Examiners' Report: Final Honour School of Mathematics Part A Trinity Term 2015

October 27, 2015

Part I

A. STATISTICS

- **Numbers and percentages in each class.**
  See Table 1.

Table 1: Numbers in each class

Range	Numbers					Percentages %				
	2015	2014	2013	2012	2011	2015	2014	2013	2012	2011
70–100	51	57	49	56	55	36.17	36.54	31.21	33.73	33.33
60–69	59	62	71	78	79	41.84	39.74	45.22	46.99	47.88
50–59	26	31	32	28	23	18.44	19.87	20.38	16.87	13.94
40–49	5	4	4	2	7	3.55	2.56	2.55	1.2	4.24
30–39	0	2	1	2	1	0	1.28	0.64	1.2	0.61
0–29	0	0	0	0	0	0	0	0	0	0
Total	141	156	157	166	165	100	100	100	100	100

- **Numbers of vivas and effects of vivas on classes of result.**
  Not applicable.

- **Marking of scripts.**
  All scripts were single marked according to a pre-agreed marking scheme which was strictly adhered to. The raw marks for papers A1 and A2 are out of 100, and for the other papers out of 50. For details of the extensive checking process, see Part II, Section A.

- **Numbers taking each paper.**
  All 141 candidates are required to offer the core papers A1, A2 and ASO, and five of the optional papers A3-A11. Statistics for these papers are shown in Table 2 on page 2.

Table 2: Numbers taking each paper

Paper	Number of Candidates	Avg RAW	StDev RAW	Avg USM	StDev USM
A1	141	73.4	14.22	66.83	11.31
A2	141	63.91	17.1	66.54	10.95
A3	67	28.88	10.22	64.52	13.02
A4	98	32.74	9.58	65.97	13.16
A5	74	35.54	6.72	67.22	10.32
A6	80	33.9	7.29	67.04	10.81
A7	55	36.87	7.08	67.2	11.71
A8	125	31.32	9.82	67.98	13.17
A9	93	33	8.84	66.6	14.29
A10	47	27.49	6.57	64.13	9.79
A11	66	39	7.26	66.7	12.14
ASO	141	35.58	7.6	66.11	10.36

B. New examining methods and procedures

This was the second year of the new Part A structure. The core papers AC1 and AC2 have been replaced with core papers A1 and A2. The cross-sectional papers AO1 and AO2 have been replaced with option papers A3-A11. In addition there is a core cross-sectional paper, ASO, examining the short option courses.

C. Changes in examining methods and procedures currently under discussion or contemplated for the future

None.

D. Notice of examination conventions for candidates

The first Notice to Candidates was issued on 3rd March 2015 and the second notice on the 12th May 2015.

These can be found at https://www.maths.ox.ac.uk/members/students/undergraduate-courses/ba-master-mathematics/examinations-assessments/examination-20, and contain details of the examinations and assessments. The course Handbook contains the link to the full examination conventions and all candidates are issued with this at Induction in their first year. All notices and examination conventions are on-line at
https://www.maths.ox.ac.uk/members/students/undergraduate-courses/examinations-assessments/examination-conventions.

Part II

A. General Comments on the Examination

The examiners would like to express their gratitude to

- Nia Roderick for her dedicated work supporting Part A examinations throughout the academic year 2014/15.
- Also Helen Lowe for running the database during the final meeting and for assisting the Chair with various enquiries during the year.
- Waldemar Schlackow for his further improvements generating reports from the examination database.
- Charlotte Turner-Smith and Academic Administration in general all help behind the scenes too with script sorting/marks entry etc..
- We also thank the assessors who set their questions promptly, took care in checking and marking them, and met their deadlines. This is invaluable help for the work of the examiners.
- All the assessors and the internal examiners would like to thank the external examiners Dr. Mark Wildon (pure mathematics) and Dr Warren Smith (applied mathematics) for their careful reading of the draft papers, thorough scrutiny of the scripts and insightful comments throughout the year. In particular Dr Wildon's coverage of all papers in the previous two years, and support for the Part A Examination as it transitioned to its new format, is greatly appreciated.

Timetable

The examinations began on Monday 15th June at 2.30pm and ended on Friday 26th June at 11.00am.

Medical certificates and other special circumstances

A subset of the Examiners attended a pre-board meeting to band the seriousness of each application of Factors Affecting Performance form received from the Proctors' office. The outcome of this meeting was relayed to the Examiners at the final exam board, who gave careful regard to each case, scrutinised the relevant candidates' marks and agreed actions as appropriate. See Section F for further details.

