Examiners' Report: Final Honour School of Mathematics Part B Trinity Term 2026

November 17, 2026

Part I

A. STATISTICS

- **Numbers and percentages in each class.**

  See Table 1.

	Numbers						Percentages %					
	2026	(2025)	(2024)	(2023)	(2022)	(2021)	2026	(2025)	(2024)	(2023)	(2022)	(2021)
I	55	(52)	(51)	(54)	(55)	(51)	41.98	(41.27)	(38.93)	(36.24)	(41.04)	(39.84)
II.1	55	(50)	(50)	(72)	(53)	(58)	41.98	(39.68)	(38.17)	(48.32)	(39.55)	(45.31)
II.2	14	(18)	(24)	(18)	(24)	(18)	10.69	(14.29)	(18.32)	(12.08)	(17.91)	(14.06)
III	3	(4)	(5)	(4)	(2)	(1)	2.29	(3.17)	(3.82)	(2.68)	(1.49)	(0.78)
P	3	(1)	(1)	(1)	(0)	(1)	2.29	(0.79)	(0.76)	(0.67)	(0)	(0.64)
F	1	(1)	(0)	(0)	(0)	(0)	0.76	(0.79)	(0)	(0)	(0)	(0.66)
Total	131	(126)	(130)	(149)	(134)	(157)	100	(100)	(100)	(100)	(100)	(100)

Table 1: Numbers and percentages in each class

- **Numbers of vivas and effects of vivas on classes of result.**

  As in previous years there were no vivas conducted for the FHS of Mathematics Part B.

- **Marking of scripts.**

  BOE Extended Essays, BSP Structured Projects and coursework submitted for the History of Mathematics course were double marked.

  The remaining scripts were all single marked according to a pre-agreed marking scheme which was strictly adhered to. For details of the extensive checking process, see Part II, Section A.

- **Numbers taking each paper.**

  See Table 7.

B. Changes in examining methods and procedures currently under discussion or contemplated for the future

None.

C. Notice of examination conventions for candidates

The Notice to Candidates Offering Coursework was issued on 25 February 2026. The first Notice to Candidates was issued on 12 March 2026 and the second notice on 11 May 2026.

All notices for 2026 examinations are on-line at
https://www.maths.ox.ac.uk/members/students/undergraduate-courses/ba-master-mathematics/examinations-assessments/examination-19.

The 2026 examination conventions are available at
https://courses.maths.ox.ac.uk/mod/book/view.php?id=65690.

Part II

A. General Comments on the Examination

The examiners would like to record their thanks to all those who helped in the preparation, administering, and assessing of this year's examinations. The chair would like to thank the academic administration team for their support of the Part B examinations.

In addition the internal examiners would like to express their gratitude to Professor Matt Tointon and Dr Ed Brambley for carrying out their duties as external examiners in such a constructive and supportive way during the year and for their thoughtful contributions during the final examiners' meetings.

For further comments on the examination process, see Section D below.

Standard of performance

The standard of performance was broadly in line with recent years. In setting the USMs, we took note of

- the Examiners' Report on the 2025 Part B examination, and in particular recommendations made by last year's examiners, and the Examiners' Report on the 2025 Part A examination, in which the 2026 Part B cohort were awarded their USMs for Part A;
- the guidelines provided by the Mathematics Teaching Committee, including its recommendations on the proportion of candidates that might be expected in each class.

Setting and checking of papers and marks processing

The internal examiners initially divided between them responsibility for the units of assessment (that is, the exam papers and projects).

Following established practice, the questions for each paper were initially set by the course lecturer, with the lecturer of a related course involved as checker before the first draft of the questions was presented to the examiners. The course lecturers also acted as assessors, marking the questions on their course(s).

Requests to course lecturers to act as assessors, and to act as checker of the questions of fellow lecturers, were sent out early in Michaelmas Term, with instructions and guidance on the setting and checking process, including a web link to the Examination Conventions.

The internal examiners met at the beginning of Hilary Term to consider those draft papers on Michaelmas Term courses, and changes and corrections were agreed with the lecturers where necessary. Where necessary, corrections and any proposed changes were agreed with the setters. The revised draft papers were then sent to the external examiners. Feedback from external examiners was given to examiners and to the relevant assessor for response. The internal examiners at their meeting in mid Hilary Term considered the external examiners' comments and the assessor responses, making further changes as necessary before finalising the questions. The process was repeated for the Hilary Term courses, but necessarily with a much tighter schedule. Before questions were submitted to the Examination Schools, setters were required to sign off a camera-ready copy of their questions.

Exams were held in-person in the Exams Schools. Papers were collected by the Academic Administration team and made available to assessors approximately half a day following the examination. Assessors were made aware of the marking deadlines ahead of time and all scripts and completed mark sheets were returned, if not by the agreed due dates, then at least in time for the script-checking process.

A team of graduate checkers, under the supervision of Charlotte Turner-Smith and Rosalind Mitchell sorted all the marked scripts for each paper of this examination, cross checking against the mark scheme to spot any unmarked questions or parts of questions, addition errors or incorrectly recorded marks. Also sub-totals for each part were checked against the mark scheme, noting correct addition. In this way a number of errors were corrected,and each change was signed by one of the examiners who were present throughout the process.

Throughout the examination process, candidates were treated anonymously, identified only by a randomly-assigned candidate number.

Timetable

Examinations began on Monday 1 June and ended on Friday 19 June.

Consultation with assessors on written papers

Assessors were asked to submit suggested ranges for which raw marks should map to USMs of 60 and 70 along with their mark-sheets, and all did so. In most cases these were in line with the assignments given by the assessors.

Determination of University Standardised Marks

The Mathematics Teaching Committee issued each examination board with broad guidelines on the proportion of candidates that might be expected in each class. This was based on the average in each class over the last four years, together with recent historic data for Part B.

We followed the Department's established practice in determining the University standardised marks (USMs) reported to candidates. Papers for which USMs are directly assigned by the markers or provided by another board of examiners are excluded from consideration.

For details on the scaling process please see the Mathematical Institute's website where it is outlined in full: https://www.maths.ox.ac.uk/members/students/undergraduate-courses/examinations-assessments/examiners-reports/scaling-algorithm.

This year a preliminary meeting of the internal examiners was held in advance of the final exam board meeting to compare the default settings produced by the algorithm alongside the reports from assessors. It was agreed that only a selection of scaling maps would be further reviewed at the final exam board, and that external examiners would be given an opportunity to review all maps prior to the meeting. Adjustments were made to the default settings as appropriate, paying particular attention to borderlines and to raw marks which were either very high or very low. Where the examiners were in doubt as to the most appropriate scaling, the preliminary scalings were held over to the final exam board meeting, where the factors considered by those in the preliminary meeting were reviewed and weighed before a final decision was made.

Table 2 on page gives the final positions of the corners of the piecewise linear maps used to determine USMs.

In accordance with the agreement between the Mathematics Department and the Computer Science Department, the final USM maps were passed to the examiners in Mathematics & Computer Science. USM marks for Mathematics papers of candidates in Mathematics & Philosophy were calculated using the same final maps and passed to the examiners for that School.

Comments on use of Part A marks to set scaling boundaries

None.

Mitigating Circumstance Notice to Examiners

A subset of the examiners (the 'Mitigating Circumstances Panel') attended a pre-board meeting to band the seriousness of the individual notices to examiners. The outcome of this meeting was relayed to the Examiners at the final exam board, who gave careful regard

to each case, scrutinised the relevant candidates' marks and agreed actions as appropriate. See Section E for further details.

The full board of examiners considered all of the notices in the final meeting, along with a number of MCEs carried over from Part A. The examiners considered each application alongside the candidate's marks and the recommendations proposed by the Part A 2025 Exam board.

Table 2: Position of corners of the piecewise linear maps

Paper	P_1	P_2	P_3	P_4	Additional Corners	Difficulty Score
B1.1	11.25;37	20.63;59.5	34.13;69.5	42.07;80	50;100	2.37
B1.2	11.62;37	21.31;59.5	34.81;69.5	42.41;80	50;100	1.69
B2.1	12.56;37	23.03;59.5	36.53;69.5	43.27;80	50;100	-0.03
B2.2	12.51;37	22.93;59.5	36.43;69.5	43.22;80	50;100	0.07
B2.3	9.08;37	16.64;59.5	30.14;69.5	40.07;80	50;100	6.36
B3.1	12.77;37	23.42;59.5	36.92;69.5	43.46;80	50;100	-0.42
B3.2	18.38;37	33.7;59.5	47.2;69.5	48.6;80		-10.7
B3.3	11.7;37	21.45;59.5	34.95;69.5	42.48;80	50;100	1.55
B3.4	12.23;37	22.43;59.5	35.93;69.5	42.97;80	50;100	0.57
B3.5	12.94;37	23.72;59.5	37.22;69.5	43.61;80	50;100	-0.72
B4.1	9.41;37	17.25;59.5	30.75;69.5	40.38;80	50;100	5.75
B4.2	10.72;37	19.66;59.5	33.16;69.5	41.58;85	50;100	3.34
B4.3	12.83;37	23.52;59.5	37.02;69.5	43.51;80	50;100	-0.52
B5.1	10.18;37	18.67;59.5	32.17;69.5	41.09;80	50;100	4.33
B5.2	12.37;37	22.68;59.5	36.18;69.5	43.09;80	50;100	0.32
B5.3	7.85;37	14.4;59.5	23;69.5	38.95;83	50;100	8.6
B5.4	13.31;37	24.41;59.5	37.91;69.5	43.96;80	50;100	-1.41
B5.5	16.58;37	30.39;59.5	43.89;69.5	46.95;80	50;100	-7.39
B5.6	12.14;37	22.25;59.5	35.75;69.5	42.88;80	50;100	0.75
B6.1	15.64;37	28.67;59.5	42.17;69.5	46.09;80	50;100	-5.67
B6.2	11.92;37	21.86;59.5	35.36;69.5	42.68;85	50;100	1.14
B6.3	12.54;37	22.99;59.5	36.49;69.5	43.25;80	50;100	0.01
B7.1	14.66;37	26.88;59.5	40.38;69.5	45.19;83	50;100	-3.88
B7.2	12.38;37	22.69;59.5	36.19;69.5	43.1;80	50;100	0.31
B7.3	13.69;37	25.1;59.5	38.6;69.5	44.3;83	50;100	-2.1
B7.4	11.78;37	21.6;59.5	35.1;69.5	42.55;80	50;100	1.4
B8.1	12.19;37	22.34;55	35.84;69.5	42.92;83	50;100	0.66
B8.2	12.91;37	23.66;59.5	37.16;69.5	43.58;80	50;100	-0.66
B8.3	12.68;37	23.25;59.5	36.75;69.5	43.38;80	50;100	-0.25
B8.4	13.79;37	25.28;59.5	38.78;69.5	44.39;80	50;100	-2.28
B8.5	15.03;37	27.56;59.5	41.06;69.5	45.53;83	50;100	-4.56
B8.6	12.89;37	23.63;59.5	37.13;69.5	43.57;80	50;100	-0.63
BSP	2000;100					0
SB1	17.4;37	31.9;59.5	49.72;69.5	57.86;80	66;100	-1.54
SB1	34;100					-1.54
SB2.1	15.05;37	27.59;59.5	41.09;69.5	45.55;83	50;100	-4.59
SB2.2	12.51;37	22.93;59.5	36.43;69.5	43.22;80	50;100	0.07
SB3.1	10.89;37	19.96;59.5	33.46;69.5	41.73;80	50;100	3.04

