Examiners' Report: Preliminary Examination in Mathematics Trinity Term 2026

November 18, 2026

Part I

A. STATISTICS

184 candidates in Mathematics and Mathematics & Statistics were awarded an overall year outcome. Candidates on both degrees submit the same assessments and no distinction is made between the two groups in this document.

Table 1: Numbers in each outcome class

	Numbers					Percentages				
	2026	(2025)	(2024)	(2023)	(2022)	2026	(2025)	(2024)	(2023)	(2022)
Distinction	58	(52)	(57)	(52)	(53)	31.52	(29.05)	(31.67)	(29.21)	(29.78)
Pass	109	(115)	(109)	(113)	(116)	59.24	(64.25)	(60.56)	(63.48)	(65.17)
Partial Pass	11	(9)	(10)	(11)	(6)	5.98	(5.03)	(5.56)	(6.18)	(3.37)
Incomplete	2	(0)	(0)	(0)	(0)	1.09	(0.00)	(0.00)	(0.00)	(0.00)
Fail	4	(3)	(4)	(2)	(3)	2.17	(1.68)	(2.22)	(1.12)	(1.69)
Total	184	(179)	(180)	(178)	(178)	100	(100)	(100)	(100)	(100)

B. NEW EXAMINING METHODS AND PROCEDURES

None.

C. CHANGES IN EXAMINING METHODS AND PROCEDURES CURRENTLY UNDER DISCUSSION OR CONTEMPLATED FOR THE FUTURE

None.

D. NOTICE OF EXAMINATION CONVENTIONS FOR CANDIDATES

The Notice to Candidates, containing details of the examinations and assessments, was issued to all candidates at the beginning of Trinity term. The Examination Conventions in full were made available at

https://www.maths.ox.ac.uk/members/students/undergraduate-courses/examinations-assessments/examination-conventions.

Part II

A. GENERAL COMMENTS ON THE EXAMINATION

Acknowledgements

First, the Moderators should like to thank the Undergraduate Studies Administration Team.

We should also like to thank Charlotte Turner-Smith for her invaluable experience with the Mitigating Circumstances Panel, and Matthew Brechin and Waldemar Schlackow for maintaining and running the examination database and their assistance during the final examination board meeting.

We should like to thank the lecturers for their feedback on proposed exam questions; the assessors for their extraordinary assistance with marking; Patrick Farrell for coordinating the Computational Mathematics assessments, and the team of graduate checkers for their rapid work checking the marks on the papers.

Timetable

The examinations began on Monday 22$^{\text{nd}}$ June and ended on Friday 26$^{\text{th}}$ June.

Setting and checking of papers

The Moderators set and checked the questions, model solutions, and mark schemes. Every question was carefully considered by at least two moderators, and feedback was sought from lecturers. In a small number of cases feedback from lecturers was not available, and those were discussed in more detail until the Board of Moderators was satisfied that all questions were appropriate.

The questions were then combined into papers which were considered by the Board of Moderators and small changes were made to satisfy the Board that the papers were appropriate. After this a final proof-reading of the papers

was completed before the Camera Ready Copies (CRCs) were produced. The whole Board of Moderators signed off the CRCs which were submitted to Examination Schools.

Marking and marks processing

The Moderators and Assessors marked the scripts according to the mark schemes and entered the marks. Small adjustments to some mark schemes were made at this stage, and care was take to ensure these were consistently applied to all candidates.

A team of graduate checkers, supervised by the Academic Admin Team sorted all the scripts for each paper and carefully cross checked these against the mark scheme to spot any unmarked parts of questions, addition errors, or wrongly recorded marks. A number of errors were corrected, with each change checked and signed off by a Moderator, at least one of whom was present throughout the process.

Mitigating Circumstances

A subset of the examiners (the 'Mitigating Circumstances Panel') attended a pre-board meeting to band the seriousness of the individual notices to examiners. The outcome of this meeting was relayed to the Examiners at the final exam board, who gave careful regard to each case, scrutinised the relevant candidates' marks and agreed actions as appropriate. See Section F for further details.

Determination of University Standardised Marks

Marks for each individual assessment are reported as a University Standard Mark (USM) which is an integer between 0 and 100 inclusive. The Moderators used their academic judgment to map the raw marks on individual assessments to USMs using a process similar to previous years. In coming to this judgement the board followed the advice from the Mathematics Teaching Committee that the percentages awarded for each USM range of the examination should be in line with recent years. This alignment can be seen in Table 1; in more detail, for Papers I–V, a piecewise linear map was constructed as follows:

1. Candidates' raw marks for a given paper were ranked in descending order.
2. The default percentages p_1 of Distinctions and p_2 of Nominal Upper Seconds were selected.
3. The candidate at the p_1 percentile from the top of the ranked list was identified and assigned a USM of 70, and the corresponding raw mark

denoted R_1.

4. The candidate at the $(p_1 + p_2)$ percentile from the top of the list was assigned a USM of 60 and the corresponding raw mark denoted R_2.

5. The line segment between $(R_1, 70)$ and $(R_2, 60)$ was extended linearly to USMs of 72 and 57 respectively, and the corresponding raw marks denoted C_1 and C_2 respectively.

6. A line segment through $(C_2, 57)$ was extended towards the vertical axis, as if it were to join the axis at $(0, 10)$, but the line segment was terminated at a USM of 37 and the raw mark at the termination point was denoted C_3.

With these data a piecewise linear map was constructed with vertices at $\big\{(0,0),\ (C_3, 37),\ (C_2, 57),\ (C_1, 72),\ (100, 100)\big\}$.

Reports from the Assessors describing the apparent relative difficulty and the general standard of solutions for each question were then considered, and the Board decided that the values of $p_1 = 31\%$ and $p_2 = 48\%$ were suitable for all papers.

In line with previous years, for the Computational Mathematics assessment the linear map with gradient 2.5 was used to map from raw marks to USMs.

The vertices of the final maps used in each assessment are listed in Table 2.

Table 2: Vertices of final piecewise linear model

Paper	Vertices				
I	0;0	37;37	59.2;57	83.2;72	100;100
II	0;0	31;37	45;57	78;72	100;100
III	0;0	38;39.5	51.2;57	90.2;72	120;100
IV	0;0	21;37	32.5;57	55;72	80;100
V	0;0	27;39.6	38;57	60.6;72	80;100
CM	0;0	40;100			

With the USMs, a provisional outcome class for each candidate was produced as follows: Write MI, MII, $MIII$, MIV and MV for the USMs on Papers I–V respectively, and CM for the USM on the Computational Mathematics assessment. Write Av_1 and Av_2 for the quantities

$$\frac{MI + MII + \frac{6}{5}MIII + \frac{6}{5}MIV + \frac{4}{5}MV + \frac{1}{2}CM}{5.3}$$

and

$$\frac{MI + MII + \frac{6}{5}MIII + \frac{4}{5}MIV + \frac{4}{5}MV}{4.8}$$

respectively, symmetrically rounded. With these auxiliary statistics the provisional outcome class was determined by the definitions:

Distinction: both $Av_1 \geq 70$ and $Av_2 \geq 70$ and a USM of at least 40 on each paper and for the Computational Mathematics assessment;

Pass: not a Distinction and a USM of at least 40 on each paper and for the Computational Mathematics assessment;

Partial Pass: not a Pass or Distinction and a USM of at least 40 on three or more of Papers I–V;

Fail: not a Partial Pass, Pass, or Distinction, and a USM of less than 40 on three or more of Papers I–V.

The scripts of those candidates at the boundaries between outcome classes were scrutinised carefully to determine which attained the relevant qualitative descriptors and changes were made to move those into the correct class.

Mitigating Circumstances were then considered using the banding produced by the Mitigating Circumstances Panel, and appropriate actions were taken and recorded.

