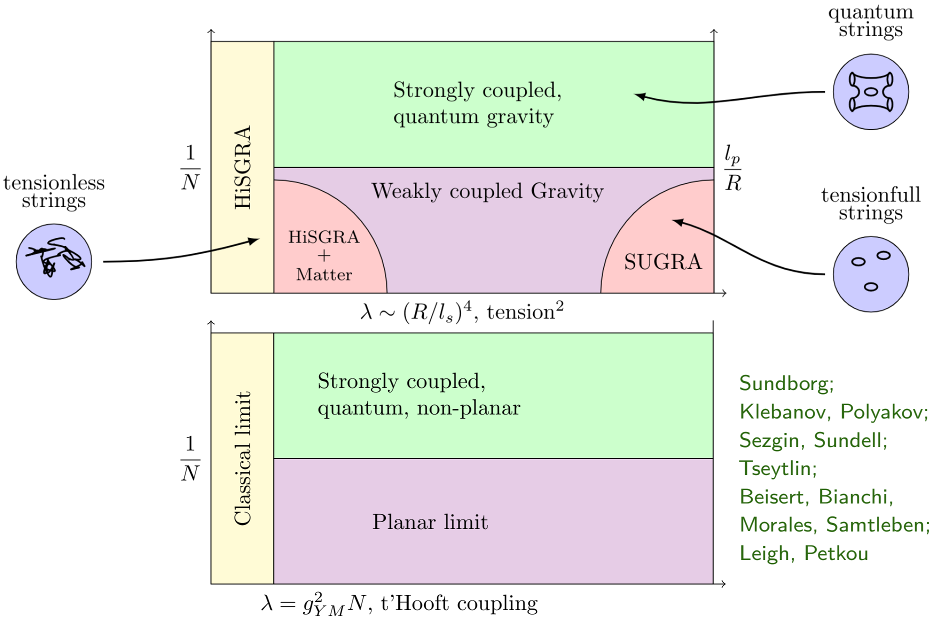
The same A_∞ can be recycled

$$\text{same } A_\infty \quad \begin{cases} 4d, & \textbf{Chiral HiSGRA} \\ 3d, & \textbf{Courant Sigma Model}\text{, matter-coupled HiSGRA,} \\ & \text{close to the conjecture by (Gaberdiel, Gopakumar)} \\ 2d, & \textbf{HS-Jackiw-Teitelboim}\text{, genuine PSM} \end{cases}$$

Can be a playground for **higher-form symmetries** since there are conserved p-forms with $p = 0, 2, 3, 4, 5, ...$ as different from (SU)GR(A)

Back to strings and HiSGRA within AdS/CFT

HiSGRA from Tensionless Strings



Sundborg;
Klebanov, Polyakov;
Sezgin, Sundell;
Tseytlin;
Beisert, Bianchi,
Morales, Samtleben;
Leigh, Petkou

Chiral HSGRA vs. Tensionless Strings

Strings on $AdS_4 \times \mathbb{CP}^3$ are dual to ABJ = Chern-Simons (k) matter with bi-fundamental matter, $N \times M$, (Chang, Minwalla, Sharma, Yin)

There is a vector-like limit $N \gg M$, where it is dual to $\mathcal{N} = 6$ $U(M)$-gauged HiSGRA, which suffers from the non-locality[1]

Inside this non-local/non-existing HiSGRA there is $\mathcal{N} = 6$ $U(M)$-gauged Chiral HiSGRA, which is local

By the same token, ABJ theory contains a "self-dual subsector", which should be dual to a "self-dual" subsector of tensionless strings

Is it possible to directly identify the Chiral subsector of tensionless strings on $AdS_4 \times \mathbb{CP}^3$?

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[1] For Vasiliev's equations see (Boulanger et al); in general, see (Bekaert, Erdmenger, Ponomarev, Sleight, Taronna) for "non-existence" of vector model duals.