B. Equality and Diversity issues and breakdown of the results by gender

Table 3: Breakdown of results by gender

Class	Number											
	2026			2025			2024			2023		
	Female	Male	Total	Female	Male	Total	Female	Male	Total	Female	Male	Total
I	6	49	55	12	40	52	7	44	51	5	49	54
II.1	18	37	55	12	38	50	14	36	50	23	49	72
II.2	6	8	14	10	8	18	9	15	24	7	11	18
III	0	3	3	1	3	4	3	2	5	2	2	4
P	1	2	3	0	1	1	1	0	1	0	1	1
F	0	1	1	0	1	1	0	0	0	0	0	0
Total	31	100	131	34	97	131	37	112	149	40	93	134

Class	Percentage											
	2026			2025			2024			2023		
	Female	Male	Total	Female	Male	Total	Female	Male	Total	Female	Male	Total
I	19.36	49	42	34.29	43.96	41.27	20.59	45.36	38.93	13.51	43.75	36.24
II.1	58.06	37	42	34.29	41.76	39.68	41.18	37.11	38.16	62.16	43.75	48.32
II.2	19.35	8	11	28.57	8.79	14.29	26.47	15.46	18.32	18.92	9.82	12.08
III	0	3	3	3	3.3	3.17	8.82	2.06	3.81	5.41	2.68	2.68
P	3.23	2	2	0	1.1	0.79	2.94	0	0.76	0	0.89	0.67
F	0	1	1	0	1.1	0.79	0	0	0	0	0	0
Total	100	100	100	100	100	100	100	100	100	100	100	100

C. Detailed numbers on candidates' performance in each part of the examination

The number of candidates taking each paper is shown in Table 7.

Table 4: Rank and percentage of candidates with this or greater overall USMs

Av USM	Rank	Candidates with this USM and above	%
89	1	1	0.76
86	2	2	1.53
85	3	3	2.29
85	4	4	3.05
85	5	5	3.82
84	6	6	4.58
84	7	7	5.34
81	8	8	6.11
81	9	9	6.87
79	10	10	7.63
79	11	11	8.4
79	12	12	9.16
78	13	13	9.92
77	14	14	10.69
77	15	15	11.45
76	16	16	12.21
76	17	17	12.98
76	18	18	13.74
76	19	19	14.5
76	20	20	15.27
76	21	21	16.03
75	22	22	16.79
75	23	23	17.56
75	24	24	18.32
75	25	25	19.08
75	26	26	19.85
74	27	27	20.61
74	28	28	21.37
74	29	29	22.14
73	30	30	22.9
73	31	31	23.66
73	32	32	24.43
73	33	33	25.19
73	34	34	25.95
73	35	35	26.72
73	36	36	27.48
72	37	37	28.24
72	38	38	29.01
72	39	39	29.77
72	40	40	30.53
71	41	41	31.3
71	42	42	32.06
71	43	43	32.82
71	44	46	35.11
71	44	46	35.11
71	44	46	35.11
71	47	47	35.88
70	48	48	36.64
70	49	49	37.4
70	50	50	38.17
70	51	51	38.93
70	52	52	39.69
70	53	53	40.46
70	54	54	41.22
70	55	55	41.98

Table 5: Rank and percentage of candidates with this or greater overall USMs

Av USM	Rank	Candidates with this USM and above	%
69	56	56	42.75
69	57	57	43.51
69	58	58	44.27
69	59	59	45.04
69	60	60	45.8
69	61	61	46.56
68	62	62	47.33
68	63	63	48.09
68	64	64	48.85
68	65	65	49.62
67	66	66	50.38
67	67	67	51.15
67	68	68	51.91
67	69	69	52.67
67	70	70	53.44
67	71	71	54.2
67	72	72	54.96
67	73	73	55.73
66	74	74	56.49
66	75	75	57.25
66	76	76	58.02
65	77	77	58.78
65	78	78	59.54
65	79	79	60.31
65	80	80	61.07
65	81	81	61.83
65	82	82	62.6
65	83	83	63.36
64	84	84	64.12
64	85	85	64.89
64	86	86	65.65
64	87	87	66.41
64	88	88	67.18
64	89	89	67.94
64	90	90	68.7
64	91	91	69.47
64	92	92	70.23
63	93	93	70.99
63	94	94	71.76
63	95	95	72.52
63	96	97	74.05
63	96	97	74.05
63	98	98	74.81
63	99	99	75.57
62	100	100	76.34
61	101	101	77.1
61	102	102	77.86
61	103	103	78.63
61	104	104	79.39
61	105	105	80.15
60	106	106	80.92
60	107	107	81.68
60	108	108	82.44
60	109	109	83.21
60	110	110	83.97
60	111	111	84.73
59	112	112	85.5
59	113	113	86.26
59	114	114	87.02
59	115	115	87.79

Table 6: Rank and percentage of candidates with this or greater overall USMs

Av USM	Rank	Candidates with this USM and above	%
58	116	116	88.55
58	117	117	89.31
55	118	118	90.08
55	119	119	90.84
54	120	120	91.6
54	121	121	92.37
52	122	122	93.13
50	123	123	93.89
50	124	124	94.66
50	125	125	95.42
48	126	126	96.18
48	127	127	96.95
47	128	128	97.71
37	129	129	98.47
34	130	130	99.24
30	131	131	100

Table 7: Numbers taking each paper

Paper	Number of Candidates	Avg RAW	StDev RAW	Avg USM	StDev USM
BSP	29	1426.59	147.48	71	8.01
B1.1	37	26.28	9.75	59.7	17.11
B1.2	43	28.48	11.3	61	20.04
B2.1	39	32	14.3	66.46	25
B2.2	26	34.04	8.47	69.5	11.64
B2.3	18	27.71	11.31	63.06	21.68
B3.1	44	33.05	7.21	67.89	8.62
B3.2	20	45.1	5.76	80.2	15.63
B3.3	13	33.15	9.44	70.92	11.93
B3.4	30	32.3	9.05	66.8	11.52
B3.5	35	35.53	8.72	68.06	17.68
B4.1	44	26.8	10.9	65.8	15.3
B4.2	34	28.71	9.13	66.71	13.53
B4.3	14	36.86	5.38	71.36	6.26
B5.1	31	26.03	12.2	61.81	18.59
B5.2	40	30.82	9.84	65.9	13.69
B5.3	29	20.14	8.13	62.69	15.17
B5.4	28	30	9.29	62.79	13.19
B5.5	30	39.27	6.81	67.7	9.24
B5.6	20	28.5	7.52	64.7	9.5
B6.1	9	40.89	4.91	71.78	9.74
B6.2	17	31.82	7.77	66.76	12.14
B6.3	5	31.8	13.9	66.2	19.01
B7.1	16	36	6.14	67.69	7.8
B7.2	15	28.33	10.35	62.73	14.47
B7.3	8	34.12	8.95	69.62	13.91
B7.4	14	25.64	10.78	59.86	16.29
B8.1	55	34.31	5.93	69.16	8.33
B8.2	30	36	8.09	70.77	11.81
B8.3	41	33	8.3	67.51	10.85
B8.4	37	32.76	9.84	63.84	14.29
B8.5	40	36.62	6.66	67.53	9.22
B8.6	5	35.8	10.38	71.4	14.57
SB1	14	35.14	10.36	71	4.26
SB2.1	16	39.69	7.5	72.31	12.56
SB2.2	24	34.58	6.1	69.79	7.05
SB3.1	41	29.39	9.03	66.17	11.63

Individual question statistics for Mathematics candidates are shown below for those papers offered by no fewer than six candidates.

Paper B1.1: Logic

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14.06	14.06	5.5	34	0
Q2	9.95	11.72	5.61	18	4
Q3	12.95	14.28	6.81	18	3

Paper B1.2: Set Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.64	16.06	6.64	32	1
Q2	12.97	13.17	5.95	30	1
Q3	12.67	13.05	5.47	22	2

Paper B2.1: Introduction to Representation Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14.24	14.24	7.84	37	0
Q2	18.31	18.74	7.33	34	1
Q3	10.75	12	9.1	7	1

Paper B2.2: Commutative Algebra

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.58	17.58	3.85	26	0
Q2	16.46	16.46	5.3	26	0
Q3	10			0	1

Paper B2.3: Lie Algebras

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14.18	14.18	6.07	17	0
Q2	10.8	12.5	5.88	8	2
Q3	14.44	14.44	7.88	9	0

Paper B3.1: Galois Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.83	19.44	5.88	27	3
Q2	14.1	14.53	4.69	19	1
Q3	15.55	15.55	3.23	42	0

Paper B3.2: Geometry of Surfaces

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	23	23	2.77	18	0
Q2	20.12	21	6.4	7	1
Q3	21	22.73	5.4	15	2

Paper B3.3: Algebraic Curves

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.43	18.43	4.43	7	0
Q2	15.45	16.9	8.21	10	1
Q3	13.82	14.78	4.96	9	2

Paper B3.4: Algebraic Number Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.42	17.42	4.16	24	0
Q2	16.26	16.26	5.63	19	0
Q3	13.74	14.24	5.75	17	2

Paper B3.5: Topology and Groups

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.38	18.96	5.44	25	1
Q2	15.19	16.58	7.17	19	2
Q3	16.88	17.46	5.24	24	1

Paper B4.1: Functional Analysis I

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14.76	14.76	5.51	34	0
Q2	12.81	12.81	6.52	37	0
Q3	10.24	11.94	5.95	17	4

Paper B4.2: Functional Analysis II

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	13.68	14.17	5.06	24	1
Q2	14.67	15	5.42	32	1
Q3	11.5	13	5.79	12	2

Paper B4.3: Distribution Theory and Fourier Analysis: An Introduction

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	19.54	19.54	2.96	13	0
Q2	17.8	17.8	4.6	5	0
Q3	17.3	17.3	1.89	10	0