Table 3 gives the rank list ordered by the average of Av_1 and Av_2 (as defined above), showing the number and percentage of candidates with USM greater than or equal to each value.

Table 3: Rank list of average USM scores

		Candidates with USM $\geq x$	
USM (x)	Rank	Number	Percentage
91.58	1	1	0.54
90.42	2	2	1.09
89.21	3	3	1.63
89.17	4	4	2.17
88.71	5	5	2.72
86.29	6	6	3.26
84.96	7	7	3.8
83.04	8	8	4.35
81.42	9	9	4.89
81.29	10	10	5.43
79.72	11	11	5.98
78.21	12	12	6.52
78.04	13	13	7.07
77.42	14	14	7.61
77.25	15	15	8.15

Table 3: Rank list of average USM scores (continued)

		Candidates with average USM $\geq x$	
USM (x)	Rank	Number	Percentage
76.75	16	16	8.7
76.38	17	17	9.24
76.17	18	18	9.78
76.13	19	19	10.33
75.83	20	20	10.87
75.58	21	21	11.41
75.5	22	22	11.96
75.38	23	23	12.5
75.17	24	25	13.59
75.17	24	25	13.59
74.63	26	26	14.13
74.5	27	27	14.67
74.13	28	28	15.22
74.08	29	29	15.76
73.88	30	30	16.3
73.67	31	31	16.85
73.33	32	33	17.93
73.33	32	33	17.93
73.25	34	34	18.48
73.13	35	35	19.02
72.63	36	36	19.57
72.58	37	37	20.11
72.54	38	38	20.65
72.5	39	39	21.2
71.79	40	40	21.74
71.63	41	41	22.28
71.5	42	42	22.83
71.38	43	43	23.37
71.04	44	44	23.91
70.92	45	45	24.46
70.79	46	46	25
70.67	47	47	25.54
70.58	48	49	26.63
70.58	48	49	26.63
70.5	50	50	27.17
70.13	51	51	27.72
70.08	52	52	28.26
69.88	53	53	28.8
69.83	54	54	29.35
69.67	55	55	29.89

Table 3: Rank list of average USM scores (continued)

		Candidates with average USM $\geq x$	
USM (x)	Rank	Number	Percentage
69.54	56	57	30.98
69.54	56	57	30.98
69.5	58	58	31.52
69.28	59	59	32.07
69.25	60	60	32.61
68.92	61	61	33.15
68.83	62	62	33.7
68.71	63	63	34.24
68.5	64	64	34.78
68.38	65	65	35.33
68.33	66	66	35.87
68.08	67	67	36.41
67.88	68	69	37.5
67.88	68	69	37.5
67.83	70	70	38.04
67.79	71	71	38.59
67.71	72	73	39.67
67.71	72	73	39.67
67.58	74	74	40.22
67.42	75	75	40.76
67.29	76	76	41.3
67.21	77	77	41.85
67.08	78	78	42.39
66.92	79	79	42.93
66.54	80	80	43.48
66.38	81	81	44.02
66.29	82	82	44.57
66.25	83	83	45.11
66.23	84	84	45.65
66.04	85	85	46.2
65.46	86	86	46.74
65.32	87	87	47.28
65.13	88	88	47.83
65.08	89	89	48.37
64.96	90	90	48.91
64.88	91	93	50.54
64.88	91	93	50.54
64.88	91	93	50.54
64.63	94	95	51.63
64.63	94	95	51.63

Table 3: Rank list of average USM scores (continued)

		Candidates with average USM $\geq x$	
USM (x)	Rank	Number	Percentage
64.46	96	97	52.72
64.46	96	97	52.72
64.25	98	98	53.26
64.17	99	99	53.8
64.13	100	100	54.35
64.08	101	101	54.89
63.92	102	102	55.43
63.75	103	104	56.52
63.75	103	104	56.52
63.58	105	105	57.07
63.33	106	106	57.61
62.92	107	108	58.7
62.92	107	108	58.7
62.7	109	109	59.24
62.42	110	110	59.78
61.88	111	111	60.33
61.83	112	112	60.87
61.79	113	113	61.41
61.71	114	115	62.5
61.71	114	115	62.5
61.66	116	116	63.04
61.63	117	117	63.59
61.58	118	118	64.13
61.42	119	119	64.67
61.4	120	120	65.22
61.29	121	121	65.76
61.25	122	122	66.3
60.79	123	123	66.85
60.75	124	124	67.39
60.67	125	126	68.48
60.67	125	126	68.48
60.54	127	127	69.02
60.43	128	128	69.57
60.29	129	129	70.11
60.21	130	130	70.65
60.17	131	131	71.2
60.08	132	133	72.28
60.08	132	133	72.28
59.83	134	134	72.83
59.71	135	135	73.37

Table 3: Rank list of average USM scores (continued)

		Candidates with average USM $\geq x$	
USM (x)	Rank	Number	Percentage
59.54	136	137	74.46
59.54	136	137	74.46
59.5	138	138	75
59.28	139	139	75.54
59.17	140	140	76.09
59.08	141	141	76.63
59.04	142	142	77.17
59	143	143	77.72
58.83	144	144	78.26
58.75	145	145	78.8
58.63	146	146	79.35
58	147	147	79.89
57.92	148	148	80.43
57.58	149	149	80.98
57.42	150	150	81.52
57.08	151	151	82.07
56.96	152	152	82.61
56.88	153	153	83.15
56.71	154	154	83.7
56.67	155	155	84.24
56	156	156	84.78
55.42	157	157	85.33
54.71	158	158	85.87
54.26	159	159	86.41
54	160	160	86.96
53.25	161	161	87.5
53.13	162	162	88.04
53.08	163	163	88.59
52	164	164	89.13
51.33	165	165	89.67
50.92	166	167	90.76
50.92	166	167	90.76
50.79	168	168	91.3
49.88	169	169	91.85
49.54	170	170	92.39
49.46	171	171	92.93
49	172	172	93.48
48	173	173	94.02
47.92	174	174	94.57
47.33	175	175	95.11

Table 3: Rank list of average USM scores (continued)

		Candidates with average USM $\geq x$	
USM (x)	Rank	Number	Percentage
45.88	176	176	95.65
45.21	177	177	96.2
39.33	178	178	96.74
38.58	179	179	97.28
23.29	180	180	97.83
11.63	181	181	98.37
0	182	184	100
0	182	184	100
0	182	184	100

B. EQUAL OPPORTUNITY ISSUES AND BREAKDOWN OF THE RESULTS BY GENDER

Table 4 shows the performances of candidates by gender. Here gender is the gender as recorded on eVision.

Table 4: Breakdown of results by gender

Outcome	Number											
	2026			2025			2024			2023		
	Female	Male	Total	Female	Male	Total	Female	Male	Total	Female	Male	Total
Distinction	5	53	58	3	49	52	4	53	57	9	43	52
Pass	35	74	109	37	78	115	35	74	109	38	75	113
Partial Pass	6	5	11	3	6	9	2	8	10	4	7	11
Incomplete	0	2	2	0	0	0	0	0	0	0	0	0
Fail	2	2	4	1	2	3	1	3	4	1	1	2
Total	48	136	184	44	135	179	41	138	180	52	126	178
Outcome	Percentage											
	2026			2025			2024			2023		
	Female	Male	Total	Female	Male	Total	Female	Male	Total	Female	Male	Total
Distinction	10.42	38.97	28.80	6.82	36.3	29.05	9.52	38.41	31.67	17.23	34.15	29.21
Pass	72.92	54.41	40.22	84.09	57.78	64.25	83.33	53.62	60.56	73.08	59.52	63.53
Partial Pass	12.5	3.68	2.72	6.82	4.44	5.03	4.76	5.8	5.56	7.69	5.56	6.17
Incomplete	0.00	1.47	1.09	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Fail	4.17	1.47	1.09	2.27	1.48	1.68	2.38	2.17	2.22	1.92	0.79	1.12

C. STATISTICS ON CANDIDATES' PERFORMANCE IN EACH PART OF THE EXAMINATION

Table 5: Numbers taking each paper

Paper	Number of Candidates	Average raw mark	Std dev of raw marks	Average USM	Std dev of USMs
I	182	71.45	13.93	64.76	12.51
II	182	61.09	17.3	63.63	13.8
III	182	73.67	18.58	65.38	12.09
IV	181	45.16	11.47	64.94	11.44
V	182	48.98	13.25	63.86	14.21
CM	183	34.44	5.26	86.21	13.28

Tables 6–11 give the performance statistics for each individual assessment, showing for each question the average mark, first over all attempts, and then over the attempts used; the standard deviation over all attempts; and finally the total number of attempts, first those that were used, and then those that were unused.

