

Examiners' Report:

Final Honour School of Mathematics Part A

Trinity Term 2026

October 27, 2026

Part I

A. STATISTICS

- **Numbers and percentages in each class.**

See Table 1.

Table 1: Numbers in each class

Range	Numbers					Percentages %				
	2026	(2025)	(2024)	(2023)	(2022)	2026	(2025)	(2024)	(2023)	(2022)
70–100	43	(50)	(44)	(44)	(59)	34.4	(34.97)	(34.59)	(30.77)	(36.65)
60–69	56	(66)	(64)	(67)	(71)	44.8	(46.15)	(48.12)	(46.85)	(44.1)
50–59	20	(19)	(22)	(25)	(22)	16	(13.29)	(16.54)	(17.48)	(13.66)
40–49	-	(-)	(-)	(-)	(-)	-	(-)	(-)	(-)	(-)
30–39	-	(-)	(-)	(-)	(-)	-	(-)	(-)	(-)	(-)
0–29	-	(-)	(-)	(-)	(-)	-	(-)	(-)	(-)	(-)
Total	125	(143)	(133)	(143)	(161)	100	(100)	(100)	(100)	(100)

- **Numbers of vivas and effects of vivas on classes of result.**

Not applicable.

- **Marking of scripts.**

All scripts were single marked according to a pre-agreed marking scheme which was strictly adhered to. The raw marks for paper A2 are out of 100, and for the other papers out of 50. For details of the extensive checking process, see Part II, Section A.

- **Numbers taking each paper.**

All 125 candidates are required to offer the core papers A0, A1, A2 and ASO, and five of the optional papers A3-A11. Statistics for these papers are shown in Table 2 on page .

Table 2: Numbers taking each paper

Paper	Number of Candidates	Avg RAW	StDev RAW	Avg USM	StDev USM
A0	125	27.91	7.58	65.82	9.98
A1	125	34.07	8.24	66.57	10.92
A2	125	63.52	18.12	67.13	12.68
A3	76	37.57	8.66	68.79	14.09
A4	114	34.14	8.81	68.57	12.51
A5	76	30.55	8.45	67.41	10.96
A6	61	33.15	7.64	66.26	10.86
A7	35	30.09	10.57	63.77	15.57
A8	120	30.5	7.88	66.38	10.69
A9	59	30.98	9.34	65.73	14.18
A10	25	35.44	7.11	67.84	10.33
A11	31	32.1	10.02	67.03	15.6
ASO	125	30	7.79	66.72	10.24
A13	30	27.83	7.91	66.37	8.84

B. New examining methods and procedures

Starting with this academic year, core papers have been set by the course lecturers, and marked by the examiners. In addition, this year the scaling algorithm used at Parts C and B was introduced to determine the University Standardised Marks for Part A students. For details on this rigorous process, see Part II, Section A.

C. Changes in examining methods and procedures currently under discussion or contemplated for the future

None.

D. Notice of examination conventions for candidates

The first notice to candidates was issued on the 9th February 2026 and the second notice on the 5th May 2026.

These can be found at <https://www.maths.ox.ac.uk/members/students/undergraduate-courses/ba-master-mathematics/examinations-assessments/examination-20>, and contain details of the examinations and assessments. The course handbook contains the link to the full examination conventions and all candidates are issued with this at induction in their first year. All notices and examination conventions are online at <https://www.maths.ox.ac.uk/members/students/undergraduate-courses/examinations-assessments/examination-conventions>.

Part II

A. General Comments on the Examination

Acknowledgements

- Hannah Ross for her work in supporting the Part A examinations throughout the year, and for her help with various enquiries throughout the year.
- Waldemar Schlackow and Matt Brechen for running the database and the algorithms that generate the final marks, without which the process could not operate.
- Charlotte Turner-Smith and Rosalind Mitchell for their help and support, together with the Academic Administration Team, with marks entry, script checking, and much vital behind-the-scenes work.
- The assessors who set their questions promptly, provided clear model solutions, took care with checking and marking them, and met their deadlines, thus making the examiners' jobs that much easier.
- Several members of the Faculty who agreed to help the committee in the work of checking the papers set by the assessors.
- The internal examiners and assessors would like to thank the external examiners, Prof Ali Taheri and Prof Mark Blyth, for helpful feedback and much hard work throughout the year, and for the important work they did in Oxford in examining scripts and contributing to the decisions of the committee.

Timetable

The examinations began on Monday 15th June and ended on Friday 26th June.

Mitigating Circumstances Notices to Examiners

A subset of the examiners (the 'Mitigating Circumstances Panel') attended a pre-board meeting to band the seriousness of the individual notices to examiners. The outcome of this meeting was relayed to the Examiners at the final exam board, who gave careful regard to each case, scrutinised the relevant candidates' marks and agreed actions as appropriate.

Setting and checking of papers and marks processing

As a change from previous years, questions for the core papers A0, A1 and A2, were set by the lecturers but marked by the examiners with the assistance of assessors. The papers A3-A11, as well as each individual question on ASO, were set and marked by the course lecturers/assessors. The setters produced model answers and marking schemes led by instructions from Teaching Committee in order to minimize the need for recalibration.

The internal examiners met in December to consider the agreed drafts for Michaelmas Term courses (A0, A1, A2 and A11). Corrections and modifications were agreed by the internal examiners and the revised questions were sent to the external examiners.

In a second meeting the internal examiners discussed the comments of the external examiners and passed on to the exam setters for action. The same cycle was repeated in Hilary term for

the Hilary term long option courses and at the end of Hilary and beginning of Trinity term for the short option courses. *Papers A8 and A9 are prepared by the Department of Statistics and jointly considered in Trinity term.* Before questions were submitted to the Examination Schools, setters were required to sign off on a camera-ready copy of their questions.

The whole process of setting and checking the papers was managed digitally on SharePoint. Examiners adopted specific and detailed conventions to help with version checking and record keeping.

Examination scripts were collected by the markers from Exam Schools or delivered to the Mathematical Institute for collection by the markers and returned there after marking. A team of graduate checkers under the supervision of Rosalind Mitchell and Charlotte Turner-Smith sorted all the scripts for each paper, cross-checking against the mark scheme to spot any unmarked questions or part of questions, addition errors or wrongly recorded marks. Also sub-totals for each part were checked against the marks scheme, noting any incorrect addition.

Determination of University Standardised Marks

The examiners followed the Department's established practice in determining the University standardised marks (USMs) reported to candidates. Papers for which USMs are directly assigned by the markers or provided by another board of examiners are excluded from consideration.

For details on the scaling process please see the Mathematical Institute's website where it is outlined in full: [Determination of University Standardised Marks](#).