Paper B5.1: Stochastic Modelling and Biological Processes

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.29	15.73	6.91	30	1
Q2	9.71	10.45	6.49	22	2
Q3	6.67	10.5	6.54	10	8

Paper B5.2: Applied PDEs

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.47	15.97	5.59	36	2
Q2	14.26	15.18	6.88	28	3
Q3	11.27	14.56	7.7	16	6

Paper B5.3: Viscous Flow

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	13.76	13.76	4.58	21	0
Q2	8.5	8.74	4.1	23	1
Q3	6.44	6.71	3.78	14	2

Paper B5.4: Waves and Compressible Flow

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	11.54	12.45	6.73	11	2
Q2	13.04	14.58	6.11	24	4
Q3	16.81	16.81	5.16	21	0

Paper B5.5: Further Mathematical Biology

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	21.41	21.41	2.65	29	0
Q2	14.05	16.06	6.41	16	3
Q3	18.94	20	5.67	15	1

Paper B5.6: Nonlinear Systems

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	9.68	13.82	6.57	11	8
Q2	12.09	12.09	3.56	11	0
Q3	15.47	15.83	4.31	18	1

Paper B6.1: Numerical Solution of Differential Equations I

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.5	18.5	2.51	6	0
Q2	20.78	20.78	3.42	9	0
Q3	23.33	23.33	2.89	3	0

Paper B6.2: Numerical Solution of Differential Equations II

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.46	15.46	3.41	13	0
Q2	15.53	15.53	5.42	15	0
Q3	16.71	17.83	4.54	6	1

Paper B6.3: Integer Programming

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14.5	18	8.58	3	1
Q2	7.25	8.67	5.32	3	1
Q3	19.75	19.75	7.93	4	0

Paper B7.1: Classical Mechanics

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14	15.14	4.87	7	1
Q2	15.4	16.77	4.6	13	2
Q3	21	21	2.52	12	0

Paper B7.2: Electromagnetism

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	10.17	10.17	5.39	12	0
Q2	15.93	15.93	4.7	15	0
Q3	21.33	21.33	2.31	3	0

Paper B7.3: Further Quantum Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	11	11	1.41	2	0
Q2	17.43	17.43	6.75	7	0
Q3	18.43	18.43	3.91	7	0

Paper B7.4: Special Relativity

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	11.75	12.86	7.32	7	1
Q2	10.3	10.78	6.46	9	1
Q3	13.23	14.33	5.88	12	1

Paper B8.1: Martingales through Measure Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	12.86	14.2	4.59	10	4
Q2	17.17	17.17	3.53	48	0
Q3	17.71	17.71	3.4	52	0

Paper B8.2: Continuous Martingales and Stochastic Calculus

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	11.4	11.4	7.6	5	0
Q2	17.83	17.83	4.46	29	0
Q3	18.78	19.46	4.99	26	1

Paper B8.3: Mathematical Models of Financial Derivatives

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.5	18.5	4.34	38	0
Q2	16.46	16.46	5.66	26	0
Q3	12.44	13.06	4.5	17	1

Paper B8.4: Communication Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.38	15.38	5.94	34	0
Q2	16.97	16.97	5.14	36	0
Q3	17	19.5	6.13	4	2

Paper B8.5: Graph Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.3	18.62	4.44	32	1
Q2	17.36	17.63	3.96	27	1
Q3	18.71	18.71	3.41	21	0

Paper B8.6: High Dimensional Probability

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.6	18	6.11	4	1
Q2	17	17	8.21	4	0
Q3	19.5	19.5	3.54	2	0

Paper SB1.1/1.2: Applied Statistics/Computational Statistics

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.71	17.71	2.33	14	0
Q2	13.33	13.33	3.23	12	0
Q3	16	16	2.4	10	0
Q4	8	8	1.15	4	0
PR	27.43	27.43	2.06	14	0

Paper SB2.1: Foundations of Statistical Inference

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	21.31	21.31	4.25	16	0
Q2	16.1	18.62	7.32	8	2
Q3	16.33	18.12	5.85	8	1

Paper SB2.2: Statistical Machine Learning

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.05	16.05	4.17	22	0
Q2	16.69	17.93	5.22	14	2
Q3	18.83	18.83	2.48	12	0

Paper SB3.1: Applied Probability

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	12.7	13.92	5.43	24	3
Q2	13.85	14.18	5.75	33	1
Q3	16.12	16.12	4.18	25	0

Assessors' comments on sections and on individual questions

The comments which follow were submitted by the assessors, and have been reproduced with only minimal editing. The examiners have not included assessors' statements suggesting where possible borderlines might lie; they did take note of this guidance when determining the USM maps. Some statistical data which can be found in Section C above has also been removed.

B1.1: Logic

Question 1 A common mistake for (a)(iii) and (b) was to claim that $\mathcal{L}_{\to}$ can express no formula which the always-false valuation evaluates to false; $((p_0 \to p_0) \to p_0)$ is a counterexample. Part (b) did not see many correct solutions. In (c), a fair few unfortunately confused completeness of a set of formulas with completeness of the proof system. Only a handful could really solve (c)(iii).

Question 2 The bookwork in (a) was mostly done well. Surprising few could put together the formal proof for (b)(ii), with many attempts giving the impression that they hoped to hit upon a proof at random, rather than understanding abstractly why the result follows and only then thinking about formalisation. Some solved (c) in a laborious way by analysing how MP could give rise to the formula, rather than exploiting soundness. There were no complete solutions to (d)(ii).

Question 3 Many neglected to consider assignments when defining logical equivalence in (a)(i). Distressingly many made the elementary logical error $(\forall x_i \phi \to \psi) \vDash \forall x_i (\phi \to \psi)$ in (b)(ii). In (c)(ii), most gave the uninteresting correct example of $(\phi \wedge \phi)$; the question should really have used max in place of sum. In (d), many implicitly assumed that Σ implied that R is an equivalence relation. A few gave correct solutions to (e), and a few more had the right general idea.

B1.2: Set Theory

Question 1 Part (a) was mostly done fairly well, though some omitted the uniqueness of n-approximations which is necessary for the application of Replacement, and disturbingly many effectively tried to use recursion to prove recursion, defining n-approximations by recursion. In (b)(i), many failed to properly engage with the definition of finiteness. In (c), some correctly recursed up to ω_1, but few could satisfactorily explain why this works under AC.

Question 2 The bookwork in (a) was done fairly well. In (c)(i), many had the right idea of using double transfinite induction, but most of those struggled to put it into practice. There were quite a few correct solutions, but few neats ones; in particular, many needlessly split into cases (0/succ/lim) in the induction, leading to much repetition.

Question 3 The part (a) bookwork was mostly done well, but many lost time by writing out unnecessary material from the lectures. In particular, quite a few took a circuitous route through comparability of well-orders, rather than using comparability of ordinals directly. Part (b) had few correct solutions. A distressingly common mistake involved imagining that $\mathbb{Q}$ is order-isomorphic to ω and $\mathbb{R}$ to 2^ω. There were only 2 correct solutions to (c)(i), and quite a few seemed happy to claim that $\kappa^{\aleph_0} \leq \kappa$ is impossible, even though in (ii) they

went on to correctly calculate $(2^{\aleph_0})^{\aleph_0} = 2^{\aleph_0}$. Many got the application of (i) in (ii), and even more the easy application of (ii) in (iii).

B2.1 Introduction to Representation Theory

Question 1 was a popular question, which was attempted by a large majority of the candidates. The performance was mixed. Part (a) was done well, but it should have been clear that given that only three marks were available for this part, a proof of Maschke's Theorem was in fact not required. Part (b) proved to be quite tricky. Those candidates who claimed without justification that "without loss of generality, $v_n \in U$" received very little of the four marks available. Part (c) was mostly done well; several candidates found the elegant alternative solution of comparing traces of the g-action on both $V(\lambda, n)$ and $V(\mu, n)$ after observing that m must be equal to n for dimension reasons. Those candidates who did not realise that the key to part (d) was the Jordan Normal Form theorem had a hard time trying to prove it by induction on n; none of these attempts were successful. Part (e) was done reasonably well.

Question 2 was the most popular question; not surprising as it consisted entirely of seen material. Nevertheless, the bookwork parts (b) and (c) were not done well by a sizeable minority of the candidates. Part (d) proved to be challenging, but of course not at all a challenge to those candidates who took care to properly revise the problem sheets before the exam.

Question 3 was the least popular question, probably because it appeared to be so unfamiliar, and was only attempted by about five candidates. Nevertheless, these people managed to get quite far with it.

B2.2: Commutative Algebra

Question 1 was attempted all but one student. In part (a) a common issue was the failure to note that $R \setminus \mathfrak{p}$ is a multiplicative set. Some students failed to prove all the facts needed in part (b) to establish the correspondence. In part (d), many students used a wrong definition of a primary ideal.

Question 2 was again attempted by all but one student. Part (a) was done well. In part (b) many students noticed a more direct proof that does not use Zorn's lemma. In part (c), only very few students realised that it can be deduced from part (b), and only one student carried out the deduction successfully. Most people used the Lasker–Noether theorem, which was fine provided that it, and all the relevant facts about primary decompositions, were proved. There were many failed attempts in part (f). A surprising number of students seem to think that the union of two ideals is always an ideal.

Question 3 was attempted by three students. Part (c) was done well by one person (the solution was actually smarter then the one I had in mind); the other parts were done well two people.

B2.3: Lie Algebras

All candidates attempted question 1, and answered it well, with part c) being the most challenging. Part d) was answered well, with a range of examples given. Question 2 was slightly less popular than question 3, and was also the most challenging, as the hint in

part b) seemed to be less helpful than intended, though quite a few candidates did realise that if a representation has a nondegenerate trace form it must be faithful. Question 3 was mostly well-answered. Part c) being the most challenging part, but a few students correctly calculated the size of the Weyl group.

B3.1: Galois Theory

Question 1

Candidates generally had no trouble with the statement and proof of Artin's lemma in part (a), or with determining the group in part (b)(i). Computing the fixed field in part (b)(ii) seemed to be more challenging, and several candidates gave a proper subfield of the correct answer. Parts (b)(iii) and (b)(iv) were omitted by quite a few candidates, but the solutions submitted were mostly correct.

Question 2

The majority of candidates did not attempt this question. Those who did generally did well on parts (a), (b), and (di), but struggled on parts (c) and especially part (dii). A common difficulty was expressing the counting problem in (dii) in terms of linear algebra, and correctly counting subspaces of finite-dimensional $\mathbb{F}_p$-vector spaces.