Table 6: Statistics for Paper I

Question Number	Average mark All	Used	Std dev	No. of Attempts Used	Unused
Q1	16.97	16.97	2.83	174	0
Q2	13.16	13.21	4.72	99	2
Q3	15.52	15.67	3.58	98	1
Q4	16.22	16.27	3.54	171	1
Q5	12.44	12.44	3.99	167	0
Q6	9.58	9.58	4.11	71	0
Q7	13.08	13.08	4.15	122	0

Table 7: Statistics for Paper II

Question Number	Average mark All	Used	Std dev	No. of Attempts Used	Unused
Q1	13.15	13.31	4.17	164	2
Q2	12.03	12.11	4.81	147	1
Q3	8.35	8.9	4.99	51	6
Q4	14.06	14.06	3.65	139	0
Q5	12.49	12.63	3.86	116	2
Q6	12.29	12.3	4.4	105	1
Q7	11.03	11.03	4.57	175	0

Table 8: Statistics for Paper III

Question Number	Average mark All	Used	Std dev	No. of Attempts Used	Unused
Q1	14.53	14.68	4.82	152	3
Q2	12.54	12.55	4.48	78	2
Q3	14.77	14.77	3.56	132	0
Q4	15.54	15.54	3.18	149	0
Q5	9.15	9.15	3.24	138	0
Q6	13.13	13.31	4.94	75	2
Q7	10.26	10.26	4.42	167	0
Q8	9.49	9.49	5.02	112	0
Q9	12.24	12.24	3.57	67	0

Table 9: Statistics for Paper IV

Question Number	Average mark All	Used	Std dev	No. of Attempts Used	Unused
Q1	11.79	11.9	4.19	124	2
Q2	12.25	12.25	4.13	134	0
Q3	9.69	9.72	3.24	102	1
Q4	7.76	7.81	3.46	108	1
Q5	12.73	12.73	4.35	160	0
Q6	13.11	13.11	3.9	87	0

Table 10: Statistics for Paper V

Question Number	Average mark All	Used	Std dev	No. of Attempts Used	Unused
Q1	11.95	11.95	4.66	167	0
Q2	13.66	13.66	5.35	172	0
Q3	10.55	10.55	4.4	22	0
Q4	14.68	14.68	4.17	167	0
Q5	10	10.01	4.36	103	1
Q6	8.86	8.86	3.87	91	1

Table 11: Statistics for Computational Mathematics

Question Number	Average mark All	Used	Std dev	No. of Attempts Used	Unused
Project A	18.69	18.69	2.52	124	0
Project B	16.71	16.71	2.92	110	0
Project C	16.52	16.52	2.82	130	0

D. COMMENTS ON PAPERS AND ON INDIVIDUAL QUESTIONS

Paper I

Question 1. Overall, candidates found this question quite straightforward, and easily recalled the definitions in (a)(i), with some candidates opting for verbal rather than symbolic definitions. The proofs in (a)(ii) were generally well done, though a few candidates got muddled and thought they were trying to show that these spaces were sub*groups*, and proceeded to try to multiply vectors. Most were able accurately to reproduce the proof of the Rank-Nullity Theorem in (a)(iii), but some lost marks for taking shortcuts over, or entirely omitting, the justification that the relevant set of vectors is linearly independent. Another misstep here was in assuming that $V = \ker T \oplus \operatorname{Im} T$ in general. The two parts of (b) were mostly done well, if occasionally lacking explanation. The word 'briefly' in the question was intended to indicate that a full proof of linearity was not required in either case, but many candidates supplied one nevertheless. Any plausible justification of linearity (invoking the linearity of the derivative in (i) and that of matrix multiplication in (ii)) received the marks. Some candidates created extra work for themselves in this part of the question by not recognising that the Rank-Nullity Theorem could be applied. When it came to finding the kernel and image in (b)(ii), the hint appears to have led some candidates astray: A is *not* the matrix corresponding to T_2, since it multiplies a matrix rather than a vector in the definition of T_2. We therefore cannot simply conclude from $\operatorname{rank} A = 3$ that $\operatorname{rank} T_2 = 3$ (which is in fact false). Many candidates were thus able to arrive at the correct result, namely that $\ker T_2 = \{0_{3\times 3}\}$ and $\operatorname{Im} T_2 = \mathcal{M}_{3\times 3}(\mathbb{R})$, but by incorrect reasoning that took $\dim \mathcal{M}_{3\times 3}(\mathbb{R})$ to be 3. Although these were not penalised, a number of notational issues were evident in answers to this question, particularly when it came to distinguishing between a set and its elements: many candidates wrote, for example, $\ker T = 0$ when they meant $\ker T = \{0\}$.

Question 2. Question 2 focused on direct sums of vector spaces. Question 2(a)(i) asked for what it means for a vector space V to be equal the direct sum $U \oplus W$ for subspaces U, W of V. The vast majority of candidates answered this correctly. 2(a)(ii) then asked to show that this equivalent to asking that any $v \in V$ can be uniquely written as $u + w$ for $u \in U$ and $w \in W$. Candidates had slightly more difficulty proving that the definition implies the statement, with some candidates missing out this implication, but overall this question was done very well.

The remaining questions introduced examples of vector spaces and given subspaces, with candidates asked to confirm whether or not they could be written as a direct sum. Question 2(b) gave the example of $V = \mathbb{R}[x]$ with

U being the space of polynomials of degree at most 1 and W being the space of polynomials vanishing at both 0 and 1. Part 2(b)(i) asked to find $U \cap W$. This was done successfully by most candidates, who found that

$$U \cap W = \{0\}.$$

Part 2(b)(ii) asked whether $V = U \oplus W$, this part caused more difficulty but again was done well by many candidates.

Part 2(c) was more challenging. It focused on $V = C[-1, 1]$, the space of continuous real-valued functions on $[-1, 1]$. Part 2(c)(i) asked to show that V can be written as the direct sum of the space of even functions and the space of odd functions. This part was done well with most candidates scoring full marks. Some candidates unsuccessfully tried to argue using Fourier theory.

Part 2(c)(ii) caused a few more problems. Quite a few candidates were able to show that $U \cap W = \{0\}$. Many candidates did not make any progress on this question. Among the candidates that made progress on showing that $V = W + U$, most completed the question successfully.

Question 3. Question 3(a)(i) asked for the definition of an inner product. Often candidates would write this in words, e.g. "a positive definite, symmetric bilinear form" without giving a definition of those adjectives. The definition of positive definite, symmetric and bilinear was needed for full marks. 3(a)(ii) asked for a description of the Gram–Schmidt process. This was generally done well.

3(b) gave an example of an inner product on $\mathbb{R}^2$. 3(b)(i) asked candidates to verify that it is an inner product. This was done very successfully. 3(b)(ii) asked to find an ONB with respect to the inner product. Starting with the standard basis, many candidates correctly tried to implement Gram–Schmidt. This was executed with varying success; often there were issues with correctly normalising vectors that led to numerical errors. 3(b)(iii) was done very well with many candidates obtaining full marks.