A preliminary meeting of the internal examiners was held in advance of the final exam board meeting to compare the default settings produced by the algorithm alongside the reports from assessors. Adjustments were made to the default settings as appropriate, paying particular attention to borderlines and to raw marks which were either very high or very low. Where the examiners were in doubt as to the most appropriate scaling, the preliminary scalings were held over to the final exam board meeting, where the factors considered by those in the preliminary meeting were reviewed and weighed before a final decision was made.

Table 3 on page 5 gives the final positions of the corners of the piecewise linear maps used to determine USMs.

In accordance with the agreement between the Mathematics Department and the Computer Science Department, the final USM maps were passed to the examiners in Mathematics & Computer Science. USM marks for Mathematics papers of candidates in Mathematics & Philosophy were calculated using the same final maps and passed to the examiners for that School.

Table 3: Position of corners of the piecewise linear maps

Paper	P_1	P_2	P_3	Additional Corners	Difficulty Score
A0	10.37;37	21;59.5	31.07;69.5	40.54;80 ; 50;100	4.28
A1	14.48;37	26.54;59.5	38.6;69.5	44.3;80 ; 50;100	-2.71
A2	25.05;37	45.92;59.5	70.04;69.5	85.02;80 ; 100;100	1.74
A3	15.85;37	29.06;59.5	41.12;69.5	45.56;80 ; 50;100	-5.23
A4	13.7;37	25.11;59.5	37.17;69.5	43.59;80 ; 50;100	-1.28
A5	11.56;37	21.19;59.5	33.25;69.5	41.63;80 ; 50;100	2.64
A6	14.25;37	26.12;59.5	37;69.5	44.09;80 ; 50;100	-2.29
A7	13.49;37	24.73;59.5	35;69.5	43.4;80 ; 50;100	-0.9
A8	12.38;37	23.7;59.5	33;69.5	42.38;80 ; 50;100	1.13
A9	13.27;37	24.33;59.5	35;69.5	43.2;80 ; 50;100	-0.5
A10	14.88;37	26;59.5	39.34;69.5	44.67;80 ; 50;100	-3.45
A11	13.53;37	24.8;59.5	36;69.5	43.43;80 ; 50;100	-0.97
A12	16.29;37	29.86;59.5	40;69.5	45.96;80 ; 50;100	-6.03
ASO	12.22;37	22.4;59.5	33;69.5	42.23;80 ; 50;100	4.22
A13	10.7;37	19.61;59.5	31.67;69.5	40.84;80 ; 50;100	1.43

Table 4 gives the resulting final rank and percentage of candidates with this overall average USM (or greater).

Table 4: Rank and percentage of candidates with this overall average USM (or greater)

Av USM	Rank	Candidates with this USM or above	%
90.3	1	1	0.8
88.5	2	2	1.6
87.8	3	3	2.4
87.7	4	4	3.2
85	5	5	4
84.3	6	6	4.8
84.1	7	7	5.6
82	8	8	6.4
81.3	9	9	7.2
80.8	10	10	8
80.5	11	11	8.8
80.3	12	12	9.6
80.2	13	13	10.4
79.6	14	14	11.2
78.4	15	15	12
78.3	16	16	12.8
77.9	17	18	14.4
77.9	17	18	14.4
77.1	19	19	15.2

Table 4: Rank and percentage of candidates with this overall average USM (or greater) [continued]

Av USM	Rank	Candidates with this USM or above	%
76.7	20	20	16
76.6	21	22	17.6
76.6	21	22	17.6
76.1	23	23	18.4
75.6	24	24	19.2
73.6	25	25	20
73.3	26	26	20.8
72.9	27	27	21.6
72.6	28	28	22.4
72.55	29	29	23.2
72.4	30	31	24.8
72.4	30	31	24.8
72.3	32	32	25.6
72	33	33	26.4
71	34	34	27.2
70.7	35	35	28
70.6	36	37	29.6
70.6	36	37	29.6
70.5	38	38	30.4
70.4	39	39	31.2
70.2	40	40	32
70	41	41	32.8
69.6	42	43	34.4
69.6	42	43	34.4
69.4	44	44	35.2
69.3	45	45	36
69	46	47	37.6
69	46	47	37.6
68.9	48	51	40.8
68.9	48	51	40.8
68.9	48	51	40.8
68.9	48	51	40.8
68.8	52	52	41.6
68.7	53	54	43.2
68.7	53	54	43.2
68.56	55	55	44
68.5	56	56	44.8
68.4	57	57	45.6
68.2	58	58	46.4
68	59	60	48
68	59	60	48
67.8	61	62	49.6

Table 4: Rank and percentage of candidates with this overall average USM (or greater) [continued]

Av USM	Rank	Candidates with this USM or above	%
67.8	61	62	49.6
67.3	63	63	50.4
67.2	64	64	51.2
66.7	65	65	52
66.6	66	66	52.8
66.4	67	68	54.4
66.4	67	68	54.4
66.3	69	69	55.2
66.2	70	71	56.8
66.2	70	71	56.8
66.1	72	73	58.4
66.1	72	73	58.4
65.7	74	75	60
65.7	74	75	60
65.2	76	76	60.8
65	77	77	61.6
64.9	78	78	62.4
64.6	79	79	63.2
64.3	80	80	64
64.2	81	81	64.8
64.1	82	82	65.6
64	83	83	66.4
63.7	84	84	67.2
63.4	85	86	68.8
63.4	85	86	68.8
63.3	87	88	70.4
63.3	87	88	70.4
63	89	90	72
63	89	90	72
62.6	91	91	72.8
62.3	92	92	73.6
62.1	93	94	75.2
62.1	93	94	75.2
61.8	95	95	76
61.5	96	96	76.8
60.2	97	97	77.6
60	98	98	78.4
59.9	99	99	79.2
59.4	100	100	80
59.3	101	101	80.8
58.8	102	103	82.4
58.8	102	103	82.4

Table 4: Rank and percentage of candidates with this overall average USM (or greater) [continued]

Av USM	Rank	Candidates with this USM or above	%
58.3	104	105	84
58.3	104	105	84
58	106	106	84.8
57.7	107	108	86.4
57.7	107	108	86.4
57.4	109	109	87.2
57.2	110	110	88
55	111	111	88.8
54.8	112	112	89.6
54.2	113	113	90.4
53.9	114	114	91.2
53.8	115	115	92
53.3	116	116	92.8
52.4	117	117	93.6
51.9	118	119	95.2
51.9	118	119	95.2
48.9	120	120	96
48.4	121	121	96.8
47.8	122	122	97.6
46.2	123	123	98.4
40.2	124	124	99.2
30.1	125	125	100

B. Equality and Diversity issues and breakdown of the results by gender

Table 5, page shows percentages of male and female candidates for each class of the degree.