Question 3

Most candidates did well on the bookwork in part (a), although some struggled to prove that splitting fields of separable polynomials are normal and separable. Part (b) proved more challenging than expected: although many candidates found partial solutions, many incorrect Galois groups were proposed, and no completely correct solution was submitted.

B3.2: Geometry of Surfaces

Question 1: This question was attempted by the majority of candidates and proved to be the most successful question on the paper overall. Parts a), b), and c) were generally answered very well, with most candidates demonstrating a sound understanding of the relevant concepts and techniques. The majority of marks lost on this question arose in part d), where candidates were asked to determine the number of non-homeomorphic surfaces obtained by pairwise identifications of the sides of a hexagon.

Question 2: This question was attempted by approximately half of the candidates and was generally answered well. Part (a) was handled successfully by most students, although some responses gave an incomplete definition of compatible complex charts by omitting the requirement that the transition maps be biholomorphisms. Parts (b) and (c) were also completed to a good standard, with most candidates demonstrating a sound understanding of the underlying theory. Part (d) proved to be the most challenging section of the question, particularly from a computational perspective. The most common errors arose in the determination of the ramification points, with a number of candidates identifying their locations incorrectly.

Question 3: Around half of the candidates attempted this question and generally performed well on it. Part (a) was completed successfully by most students, although some

responses stated the Gauss–Bonnet Theorem without correctly specifying all of the necessary assumptions. Part (b) was also answered well by the majority of candidates, with the main weakness being imprecise or only approximate statements of the Theorema Egregium. Part (c)(ii) proved to be the most demanding part of the question, particularly from a computational perspective. While many candidates made substantial progress, only the strongest students were able to compute both the Gaussian curvature and the mean curvature completely and correctly.

Summary: Most candidates successfully answered a substantial proportion of the questions they attempted. A significant number achieved scores close to full marks, reflecting a strong overall performance. Candidates' marks were generally well balanced across the two questions they attempted, with little variation in performance between them.

B3.3 Algebraic Curves

Q1 and Q2 were the favourite questions, but Q3 was also chosen by quite a few students. Nobody managed to answer all of Q1 (c), and Q1 (b) (iv) proved difficult for many. Q3 (c) was answered correctly by very few students. To answer Q3 (c), one has to notice that $Q_x(a, b, c)$ does not vanish if $[a, b, c]$ is not a singular point of $\mathrm{Z}(Q)$ (this follows from the assumptions), and this fact was overlooked by most.

B3.4: Algebraic Number Theory

Here is a polished version in LaTeX, keeping the tone consistent with your previous examination reports.

‘‘‘

Questions 1 and 2 were generally handled well by the candidates. Fewer candidates attempted Question 3, and some experienced difficulties with parts (b) and (c), particularly by not making effective use of the hint that

$$\Delta_2(M) = [\mathcal{O}_K : M]^2 \Delta_2(\mathcal{O}_K).$$

B3.5 Topology and Groups

Q1 ai, aii, bi, bii: were all well done.
Q1 biii: a variety of explanations were given, some less convincing than others.
The rest of Q1 was done reasonnably well, with the notable exception of cvii, which was only treated correctly by a handful of students.

Q2 ai: many misinterpreted the picture to mean that the vertical edges were glued to the horizontal edges. But the surrounding text (in particular stating the space is a genus two surface) should have made it clear that this was not the case.
Q2 aii and aiii were done ok.
Q2 aiv: the justifications provided were rarely at the expected level of rigor.
The rest of Q2 was done reasonnably well, with the notable exception of biii, were many people attempted proofs by induction, which had of course no chance of being correct.

Q3 a: was done overall very well.
Q3 b: was hard for many, even though this exact proof was presented in class, and then

again discussed extensively during revision.
Q3 ci, cii: were done well. ciiiα: many were either unable or forgot to provide a homeomorphism between $\partial\Delta^{n+1}$ and S^n.
Q3 ciiiβ was hard, but a fair number of people managed it, but Q3 ciiiγ turned up to be very hard, with only one or two students managing to solve it.

B4.1: Functional Analysis I

Question 1

This was a popular question. (a) and (b) were typically well answered (though some candidates didn't realise they needed to use a 1-norm to get surjectivity in (b)). In part (c) (and for the non-degeneracy of the norm in (d)(i)) a standard mistake was to claim incorrectly that if (f_n) is convergent to f in L^p then $f_n \to f$ almost everywhere. This is false, and instead there is a subsequence (f_{n_k}) converging to f almost everywhere. Care was taken to avoid penalising candidates twice if they repeated this error. (d)(i) was also broadly well done, though in addition to the convergence error above, a number of candidates claimed that if $\|f\|_Y = 0$, then $f = g + h$ with $\|g\|_{q_1} = \|h\|_{q_2} = 0$, rather than using the approximation property of the infima. Part d(ii) was found to be harder - many candidates didn't look to apply the result in (a), though those that did normally did this well. Part (e)(i) attracted some good answers, but only a small number of candidates realised they would need to apply the Hahn-Banach theorem to use the results of (b). The solutions that achieved this showed very impressive functional analysis skills.

Students taking the course might be interested to know that $L^{p_1} \cap L^{p_2}$ is an Orlitz space L_ϕ for $\phi(t) = t^{p_1} + t^{p_2}$ (though this isn't important in terms of answering the question).

Question 2

This was also a relatively popular question.

Parts (a)(i) and (ii) were book work which was typically done well, through with many overly long answers for (ii). A few answers to (i) only noted the isomorphism of vector spaces and neglected to show T is bounded with bounded inverse. In (ii) it would have been fine to say that F is isomorphic to $\mathbb{R}^n$ (by i) and so complete: It was not necessary to prove that completeness was preserved by an isomorphism of normed spaces. Many candidates solved the deduce part of (iii) (though there were some false claims that a bounded sequence in a finite dimensional space must converge, rather than must have a convergent subsequence), but the main claim was harder, though saw a range of creative and correct solutions.

(b) (i) was generally very well done, but a few candidates forgot that subalgebras are also subspaces. In (ii) many candidates realised how to use (a)(iii) to show that $A + \mathrm{Span}(1)$ is *closed*, and also a subalgebra so that they could apply Stone-Weierstrass. (iii) was found hard, though with some nice answers and attempts. A common error in a good attempt was to build polynomials p_n approximating h and claim that $p_n(g) \in A$. This is not necessarily true as A does not contain constant functions, and the p_n may have constant terms. Instead – and well done to the few candidates who picked up on this – $p_n(g)g \in A$, as $p_n(g)g$ is a polynomial in g with no constant term, and $p_n(g)g \to 1$ uniformly. The last part (iv) was a little bit easier again, but still challenging. Those candidates who picked up on the compactness argument typically did it well (though very few candidates realised they could directly apply the finite intersection version of compactness; it was much more popular

to go back to the open cover formulation, which essentially involved reproving the finite intersection property). A few candidates found a t_0 to get $A \subset \{f \in C_{\mathbb{R}}(K) : f(t_0) = 0\}$ but then failed to use the decomposition in (b)(ii) to get the reverse inclusion.

Question 3

This was the least popular question. 3(a)(i) and (ii) where typically well done, but with some occasional missing details. The method for calculating the norm in (a)(iii) was apprecaited by most candidates, though quite a few calculational errors ensued. 3(b) was also typically well done (though some candidates struggled to get the bound in (i), and the justifications for why $V + V^*$ is a projection didn't always quickly go through 3(a)(ii)).

Part (c) was found difficult; while quite a few candidates realised that (c)(i) would likely use a Neumann series argument, it wasn't always implemented correctly. A few candidates gave a nice argument for invertibility of T, but then didn't go on to show that U was a unitary. Attempts at (d) were rare, and candidates did not realise that this could be reduced to a 2 dimensional problem, which is essentially solved in (a)(iii).

B4.2: Functional Analysis II

Question 1 was solved by more than half of the students and most answers showed a good understanding of the topics tested in this question, i.e. Fourier series, spectral theory and compact operators. As expected, part b) proved to be more difficult than other parts, as it required the students to bring together different parts of the course to establish that the Fourier series defines a continuous function which agrees with f a.e.. In part c)ii) quite a few students forgot to explain why the function Sf is indeed continuous, but most successfully showed that the operator is not compact either by constructing sequences for which the image converges pointwise to a discontinuous function or appealing to the Theorem of Arzela-Ascoli and disproving equicontinuity. For Part c)iii) most implications were discussed well, but the proof of continuity caused more problems than expected, with only a few students observing that for linear operators for which the image of the unit ball is precompact continuity follows immediately as compact sets are always bounded and as bounded linear operators are continuous.

Question 2 was solved by nearly all students. Part a) on the uniform boundedness principle was solved well by most students, while part b), which could either be approached using uniform boundedness principle and characterisation of duals or directly using Hölder's inequality, DCT and specific examples, caused more difficulties. Not all students realised that the easiest way to argue $(i) \Rightarrow (ii)$ in c) is via Inverse function theorem, but essentially all students gave a good proof of d)i). The final part of the question required students to realise that if they wanted to obtain sets which are closed wrt C^0 convergence, then they needed to use conditions that did not involve any derivatives, but instead e.g. Lipschitz bounds, and quite a few students successfully argued this way.

Question 3 was solved by a bit less than half of the students. The first part was well solved and students generally followed the hint in b) and tried to use Gelfand's formula to determine the spectral radius r_σ, though not all of them combined that with an analysis of the point spectrum, which in analogy to the standard left shift is the open disc with radius r_σ, to establish that the spectrum is indeed the closed 2 disc. Part (i) of c) was very well solved, while part (ii) was more difficult and required students to realise that weak

convergence of f_n in L^2 implies weak convergence of $\sqrt{t}f_n$, as weakly convergent sequences cannot be expected to converge pointwise. While Part (iii) could be solved using ideas used in similar proofs of the lecture, a full solution requires a careful argument so it was not surprising that this part caused more difficulty than other parts. Many students realised that the fact that orthonormal sequences converge weakly to zero can be used to provide an easy example for (iv) while several different correct counterexamples were constructed for the last part of the question.

B4.3: Distribution Theory

Question 1 All three questions were attempted.

Question 1: The overall level of answers was good, although no candidate obtained full marks. Most candidates gained full marks in Parts (a)–(b), which consisted largely of bookwork, direct corollaries, and examples seen before. Part (c) was a variant of bookwork. Parts (d)-(e) were a combination of unseen, similar, and new riders, requiring not only careful calculations but also a solid understanding of the fundamental concepts and properties of distributions and their orders. These parts were generally handled reasonably well, although some candidates struggled with the more delicate calculations.