Question 4. The bookwork in part (a) was generally done well. Two common difficulties were forgetting that eigenvectors should be non-zero and that diagonalisable is not the same as orthogonally diagonalisable. The main difficulties encountered in part (a) were related to the proof that eigenvectors of different eigenvalues are linearly independent. Here, some candidates proved this only for symmetric matrices using inner products and some forgot to sufficiently justify their steps, e.g. by saying that eigenvectors are non-zero. A few candidates proved the more general statement for n eigenvectors using induction which was unnecessarily long. The new parts (b) and (c) were also generally done well. In part (b), candidates generally

knew how to determine diagonalisability over $\mathbb{R}$ and $\mathbb{C}$, while some difficulties arose in relation to the question about matrices over $\mathbb{Z}_2$. If mistakes were made in parts (b) and (c), these tended to be in the computations of characteristic polynomials.

Question 5. This question was attempted by most candidates. Part ai was well done apart from occasional lapses, such as omitting to show that the group is the union of cosets. In part aii most students failed to show that the order of the group is equal to the order of the element and did not receive full marks. Part aii was completed by nearly everyone. For part bi many students failed to show that the intersection of two distinct subgroups of order 11 is trivial, while some claimed it without proof. Many students opted to prove and use Cauchy's theorem instead of using Lagrange and they received full marks. A common mistake in part bii was to claim that the set of conjugates forms a subgroup. Many students solved this question using group actions rather than cosets of the centralizer, and they received full marks.

Question 6. The bookwork in part (a) was generally done well with difficulties essentially restricted to the proof that the even permutations A_n have index two in S_n. Here, many assumed without proof that a permutation is always either even or odd. When showing that any two transpositions are conjugate, some candidates used unnecessarily long case-by-case proofs. Part (b) was clearly seen as harder with some not attempting it at all and most not attempting the last part of finding a copy of S_{n-2} in A_n. For the first part of (b), most candidates found the right approach but many did not justify the fact that all transpositions map to the same element. There were several attempts at showing that A_n is the only index 2 subgroup of S_n, but only a few correct solutions using the first part.

Question 7. For the most part, candidates found parts (a) and (b) of this question quite routine, but had more difficulties with (c). The definitions in (a)(i) and (a)(ii) were usually given accurately, and the proof in (a)(iii) was mostly done well, although the existence of inverses proved tricky for some candidates, who thought that we can conclude immediately from $g \cdot x = x$ that $x = g^{-1} \cdot x$, whereas some intermediate justification is required. Marks were frequently lost in part (b)(ii) for not recognising that there is a separate case to consider when the second coordinate of a point is 0, and that similarly there are two cases in (b)(iii). A number of candidates misread (b)(ii) as 'describe the action geometrically'. The interpretation of the action in (c)(ii) caused difficulties for many candidates — this was a little different from the other orbit counting questions that candidates had seen previously. As a result, many candidates tried to force the problem into the mould of previously seen problems by permuting, labelling, or colouring the small squares, none of which approaches were indicated in the question.

The effect of these efforts was to significantly overcomplicate matters.

Paper II

Question 1. Candidates generally knew the bookwork well enough to answer part (a) correctly in full.

For part (b), a common (and quite critical) mistake is to claim that (b_n) is a *subsequence* of (a_n), which is indeed not true (sup generally is different from max) and leads to an oversimplification of the proof. Also, several candidates apparently did not realise that subpart (b)(ii) asks to prove that (a_n) converges to the limit ℓ of (b_n) that is defined in subpart (b)(i); so they ended up only proving that (a_n) converges to some limit ℓ', without showing further that $\ell' = \ell$ (this is objectively the main part of the proof).

For subpart (c)(i), a common oversight is only to apply the estimate $\frac{k\cos k}{4^k} \leq \frac{k}{4^k}$ without taking into account the absolute value of the left-hand side. This is an important part of the proof because $\frac{k\cos k}{4^k}$ is negative for infinitely many $k \geq 1$ and the Comparison Test cannot be applied directly.

Subpart (c)(ii) appears more challenging and most of the candidates were unable to figure out the right approaches. A good start would be to observe that $a_{n+1} - a_n = -\frac{1}{n+1}(a_n - a_{n-1})$, then either deduce the expansion of a_n or prove directly that (a_n) is Cauchy.

Question 2. Subpart (a)(i) was generally done well. For subpart (a)(ii), a number of candidates stated the Comparison Test (essentially) that '*Let* (a_n) *and* (b_n) *be two sequences of real numbers such that* $a_n \leq b_n$ *for all* n. *Then* ...' without emphasising the non-negativity of the terms a_n.

For subpart (a)(iii), there are two common mistakes. Firstly, when stating the hypotheses of the Alternating Series Test, many candidates did not include the limit 0 of (a_n), or the non-negativity of the terms a_n, or the monotone decreasing property of (a_n). Without any of these conditions, the series $\sum_{n=0}^{\infty}(-1)^n a_n$ need not converge; indeed:

- *If* (a_n) *is decreasing and converges to some positive number*, then one can take $a_n = 1$ for all $n \geq 0$; in this case $\sum_{n=0}^{\infty}(-1)^n a_n = \sum_{n=0}^{\infty}(-1)^n$ diverges.
- *If* $a_n \to 0$ *but there can be infinitely many negative terms in* (a_n), then one can take $a_n = \frac{(-1)^n}{n+1}$ for all $n \geq 0$; in this case $\sum_{n=0}^{\infty}(-1)^n a_n = \sum_{n=0}^{\infty}\frac{1}{n+1}$ diverges.
- *If each* a_n *is nonnegative and* $a_n \to 0$ *but* (a_n) *may not eventually decreasing*, then one can take $a_{2n} = \frac{1}{n+1}$ and $a_{2n+1} = \frac{1}{(n+1)(n+2)}$ for all $n \geq 0$; in this case $\sum_{n=0}^{k}(-1)^n a_n = \sum_{n=0}^{k}\frac{1}{n+2}$ tends to ∞ as

$k \to \infty$.

Secondly, after defining $s_k := \sum_{n=0}^{k}(-1)^n a_n$ for all $k \geq 0$, many candidates incorrectly stated that the subsequence (s_{2k}) is *increasing* (whereas it is actually *decreasing*) and only used an upper bound on (s_{2k}) (a lower bound should be used instead).

For subpart (a)(iv), a common oversight is to directly handle $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$ without considering the partial sums $\sum_{n=1}^{k} \frac{1}{n(n+1)} = 1 - \frac{1}{k+1}$. Several candidates also applied the Integral Test to conlcude that $\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = \ln 2$, which is incorrect.

For subpart (b)(i), a common oversight is to apply the estimate $\frac{n+2^n \sin n}{n^2+3^n} \leq \frac{n+2^n}{n^2+3^n}$ without taking into account the absolute value of the left-hand side. Similar to subpart (c)(i) of Q1, this is an important step in the proof because $\frac{n+2^n \sin n}{n^2+3^n}$ is negative for infinitely many $n \geq 1$ and the Comparison Test cannot be applied directly. In fact, some candidates even incorrectly claimed that $\frac{n+2^n \sin n}{n^2+3^n}$ is *always* non-negative!

Subpart (b)(ii) is the only subpart with a divergence answer by comparing with the harmonic series; however, a number of candidates instead attempted to show convergence via the Ratio Test (which of course cannot work because $|a_{n+1}/a_n| \to 1$ in this case). On the other hand, most of the candidates correctly applied the Alternating Series Test to subpart (b)(iii); however some claimed that $\left(\sqrt[n]{2} - 1\right)$ is decreasing without giving explanation (a remark that $\sqrt[n]{2} > \sqrt[n+1]{2}$ would suffice).