Table 5: Breakdown of results by gender

[illegible]

C. Detailed numbers on candidates' performance in each part of the exam

Individual question statistics for Mathematics candidates are shown in the tables below.

Paper A0: Linear Algebra

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14.86	15.32	4.71	82	4
Q2	12.6	12.86	3.92	88	4
Q3	13.63	13.76	5.12	80	1

Paper A1: Differential Equations 1

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.56	16.56	6.37	41	0
Q2	16.75	16.75	3.79	106	0
Q3	17.51	17.51	5.21	103	0

Paper A2: Metric Spaces and Complex Analysis

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	20.06	20.15	5.05	118	1
Q2	17.26	17.26	6.12	108	0
Q3	10.57	11.83	6.44	24	4
Q4	12.3	12.44	5.76	85	3
Q5	14.32	14.93	6.09	94	5
Q6	13.81	14.45	6.87	66	6

Paper A3: Rings and Modules

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.72	17.97	5.19	63	1
Q2	19.29	19.25	5.4	67	1
Q3	19.3	19.68	3.14	22	1

Paper A4: Integration

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.54	18.25	5.36	61	4
Q2	13.94	13.94	5.84	67	0
Q3	18.24	18.45	4.46	100	2

Paper A5: Topology

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.16	16.34	5.05	74	1
Q2	13.9	13.9	4.85	59	0
Q3	14.8	15.42	4.58	19	1

Paper A6: Differential Equations 2

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14.98	15.38	4.15	55	3
Q2	17.38	17.48	3.74	25	1
Q3	17.47	17.6	5.24	42	1

Paper A7: Numerical Analysis

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	13.82	13.82	5.67	33	0
Q2	16.44	16.44	5.36	32	0
Q3	14.2	14.2	7.5	5	0

Paper A8: Probability

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.08	15.29	4.1	101	2
Q2	15.53	16.46	5.6	48	5
Q3	14.3	14.57	5.12	91	5

Paper A9: Statistics

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14.79	15.28	5.88	40	2
Q2	16.77	16.77	4.47	52	0
Q3	11.45	13.27	7.52	26	5

Paper A10: Fluids and Waves

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	19.29	19.29	4.07	14	0
Q2	19	19	3.22	24	0
Q3	13.33	13.33	3.42	12	0

Paper A11: Quantum Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14.48	14.85	5.69	26	1
Q2	16.32	16.9	5.67	29	2
Q3	17	17	6.63	7	0

Paper ASO: Short Options

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	12.06	12.68	7	31	2
Q2	14.17	14.45	4.47	33	3
Q3	15.06	15.31	4.47	62	2
Q4	11	11.59	6.11	27	3
Q5	17.03	17.03	3.44	62	0
Q6	16.06	16.06	4.18	35	0

D. Comments on papers and on individual questions

The following comments were submitted by the assessors.

Core Papers

A0: Algebra 1

Question 1: Candidates did well on part (a). A common mistake was to state that a linear map admits a Jordan canonical form with two Jordan blocks if and only if the minimal polynomial is $(x - \lambda_1)(x - \lambda_2)^2$ with $\lambda_1 \neq \lambda_2$. This omits the cases where the map has two Jordan blocks with the same eigenvalue, which gives minimal polynomial $(x - \lambda)^2$.

Part (b)(i) was straightforward (given the condition recalled in part (a)), but many candidates computed the discriminant of the polynomial $x^2 + 6ab + 3b^2$ as $(6ab)^2 - 12b^2$. Part (b)(ii) received many good answers. There were some computational errors, and some candidates omitted the case $a = b = 0$. A few candidates described the Jordan canonical form in terms of the minimal polynomial, without making explicit which values of a, b gave which minimal polynomial. Such answers received partial credit. In part (b)(iii) a very common issue was that candidates asserted that the only field of characteristic 2 was the finite field $\mathbb{F}_2$. Partial credit was given for answers which only considered $\mathbb{F}_2$, often treating the four possible values of (a, b) case-by-case. Quite a few candidates skipped (b)(iv). Only a small number of candidates noticed that the answer should be given by 7 linearly independent functionals (i.e. a basis for the annihilator U°), with many answers giving a list of 8 functionals spanning the annihilator (usually the column, row and diagonal sums).

Question 2: Part (a)(i), a simple form of the primary decomposition theorem, received good answers. In part (a)(ii) it was only implicit in the question that n is the dimension of V . Credit was given for any valid application of rank-nullity. Part (a)(iii) received many correct answers applying (a)(ii). Some candidates assumed that $f(x)$ and $g(x)$ were coprime throughout part (a).

In part (b)(i) most candidates applied Bézout's lemma as expected. It was common for candidates to prove that $\text{Im } h(T) \subset \text{Im } f(T) + \text{Im } g(T)$ but then assert that they had proved equality of the two spaces. In part (b)(ii) many candidates were able to deduce from part (b)(i) that $\text{Im } h(T)^2 = \text{Im } f(T)h(T) + \text{Im } g(T)h(T)$. Only a few candidates were able to establish that the sum is direct. Part (b)(iii) also only received a few correct answers. Those candidates who had already considered the factorisation of $f(x)g(x)$ as the product of $h(x)^2$ and two coprime polynomials were more likely to make progress here. In some cases it was asserted that $h(x)$ is coprime to both $f(x)/h(x)$ and $g(x)/h(x)$, which is not necessarily the case.

Almost no candidates gave complete answers to part (c). In a few cases there were attempts to show that part (b)(iii) holds in the infinite dimensional case, not part (a)(iii) as requested. Partial credit was given for showing that the subspace W defined in the hint is an invariant subspace. Candidates who got this far often asserted that $W \cap (\ker f(T) \oplus \ker g(T)) = \ker f(S) \oplus \ker g(S)$ without justification.

Question 3:

The majority of candidates obtained full marks on (a)(i), although some marks were dropped

because the wrong implication ($E_\lambda(k) = \{0\} \Rightarrow \lambda \notin Y$) was proved. Furthermore, candidates rarely got full marks on (a)(ii) because they did not state the theorems used from the course correctly/clearly (commonly not justifying clearly why $m_T|\chi_T$). Almost all candidates struggled on (a)(iii) and very few got full marks or close to full marks. Partial credit was awarded for showing the spaces $E_\lambda(n)$ spanned a subspace of V ; checking $E_\lambda(n) \cap E_\mu(n) = \{0\}$ for $\lambda \neq \mu$ (even though this is not sufficient for showing the subspace is a direct sum in general); and applying the primary decomposition theorem to obtain a direct sum of the form $\bigoplus_\lambda E_\lambda(k_\lambda)$ for some $k_\lambda \leq n$. Very few spotted the correct solution, which was to consider the action of the operators $\prod_{\lambda \neq \mu} (T - \lambda I)^n$. Additionally, candidates often dropped marks on (a)(iv) because they only gave a sufficient condition on $\mathbb{F}$, such as $\mathbb{F}$ being algebraically closed.