Setting and checking of papers and marks processing

As is usual practice, questions for the core papers A1 and A2, were set by the examiners and also marked by them. The papers A3-A11, as well as each individual question on ASO, were set and marked by the course lecturers. The setters produced model answers and marking schemes led by instructions from Teaching Committee in order to minimize the need for recalibration.
The internal examiners met in December to consider the questions for Michaelmas Term courses (A1, A2 and A11). The course lecturers for the core papers were invited to comment

on the notation used and more generally on the appropriateness of the questions. Corrections and modifications were agreed by the internal examiners and the revised questions were sent to the external examiners.
In a second meeting the internal examiners discussed the comments of the external examiners and made further adjustments before finalising the questions. The same cycle was repeated in Hilary term for the Hilary term long option courses and at the end of Hilary and beginning of Trinity term for the short option courses. *Papers A8 and A9 are prepared by the Department of Statistics and jointly considered in Trinity term.* Before questions were submitted to the Examination Schools, setters were required to sign off a camera-ready copy of their questions. Examination scripts were collected by the markers from Ewert House or delivered to the Mathematical Institute for collection by the markers and returned there after marking. A team of graduate checkers under the supervision of Nia Roderick and Jan Boylan sorted all the scripts for each paper, cross-checking against the mark scheme to spot any unmarked questions or part of questions, addition errors or wrongly recorded marks. Also sub-totals for each part were checked against the marks scheme, noting any incorrect addition. An examiner was present at all times to authorise any required corrections.

Determination of University Standardised Marks

The examiners followed the standard procedure for converting raw marks to University Standardized Marks (USM). The raw marks are totals of marks on each question, the USMs are statements of the quality of marks on a standard scale. The Part A examination is not classified but notionally 70 corresponds to 'first class', 50 to 'second class' and 40 to 'third class'. In order to map the raw marks to USMs in a way that respects the qualitative descriptors of each class the standard procedure has been to use a piecewise linear map. It starts from the assumption that the majority of scripts for a paper will fall in the USM range 57-72, which is just below the II(i)/II(ii) borderline and just above the I/II(i) borderline respectively. In this range the map is taken to have a constant gradient and is determined by the parameters C_1 and C_2, that are the raw marks corresponding to a USM of 72 and 57 respectively. The guidance requires that the examiners should use the entire range of USMs. Our procedure interpolates the map linearly from $(C_1, 72)$ to $(M, 100)$ where M is the maximum possible raw mark. In order to allow for judging the position of the II(i)/III borderline on each paper, which corresponds to a USM of 40, the map is interpolated linearly between $(C_3, 37)$ and $(C_2, 57)$ and then again between $(0, 0)$ and $(C_3, 37)$. It is important that the positions of the corners in the piecewise linear map are not on the class borderlines in order to avoid distortion of the class boundaries. Thus, the conversion is fixed by the choice of the three parameters C_1, C_2 and C_3, the raw marks that are mapped to USM of 72, 57 and 37 respectively.
The examiners chose the values of the parameters as listed in Table 3 guided by the advice from the Teaching Committee and by examining individuals on each paper around the borderlines.

Table 3: Parameter Values

Paper	C1	C2	C3
A1	83	57.3	32.9
A2	79	41.5	23.8
A3	38.8	19	10.2
A4	41.6	22.1	12.7
A5	41	27	15.5
A6	39.8	25	15.1
A7	42.8	29.3	16.8
A8	38.5	18	10.8
A9	39	25	14.4
A10	33.6	21.6	12.4
A11	44	32.3	12
ASO	43.2	26.7	15.3

Table 4 gives the resulting final rank and percentage of candidates with this overall average USM (or greater).

Table 4: Rank and percentage of candidates with this overall average USM (or greater)

Av USM	Rank	Candidates with this USM or above	%
90	1	1	0.71
86	2	5	3.55
84	6	6	4.26
83	7	9	6.38
82	10	13	9.22
78	14	14	9.93
77	15	19	13.48
76	20	22	15.6
75	23	28	19.86
74	29	32	22.7
73	33	39	27.66
72	40	43	30.5
71	44	46	32.62
70	47	51	36.17
69	52	56	39.72
68	57	61	43.26
67	62	64	45.39
66	65	74	52.48
65	75	83	58.87
64	84	92	65.25
63	93	100	70.92
62	101	102	72.34

61	103	105	74.47
60	106	110	78.01
59	111	117	82.98
58	118	121	85.82
57	122	126	89.36
56	127	127	90.07
55	128	129	91.49
54	130	130	92.2
52	131	131	92.91
51	132	133	94.33
50	134	136	96.45
47	137	137	97.16
46	138	138	97.87
45	139	139	98.58
44	140	140	99.29
41	141	141	100.00

B. Equal opportunities issues and breakdown of the results by gender

Table 5, page 7 shows the performances of candidates broken down by gender.