For part (c), many candidates attempted to prove that $\sum_{n=1}^{\infty} \sqrt{a_n a_{n+1}}$ converges provided the weaker assumption that (a_n) converges, which is not true as can be seen by taking $a_n = 1/n$ for all $n \geq 1$ (similar to subpart (b)(ii)). Other candidates correctly applied an AM-GM/maximality argument.

Summary: Candidates generally demonstrated a good knowledge of the subject and made serious effort to answer the questions.

Question 3. A relatively small number of candidates attempted this question.

Part ai was well done by many candidates, most of them applying Cauchy's criterion and a smaller number reducing to the real case. Most candidates gave the right definition for aii, however some gave as a definition instead the corollary proven in aiii and missed a mark. Part aii was well done, including the counterexample. Some students gave just the counterexample and received partial marks for this part. Many candidates realized that the proof of bi was identical to that of aiii. A common mistake in parts bii, biii was to assume that the radius of convergence is always given by the ratio test, and candidates that used this approach received no marks. A

fair number of candidates answered at least one of bii,biii and fewer both of them.

Question 4. This was one of the more popular questions, with a fairly high average mark. A recurring issue was imprecision regarding the distinction between strict and weak inequalities and between open and closed intervals. For example, while part (a) was generally done well, candidates often stated the IVT in the form $f(a) < c < f(b)$ implies $\exists \xi \in (a, b)$ with $f(\xi) = c$, but then proved only that $\xi \in [a, b]$. Others assumed $f(a) \leq c \leq f(b)$, and then incorrectly stated $\exists \xi \in (a, b)$. It is generally easier to state and use IVT with weak inequalities and closed intervals. There were several possible proofs for the IVT, with most candidates opting for the 'binary search' proof.

Part (b) was generally fine, but some candidates stated that the continuous image of an interval is an interval, not realising that much of the point of the question was to prove this.

Part (c) caused more difficulties. Candidates often relied on extensive case-by-case analysis, unnecessarily splitting according to whether $f(a)$ was a sup or inf of f. There is a case analysis, but it is whether $f(a) \leq y$ or $f(a) \geq y$. Quite a few candidates split according to $f(a) < y$ or $f(a) > y$, and forgot the case $f(a) = y$ (often due to using strict inequalities in their statement of the IVT).

In part (d), candidates often missed that we need to take $y \neq f(a)$ in part (c), or failed to mention the case when $\inf f = \sup f$. Because this part carried three marks, a complete justification rather than "Follows immediately from part (c)" was expected.

Part (e) was the hardest part and the majority of candidates failed to answer it correctly. Quite a common error was to assume the preimage was always of size 0 or 2 even when the function was restricted to the interval between, say, the two points at which the maximum is attained. Many candidates, however, did not make any substantive progress with this part.

Question 5. This was question was quite popular. Almost all candidates correctly wrote down the definition of uniform convergence with a few missing the require statement that it holds for all $x \in [a, b]$. A few candidates substituted $[a, b]$ with the letter E which I assume comes from the lectures.

Most candidates correctly answered (b)(i) showing that f_n uniformly converges. Many candidates computed derivatives of f before realising that it was not required. For (b)(ii), the sequence $x_n = 1 - 1/n$ was used by most. A few chose $x_n = 1 - 1/(2n)$ or similar. Others chose $x_n = (1/2)^{1/n}$ (or similar) which also converges to 1. These candidates and a few others who chose the intended sequence, often did not explicitly state the limit on x_n^n (or assumed it converged to 1). The candidates who correctly answered the

first two parts of (b) then tended to get the final part correct.

In part (c) almost everyone correctly stated the definition of uniform continuity. (d) part (i), constituted the hardest part of the question for candidates who attempted to answer it. Few realised that showing f is Lipschitz is the correct path. However, most candidates chose the correct sequences for (d)(ii) and subsequently got the final part correct as well.

Question 6. This was slightly less popular than question 4. The average mark was also slightly lower, but still fairly good. Part (a) was generally done well, although quite a few candidates did not state the conditions for Taylor's theorem correctly. Some, rather worryingly, just claimed that Taylor's theorem implies that every function is represented by an infinite Taylor series.

In part (b) quite a few candidates tried to use the MVT to show increasing implies $f' \geq 0$. This does not work as it only shows $f'(\xi) = \frac{f(y)-f(x)}{y-x} \geq 0$ for some $\xi \in (x, y)$. It is not clear how to get the result for all ξ. Similarly, quite a few candidates tried to use the definition of $f'(x) = \lim_{y\to x} \frac{f(y)-f(x)}{y-x}$ to show that $f'(x) \geq 0$ implies f increasing. But again, that only shows $f(y) \geq f(x)$ for $y \geq x$ sufficiently close to x, and does not prove this inequality globally.

Many candidates had trouble with (c)(i). A few proved the result assuming f'' existed, and obtained partial credit for this. Part (d) also often caused difficulties. Many candidates just showed the power series satisfied the conditions. A significant number of candidates claimed that because f is infinitely differentiable, it must be given by its infinite Taylor expansion. The lecture course spent some time discussing a counterexample to this! Quite a few tried using the constancy theorem to prove any two solutions must be equal. Unfortunately, it seems hard to make this approach work.

Question 7. This question was compulsory and thus attempted by (nearly) everyone. Most candidates knew the definition of Riemann integrable although only a handful of candidates remembered to include that f should be bounded. Many used the notation $I(\cdot)$ without defining what I is.

In general, many candidates struggled with (b). Many candidates swapped limits, derivatives, and integrals without justification. For those who got it correct, the attempts were split between applying MVT or L'Hôpital's rule. Often the route via MVT was not fully rigorous. Many candidates used Taylor's theorem, often assuming that f possesses a second derivative.

Most candidates got full marks in part (c) (i) and (ii). A few forget to check the special case $x = 1$ in (i). Most candidates got (c)(iii) correct, however, often this was done by explicit calculation. Surprisingly few realised that they could leverage (i) and (ii) to immediately deduce that f_n does not con-

verge uniformly. Question (c) (iv) was correctly answered by only a handful of candidates and often the derivation was a bit messy. Many candidates applied MVT to g or attempted an integration by parts but then could not justify further.

Paper III

Question 1. This question was very popular. Almost all candidates could show that $y = x^3$ satisfies the given differential equation, but finding the general solution proved harder for some. Multiple candidates wrote the solution in terms of the transformed variable only instead of giving their final answer as $y = f(x)$. The most common mistakes in part (b) were typos or incorrect signs, but it was generally done well. Part (c) was definitely more challenging for the candidates, with several of them unsure on how to even begin. Others got lost in lengthy computations or found the correct integrand but did not integrate it correctly.

Question 2. This question was less popular than Question 1. Almost all candidates, who attempted this question, were able to correctly apply the chain rule in part (a)i. to verify that $x\partial_x u + y\partial_y u = 2\phi\partial_\phi u$ under change of variables. Several candidates in part (a)ii. became confused when applying the chain rule to second derivatives, likely because of the lengthy computations involved. For those that managed to show that $u(\phi, \psi)$ satisfies $\frac{\partial^2 u}{\partial \phi^2} = 0$, they forgot to answer the last part of the question that required them to find the general solution. Part (b)i. was somewhat difficult for the candidates. The majority could identify the correct region R in ϕ, ψ coordinates, although some had trouble with the interval of ϕ. Other common mistakes included the incorrect use of the Jacobian when transforming the integral from x-y coordinates to ϕ-ψ coordinates. Part (b)ii. seemed challenging for the candidates. Many attempted to compute the integral, but did not manage to do so correctly, unable to show that the result would be independent of C.