Part (b)(i) was easily solved by almost all candidates, and candidates were also quite successful with (b)(ii) especially with spotting the inductive argument (although some marks were dropped because candidates did not fully justify why $T - \lambda I$ was self-adjoint). Good solutions to parts (b)(iii) and (b)(iv) were provided, although some candidates erroneously assumed all $E_\lambda(1)$ were 1-dimensional. Roughly half of the candidates managed to find a solution to (b)(v), often using the reasoning that having distinct eigenvalues implies the matrix is diagonalisable.

A1: Differential Equations 1

Question 1. This question covered aspects of the general theory of ordinary differential equations, including the reduction of higher-order equations to first-order systems, a variant of Grönwall's lemma, and an application to continuous dependence on initial conditions.

- **Part (a)** concerned the reduction of a higher-order ODE to a system of first-order ODEs and the derivation of Lipschitz estimates for the resulting system. This was largely bookwork and was answered well by most students.
- **Part (b)** concerned a system of two second-order ODEs. The first part was standard and was answered well by most students, while the second part proved more challenging and was answered successfully by approximately half of the students.
- **Part (c)** concerned a variant of Grönwall's lemma. Approximately half of the students produced complete solutions, while a further quarter made substantial progress.
- **Part (d)** required students to deduce continuous dependence on initial conditions using the result established in part (c). Approximately half of the students produced complete solutions, while a further quarter answered the question well.

The question was chosen by approximately one third of the candidates. The scripts were somewhat polarised, with many solutions of excellent quality and many only just above pass standard. There were relatively few scripts below pass standard and comparatively few of intermediate quality.

Question 2. This question concerned the qualitative theory of dynamical systems in the plane.

- **Part (a)** was bookwork, requiring a proof of the Bendixson–Dulac theorem. This was answered very well by approximately two thirds of the students. A common omission

was a failure to state or correctly use the assumption that the region under consideration must be simply connected.

- **Part (b)** involved first an application of the Bendixson–Dulac theorem to a given dynamical system. Students were then required to determine the critical points, perform a linear stability analysis, establish the existence of a first integral, and apply it to study the system. This part was answered very well by most students. A common misconception was to assume that sharing the same value of the first integral is always sufficient for two points to lie on the same trajectory. While it is a necessary condition, it is sufficient only when the corresponding level set contains no critical points.
- **Part (c)** required a phase-plane analysis of the dynamical system from part (b), together with questions concerning the qualitative behaviour of solutions. The quality of responses varied considerably. Most students produced an essentially correct phase portrait, but relatively few provided rigorous arguments to justify their conclusions regarding the qualitative behaviour. A common imprecision was to treat the set S as a single trajectory, whereas it consists of four distinct trajectories: two equilibrium solutions corresponding to the critical points $(-1, 0)$ and $(1, 0)$, and two heteroclinic trajectories connecting them.

This was the most popular question and was attempted by almost all candidates. Most solutions were of very good or excellent quality. Relatively few scripts were at pass level, and very few were below pass standard.

Question 3. This question covered the method of characteristics for a first-order quasilinear PDE, the maximum principle, and the classification and reduction to canonical form of a second-order PDE.

- **Part (a)** required candidates to solve a first-order quasilinear PDE using the method of characteristics. Many candidates correctly derived the characteristic ODEs for x , y , and z , and solved them subject to the prescribed initial data. Most were then able to eliminate the characteristic parameters and obtain an explicit expression for z in terms of x and y . A common weakness was in the geometric interpretation of the solution: many sketches were inconsistent with the characteristic projections that had been derived analytically. While most candidates correctly observed that the initial data are non-characteristic, this alone is not sufficient to justify existence and uniqueness throughout the half-plane $x > 0$. The correct argument was based on a careful analysis of the characteristic parametrisation, showing that each characteristic projection intersects the initial data line exactly once and that the family of characteristics foliates the entire region $x > 0$.
- **Part (b)** concerned the maximum principle and its application to uniqueness. This part was answered well by most candidates. The majority correctly applied the maximum-principle argument and used it to establish uniqueness by considering the difference of two solutions.
- **Part (c)** concerned the determination of characteristic variables and the reduction of a second-order PDE to canonical form. Many candidates identified the correct characteristic variables and made substantial progress towards obtaining the canonical form. In the case $x > 0$, the most common errors arose in the application of the chain rule

when transforming the second-order derivatives. In the case $x < 0$, the difficulties were more conceptual. Several candidates attempted to apply the hyperbolic construction from the region $x > 0$ without modification, rather than recognising that the equation is elliptic for $x < 0$ and adapting the method accordingly, as discussed in the lectures.

Together with Question 2, this was the most popular question and was attempted by the majority of candidates. Overall, the standard of the solutions was very good. Strong scripts demonstrated a sound grasp of both the analytical and geometric aspects of the material. Weaker scripts often revealed gaps in the underlying bookwork and a limited familiarity with some of the standard techniques developed during the course.

A2: Metric Spaces and Complex Analysis

Question 1 This was the most popular question attempted by all except a small minority of candidates. There were many full solutions.

Part (a) was straightforward. In (b) some candidates quoted without proof that uniform limits of continuous functions are continuous but this was the main task of (b), so did not earn marks. Part (c) was done well by many candidates which then went on to answer (d)(i) very efficiently by observing that $g_x = f_x - f_{x_0}$ and hence is continuous by (c).

Question 2 This was also a popular question with good results. In (a) some candidates ignored the rubric and used that compactness in metric spaces is equivalent with sequential compactness, thus losing some marks. Part (b)(ii) was surprisingly challenging to many candidates. A common mistake was to assume that $|d(x, B) - d(y, B)| \leq \inf_{b \in B} |d(x, b) - d(y, b)|$ which is false in general. Part (c) had many good solutions following the suggested steps, together with (b)(iii).

Question 3 This question was not popular. The bookwork in part a) was done well, but only counted towards one mark. Students did not always see how to apply the MVT in the second part of part ii). The hints in part b) did help the students, but not all thought to use Cauchy-Schwartz. Most people had an idea for b) ii) but did not fully work it out. Most students got full marks for part c) i), and c) ii) went relatively well too (but this was bookwork). Most people struggled with c) iii), but did understand they could use previous parts of the question.

Question 4 Most candidates made good progress a). For b) most attempts guessed the right contour, fewer managed to complete the calculation. For c) few candidates make progress or give a correct argument. A common mistake for c) was to make the change of variable $x = \sqrt{i}y$ (or similar) but completely ignore the resulting contour rotation. Although such approaches are not justified they can coincidentally give the correct result.