Question 2: The overall standard of answers was slightly lower than for Question 1. Again, no candidate obtained full marks. In particular, marks were frequently lost in Parts (c)(iii)–(iv), which concerned the extension and support of distributions as linear functionals on spaces of continuously differentiable functions; these parts involved new examples and a new rider, which proved difficult for several candidates. Parts (a)–(b), which consisted of a mixture of bookwork, similar, and new examples, were generally answered well.

Question 3: Overall performance on this question was weaker than on Questions 1-2. Parts (a)–(b) consisted mainly of bookwork and its variants, but required a sound understanding of distributional partial derivatives, as well as the notions of order, restriction, and support of distributions; these parts were generally completed successfully. However, no candidate obtained full marks on Part (c). In particular, several candidates encountered difficulties with Parts (c)(ii)–(iii). The marking was carried out strictly in accordance with the mark allocation specified in the solutions for Question 3.

B5.1: Stochastic Modelling and Biological Processes

Question 1 was very popular (all candidates attempted it) and relatively easy (mean mark was 15.2, and 7 out of 37 attempts gained 24 or 25 marks).

Questions 2 and 3 were similar in difficulty, with 30 candidates attempting Q2 with an 8.6 mean mark and 23 candidates attempting Q3 with a 7.4 mean mark. There were 16 candidates who attempted all three questions, of whom 10 scored 6 or less on both. This suggests that a substantial proportion of candidates were undecided between Questions 2 and 3, and would likely have performed better had there been only two questions and no choice in the examination.

Question 1 The first 10 marks were bookwork and easy to get. Only candidates who were unable to determine the kernel of a 3×4 matrix (required to find the conservation

laws) who struggled to get those 10 marks (parts (a) to (c)). In part d), some candidates showed that the density given was stationary by checking detailed balance and its norm; others derived it using the forward Kolmogorov equation. For part e), some candidates failed to consider the two cases ($C = 0, 1$) and use the result in d), and instead reverse-engineered the required result. Part (f) was easier than anticipated, despite being unseen in this course, but several candidates obtained the results in a very straightforward way using Markov Chain theory (expected time for a pure birth process with rates given in (e)). It would be interesting to see how many of the high scorers on this question also took SB3.1 Applied Probability.

Question 2 I was surprised by how many candidates failed to get most of the marks in parts (a) and (b). In particular, part a) was about a simple Brownian motion and asked to write $dX(t) = \sqrt{2D}dW(t)$ and identify and solve the backward equation $D\tau'' = -1$, which was covered repeatedly in the course. The last part of (b) required solving this simple equation and noticing it was not possible to find a positive solution satisfying the boundary condition $\tau(0) = 0$. Many candidates missed this. Only a couple of candidates did parts (b) and (c) correctly. Part b) required identifying that this was a hybrid process with the same continuous part as in (a) and a jump part (the resetting). Unfortunately, without solving part (b), it was difficult to make progress on part (c). However, some marks were given to candidates who got (b) wrong but attempted (c) nonetheless. Many candidates jumped from (a) to (d) directly without attempting them. Parts (d) and (e) were more successful and serious attempts in Q2, and often received full marks. In hindsight, Question 2 might have been better structured as parts (a), (d), (e), (b), and (c).

Question 3 This question had an unusual pattern of performance, with 16 attempts with 8 or fewer marks, and only 5 attempts with a score of 18 or more. Part (a) required applying the Itô formula, and surprisingly, many candidates completely missed this. A couple of candidates applied the formula to the three-dimensional difference vector (instead of the scalar $R(t)$ which was given in the question statement); this made the calculations unnecessarily cumbersome and lengthy. Despite part b) not requiring a) (since the SDE for $R(t)$ was provided), many candidates scored poorly on this part because they did not write down the generator $\mathcal{L}$ and the equation $\mathcal{L}h = 0$ for the hitting probability. Without this equation for h, candidates struggled with parts c) and d). Candidates did well, and most achieved full marks in the pseudocode asked in part e).

B5.2: Applied PDEs

Question 1 This was the most popular question. Part (a) was straightforward and generally done well apart from some careless sign errors. In part (b)(i), not many candidates managed to exploit the homogeneity of f to simplify and integrate Charpit's equations. Several incorrectly asserted that f_p and f_q must be constant along rays, and very few candidates realised that we don't need to integrate the ODE for u, because u is given as a function of p and q. Many candidates didn't notice that part (b)(ii) is a special case of part (b)(i) and instead started again from scratch. Solutions were often over-complicated and thus prone to algebraic slips. For those who made it to the end, the sketching and identification of the domain of definition were generally done well.

Question 2 This question was also popular. Part (a) was straightforward. In part (b), many candidates were unable to find the left eigenvectors (for example, calculating the right eigenvectors instead) and even those who found the correct ODEs satisfied along characteristics often couldn't work out how to integrate them. There were a lot of fallacies based on fundamental misunderstandings about the difference between total and partial differentiation, correct application of the chain rule, etc. In parts (c) and (d), several candidates managed to derive some of the required results directly from the PDE without directly using the Riemann invariants from part (b).

Question 3 This was by far the least popular question. Parts (a) and (b) were both quite standard applications of similarity solutions. Most candidates were able to obtain the correct exponents and the correct ODEs, though there were a lot of careless errors in integrating the ODEs. The basic idea of part (c), to calculate the relevant integral kernels from known similarity solutions, was covered in problem sheet 4. Those who didn't remember the approach generally tried to use some kind of Green's function argument but then got lost in the details. Few candidates made serious progress with part (d), which relied on having correctly calculated g and h in part (c) and then figuring out how to use them.

B5.3: Viscous Flow

Part (a) of question 1 was generally well done. Some candidates lost marks in the early parts by not defining quantities that they introduced. Others lost marks for using a corollary to Reynolds Transport Theorem without justification. Some candidates did not correctly remember how to write down the heat flux through the boundary of the material volume, or the rate of working of the stresses on the surface. Part b(i) was also generally well done, although some candidates retained u_r rather than setting it to zero, and others struggled to justify setting $\partial p/\partial\theta = 0$. There were a fair few algebraic slips. Candidates were not penalised for not making the argument of the log term in the pressure dimensionless. Only a handful of candidates tried parts b(ii) and b(iii). Of those, not all spotted that they needed to convert the viscous heating term from cartesian to polar coordinates. There was an error in the stated formula at the end of b(iii); it should have been

$$\mu > \frac{k(r_1^2 - r_0^2)^2}{r_1^2 U^2} \left(\frac{T_0 - T_1}{r_0^2 \left(2\log\left(\frac{r_1}{r_0}\right) - 1 + \frac{r_0^2}{r_1^2} \right)} \right)$$

The one candidate that derived the result as stated on the paper was not penalised.

Question 2 was the most popular question. Some candidates lost marks in a(i) for not writing down the solution for the pressure as well as for the velocity. Despite a(ii) being completely bookwork, there were more errors than anticipated, with some candidates introducing the stream function incorrectly, and others guessing α and β to make the answer work rather than asserting what α and β have to be to turn the system into an ODE system. Some candidates incorrectly stated that they needed $v \to 0$ as $Y \to \infty$. In a(iii), most candidates missed the point that v in the inner solution is $O(1/\sqrt{\mathrm{Re}})$ and that this generates a correction of that order in the outer flow. In part b(i), some candidates did not spot that they needed to justify setting an arbitrary function of y to zero. Others did not justify the boundary conditions on ψ. Only a handful of candidates tried b(ii); they spotted that the complex

solution was $\psi = -\lambda\sqrt{x + \mathrm{i}y}$, but only one was able to find u_1 and v_1. There were no attempts at part b(iii).

Question 3 was attempted by fewer candidates. Many struggled to correctly write down that the load had to be equal to the integral of the pressure across the top surface. Solutions were marred by algebraic manipulation errors. Many candidates did not manage to derive the (standard) Reynolds equation, and only a couple got the equation for h'. Only one candidate made any significant headway with parts (c); no one tried part (d).

B5.4: Waves and Compressible Flow

Question 1 There were two typos in the question. In the preamble, $\mathbf{u} = U\mathbf{u}_z + \epsilon\mathbf{u}_1$ should have read $\mathbf{u} = U\mathbf{u}_x + \epsilon\mathbf{u}_1$. This error was easy to spot (since the correct form was given two lines above) and no candidate was seriously put off by it. The other typo was in the hint, where the mass conservation equation should have read $\boldsymbol{\nabla} \cdot (\rho\mathbf{u}) = 0$ instead of $\boldsymbol{\nabla} \cdot \mathbf{u} = 0$. Any candidates who were misled by this error were given full credit for correctly using the given information.

This was both the least popular question and the one that attracted the lowest marks. In part (a), many candidates failed to show that the flow is irrotational (and may thus be described using a velocity potential). Very few argued convincingly why $\boldsymbol{\nabla}\phi$ has to be continuous except across the wing. Part (b) was generally done quite well, and most candidates managed to derive the given boundary conditions. Part (c) caused a lot of problems, not least for those who thought the characteristics were given by $y \pm Bx =$ constant instead of $x \pm By =$ constant.

Question 2 This was the most popular question and generally done quite well. Many candidates seemed to lack confidence with vector calculus and so resorted to taking components. In part (a), some couldn't derive the log term, for example just assuming that $(\boldsymbol{\nabla}p)/\rho = \boldsymbol{\nabla}(p/\rho)$. Many struggled with the linearisation in part (b), for example expanding $\log(a+b) = \log(a)+\log(b)$ or even $\log(a+b) = \log(a)\log(b)$! In part (c), some candidates couldn't see how to use the hint (to solve for $\partial\phi/\partial z$ instead of ϕ), and the solution of the simple quadratic auxiliary equation caused a surprising number of problems.

Question 3 This was a popular question and attracted some very good solutions. The calculations in part (a) were mostly successful, though often overcomplicated (not exploiting, for example, $\partial p/\partial x = c^2\partial\rho/\partial x$). Part (b) was also bookwork and generally ok apart from some incomplete explanations. Part (c) was difficult for most. Even those who realised that they should use the chain rule often struggled with the simple exercise of calculating $\partial x/\partial\tau$. For the final calculation, one can evaluate t_* by finding where the denominator of $\partial u/\partial x$ is zero and then minimising over τ; a handful of candidates found the alternative route of expressing the denominator as a quadratic in τ and asking when it has real roots.

B5.5: Further Mathematical Biology

Question 1: This question was attempted by almost all candidates and was answered very well. Candidates demonstrated a strong understanding of the material. A few specific observations are worth noting.

(a)(i) Many candidates did not define the meaning of each parameter.