Celestial holography

Chiral HiSGRA for celestial holography

(Ren, Spradlin, Yelleshpur Srikant, Volovich) found that Chiral HiSGRA is a solution to celestial OPE associativity

$$\sum_{\lambda_i} \frac{\kappa\, l_{\mathrm{Pl}}^{\lambda_1+\lambda_2+\lambda_3-1}}{\Gamma(\lambda_1+\lambda_2+\lambda_3)} [12]^{\lambda_1+\lambda_2-\lambda_3} [23]^{\lambda_2+\lambda_3-\lambda_1} [13]^{\lambda_1+\lambda_3-\lambda_2}$$

Everything that works for self-dual theories should also be true + **extra**, e.g. Twistorial anomaly (Tran), chiral algebra (Tran)

There is an analog of vector model AdS/CFT duality (Ponomarev):

$\Box\phi = 0$, single-trace operators $J_s \sim \phi\partial^s\phi$ are dual to HS bulk fields

AdS_4 algebra contracts to the Maxwell algebra (almost Poincare). There are representations $S^\pm$ and $S^+ \otimes S^- \sim$ massless HS fields in flat space.

(Distributional) amplitudes are invariants of the algebra like in AdS/CFT

Wish list

- Some sensible examples of HiSGRA: yes, but an extension to the parity invariant theory and nonlocalities are yet to be tamed
- Cancellation of UV divergences: yes, but $(A)dS$-extension and higher loops require new tools, e.g. Chern-Simons on twistor space (Tran; Mason, Sharma)
- Applications to physics: yes and it would be great to make 3d bosonization be direct consequence of Chiral HiSGRA; direct applications of HS-symmetry (3d Virasoro) to CFT's; self-dual CFT's and subsectors of vector models
- Beautiful math to explain all of that: Existence of Chiral HiSGRA goes beyond Kontsevich formality; equations = (Type-A) Poisson sigma-model, but the action is less understood
- M-theory/Strings: self-dual subsector via ABJ; celestial holography

Thank you for your attention!

... backup slides ...

“Flat” limit of the free scalar

AdS_4-algebra is realized by two pairs of $a_i^\dagger$, a_i, $i = 1, 2$, out of which one pair becomes commutative in the flat limt

$$\bar{J}_{\dot{1}\dot{1}} = a^2, \qquad \bar{J}_{\dot{2}\dot{2}} = -\frac{\partial^2}{\partial a^2}, \qquad \bar{J}_{\dot{1}\dot{2}} = -i(a\frac{\partial}{\partial a} + \frac{1}{2}),$$

$$P_{\alpha\dot{1}} = -\frac{1}{\sqrt{2}}\lambda_\alpha a, \qquad P_{\alpha\dot{2}} = \frac{i}{\sqrt{2}}\lambda_\alpha\frac{\partial}{\partial a},$$

$$L_{\alpha\alpha} = \frac{1}{2}\lambda_\alpha\lambda_\alpha,$$

$$J_{\alpha\alpha} = 2i\lambda_\alpha\frac{\partial}{\partial\lambda^\alpha},$$

acting on $c(\lambda, a)$.

$$J \equiv S^+ \otimes S^- = \sum_s \text{massless spin } s \text{ field in 4d Minkowski.}$$

Chiral HiSGRA: math we did not want to know

Global symmetry on the CFT side $\rightarrow$ Gauge algebra on AdS side

Let's take a free CFT, e.g. free scalar $\Box\phi = 0$.

Miracle: HS-algebra $\mathfrak{hs}$ is ∞-dim **associative** $\ni so(d,2)$; symmetries

$$J_s \sim \phi\partial^s\phi \quad \Longrightarrow \quad S(x,\partial)$$

can be multiplied (Eastwood). $\mathfrak{hs} \sim gl(|\phi\rangle)$.