Question 5 Most candidates made good progress on the main part of a) although a common loss of marks was to not spell out the precise assumptions of the theorem. Many good attempts for b) though many attempts (wrongly) claimed that the singularity at $z = 0$ is removable. In contrast, nearly everyone attempting c) got high marks and similarly for d).

Question 6 This question was more popular. The bookwork in part a) was mostly done well, some students lost marks on omitting or incorrectly stating the assumptions in the statements. Part b) was a mix, I think the majority did this correctly but some struggled or had no idea. Part c) was where most students got quite far, but not always giving a full

proof. Quite a few students did get full marks despite this. Part d) seemed to be the most difficult one – some students did not attempt this part. Students who did attempted were able to obtain some marks.

Long Options

A3: Rings and Modules

Question 1 was on ideals and proved popular. Part (b), essentially on the definition of the radical of an ideal, was quite well done, with many candidates using the binomial theorem to show the radical was closed under addition. Part (c) proved the most difficult—many people thought the condition of the radical being equal to the original ideal was equivalent to prime, when in fact it is substantially weaker. However many candidates did correctly argue that the relevant ideals in the ring of integers were generated by products of distinct primes. Part (d), on how the radical behaved for products and intersections, was very well done

Question 2 This question, on ring homomorphisms, characteristic and field extensions proved popular and was generally well done. The earlier sections were generally attempted successfully, though quite a few people omitted to show the isomorphism map in (a) was well-defined. Some people had trouble with finding a suitable irreducible polynomial in part (e).

Question 3 This question, on modules, was less popular. Most people got the bookwork in part (a) right, though some omitted the condition that the ring should be a PID or that the module should be finitely-generated. A few confused the cyclic and primary decomposition theorems. Parts (b) and (c) were very well done, with candidates showing familiarity with primary decomposition and Smith normal form. As expected, part (d) proved much more challenging, but a few people got the idea of bounding the number of roots of $x^d = 1$ by the Euclidean algorithm and then applying the cyclic decomposition of (a) to get a contradiction if the number of summands was greater than one.

A4: Integration

Question 1 (a) [5 marks] Standard bookwork, there are still a lot of students who lose marks when proving the outer measure $m^*(A)$ is positive.

(b) (i) [3 marks] this part is a natural extension of (a). The key point is based on the countable additivity of measure. Merely using subadditivity, the result is only $\leq \infty$, which is not the desired result. This is the main reason that some students lose marks.

(ii) [5 marks] The majority of the students directly write the open interval $I = (a, b)$, with a, b finite numbers. This needs justification since an open interval can also be unbounded. One may use monotonicity of the measure μ to deduce the existence of (a, b) with finite, nonzero measure.

Among all the students who attempted this problem, only three students use Heine-Borel to justify the bounded set can be covered by finite many open intervals shifted from I . Most students who attempted the question manage to get the finite cover by shifting (a, b) numerically, which is also acceptable.

(c) (i) [5 marks] The argument then follows from applying the hint, with a bit justification about the fact that E_2 is non-empty.

(ii) [4 marks] Most of the students who attempted got it. Some said E_2 could be anything, without specifying a particular choice of E_2 , which loses some marks.

Question 2 (a)(i) [1 mark] Only several students lose the mark.

(ii) [4 marks] It is important to find the controlling function g in DCT, independent of R . There occur some issues: some still use functions depending R ; some are reluctant to switch notation from n to R ; some tries to justify the integrability in $(0, \infty)$ for the their g function which blows up like $1/t$ when approaching 0.

[3 marks] the $[R, \infty)$ part of the integral is controlled already in (ii). So it suffices to consider the part near $(0, 1)$. As $t \rightarrow 0$ both parts of the integrand blow up, nevertheless their difference is not singular. Quite a lot of students treated two singular parts separately, which is the main reason of losing marks.

(iv) [7 marks] One could either use Fubini and Tonelli, or use differentiation theorem. The conditions of the theorems should be properly checked. The most serious problems are integrating the two parts of integrand directly with a “formal” formula. This is seriously wrong.

(b) (i) [5 marks] Standard application of Fubini and Tonelli. The proof needs careful justification of the assumptions of the theorems. Some students mistook the integrability of $f(s)g(t)$ as for $f(s)g(s)$. The majority of the students did well here.

(ii) [5 marks] Some students apply (i) carefully to get the final result by direct calculation. Only a few drew the figure for the left hand-side, the result then follows immediately from using (i). This is actually the preferred method. Some students add more assumptions to f and g to make their arguments through, such as $f, g \geq 0$, or f and g are continuous. They lose marks due to such alterations.

Question 3 a) (i) was merely a definition, so very well solved.

And (ii) was part statement of a theorem, which was well done. The other part was giving a counterexample. This was much harder and either not solved at all, or solved very well.

(iii) Was a proof they had seen in the lectures, again very well solved.

b) (i) and (ii) were very well solved.

(iii) Gave most points and was hard for most students. Most missed some parts of it. Some didnt know how to tackle the question at all, but most tried and missed some details.

(iv) Was very hard and mostly remained unsolved. Some realised the idea and could reprove it.

A5: Topology

Question 1.

Almost all students attempted this question.

Part ai was generally very well done.

In part aii , most students managed to prove that Z is not Hausdorff, but struggled with the part of the question relative to connection. Many candidates stated that all subset are disconnected, not realising the need of the infiniteness of the set for the argument to work. It was common for the correct definition of connectedness to be stated in ai , an the wrong

definition to be used in *aii*

Part *aiii* was solved by the majority of the students, as well as part *aiv*.

Part *bi* found a sizeable amount of students struggling to find the correct path, both for the proof and counterexample.

Part *bii* was the one with fewest attempts and fewest solutions. Many incorrect proofs did not use the fact that X was connected, and were therefore wrong, as this is a necessary hypothesis.

Question 2.

The majority of candidates attempted this question. Parts *ai,aii* were generally well done. The definition of the standard topology in *aiii* was given by most candidates but some did not manage to prove that the two topologies are the same, often proving only one inclusion or giving insufficient or too informal arguments and obtaining only partial credit. Part *aiv* was generally well done. Many candidates solved *bi* using sequential compactness. Part *bii* was done by most candidates, quite a few who did not attempt *bi* solved the problem assuming the result of *bi* and received full marks for this part. A common mistake for *bii* was to state that the set of all sequences with rational terms is countable. Only few candidates managed to solve completely *biv* using *biii* and a diagonal argument, but some obtained partial credit for having the right idea. Question 2 a) was marked out of 13, and 2 b) was marked out of 12.

Question 3.