(b)(ii) Some candidates did not recognise that the steady-state value of A is arbitrary and that its value of 1 is determined by the initial condition. The choice of steady states for the travelling wave was also not clearly justified by most candidates. Far behind the wave, the introduced cells reach their carrying capacity ($N = 1$) and consume all the attractant ($A = 0$). Far ahead of the wave, no cells have arrived ($N = 0$), so the chemoattractant remains at its initial steady-state value ($A = 1$). The condition $V = 0 = W$ corresponds to a stationary profile, in which the cell density and chemoattractant concentration are spatially uniform.

(c)(i) A few candidates identified the direction of the waves incorrectly.

Question 2: This question was attempted by approximately half of the candidates and was considered the most difficult question by many. Nevertheless, candidates demonstrated a good understanding of the material.

(a) Many candidates did not explain that, for a spatially uniform steady state, all derivatives must vanish. Some also failed to recognise that a steady state must satisfy the boundary conditions. The linear stability analysis was generally carried out reasonably well, but many candidates did not state that stability or instability is determined by the sign of the real part of the eigenvalue. Furthermore, when determining the minimum of the function $h(k^2)$, candidates often failed to specify that this minimum must be positive, since $k^2 \geq 0$ for real k.

(c) In this case, the eigenfunctions are products of sine functions. For long, thin stripes, this means that no pattern forms: the eigenfunctions are

$$\sin\left(\frac{n\pi x}{L_x}\right)\sin\left(\frac{m\pi y}{L_y}\right).$$

As L_y becomes large, the relevant mode has $m = 0$, making the eigenfunction identically zero. Several candidates missed this distinction from part (b)(iii).

Question 3: This question was attempted by approximately half of the candidates and was answered very well.

(a) The key point is that, if a particle is at site n and moves to the right, it must have been at site $n - 1$ at the previous time step. A number of candidates did not recognise this.

(b) Many candidates neglected the higher-order terms at an early stage. Consequently, their arguments strictly established that the equations were correct only to first order. However, if the second-order terms are retained, then dividing by Δt and taking the limit shows that these higher-order terms vanish.

(c)(i) Some candidates did not state the assumption that the limit D exists and is bounded.

B5.6: Nonlinear Systems

Candidates demonstrated a good understanding of the course material and generally received marks for the bookwork components of each question. A couple of candidates produced almost perfect solutions; however, the majority lost marks on the most challenging parts, which required careful consideration of the most appropriate solution approach. **Question 1** investigated a planar system of ordinary differential equations, drawing on

material from course B5.6, which builds on the Part A course Differential Equations 1. Question 1(a) required candidates to apply the Part A concepts of finding and classifying critical points, and most candidates performed well. Question 1(b) involved analysing Hopf bifurcations and was answered with mixed success. The strongest responses came from candidates who recognised not only that there were two Hopf bifurcations, but also that some calculations needed to be carried out only once, since the corresponding matrices differed only by a sign and therefore had the same eigenvectors. Only a few candidates attempted Question 1(c).

Question 2 covered the material on discrete-time dynamical systems (maps). Questions 2(a) and 2(b) were generally answered well, with most candidates demonstrating the ability to find fixed points and 2-cycles, and determine their stability. Question 2(c) required candidates to show that the set of periodic points of F is dense in $[0, 4]$. This could be proved either directly or by showing that the map F is conjugate to the tent map; that is, $g^{-1} \circ F \circ g$ is the tent map, where $g(y) = 4y^2$. The latter approach could also be used in Question 2(d) to find a periodic point with period 7 as $g(z) = 4z^2$, where $z = 2/(2^7 + 1)$ is a periodic point with period 7 of the tent map.

Question 3 introduced a system of three chemical species undergoing six chemical reactions. Almost all candidates were able to write down the corresponding reaction rate equations in Question 3(a). Many also answered Question 3(b) successfully by finding and classifying the two critical points. In Question 3(c), a number of candidates correctly hypothesised, based on their results from Question 3(b), that the system undergoes a transcritical bifurcation. However, some were unable to use the extended center manifold to classify the bifurcation and consequently lost marks for failing to calculate the quadratic approximation to the center manifold.

B6.2 Optimisation for Data Science

The students performed well on the exam. Question 2 was done particularly well. Question 1 contained slightly more unusual material so the students had to really understand the course to approach full marks. Question 3 covered two distincts parts of course material (one from the first half and one from the end of the course) so it really needed students to have learnt the material. Thus, I consider that the students have done very well, considering the scope and breadth of the questions.

B6.3: Integer Programming

No comment.

B7.1: Classical Mechanics

1. This question was found hard and was relatively unpopular both because the approximation to write the Lagrangian was difficult and because the second part was a somewhat nonstandard extension beyond the more standard normal modes material. The normal modes were however done very well.

2. The first half was easily done well by most candidates. Showing that θ is constant for the second half was found hard by nearly everyone despite being a matter of equating $\mathbf{L} \cdot \mathbf{e}_3$ in the body frame and the inertial space-frame.

3. Despite the complexity of the calculation required in the second part, this question attracted many strong solutions although some got lost in the complexity of showing that $\mathbf{J}$ is constant.

B7.2: Electromagnetism

All candidates but chose Question 2. About 3/4 of the candidates chose Questions 1 and 1/4 chose Questions 3. Candidates were well-prepared to apply even the most advanced concepts if they arose in seen before setups, but most struggled to adapt their knowledge to setups that were slight variations of familiar ones.

Question 1

The average mark on this question was low. Part a) was designed to be elementary, with the Poisson equation taking the schematic form $\phi''(z) = 0$ and $\phi''(z) = 1$ on different intervals and continuous differentiability or Dirichlet boundary condition imposed at the boundaries of intervals. It seems that the majority candidates had difficulty translating the problem's setup into the language of differential equations. This is despite having solved similar problems in more complicated geometries (cylinder, sphere) in problem sheets.

Part b) asked for applying the image charge method for a thin charged wire. While several candidates correctly determined the potential of an isolated wire, only a few set up the image charge computation correctly, and only one succeeded in solving the problem completely. Many candidates had difficulty in computing the distance between points on the circle and a fixed point representing the (image) wire. This is despite the fact that the almost identical problem of the image charge for a sphere was discussed extensively during the course.

Question 2

Part b) of this question proved to be the easiest on the exam, with almost all candidates correctly finding the gauge transformations of the potentials and then manipulating Maxwell's equations correctly. The familiar computations in part a) caused some difficulty to candidates and very few gave the answer $\mathbf{A}(\mathbf{r}) = \mathbf{0}$ on the z axis. Several candidates then drew the incorrect conclusion that this would imply that $\mathbf{B} = \nabla \times \mathbf{A}$ would also be zero. (This is incorrect, since one cannot compute all partial derivatives, if one only knows a function on a line.) This however only led to few lost marks. The computation of the dipole field also caused some difficulty.

Part c) was unfamiliar. Many candidates still managed to obtain all conditions on the constants in part i), but only a few made progress in ii). A few candidates tried to find physically inequivalent solutions to the equations in part ii), instead of looking for gauge transformations. Only one candidate found that the number of independent parameters in α, β is two.

Question 3

The average mark on this question was very high, despite the problem going through the most challenging proof in the course, but adapted to two dimensions. Candidates had a good command of the Fourier transform, explained the sign choice of ω, applied contour integral methods confidently, and were precise in their algebra. The only common mistake was a mistaken sign in the residue theorem for a clockwise contour. In part d) only one candidate obtained the correct support of the Green's function.

B7.3: Further Quantum Theory

Question one was the least popular question by a wide margin, and most attempts did not make it much beyond the bookwork and easy spectral problem in parts (a) and (b). The actual calculations in part (c) and the computational part of (d) were very simple when performed in the right basis, but this was not picked up on.

Question two was attempted by nearly all candidates. The bookwork in parts (a) and (b) was done reasonably well by most. In the computations in the remaining parts, many candidates had the right idea but only a few were able to carry through to completion without significant slips or getting stuck.

Question three was also very popular and was generally answered well, especially the bookwork in part (a) and the calculations in part (b) (though forgetting to normalize the trial wave function/include normalization in the Rayleigh quotient was a recurring issue here). Very few identified the key idea in part (c), where the result of the virial theorem held in spite of the trial wave function not being an exact energy eigenstate because the trial family of wave functions is precisely the one that is extremised over in the derivation of the virial theorem.

B7.4: Special Relativity

In summary, the questions worked really well. They were well balanced, both in terms of amount of students attempting them, as well as in terms of average marks (all three averages were in the 10-15 range). Furthermore, the marks were well spread. The total marks for every part and subpart were not modified.

B8.1: Probability, Measure and Martingales

Question 2 and Question 3 were the most popular questions, with very few candidates choosing Question 1.

In Q1(a)(ii), very few candidates realized that a non-negative martingale is a supermartingale and is therefore bounded by its given initial value, so that Doob's convergence theorem applies. Few candidates made significant progress on Q1(c).

For Q2, a surprising number of candidates struggled with Q2(a)(ii), and some gave very convoluted arguments for Q2(b)(i). There were some good attempts at Q2(c), but few candidates realized that one can directly compute the quadratic variation of the discrete stochastic integral W.

A common source of lost marks in Q3(b) was to not verify measurability with respect to the tail σ-algebra.

B8.2: Continuous Martingales and Stochastic Calculus

Question 1. This is less liked question, only small portion of candidates attempted it. Part (a) Only several candidates didn't answer the book work part (a)(i), (ii) and (iii) correctly, most candidates got the deserved mark for this part. Even with a hint, most candidates how to prove (a) (iv) which is quite surprising to the setter. The appropriate process the candidate should look at is the couple the left limit process of X and X, a process valued in the plane, and look at the hitting time to the diagonal. Part (b). Most candidates know

how to identify the deterministic processes, and were able to provide good arguments and computation to justified, while candidates generally lacked a sense how to settle (b)(ii) with a rigorous argument. Part (c) is new and unseen in the lectures, but similar arguments appear in the discussion of square integrable martingales, covered in lectures and lecture notes. Very few candidates were able to solve this part and thus lose most oof marks assigned for this part.