Question 3. Candidates had few problems on their bookwork; nearly all candidates were able to correctly write down the second-order Taylor expansion and relate it to the form given in part (ii) by either direct manipulation of the Taylor expansion or expanding the expression given in part (ii) and relating it to the Taylor expansion. For part (iii), most students were able to successfully argue for the expected behavior when the determinant of the Hessian was nonzero; no points were awarded for simply stating the characterisation of the critical point by the sign of the determinant of the Hessian. Far fewer were able to correctly identify that degeneracy of the Hessian at a critical point indicated the critical point was not isolated, with most stating that a higher-order expansion was required. Nearly all candidates were able to successfully set up the optimization problem for part (b) via Lagrange

multipliers, although there were several candidates that tried to combine the two constraints into a single constraint, e.g. by simply adding the constraints. Many candidates were not able to successfully solve the resulting system of equations; besides algebraic errors, the most common error was failing to verify their putative solution solved one or both of the constraints. Some candidates recognized the symmetries of the problem but few were unable to correctly implement them, e.g. identifying the permutation symmetry of the problem and then asserting the point must be invariant under the action instead of fitting into a nontrivial orbit.

Question 4. Among the 166 non-blank attempts I marked, Question 4 produced a good spread of marks. The median mark was about 16 out of 20, with a sizeable number of very strong answers and relatively few very low-scoring attempts. Overall, candidates who attempted the question often had the right general ideas, but marks were frequently lost through imprecise statements of bookwork in part (a) and through overcounting or incorrect independence assumptions in part (b).

Part (a)(i) was generally well answered. Part (a)(i)(α) was answered very well by almost all candidates. Part (a)(i)(β) was also generally good, although some candidates gave unnecessarily long or messy arguments, and some answers became confused about how to decompose the union into disjoint pieces.

Part (a)(ii) was also often handled successfully, but many solutions were considerably longer than necessary. A common pattern was that candidates made confused intermediate expansions which were then corrected, sometimes without a fully coherent derivation. Some candidates also used finite additivity for three or more pairwise disjoint sets without explaining how this followed from the stated two-set axiom.

Part (a)(iii) was usually answered correctly. The most common error was to state only finite additivity, with a union and sum up to n rather than over a countable sequence. Some candidates omitted the disjointness condition, replaced pairwise disjointness by the condition that the intersection of all the events is empty, or gave an inclusion-exclusion formula rather than countable additivity.

In part (a)(iv), the most frequent errors were giving only pairwise independence, or stating only the full-intersection condition for all n events rather than requiring the product formula for every subcollection. Other common errors were replacing intersections by unions, or giving a conditional-probability formulation which did not capture mutual independence. Rarer errors included giving only the two-event definition or confusing independence with disjointness.

Part (b) was generally harder. Many candidates chose a sensible strategy,

either using inclusion-exclusion on the events that a given season is missing, or counting assignments with season-count pattern $(2, 1, 1, 1)$. However, counting errors were common. Some candidates treated events such as no students being born in spring and no students being born in summer, or events specifying the numbers of students in different seasons, as disjoint or independent. Others assumed that the possible season-count vectors were equally likely. There were also frequent missing combinatorial factors, for instance not accounting for which students occupied the repeated season, and occasional confusion about the size of the sample space.

Question 5. The average score for this question was quite low. In part (a), most of the candidates lost a point for failing to mention that the partition of the probability space had to be countable (or finite). Apart from this, most candidates stated and proved the Law of Total Probability correctly. In part (b), most candidates lost one point for not stating the values of s for which the generating function $G_X(s)$ is defined for, and lost another point for not checking that this domain includes 1. Various arithmetic and algebraic mistakes were also relatively common in (b)(ii) and (b)(iii). However, most of the candidates knew how to approach this question. The vast majority of candidates struggled with part (c). Most candidates stated that independence of B, Y, and Z implies $G_W = G_B G_{2Y} G_{2BZ} G_{-2BY}$, which is false. A few candidates gained 1 or 2 points for (c)(ii) if they did not use the incorrect formula for G_W, which they obtained in (c)(i), but instead used the linearity of expectation directly.

Question 6. The average score for this question was quite high. In part (a), most of the points were deducted for errors in (or omissions of) domains. Apart from that, results for this question were good. In part (b)(i), most candidates were able to obtain the desired equality by integrating by parts. In (b)(ii) most candidates lost a point for not discussing the domains. In part (b)(iii) many candidates were deducted a point for not stating explicitly that they are using (b)(ii). In (b)(iv), the most common mistake was to estimate $\int_0^1 x^2 - 2x \leq 0$, which led only to a weaker conclusion. Part (b)(v) was done correctly by almost everyone who did (b)(iv).

Question 7. Most candidates chose this question. However, it has a very wide spread of marks because a significant number of candidates got muddled with their bookwork in (a) and had further difficulties with the expectation calculations, while others gave very confident answers possibly excluding the last part of (c).

More precisely, in (a) most candidates knew the definitions of bias and MSE, and I marked generously the many vague statements about statistics and estimators, as long as they mentioned data and parameters in a meaningful way.

In (b)(i), a significant number of candidates lost a mark if they assumed that the MLE is a function of $X_1 + \cdots + X_n$ or if they did not check that their stationary point is a maximum, or if their final ratio was upside down.

In (b)(ii), a significant number of students were either uncomfortable with continuous random variables or did not know what to do with the integrals. Of those who did complete this part, about half the students chose an inductive method rather than identifying the integrand as $f_{n-1,\lambda}$ up to a constant factor.

In (c)(i), some candidates struggled with the constraint on α, partly due to poor notation (writing x instead of x_i), and many did not justify why the maximum likelihood is achieved at $\min\{x_1, \ldots, x_n\}$. Others lost a mark for not replacing α by $\hat{\alpha}$ in the expression for $\hat{\lambda}$.

In (c)(ii), those who attempted this often scored a mark for correctly relating

$$P(\hat{\alpha} > x)$$

to the distribution of the X_i.

Question 8. More than half the candidates chose this question, but not all were comfortable with both the regression part and the bookwork (let alone application) of the multivariate normal distribution, so there was again a rather wide spread of marks, as on the one hand quite a few answers were very partial answers, while others were perfect or near-perfect or indeed lost a variable number of marks particularly in the unseen parts of (b)(iii) and (c)(ii).

More specifically, quite a few students lost a mark for mistakes in the joint pdf and in (a)(ii) for getting the dimension wrong or the placing of transposes (not inverses! B isn't even a square matrix, in general!).

(b)(i)–(ii) were generally done well, occasionally with a mark lost for skipping the argument that gets us from MLE to LSE. Some wasted time here by writing everything twice rather than pointing at symmetry for $\hat{\beta}_2$.

In (b)(iii) many partial marks were awarded to those who attempted this, and some did derive all parameters of the bivariate normal distribution.

(c)(i) was done well by many, whether or not (b)(iii) was completed. Full marks were given where candidates applied the argument in reverse to identify the distribution of $\hat{\beta}_1$.

Few candidates made meaningful progress on (c)(ii), but about a dozen found either rectangular or elliptical confidence regions, not always at the correct confidence level.

Question 9. Fewer than half the candidates chose this question. The question has a good spread of marks averaging significantly higher than the

other two questions. Many students lost a mark or two in (a)(i), mainly for lack of mathematical precision. The algorithm in (a)(ii) was usually stated well, except where the choice of K or a clear rule for the initial cluster assignment were missing. Fewer candidates both noted that the objective function is strictly decreasing at each change in an iteration of the algorithm so that the algorithm ends in a local (not, in general, global!) minimum.

In (b)(i), a number of candidates missed the misallocated diamond, others claimed a three-cluster output is impossible, while this is in fact only very unlikely, as it is normal that a random allocation to clusters produces initial cluster means that are all in the same gap between data points and there may be one (in the middle) that is not nearest to any data point.