Relatively few students attempted this. For *ai* some candidates did not give the topology of the quotient space. Part *aii* was generally well done but quite a few candidates apparently forgot to do the last part. There were a few mistakes in *aiii* as many candidates did not realize that they had to use the quotient topology. Most candidates defined the star correctly but quite a few missed the relationship to the previous part and some asserted that the star is open by definition as a union of open sets, which is clearly false. Several students realized that they could take a cover by stars of vertices and contradict compactness for *av* and did the first part, but the arguments given for edges were generally incorrect. Parts *bi,bii* were generally well done. Many candidates solved the first part of *biii* and most candidates that managed to represent the surface as a polygon with a *cssi* gave a complete answer to *biii* and received full marks.

A6: Differential Equations 2

Question 1: This question was the most popular, attempted by nearly every candidate. Part (a) was generally answered well, though a few candidates made algebraic errors when calculating the adjoint boundary conditions in the first part. These candidates did not seem to spot that their solutions did not admit nontrivial solutions for $a = 3$ (as highlighted in the second part of (a)), and so this error often led to struggles with this question in general. A surprising number of candidates were unable to answer part (b) in full. A common error was to choose y_1 and y_2 that did not satisfy the boundary conditions, and then later fail to ensure that the boundary conditions were satisfied. Several students tried to use the delta function formulation despite the question asking for variation of parameters. Many students ended

up deriving the formulae instead of quoting them as standard results (as highlighted in the question). In part (c), many students did not seem to realise that they could use part (b) [or even the solution given therein] to evaluate the result. Generally, those who did were able to answer this part fully (notwithstanding a small number of candidates struggling to integrate $(s-1)/(s+1)$). Not many students attempted part (d); those that did were generally happier with the first part than the second. Some students incorrectly interpreted FAT as using the generic zero eigenfunction i.e. without imposing boundary conditions, and hence their version of FAT generated two inconsistent conditions for b .

Question 2: This question was the least popular, but was answered fairly well by those that gave it a serious attempt. The only recurrent issue with part (a) was the common neglect of the special $c = 0$ case. Part (b) was generally fine, though several students used the lower indicial value to calculate the first Frobenius series, with predictable issues arising. A few students were not able to evaluate their calculated series in closed form. Part (c) was generally not answered well, with the special logarithm and terminating cases rarely determined correctly in the first part. No one spotted the special case at $\alpha = 1$ that arises from the second Frobenius series in the second part.

Question 3: This question was generally answered well. In part (a), most candidates were comfortable with the first part, with occasional algebraic errors when determining the first correction term. In the second part, a surprising number of candidates failed to consider the additional roots that arose, though most were able to understand and articulate that the same roots from the first part would arise here. There was a wide range of answers to part (b). While several candidates were able to give full solutions to this part, many struggled with making the correct transformations due to the boundary layer being at $x = -1$ (rather than $x = 0$). Several candidates tried to (incorrectly) justify a boundary layer in the middle of the domain at $x = 0$, which inevitably led to inconsistencies.

A7: Numerical Analysis

Question 1 Q1 was attempted by most candidates. Discussion on uniqueness is needed in (a)(ii). (iv) is not difficult but needs an argument that shows $\|f - \tilde{f}\|_\infty \leq \delta$. (b) seemed mostly fine. (c) (i) many candidates failed to fully justify the eigenvalues of $A^T A$ are positive. (ii) Many attempted to start from $A^T A$, but this is not the simplest approach.

Question 2 (a) Most students correctly provided the theory for the Gershgorin Theorems, and correctly obtained a loose bound. Few realised the need to use a similarity transform to sharpen the bounds, and even fewer carried it out correctly, with the common mistake being to introduce an overlap with other discs.

(b) Parts (i) and (ii) were done well by most students, except for errors in performing the required integrals and arithmetic. Part (iii) was also done well by most students.

Question 3 Not many students did this question. Parts (a) and (b) were done best, with the other parts being roughly equally difficult.

A8: Probability

See Mathematics and Statistics report.

A9: Statistics

See Mathematics and Statistics report.

A10: Fluids and Waves

Question 1: This question was quite popular, and generally quite well done. Part (a) was generally well tackled, though some students struggled with the vector calculus required to determine the correct coefficient in $\phi \propto 1/r$. In part (b), a surprising number of students tried to resolve the uniform velocity into the spherical polar coordinates (with differing levels of success), rather than simply noting that the relevant potential must be $Uz = Ur \cos \theta$.

Question 2: This question was extremely popular, being attempted by almost all candidates. This question was also very well done on the whole. The most common errors here were in part (c)(iii) where the ‘instantaneous velocity of the vortex’ was found in the conformally mapped plane (rather than using the chain rule to determine the physical velocity).

Question 3: This question was found to be quite difficult, with only a few good attempts at part (b). Part (a), was quite well done, though only one or two candidates realised that in this case it is as if there is a single fluid (since the densities are equal and surface tension is absent); hence the fact that the ratio $B/A = \exp(-kh)$ reflects the exponential decay of the flow away from the free surface at $z = 0$. Part (b) was significantly more complicated with the change to the pressure boundary condition at $z = 0$ frequently missed while the sign of the pressure jump at $z = -h$ was often incorrect.

A11: Quantum Theory

Questions 1 and 2 were the most popular, while only 1/5 of students chose Question 3. The average marks were remarkably close on all three questions. Bookwork and already seen mechanistic calculations were performed to a high standard by most students. However, only the strongest students gave the correct interpretation of several of these calculations.

Question 1

Part a) was done well. Some students seemingly struggled with complex exponentials, but many ultimately succeeded in solving the problem by converting to cosine and sine. In part b) the decomposition of Ψ into eigenstates was done well, but the normalisation $\mathcal{N}$ was often not treated with enough care. A few students went through pages of integrals instead of using the orthonormality of eigenstates. The negative eigenvalues of P were missed by several students.

In part c) the unseen commutator was computed accurately by most students. Only few used that $\Delta_{\Psi}(P) = 0$ because Ψ is an eigenstate; instead students went through explicit computation.

Question 2

Part a)(i) was done very well. In a)(ii) many students started with the rewriting $(a_+)^2(a_-)^2 = a_+Na_-$, which was not ideal. Very few seemed to recall the standard formula $[N, a_{\pm}] = \pm a_{\pm}$, although some have derived it in the process of computing the commutator of Hamiltonians. Surprisingly, very few students used the basic fact that the eigenvalues of the linear operator N^k are n^k ($n \in \mathbb{N}$), and several made the erroneous claim that commuting operators have the same eigenvalues. Some have given very long (and correct) arguments establishing the

eigensystem of N , which was however not required.

In part b) students were doing correct algebraic manipulations, but often somewhat aimlessly. Many have discovered that $(b_{\pm})^2 = 0$ during their explorations, but failed to appreciate how much this simplifies the entire problem. The strongest students gave elegant and succinct solutions.

Question 3

Part a) was bookwork and was done very well except for keeping track of powers of $\hbar$.