Free CFTs/Large-N vector models $=$ Associative algebra

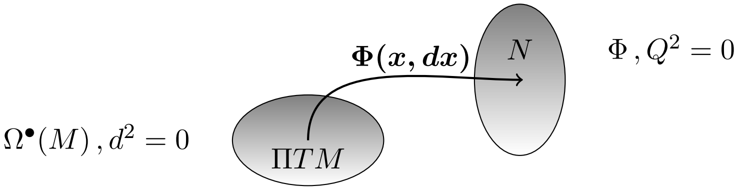
We deal with $\mathrm{Lie}(\mathfrak{hs} \otimes \mathrm{Mat}_N)$ since ${J_s}^i{}_j \sim \phi^i \partial^s \phi_j$ can be colored

$$\boldsymbol{L_\infty}/\textbf{Chevalley-Eilenberg} \xrightarrow{\text{large matrices}} \boldsymbol{A_\infty}/\textbf{Hochschild}$$

Conclusion: Underlying structure is $\boldsymbol{A_\infty}$! (almost a unique example in physics, save for SFT). In our case it is NC-Poisson (Kontsevich et al)

Formal equations

Let us be given a Q-manifold (view it locally as an L_∞-algebra)



then we can always write a sigma-model:

$$\boldsymbol{d\Phi = Q(\Phi)}$$

Any PDE can be cast into such a form ... (Barnich, Grigoriev)

Other names: Free Differential Algebras (Sullivan), in physics: (van Nieuwenhuizen; Fre, D' Auria); FDA=unfolding (Vasiliev), AKSZ (AKSZ); gauged PDE (Grigoriev, Kotov); string field theory (Zwiebach)

Poisson structure awakens

The Poisson structure is on $C^i = C(y)$, which is ∞-dim.

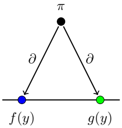
$\pi^{ij} = 0 + \pi^{ij}_k C^k + \ldots$ and π^{ij}_k is the Moyal-Weyl commutator

Any linear Poisson structure is just a Lie algebra

$$\begin{array}{ccccccccc} \mathbb{R}^2 & & f(y) \in C[\mathbb{R}^2] & & \boldsymbol{A_1} & & \mathrm{Lie}(\boldsymbol{A_1}) & & \\ & \rightarrow & & \rightarrow & & \rightarrow & & \rightarrow & \pi^{ij}_k \\ \epsilon^{AB} & & \{f, g\} & & f \star g & & [f, g]_\star & & \end{array}$$

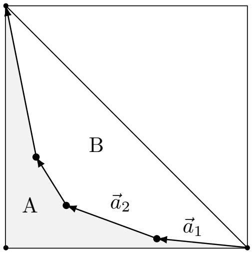
$$\{f, g\} = \partial_A f \, \epsilon^{AB} \, \partial_B g$$

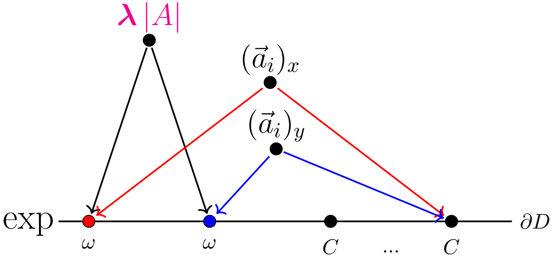
$$f \star g = \exp$$



Convex geometry, explicit maps

Explicit answer for all maps, e.g. $\boldsymbol{S}(\omega, \omega, C, ..., C)$:





(Sharapov, E.S., Van Dongen) the configuration space is of convex polygons B or swallowtails A, related to Grassmannian of two-planes.

The proof of $\boldsymbol{A}_\infty$ is by Stokes theorem: $\int_C d\Omega = \int_{\partial C} \Omega$

Deformations of Poisson Orbifold: Weyl Algebra

Weyl algebra $\boldsymbol{A_1}$ gives a **fuzzy-sphere** at a particular radius

$$x^i = (q^2, p^2, \tfrac{1}{2}[qp+pq]) \qquad\qquad C_2 = r^2 = -\tfrac{3}{4}$$

$\boldsymbol{A_1}$ is a rigid ∞-dim associative algebra that contains $sp(2)$.