Question 2. Almost everyone did this question and most candidates achieved a good mark. Part (a). Still some candidates do not know what is a Gaussian process – which should be a process whose finit dimensional marginal distributions are normal, but a few candidates thought one dimensional marginal distributions are Gaussian. Even several candidates confused with processes with independent increments. Part (a)(iii) is a special case of the reflection principle, which should be verified directly using Gaussian nature of Brownian motion. Many candidates reproduced a proof in the lecture notes (which is okay but too much for this simple case). Part (b). Most candidate did well for the first part (i) of proving the martingale property but failed to indicate a proper right continuous filtration, or justify the martingale property with respect a right continuous filtration. A few candidates didn't know what is a stopped martingale (which is a convention that it means a martingale stopped at a stopping time, rather than a new class of stochastic processes). Most candidates give a definition of Doob's optional stopping theorem (in this case full marks are given for this part, as OST implies (ii)). Part (b) (iii) meant to be a challenging part, and it is. Still I received a few very good and complete solutions. The candidate needs to identify a family of exponential martingales, and the key is to use the stopped martingale theorem (Part (b)(ii)) and to justify the limit as time parameter tending to infinity. Most candidates are able to apply OST to a proper martingales, but a few candidates lose marks for justifying the use of dominated convergence theorem.

Question 3. Like Question 2, most candidates attempted this question. Part (a). Most candidates can state Ito's formula which is good. Several candidates couldn't state Levy's martingale characteristic, but most candidates did well for this bookwork part. For part (a)(iii) one should to use the Ito's isometry and compute the bracket process, and the local martingale property of Ito's integrals, then use Levy's theorem (a)(ii). Part (b) (i)(ii) are essentially bookwork, and I expected careful arguments using Ito's formula. Most candidates did well. Surprisingly very few people were able to prove the exponential inequality. One should apply Doob's maximal inequality for a family of exponential martingales then optimize Doob's bound to exponential decay.

B8.3: Mathematical Models of Financial Derivatives

Question 1

(a) This was a standard arbitrage problem requiring candidates to construct a portfolio and demonstrate the existence of an arbitrage opportunity. Many candidates handled this well, although some were unclear about long and short positions, resulting in sign errors in the portfolio. Others failed to construct an appropriate portfolio.

(b) Most candidates did reasonably well, although a common omission was the no-arbitrage condition implying that $\mu = r$.

(c) Most candidates did well, as this was largely bookwork. However, some candidates

forgot to include the dividend.

(d) This question was relatively challenging. Some candidates calculated the value at earlier times and considered the limit as $S \to \infty$ to demonstrate the existence of an exercise region. The BSPDE and boundary conditions were often incomplete, and only a few candidates included the smooth-pasting condition.

(e) This question was generally well answered. Candidates showed that the payoffs are equal and pointed out that A offers greater flexibility in exercising.

Question 2

(a) Most candidates did well, as this was a routine calculation.

(b)(i) and (ii) Most candidates did well. A common mistake was to use the discount factor $e^{-r(T-t)}$. Since r is a function in this case, the correct discount factor is $e^{-R_{t,T}}$.

(b)(iii) Most candidates obtained the correct answer, namely that the two quantities are equal because their integrated variances are the same.

(c) Most candidates did reasonably well by applying Itô's formula. A few candidates did not use the correct initial condition for X_0.

Question 3

(a)(i) This part was generally well answered, with candidates applying differentiation to the BSPDE.

(a)(ii) This part was more challenging. Some candidates derived the PDE for the vega, but only a few obtained the correct solution.

(b)(i) Most candidates did well.

(b)(ii) Most candidates correctly obtained the solution on $(T_2, T_3]$, and some also solved the interval $(T_1, T_2]$ correctly. Fewer candidates handled $[0, T_1)$ correctly, mainly because of the application of the chain rule in the vega calculation.

(c)(i) This was a bookwork question and was generally well answered.

(c)(ii) This was a very challenging question. Only a few candidates clearly demonstrated that w satisfies the BSPDE and the required boundary condition. Very few candidates worked out the values for the binary and vanilla call cases.

B8.4: Information Theory

The examination was well done by many. The first two questions were very popular and saw a few candidates able to obtain full marks on them. Those that did question 3 usually did it very well.

Question 1

There were many good attempts at this question, and the bookwork at the beginning was generally well done. Marks were lost by candidates who assumed that Y and $\mathbb{I}_{\{X \neq Y\}}$ were independent in their proof of Fano's inequality. Part (b)(iii) proved challenging, as many candidates incorrectly assumed that mutual information decreases upon conditioning. In

part (c)(i), many candidates did not use the data processing inequality to bound $I(W;\hat{W})$. The final part was argued successfully by only a few candidates.

‘‘‘

Question 2 : The first parts were standard bookwork. Parts b(ii) and (iii) required similar arguments but (iii) was more straightforward. Many attempts were not well argued, often not discussing or showing optimality carefully enough. For part (c) many lost marks for the first part by not mentioning how to construct the code for each symbol. For the second part there were only a few correct answers with many not able to extend the ideas from b(iii) correctly.

Question 3 This was the least popular question with not many attempts. The question was quite straightforward for those that knew this material and the marks were generally high. The final part did not really require the Wonham filter and could have been reasoned directly.

B8.5: Graph Theory

Question 1 was with hindsight a little too easy; at least, it didn't distinguish that well at the higher end, though there were also a number of rather low marks.

Part (a) was generally well done, with a surprising variety of arguments for (iii). Part (b) turned out to be the trickiest: it's asking for part of the argument in lectures proving Brooks' Theorem. Although the overall proof is by induction, this part is not (i.e., does not require the induction hypothesis). Quite a few candidates tried to prove the given assertion by induction, which does not work easily, since any smaller graphs arising are not necessarily 3-connected. Part (c) was generally ok, though there were some incorrect, and many long, arguments to show that $\kappa(Q_3) = 3$. For the lower bound it's quickest just to consider the three possible cases for deleting two vertices and draw them!

Question 2 seemed to distinguish fairly well.

Part (a) was mostly well done, though disappointingly few definitions of planar graphs were completely correct. In Part (b) candidates often failed to distinguish between the number of crossings in a particular drawing, and the minimum over all drawings. (b)(ii) (a double counting argument, considering pairs $(H, \{e, f\})$ consisting of a subgraph H and two crossing edges e, f both included in H) turned out to be tricky.

Question 3 was least popular, but those who attempted it generally did well, with only part (d) causing difficulty. For a graph of the given form, it has no matching by Tutte's Theorem taking $S = V_0$. When adding an edge, to check that there is a matching it is quicker to argue directly than to try to apply the theorem. For the converse, just start from some S with $q(G - S) > |S|$, argue that $q(G - S) \geq |S| + 2$ by parity, and keeping S fixed argue that adding any edge not already present must join two odd components.

B8.6: High Dimensional Probability

Question 1. A few candidates chose to do this question, and those who attempted find the question challenging. Part (a) (i) is standard and similar example (Gaussian case) has been discussed throughly in the lectures. Part (a)(ii) is new and challenging. Most candidates were able to follow the hint and produced good proofs, including more careful estimates

giving slightly better and precise bounds, while some candidates could not go further after reproduced the hint in the context, and thus lose most of the marks for this part. Part (b) is about Cramer's large deviation principle. (b)(i) is standard book work for computing the rate function for the given three point distribution. Mot candidates did know the definition of the rate function, but are unable to work out the explicit formula which is needed in the second part. On the other hand, candidates, most of them, did know well Cramer's LDP and were able to work out the asymptotic probabilities in terms of rate function.

Question 2. This is a popular question, and most of the candidates did this question. Part (a) is book work, but still some of those attempted could not state Slepian's inequality correctly. Part (b) (i) is also a book work, and most candidates are able to answer this part and give a correct formula for the transition probability function of the OU semi-group. Part (b)(ii) can be answered in various ways, including the one in the lecture. Most candidates carried out a computation via integration by parts, while few candidates noticed the second part follows the first part of (ii), applying to fg immediately. Part (b)(iii) can be proved by using Hölder's inequality, and most candidates, if attempted, did well for this part. Part (c) is new but can be solved by the method developed in the lecture. Ideally an approximation procedure should be used, while few candidates felt it is needed, but this technique is not strictly required for this question. Most candidates understand that one needs to apply the standard method with the p-th power function, so earned rightly the marks for (i). Most candidates who attempted were able to carry on and tried to answer (ii) following the method developed in the course, bur few were able to give a complete solution.

Question 3. Those who attempted were able to follow the methods developed in the lecture to attack this question, but few are able to solve the question substantially, unfortunately. Part (a) (i)(ii) are book work and answered well. Candidates who attempted are able to evaluate the limit via the hint. Most candidates are able to bring the method developed in the lectures and try to solve (a)(iv), but few are able to implement the argument fully. Part (b) is a variation of the Poincare inequality. Candidates are able to carry out the computation to write the left hand side as an integral, while only few candidates are able to go further, and apply Cauchy-Schwartz inequality to complete the solution. Overall candidates understand the methods developed in the lecture, but in general have difficulty to provide the details required and thus implement methods fully.

BO1.1: History of Mathematics

Both the extended coursework essays and the examination scripts were blind double-marked. The marks for the essays and the examination were reconciled separately. The two components carry equal weight in determining a candidate's final mark. The first half of the examination paper (Section A) consists of six extracts from historical mathematical texts, from which candidates must choose two to comment on. The second half (Section B) offers a choice of three essay topics, from which candidates must choose one. The Section B essay accounts for 50% of the overall examination mark, while each Section A answer accounts for 25%.

Throughout the course, candidates were invited to analyse historical mathematical materials in terms of their 'context', 'content', and 'significance'. These were the three aspects that candidates were asked to consider when discussing the extracts in Section A of the

examination paper. A number of candidates used these terms as subheadings in their answers. The term 'significance' was used consistently throughout the course to capture a broad sense of where a given source sits within the historical development of mathematics. This usage was repeatedly emphasised. Some candidates were penalised, however, for interpreting significance only in the narrower sense of 'importance'. Relatedly, some candidates were penalised for taking an overly presentist approach to the extracts: historical mathematics ought to have been treated on its own terms, rather than being compared unfavourably with modern practice. This observation also applies to the extended essays (see below).

Some answers to this year's questions tended towards vagueness. In Question 6, for example, candidates rightly noted that earlier definitions of continuity existed, such as Cauchy's, but failed to state explicitly what those definitions were. Similarly, examples cited in answers to the Section B questions often lacked the necessary detail.

The numbers of candidates who attempted Section A Questions 1–6 were 3, 7, 6, 1, 2, and 10, respectively.