In (b)(ii), most identified the top left as the best, but few justified this with reference to the objective function.

In (b)(iii), most just pointed at the possibly substantial additional variation in the other PC directions, while others interpreted "good" in a more general sense and pointed out that PC1 is not necessarily a good choice—an example being the crabs dataset. Both marks were awarded to those mentioning both or other plausible reasons.

Paper IV

Question 1. This question received a wide range of answers. Some candidates made little progress after part (a) while others produced complete or near-complete solutions.

Almost all candidates obtain full marks for part (a), though a few made sign errors in their expression for the vector product. Many spent too long establishing (a)(iii) in components instead of using (a)(ii) or $|\mathbf{a}\wedge\mathbf{b}| = |\mathbf{a}||\mathbf{b}|\sin\phi$.

Some candidates wrote an unnecessary page to establish $\mathbf{v} = \mathbf{v}_\parallel + \mathbf{v}_\perp$ with $\mathbf{v}_\parallel = \mathbf{n}\,(\mathbf{n}\cdot\mathbf{v})$ and $\mathbf{v}_\perp = \mathbf{v} - \mathbf{v}_\parallel$. Some confused the rotation angle θ with the angle between $\mathbf{n}$ and $\mathbf{v}$ in the dot product $\mathbf{n}\cdot\mathbf{v}$. The rotation leaves $\mathbf{v}_\parallel$ unchanged, while $\mathbf{v}_\perp$ rotates to $\mathbf{v}_\perp\cos\theta + \mathbf{n}\wedge\mathbf{v}_\perp\sin\theta$. Together these give

$$\mathbf{v}' = \mathbf{v}_\parallel + \mathbf{v}_\perp\cos\theta + \mathbf{n}\wedge\mathbf{v}_\perp\sin\theta = \mathbf{v}\cos\theta + (\mathbf{n}\wedge\mathbf{v})\sin\theta + \mathbf{n}\,(\mathbf{n}\cdot\mathbf{v})(1-\cos\theta)$$

using $\mathbf{n}\wedge\mathbf{v}_\perp = \mathbf{n}\wedge\mathbf{v}$. The three vectors $\mathbf{v}_\parallel$, $\mathbf{v}_\perp$, $\mathbf{n}\wedge\mathbf{v}_\perp$ in the first expression for $\mathbf{v}'$ are orthogonal, but the three vectors $\mathbf{v}$, $\mathbf{n}\wedge\mathbf{v}$, $\mathbf{n}$ in the second expression for $\mathbf{v}'$ are *not* orthogonal. To calculate $|\mathbf{v}'|^2$ for part (c) from the second expression we must include the $2(\mathbf{n}\cdot\mathbf{v})^2\cos\theta(1-\cos\theta)$ cross term. Many candidates forgot this term. Alternatively, the first expression gives

$$|\mathbf{v}'|^2 = |\mathbf{v}_\parallel|^2 + |\mathbf{v}_\perp|^2\cos^2\theta + |\mathbf{n}\wedge\mathbf{v}_\perp|^2\sin^2\theta = |\mathbf{v}_\parallel|^2 + |\mathbf{v}_\perp|^2 = |\mathbf{v}|^2,$$

as $|\mathbf{n} \wedge \mathbf{v}_\perp| = |\mathbf{v}_\perp|$. For part (d) the matrix N is

$$N = \begin{pmatrix} 0 & -n_z & n_y \\ n_z & 0 & -n_x \\ -n_y & n_x & 0 \end{pmatrix},$$

so $N\mathbf{v} = \mathbf{n} \wedge \mathbf{v}$ and $N^2\mathbf{v} = \mathbf{n} \wedge (\mathbf{n} \wedge \mathbf{v}) = \mathbf{n}\,(\mathbf{n} \cdot \mathbf{v}) - \mathbf{v} = -\mathbf{v}_\parallel$ using part (a). We then get $N^3 = -N$ as $\mathbf{n} \wedge \mathbf{n} = 0$, so $N^4 = -N^2$. This is enough to show that $R^\mathsf{T} R = I$.

A few candidates asserted that rotating axes gives

$$R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix} \text{ and } N = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix},$$

However, the question asked for a "direct calculation" from R as given, and $\mathbf{v}_\parallel$, $\mathbf{v}_\perp$, $\mathbf{n} \wedge \mathbf{v}_\perp$ are orthogonal but not orthonormal. One needs a rescaling as well as a rotation. This argument received some but not full marks.

Question 2. This question was broadly well answered overall, with most dropped marks being in part (d). Part (a) was broadly well done, with most candidates recalling the relevant definitions for trace and for a change of basis, and recalling or recognising that part (i) could be used to solve part (ii) for the trace.

Part (b) was similar to a common question for the specific case of $n = 3$. While most candidates recognised that the same argument could be used for n odd in general, there were several who instead solved that specific case. An extremely common cause of dropped marks in the case of n even was also providing a counterexample for a specific n (almost always $n = 2$), rather than for the general case of arbitrary even n. Few candidates therefore scored full marks on this section, despite the proof being very well handled.

Part (c) was bookwork, and most candidates scored highly here. The most common cause of dropped marks, though rare, was a lack of attention to detail in distinguishing between A and A' (usually expressing one in the basis of the other). Part (d) caused by far the most problems, with many candidates not attempting the question. Many recognised that the axis of rotation should be invariant under transformation T, but struggled to identify it. The most common issue was assuming that this axis must go through the origin, or through $\mathbf{b}$, rather than identifying some point $\mathbf{v}_0$ which solves $(I - A)\mathbf{v}_0 = \mathbf{b}$.

For those candidates who attempted the specific case, most were able to correctly find the correct angle of rotation. More challenging (but with reasonable success) was finding the correct vector for the axis of rotation,

while finally associating this vector with a point $\mathbf{v}_0$ was accomplished by a very small number of candidates.

Question 3. Part (a), on the standard formulae for the length of a smooth curve, the area of a smooth parametrised surface, and the area of a surface of revolution about the x-axis, was very well done on the whole, with nearly all candidates scoring full or near-full marks. Part (b), concerning the catenary-type curve $(x(t), y(t)) = (t - \tanh t, \operatorname{sech} t)$, and the associated arc length and surface area integrals, was quite well done. Many candidates, however, struggled to manipulate the hyperbolic functions efficiently or accurately in the line and surface integrals, or to make full use of the symmetry of the curve about the y-axis. The limiting surface area as the endpoints tend to $\pm\infty$ was usually obtained correctly once the earlier steps had been completed.

There were very few attempts at part (c), on the intersection of two right circular cylindrical surfaces of unit radius whose axes meet at a point and make an angle 2α. Nevertheless, a significant minority made some good progress. There were also a handful of complete solutions, and the best of these identified the two elliptical cross-sections and their eccentricities correctly.

Question 4. This question was found very challenging, although questions on this topic used to be set regularly.

Many candidates started part (a) by writing $\dot{\mathbf{r}} = \dot{r}\mathbf{e}_r + r\dot{\theta}\mathbf{e}_\theta$. This already assumes that the particle's trajectory lies on a plane through the origin, so the argument is circular. Instead, one should show that $\mathbf{L} = \mathbf{r} \wedge \dot{\mathbf{r}}$ is constant without introducing coordinates.

Many candidates started part (b) by treating $1/r$ as constant when calculating $\dot{\mathbf{A}}$. Most then also assumed that $\dot{\mathbf{r}} \cdot \mathbf{r} = 0$ to make $\dot{\mathbf{A}}$ vanish. Both assumptions are only true when the orbit is circular.

A few candidates could do the second part of (b). They all argued that the major axis passes through the points where $\dot{r} = 0$. The dot product $\dot{\mathbf{r}} \cdot \mathbf{r} = r\dot{r}$ is indeed zero at these points, so $\mathbf{A}$ is parallel to $\mathbf{r}$ at these points.