Orbifold $\mathbb{R}^2/\mathbb{Z}_2$ admits 'second' quantization on top of the Moyal-Weyl $\star$-product, (Wigner; ...; Pope et al; Vasiliev; Madore; Bieliavsky et al; Joung, Mrtchyan; Korybut; Basile et al; Sharapov et al), freeing radius r

$$[q,p] = i\hbar + i\nu \boldsymbol{R} \qquad\qquad \boldsymbol{R}\, y_A \boldsymbol{R} = -y_A\,, \quad y_A = (q,p)$$

The first deformation is given by some two-cocycle ϕ_1. This is thanks to $\boldsymbol{HH^2(A_1, A_1^*) = \mathbb{C}}$ and the cocycle was obtained (Feigin, Felder, Shoikhet) from Shoikhet-Tsygan-Kontsevich formality

Higher spin symmetry and bosonization duality

Unbroken Higher spin symmetry

In free theories we have ∞-many conserved $J_s = \phi\partial...\partial\phi$ tensors.

Free CFT = Associative (higher spin) algebra

Conserved tensor → current → symmetry → invariants=correlators.

$$\partial \cdot J_s = 0 \quad \Longrightarrow \quad Q_s = \int J_s \quad \Longrightarrow \quad [Q,Q] = Q \quad \& \quad [Q,J] = J$$

HS-algebra (free boson) = HS-algebra (free fermion) in $3d$.

Correlators are given by invariants (Sundell, Colombo; Didenko, E.S.; ...)

$$\langle J...J \rangle = \mathrm{Tr}(\Psi \star ... \star \Psi) \qquad\qquad \Psi \leftrightarrow J$$

where Ψ are coherent states representing J in the higher spin algebra

$$\langle JJJJ \rangle_{F.B.} \sim \cos(Q^2_{13} + Q^3_{24} + Q^4_{31} + Q^1_{43}) \cos(P_{12}) \cos(P_{23}) \cos(P_{34}) \cos(P_{41}) + ...$$

Slightly-broken Higher spin symmetry is new Virasoro?

In large-N Chern-Simons vector models (e.g. Ising) higher spin symmetry does not disappear completely (Maldacena, Zhiboedov):

$$\partial \cdot J = \frac{1}{N}[JJ] \qquad\qquad [Q,J] = J + \frac{1}{N}[JJ]$$

What is the right math? We should deform the algebra together with its action on the module, so that the currents can 'backreact':

$$\delta_\xi J = l_2(\xi, J) + l_3(\xi, J, J) + \dots\,, \qquad\qquad [\delta_{\xi_1}, \delta_{\xi_2}] = \delta_\xi\,,$$

where $\xi = l_2(\xi_1, \xi_2) + l_3(\xi_1, \xi_2, J) + \dots$. This leads to L_∞-algebra.

Correlators = invariants of L_∞-algebra and are unique (Gerasimenko, Sharapov, E.S.), **which proves $3d$ bosonization duality at least in the large-N**. Without having to compute anything one prediction is

$$\langle J \dots J \rangle = \sum \langle \text{fixed} \rangle_i \times \text{params}$$

Local HiSGRA in 2025

3d massless, conformal and partially-massless (Blencowe; Bergshoeff, Stelle; Campoleoni, Fredenhagen, Pfenninger, Theisen; Henneaux, Rey; Gaberdiel, Gopakumar; Grumiller; Grigoriev, Mkrtchyan, E.S.; Pope, Townsend; Fradkin, Linetsky; Lovrekovic; ...), $S = S_{CS}$ for a HS extension of $sl_2 \oplus sl_2$ or $so(3,2)$

$$S = \int \omega\, d\omega + \tfrac{2}{3}\omega^3$$

4d conformal (Tseytlin, Segal; Bekaert, Joung, Mourad; Adamo, Tseytlin; Basile, Grigoriev, E.S.; ...), higher spin extension of Weyl gravity, local Weyl symmetry tames non-localities

$$S = \int \sqrt{g}\,(C_{\mu\nu,\lambda\rho})^2 + ...$$

4d massless chiral (Metsaev; Ponomarev, E.S.; Ponomarev; E.S., Tran, Tsulaia, Sharapov, Van Dongen, ...). The smallest HiSGRA with propagating fields.

IKKT model for fuzzy H_4 (Steinacker, Sperling, Fredenhagen, Tran)

The theories avoid all no-go's, as close to Field Theory as possible