The setup of part b) was unfamiliar, but this has caused no issue to students. In the first step only L_3 , but not L^2 was measured, correspondingly in case of the outcome $0 \cdot \hbar$ one needed to project onto a two-dimensional subspace and renormalise the wavefunction appropriately. A common mistake was ignoring wavefunction collapse and acting with L_1 on the original wave function χ . Most students inverted the relations given in the problem to express ψ in the φ basis, instead of computing overlaps, e.g. $P_{\psi}(L_1 = m\hbar) = |\langle \varphi_{1,m} | \psi \rangle|^2$, which would have been much simpler. Only the strongest students obtained the correct probabilities of the sequence of measurements.

A13 Geometry

Question 1. This question went well with a good range of marks. It was the most popular question, with most students attempting it.

- (a) This was bookwork and mainly done well. The common error was not to give the characterisation of the tangent spaces in terms of the differential (which is clearly needed for (b)).
- (b) This was similar to an example in the lectures and was usually done quite well. Common errors were related to not verifying the conditions of the Regular Value Theorem: i.e. that the map defined is smooth and maps between vector spaces (thus submanifolds). Almost all students showed that $O(n)$ is a submanifold but some failed to argue correctly that $SO(n)$ is also a submanifold with the same tangent spaces.
- (c) This part was done well. The common error was not to use the fact that A was an isometry (as given in the hint) to argue that the tangent space to the sphere at Ae_1 is spanned by $\{Ae_2, \dots, Ae_n\}$.
- (d)
 - (i) Most students understood to use the Regular Value Theorem to get the dimension. The main error was not to show that M_x is non-empty (by extending x to an oriented orthonormal basis, for example).
 - (ii) Many students did not attempt this part. If they did, then most students understood what left coset M_x should be but did not provide a full solution.

Question 2. The question also went well with a good spread of marks, and was the second most popular question.

- (a)
 - (i) This was standard bookwork and answered correctly by all students.
 - (ii) This bookwork question was almost always answered correctly. The only common error was to forget the assumption that N is connected.
- (b) This part provoked a mixed response. Most students that attempted this part under-

stood the idea, namely that being a regular value of $h|_M$ was equivalent to having a vector in $T_p M$ whose projection onto its n th component was non-zero, for all $p \in M \cap P_c$. However, many failed to give a complete argument.

- (c) This part proved challenging for students, though almost all attempted using (b). The main difficulty was to write $h|_M$ as a function of (θ, ϕ) , do the differential and find when it was zero correctly. Several also forgot that points where submanifolds do not meet are automatically transverse intersection points vacuously. A few students correctly understood the geometric picture and obtained the correct values for c .
- (d) Many students realised that the degree is zero, but failed to justify it fully. Most students failed to show that the degree was well-defined or that the point picked in $\mathcal{S}^2$ was a regular value.
- (e) Most students had the right idea, i.e. to wind around the ϕ direction m times. The most common error was to fail to argue why the degree is negative for $m < 0$ and zero for $m = 0$.

Question 3. The question went quite well, with a spread of marks from low to high. It was the least popular question, but still a good proportion of the students attempted it.

- (a) This was standard bookwork, but a common mistake was to omit the assumption that the linear map is injective.
- (b) This was similar to a problem sheet question and was usually done well. The most common error was not to argue why when one writes $v_{n+1} = \sum_{i=0}^n \lambda_i v_i$ for $\lambda_i \in \mathbb{F}$ that $\lambda_i \neq 0$ for all i using the general position hypothesis.
- (c)
 - (i) The most common mistake here in this bookwork question was to omit the description of the orientation-preserving isometries as projective linear transformation, and many descriptions given were incorrect.
 - (ii) This was a straightforward computation question and done correctly whenever it was attempted.
- (d)
 - (i) This proved challenging. As expected, the most common issues occurred in finding the centre and radius of $C(i, r)$, the most common error being to assume that the centre was i . Other typical errors included not observing that T was an isometry and not to argue correctly why $C(w, r)$ is a Euclidean circle.
 - (ii) Many students had good ideas for the first part of this question, but common errors were not to argue adequately why the points were unique or must lie on a geodesic through w . Very few students attempted the final part of the question.

Short Options

ASO: Q1. Number Theory

The question had 4 parts which were mostly unrelated. This led to a reasonable spread of marks, with some excellent complete solutions on the one hand, and some patchy attempts gaining partial marks on the other.

For (c) some candidates found the following shorter solution. Suppose 2027 is not prime and that $q \leq \sqrt{2027}$ is a prime factor. Then $d := \text{ord}_q(2)$ divides both 2026 (since $q \mid 2027 \mid$

$2^{2026} - 1$ and $q - 1$ (FLT). Since $2026 = 2 \times 1013$ and $q < 50$, this forces $d = 2$, so $q = 3$. But one can trivially check that 3 is not a factor of 2027.

ASO: Q2. Group Theory

There were 36 scripts. The average mark was 14.3 and the median 14. The highest mark was 23/25.

All but one candidate received 6 full marks for Question a), which was bookwork.

Question b(i) should have been clear to everyone (its proof was even discussed in class), but a number of students failed to reproduce the simple argument. Question b(ii) was more tricky and was done well in approximately a third of the scripts.

Question c) was well treated overall and more than two third got at least 6 marks out of 8.

Question d(i) and ii) were generally well done, and when they were not it revealed some serious misunderstanding.

The last question d(iii) was the hardest and worth 4 marks. No one managed to get the full 4 marks on this question, which came at the end of a long question and required time to treat properly.

The exam's markscheme attributed a lot of marks to bookwork and to questions that had been discussed in class (roughly 8 marks in total). Overall I was really pleased by the performance of the cohort's top half.

ASO: Q3. Integral Transforms

The question had two parts, part (a) on distributions and part (b) on integral transforms.

Part (a)(i) asked students to prove a version of the product rule, which was generally well done. This section contained a mistake. The product rule was stated as

$$(FT)' = f'T + Tf';$$

, where one of the dashes should be moved to the other factor. This mistake was taken into account when marking.

Part (a)(ii) asked to obtain general n -th order derivatives of $x^k H(x)$. Only a relatively small number got all parts of the result and the proof. The main difficulty seemed to be that students ignored the hint to use a particular inductive approach, which would have led them to consider cases with small k and n . Hence these candidates struggled to formulate a conjecture for the answer and got lost in algebraic complications.

Part(b)(i) was again done correctly by nearly all students who attempted this part. A very small group of students misread the question and their derivation gave the wrong result. For (b)(ii), some students got the complete answer. The difficulty here was to use the hints to get a closed form for the series expansion and then use further hints to obtain the residues from this expression. A small but sizeable group of students chose unsuitable closures for the complex contour integration.