Another approach starts from $\mathbf{A} \cdot \mathbf{r} = \mathbf{r} \cdot (\mathbf{p} \wedge \mathbf{L}) - mkr$ and rearranges it into $(|\mathbf{A}| \cos\theta + mk)r = L^2$, where θ is the angle between $\mathbf{r}$ and $\mathbf{A}$. Hence r is smallest when $\cos\theta = 1$, and largest when $\cos\theta = -1$.

Noone made any progress with part (c). One starts with $|\mathbf{p} \wedge \mathbf{L} - \mathbf{A}|^2 = k^2$ and shows that this is the equation of a circle.

Many candidates could show that $\mathbf{B} \wedge \mathbf{L} = \mathbf{A}$. However, most then incorrectly asserted that $\dot{\mathbf{B}} \wedge \mathbf{L} = 0$ implies $\dot{\mathbf{B}} = 0$. It only implies that $\dot{\mathbf{B}}$ is parallel to $\mathbf{L}$. The vector $\mathbf{B}$ is perpendicular to both $\mathbf{L}$ and $\mathbf{A}$, and so parallel to the minor axis of an elliptical orbit.

Question 5. The first two parts, (a) and (b), were mostly fine. In part (b), some students did not explain why the angular velocity was constant, or did not make explicit the temporal derivatives of z in terms of r and $f(r)$. Part (c) was answered correctly using a linearisation argument by only a few students. Those who attempted it often tried to define an effective potential and prove its convexity, sometimes without explaining why the problem could be reduced to a potential equation using the positivity of the "mass" factor. However, many students who adopted this approach did not include the angular component from h in the effective potential. Many others gave conceptually incorrect solutions, for example stating that at equilibrium ($r'' = 0$) but not necessarily ($r' = 0$), or vice versa.

Question 6. This question appeared to be easier than the previous one. The first three parts were generally well answered, although some students had algebraic difficulties in part (b)(ii). Part (b)(iii) was more challenging, with some students running out of time. Many of the students who made progress used a small-angle approximation, probably because they were confused by the phrase "equation ... of a simple pendulum". Some students who otherwise gave good answers also struggled to express the effective gravity as a vector. A clearer wording might have been: "Determine the magnitude and direction of the effective gravity."

Summary: I cannot compare with previous years, but the quality of writing and presentation was particularly poor. In many cases, I had to infer what students were trying to do, which was sometimes difficult given that, in several parts of the questions, the final answer was provided and students were asked to prove it. For instance, there were a few cases where students wrote statements such as "simplifying the algebra, we get [the final answer from the problem statement]", even though the preceding expressions were incorrect. Technically, I felt that the stability argument in Question 5 was definitely challenging, and only a few students showed a genuine understanding of the method.

Paper V

Question 1 and **Question 2**.

In summary, the questions worked really well. They were well balanced, both in terms of amount of students attempting them, as well as in terms of average marks (both questions were around the 12-13 range). Furthermore, the marks were relatively well spread. The total marks for every part and subpart were not modified.

Question 3. Part 3a (i) was, on the whole, answered well by the candidates, as they were able to state the Gauss' Flux Theorem and use the divergence theorem to prove that it follows from Poisson's equation. In their solutions

to part 3a (ii), candidates struggled to apply the divergence theorem to the vector field $\phi\nabla\phi$ to obtain $\iint_{\partial B_R} \phi\,\nabla\phi\cdot\mathbf{n}\,\mathrm{d}S$, and to simplify the integrand of the surface integral to $\phi\,\frac{\partial\phi}{\partial r}$. Very few were able to deduce mathematically that $\iiint_{\mathbb{R}^3}|\nabla\phi|^2\,\mathrm{d}V = 0$ by taking the appropriate limit and applying the relevant bounds. Question 3b (i) was answered well by most candidates as they were able to apply Gauss' Flux Theorem to the given density ρ. However, a few made mistakes when evaluating the integrals. In answering part 3b (ii), most of the candidates seemed to have difficulty understanding the physical significance of the result: namely, that the computed exterior field is the same as that produced by a point mass of magnitude $M_{\mathrm{tot}} = M\big(a(1+\delta)\big) = \frac{4\pi}{3}\,\frac{\sigma}{\delta a}\,a^3\left((1+\delta)^3-1\right)$, located at the origin. Only a handful were able to explain the result of part 3b (iii) in terms of Gauss' Flux Theorem.

Question 4. Part (a), concerning the derivation of the Fourier sine coefficients and Parseval's identity, was answered very well by the majority of candidates, with many obtaining full marks. Only few candidates lost marks through minor algebraic slips or incomplete derivations.

Part (b), requiring the Fourier sine series for a linear function and sketches of the resulting periodic extension, was generally well answered. Most candidates obtained the correct series, though a noticeable minority lost marks through inaccurate sketches of the limiting function. In particular, insufficient attention was paid to the values of the function at the discontinuities, especially at $x = \pi$, 2π, and 3π, leading to incorrect representations of the convergent Fourier series.

Part (c) proved more challenging. While many candidates recognised the connection with the results obtained in part (b), a significant minority failed to exploit it effectively. Marks were also lost through numerical and algebraic errors in the evaluation of the series and the final identity, even where the overall method was sound.

Question 5. Part (a), involving the separation of variables solution of the heat equation with Neumann boundary conditions, was answered well by most candidates. Only a few lost marks through minor errors, particularly missing some use cases for the possible values of lambda.

Part (b) was attempted successfully by only a small minority of candidates and proved considerably more difficult than anticipated. Many candidates struggled to identify an appropriate transformation of the inhomogeneous equation, and relatively few obtained a complete series solution. Consequently, little progress was made on the later parts concerning uniqueness and the long-time behaviour of the solution, with only the strongest candidates producing complete and coherent arguments.

Question 6. The bookwork in part (a), on the derivation of the general solution of the wave equation, was very well done on the whole, with only a handful of candidates losing marks for an incorrect application of the chain rule. The imposition of the initial conditions in part (b)(i), to obtain the specialisation of D'Alembert's solution, was also well done on the whole, though many candidates lost a mark for not accounting correctly for the arbitrary integration constant that appears on integrating $F' = G'$. The sketches in part (b)(ii), for the Gaussian initial profile, were done well by a significant minority, though many candidates lost marks through incorrect sketches arising from an inaccurate representation of the initial profile.

The application of the initial and boundary conditions in part (c)(i) caused substantial difficulties for all but a small minority of candidates, mainly because of a failure to keep track of the ranges of the arguments of F and G for which the various expressions are valid. Only a handful of complete solutions were obtained, and those candidates who handled efficiently the restriction of the domains of definition of F and G made good progress on the tail in part (c)(ii). Overall, the generalisation of the derivation in part (b) to the IBVP in part (c) proved much harder than anticipated, and would have benefited from a little more signposting in part (c)(i).

E. COMMENTS ON PERFORMANCE OF IDENTIFIABLE INDIVIDUALS

Prizes

The Departmental Prize was awarded to:

Shikhar Gupta, Pembroke College.

F. MODERATORS AND ASSESSORS

Moderators: Prof. Paul Balister (Chair), Prof. Fernando Alday, Prof. Paul Dellar, Prof. Jim Oliver, Prof. Panos Papazoglou, Prof. Chris Hollings, Prof. Matthias Winkel.

Assessors: Dr William Hide, Dr Konstantin Recke, Dr Tung Nguyen, Dr Ioannis Papadopoulous, Dr Josh Bull, Dr Carles Falco i Gandia, Dr Christiana Mavroyiakoumou, Dr Niklas Garner, Dr Peleg Michaeli, Dr Ilya Losev, Dr Lida Kanari, Dr Fiyanshu Kaka.