- **Question 1:** This question was linguistically challenging and may not have provided the easiest starting point for a discussion of the parallel postulate. Answers were improved by the inclusion of a diagram illustrating the postulate. It was important to recognise that the question was not solely about axiomatisation: the shift in attitudes towards geometry during the nineteenth century was also significant.
- **Question 2:** A common omission in answers to this question was a failure to provide a full description of Fermat's method. Without referring directly to the mathematical details, candidates found it difficult to address the question of significance adequately. The popularity of this question may have reflected the prominent place Fermat occupied in this year's reading course, which gave candidates relevant knowledge to draw upon. However, some candidates spent too much time discussing Fermat and his wider mathematical contributions, rather than focusing specifically on the extract.
- **Question 3:** The appearance of the word 'synthetic' led some candidates to discuss Pappus's notions of 'analysis' and 'synthesis'. While this was not entirely irrelevant, some candidates devoted too much attention to this aspect at the expense of other points. An important point missing from some answers was that challenge problems such as the one presented in the extract helped to justify and consolidate the Leibnizian calculus during the early decades of the eighteenth century.
- **Question 4:** Although the extract was rather convoluted, Newton's identities were reasonably clear. Nevertheless, candidates may have avoided this question because the expected focus was not immediately apparent.
- **Question 5:** A key aspect of this question was to consider not only what Hamilton was writing, but also why he was writing it.
- **Question 6:** This was the most straightforward and mathematically familiar of the Section A extracts. Consequently, some answers tended towards presentism. Otherwise, the question was handled reasonably well, although some candidates ranged too widely over the history of the rigourisation of analysis instead of focusing specifically on the concept of continuity. Another shortcoming in some answers was the failure to state explicitly what earlier definitions of continuity had been.

Each of the Section B questions (Questions 7–9) was attempted by the same number of candidates.

- **Question 7:** Euler's contributions to analysis were expected to provide the main thread of answers to this question. However, discussing his use of the concept of a function, which had been covered in detail during the course, was not sufficient on its own. The stronger answers went beyond analysis, drawing, for example, on Euler's contributions to number theory. Here, too, some answers were vague: candidates referred to Euler's contribution to the introduction of the concept of a function without specifying his definition or definitions.
- **Question 8:** Two central themes in answers to this question were the development of the understanding of complex numbers and the emergence of a rigorous definition of real numbers. However, candidates were also expected to address other relevant aspects. Some devoted too much attention to complex numbers at the expense of the real numbers. Candidates rarely discussed adequately the attitudes towards numbers at the beginning of the seventeenth century. Some reference should have been made to the persistence, albeit brief, of the ancient Greek distinction between the discrete and the continuous. Few candidates discussed the emergence of the concept of transcendental numbers. Drawing again on the reading course, some candidates included lengthy discussions of number theory, which largely distracted from more relevant issues. Number theory could have been incorporated effectively if framed in terms of attitudes towards calculation versus theory, but this was rarely done.
- **Question 9:** This question was generally handled well. In some answers, however, greater detail would have been helpful regarding the successive definitions of finite groups under discussion.

The extended (coursework) essays were generally of a good standard. Some candidates, however, dived straight into the 17th-century material without giving adequate background on Diophantus's Arithmetica. In a similar vein, the source of Euler's interest in number theory was not always covered explicitly. There was occasionally some disjointedness in how the essays moved from Fermat to Euler to Germain to Kummer. Lots of negative reasons were cited for most 17th-century mathematicians not having studied number the3ory, but it ought to be remembered that there were also positive reasons, such as the distraction provided by the advent of calculus. The stronger essays were those that went beyond the broad assertion that mathematical development is influenced by historical context, which should have been obvious, since it is a theme of the course overall. A few essays leaned a little too heavily on secondary sources, including some slightly disreputable ones.

Statistics Options

Reports of the following courses may be found in the Mathematics & Statistics Examiners' Report.

SB1.1: Applied Statistics

SB1.2: Computational Statistics

SB2.1: Foundations of Statistical Inference

SB2.2: Statistical Machine Learning

SB3.1: Applied Probability

Computer Science Options

Reports on the Computer Science courses may be found in the Mathematics & Computer Science Examiners' Reports.

D. Comments and Recommendations from the Examination Board

The relationship between rescaled marks, grade boundaries and qualitative marking descriptors should be reviewed, with particular attention given to the 2.2/2.1 boundary, where the current 13.5 raw-mark gap creates a significant mismatch. The review should consider whether the existing descriptors adequately reflect the outcomes produced by the scaling process and whether they continue to support qualitative academic judgement rather than placing excessive emphasis on comparative cohort performance. Consideration should also be given to introducing a formal moderation stage to reconcile descriptor-based judgements with algorithmic scaling outcomes.

The examiners would encourage Teaching Committee to reconsider the assessor reporting requirements in the assessors reports. Explicit guidance on appropriate grade boundaries would be helpful to assist assessors when identifying any significant discrepancy between the observed standard of candidates' work and the published descriptors. It was noted that the scaling processes were hampered in cases where assessors had not completed the suggested scaling section of the report.

The standard and difficulty of papers B3.2, B5.3 and B4.2 should be reviewed in light of the substantial moderation required this year. Future papers should be scrutinised carefully to ensure that their content is set at the correct level and is broadly consistent with the expected standard. The checking process should include an explicit assessment of whether a paper is materially easier or harder than intended, informed where appropriate by evidence from previous years, including mark distributions and scaling requirements.

The reasons why the relative ease of B3.2 was not identified during the checking process should also be examined, and the findings should be used to strengthen future scrutiny procedures. Version-control arrangements for examination papers should be improved so that comments from setters and checkers are retained, traceable and demonstrably addressed. A clear audit trail should show how each substantive comment has been resolved.

The number, nature and impact of examination-paper errors should be recorded systematically and monitored over several years. This will allow the Board to determine whether the unusually high volume of errors this year was an isolated occurrence or evidence of a recurring problem.

Exam setters and assessors who are new to the Oxford examination system should be identified at an early stage and required to complete an appropriate induction before undertaking examination duties. New setters should also be given access to experienced mentors, exemplar papers, marking conventions and relevant procedural guidance, with additional support available throughout the examination cycle.

The current approach to examiner appointments and terms of office should be reviewed to reduce the risk of several experienced examiners leaving at the same time. A more staggered or rolling system of rotation should be adopted where possible to preserve continuity. Sufficient overlap between outgoing and incoming examiners should be built into the process.

E. Comments on performance of identifiable individuals

1.Aggregation of marks for the award of the classification on the successful completion of Parts A and B

Classification for a candidate was determined through the following method:

- 10 units at Part A (counting A2 as a double-unit and, for candidates offering 6 long options, two of the long option papers as half-units)
- 6 units (or equivalent) at Part B.

The two average USMs will be:

1. The relative weightings of the Parts is as follows:
   (a) The weighting of Part A is 40%.
   (b) The weighting of Part B is 60%.
2. The relative weightings of the Parts is as follows:
   (a) The weighting of Part A is 100%.
   (b) The weighting of Part B is 0%.

The first class Strong Paper Rule says that to get a first class degree the candidate must have:

(a) average USM ≥ 69.5;

(b) at least 6 units in Parts A and B with USMs ≥ 70;

(c) at least 2 units in Part B with USMs ≥ 70.

The analogous rules apply for II.1 and II.2 degrees. The examiners considered all candidates near each borderline who had been caught by the Strong Paper Rule, that is, who satisfied (a) but failed (b) or (c), and so were due to receive the lower degree class. For two such candidates at the I/II.1 borderline the examiners decided to suspend the examination conventions, and placed the candidates in the first class.

2. Mitigating Circumstance Notices to Examiners

The full board of examiners considered 30 notices, in addition to notices passed on from the Part A Examiners in 2025.

In two cases, papers were disregarded as a result of the notice, with candidates classified on the basis of their remaining work. For one case the Board agreed to allow the candidate to progress to Part C. The Board agreed to review the classification outcome requirements for one candidate, awarding them a 1st. In all other cases the decision made was to take no action.

3. Prizes

Prizes were awarded as follows.

Gibbs Prize £500: Demi Lang, St John's College
Gibbs Prize £200: Jiongjie Hua, St Catherine's College

Part B Junior Mathematical Prize £240: Shing Fung Chan, University College
Part B Junior Mathematical Prize £240: Nathan Adlam, Balliol College

IMA Prize: Wilfred Teasdale, St Hilda's College

F. Names of members of the Board of Examiners

- **Examiners:**

Prof. Alain Goriely (Chair)
Prof. Ben Green
Dr Neil Laws
Prof. Radek Erban
Prof. Xenia De La Ossa
Prof. Gui Qiang Chen

Prof Matt Tointon (External)
Dr Ed Brambley (External)

- **Assessors:**

Prof. Andre Henriques
Prof. Alan Lauder
Prof. Andrea Mondino
Prof. Ben Hambly
Dr. Cameron Gates Rudd
Dr Catherine Wilkins
Prof. Christopher Beem
Prof. Christopher Breward
Prof. Christopher Douglas
Prof. Christopher Hollings
Prof. Christoph Reisinger
Prof. Coralia Cartis
Dr. Corentin Bodart
Prof. Damian Rössler
Prof. Dawid Kielak
Prof. Derek Moulton
Prof. Dmitry Belyaev
Prof. Eamonn Gaffney
Dr. Edgar Sucar
Prof. Emmanuel Breuillard
Prof. Endre Suli
Dr. Fedor Pavutnitskiy
Prof. Fernando Alday
Dr. Giovanni Italiano
Prof. Harald Oberhauser
Prof. Hulya Arguz
Prof. Ian Hewitt
Dr. Jakob Zeitler
Prof. James Maynard
Prof. James Newton
Prof. James Sparks
Prof. Jamshid Derakhsan
Prof. Jan Kristensen
Prof. Jared Tanner
Dr. Jaroslav Fowkes
Dr. Jasmina Panovska-Griffiths
Prof. Jason Lotay
Prof. Jon Keating
Prof. Jose Carrillo de la Plata
Prof. Kevin McGerty
Prof. Kobi Kremnitzer
Prof. Konstantin Ardakov
Prof. Lionel Mason
Dr. Luciana Basualdo Bonatto
Dr. Lukas Brantner
Prof. Maria Bruna
Prof. Mark Mezei
Dr. Martin Bays
Prof. Massimiliano Gubinelli
Prof Melanie Rupflin
Prof. Mohit Dalwadi
Dr. Nathaniel Pritchard
Dr. Nazem Khan
Prof. Oliver Riordan
Dr Paolo Marimon
Prof. Patrick Rebeschini
Prof. Paul Dellar
Dr. Paz Fink Shustin
Prof. Peter Howell
Dr. Philip Beeley
Prof. Philip Maini
Prof. Radek Erban
Prof. Reuben Binns

Dr Robert Hinch
Prof. Robin Thompson
Prof. Sam Cohen
Prof. Stuart White
Prof. Victor Flynn
Dr. Victor-Alexandru Darvariu
Prof. Vidit Nanda
Dr. Vukasin Stojisavljevic
Dr. William Hart
Dr. William Hide
Dr. Xinyu Li
Dr. Yangchen Pan
Prof. Yuji Nakatsukasa
Prof. Zhongmin Qian