ASO: Q4. Calculus of Variations

Many answers showed little familiarity with standard results from the course.

Part (a) has been set many times before, but was still found challenging. Not all candidates remembered that an Euler–Lagrange equation holds for each pair $q_i, \dot{q}_i$ individually, not just as a sum over i . Many candidates could not show that H is constant.

Part (b) is another result from the lecture notes. The Euler–Lagrange equations for J are (for $i = 1, \dots, n$)

$$0 = \frac{d}{dt} \left(\frac{\partial f(L)}{\partial \dot{q}_i} \right) - \frac{\partial f(L)}{\partial q_i} = f''(L) \frac{dL}{dt} \frac{\partial L}{\partial \dot{q}_i} + f'(L) \left(\frac{d}{dt} \frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} \right),$$

and $f'(L) > 0$. From the chain rule we get the total time derivative dL/dt , not the partial time derivative $\partial L/\partial t$ that we already know vanishes. However, the Lagrangian L is a homogeneous quadratic function of $\dot{q}_1, \dots, \dot{q}_n$ so

$$H = \sum_{i=1}^n \dot{q}_i \frac{\partial L}{\partial \dot{q}_i} - L = 2L - L = L.$$

L itself is constant along a path by part (a), so $dL/dt = 0$. This eliminates the $f''(L)$ term. Most candidates did not notice that there should be a $f''(L)$ term. Some incorrectly assumed that L has no dependence on $q_1, \dots, q_n$.

Some candidates tried to do this from first principles instead of just replacing L with $f(L)$ in the Euler–Lagrange equations as above. This would work, but most candidates did not notice that integrating by parts to eliminate the time derivative $\dot{\eta}_i$ of the variation introduces an $f''(L) dL/dt$ term.

The cone in part (c) is the surface $z = r \cot \alpha$ in cylindrical polar coordinates. The geodesics are the curves $\mathbf{r}(t)$ that minimise the integral of $|\dot{\mathbf{r}}|$. By part (b) the geodesics also minimise the integral of $L = |\dot{\mathbf{r}}|^2 = \dot{r}^2 \operatorname{cosec}^2 \alpha + r^2 \dot{\theta}^2$.

Many candidates did not think to use part (b) to simplify the variational problem. Others included a gravitational potential energy. The particle trajectories in the mechanical problem with gravity are not geodesics. Some candidates confused the θ coordinate with the (constant) semi-angle α , or had the wrong relation between z and r .

The θ coordinate is ignorable, so $h = \partial L/\partial \dot{\theta} = r^2 \dot{\theta}$ is constant along a geodesic. Substituting $\dot{\theta} = h/r^2$ in the Euler–Lagrange equation for r gives the result in (c)(i).

As in Prelims Dynamics, we put $r(t) = 1/u(\theta)$ and use the chain rule: $\dot{r} = -(1/u^2)(du/d\theta)\dot{\theta} = -h du/d\theta$ and $\ddot{r} = -h^2 u^2 d^2u/d\theta^2$ to get

$$\frac{d^2u}{d\theta^2} + (\sin^2 \alpha)u = 0.$$

The general solution is the displayed equation in (c)(ii).

There were few attempts at part (d). Unrolling creates an isometry from the cone to the plane, so

$$|\dot{\mathbf{r}}|^2 = \operatorname{cosec}^2 \alpha \dot{r}^2 + r^2 \dot{\theta}^2 = \dot{R}^2 + R^2 \dot{\phi}^2$$

in plane polar coordinates with angle $\phi = \theta \sin \alpha$, and radius $R = r \operatorname{cosec} \alpha$. $R = \sqrt{r^2 + z^2}$ is also the spherical polar radius of a point on the cone. The equation in part (c)(ii) becomes the equation for a straight line:

$$X = R \cos(\phi - \phi_0) = 1/(A \sin \alpha) \text{ is constant.}$$

The previous $\cos(\sin \alpha (\theta - \theta_0))$ with $\sin \alpha$ inside the argument of the cosine becomes $\cos(\phi - \phi_0)$ because unrolling the cone rescales the angle.

ASO: Q5. Graph Theory

ASO: Q6. Mathematical Biology

Overall, this question was answered well, with part (a) generally answered better than part (b). Parts (a)(i) and (a)(ii) were mostly well done. In part (a)(i), the most common error was to ignore the given model and instead carry out a linear stability analysis for the standard model $dN/dt = f(N)$, without either defining a new function “ $f(N)$ ” or relating the result back to the model given in the question. Part (a)(ii) was answered correctly by most candidates, with lost marks arising from minor calculation errors. A small number of candidates attempted to construct a cobweb diagram despite the model being in continuous time.

Part (a)(iii) was generally answered well, although some candidates did not explicitly consider the intersection of relevant graphs as directed in the question, and some reached the wrong conclusion regarding the possibility of hysteresis. Most candidates who sketched the correct bifurcation diagram in part (a)(iii) correctly identified the maximum value of E for which a positive yield could be obtained in part (a)(iv).

Part (b)(i) was answered correctly by almost all candidates. Most candidates attempting part (b)(ii) gained some marks, although calculation errors were common, and some candidates were confused by the combination of logistic growth continuously within each year and the discrete harvesting event at the end of each year. As expected, part (b)(iii) proved to be the hardest part of this question. Some candidates made no attempt to answer part (b)(iii), possibly because of time pressure, but there were many good attempts for which candidates gained some credit. However, no candidate produced a “perfect” solution.

E. Recommendations for Next Year's Examiners and Teaching Committee

It was noted that in future, when calculating paper difficulty, the ASO paper should not be included in the calculation due to each of the questions in the paper being set by a different lecturer.

F. Comments on performance of identifiable individuals

1. Prizes

The Gibbs Prizes for Mathematics Part A were awarded to:

Aakash Shankar, Mansfield College

Zizhou Jin, Worcester College

G. Names of members of the Board of Examiners

• Examiners:

Prof. Nikolay Nikolov (Chair)

Prof. James Newton

Prof. Harald Oberhauser

Dr Neil Laws

Prof. James Martin

Prof. Andrea Mondino

Prof. Mark Blyth (External Examiner)

Prof. Ali Taheri (External Examiner)

• Assessors:

Dr Samuel Johnson

Dr Amelie Loher

Prof. Andreas Muench

Prof. Andrew Dancer

Dr Andrew Graham

Prof. Ben Green

Prof. Dominic Vella

Prof. Emmanuel Breuillard

Dr Giovanni Italiano

Dr Hanneke Wiersema

Prof. Jason Lotay

Prof. Mark Mezei

Prof. Mike Giles

Prof. Mohit Dalwadi

Prof. Oliver Riordan

Prof. Panos Papazoglou

Prof. Paul Dellar

Prof. Qian Wang

Prof. Robin Thompson