Table 5: Breakdown of results by gender

Range	Total		Male		Female	
	Number	%	Number	%	Number	%
70–100	51	36.17	41	40.59	10	25
60–69	59	41.84	42	41.58	17	42.5
50–59	26	18.44	16	15.84	10	25
40–49	5	3.55	2	1.98	3	7.5
30–39	0	0	0	0	0	0
0–29	0	0	0	0	0	0
Total	141	100	101	100	40	100

C. Detailed numbers on candidates' performance in each part of the exam

Individual question statistics for Mathematics candidates are shown in the tables below.

Paper A1: Algebra 1 and Differential Equations 1

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14.91	15.35	5.84	86	4
Q2	15.01	15.48	5.17	93	4
Q3	19.78	20.17	5.21	103	3
Q4	19.63	19.63	2.90	130	0
Q5	21.23	21.23	3.88	117	0
Q6	14.58	14.91	4.21	32	1

Paper A2: Metric Spaces and Complex Analysis

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.06	16.25	6.90	119	2
Q2	16.12	16.19	5.18	118	1
Q3	14.29	14.96	6.17	89	6
Q4	13	14.55	6.66	74	14
Q5	16.86	17.36	5.86	113	7
Q6	14.82	15.63	4.38	51	5

Paper A3: Algebra 2

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	13.38	13.38	4.61	65	0
Q2	19.87	21.29	7.78	14	1
Q3	14	14.20	5.39	54	1

Paper A4: Integration

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	19	19.28	5.64	68	1
Q2	13.19	13.33	5.05	86	2
Q3	17.27	17.88	7.59	42	2

Paper A5: Topology

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.81	18.81	3.65	68	0
Q2	16.58	16.58	3.63	59	0
Q3	17.76	17.76	4.75	21	0

Paper A6: Differential Equations 2

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.31	17.42	3.50	69	1
Q2	14.80	15.76	5.33	49	7
Q3	16.21	17.57	6.22	42	5

Paper A7: Numerical Analysis

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.16	16.43	5.23	30	1
Q2	15.28	17.46	6.07	28	8
Q3	20.12	20.12	3.45	52	0

Paper A8: Probability

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.02	15.61	6.22	98	5
Q2	15.46	15.53	5.13	98	1
Q3	15.31	15.98	5.65	54	4

Paper A9: Statistics

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.54	15.69	4.36	67	1
Q2	16.24	16.39	5.54	77	1
Q3	17.06	18.00	5.81	42	5

Paper A10: Waves and Fluids

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	8.95	9.42	4.06	19	2
Q2	17.30	17.30	4.35	46	0
Q3	10.48	10.93	4.03	29	2

Paper A11: Quantum Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	20.52	20.87	3.30	53	1
Q2	19.28	19.28	4.70	58	0
Q3	16.67	16.67	7.35	21	0

Paper ASO: Short Options

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14.43	14.54	3.98	41	1
Q2	17.73	17.73	2.74	15	0
Q3	16.57	19.00	7.55	6	1
Q4	17.71	18.60	6.37	20	1
Q5	18.69	18.69	2.74	90	0
Q6	19.98	20.20	5.58	54	2
Q7	15.55	16.00	5.38	52	3
Q8	14.20	16.00	7.16	4	1

D. Recommendations for Next Year's Examiners and Teaching Committee

It has already been agreed by Teaching Committee that the 3-hours-long Paper A1 be split into separate 1.5-hours-long papers on Algebra and Differential Equations. This will eliminate any concerns that the Examiners might have about differences in the difficulties between the two halves, and potential questions for rescaling A1(CP) differently to A1.

The Examiners felt that the cross-sectional ASO paper worked well this year, but would like to stress the need for questions to be of a comparable difficulty as the individual questions cannot be scaled. The guidance to short option assessors might be strengthened to highlight this; also a more granular mark scheme allows assessors more flexibility in generously/strictly marking overly hard/easy questions.

On some few of the papers, it was noted that there was rather shallow scaling in the middle range of marks – i.e. large differences in raw marks led to small changes in the USMs. It was pointed out that this could be addressed by having greater granularity in the mark scheme, so that partial attempts of some parts might at least accrue some marks.

Part A 2015: Comments on sections and on individual questions

The following comments were submitted by the assessors.

Core Papers

A1: Algebra 1 and Differential Equations 1

Q1: There were some good attempts at this question, but many candidates encountered surprising difficulties.

In part (a), some candidates asserted that $V = \dim T \cup \ker T$. Many missed the special cases when T is the identity or the zero transformation. Some missed that 0 could be an eigenvalue of a projection.

In part (b), some answers relied on the false assumption that the ring of linear transformations on V has no zero-divisors; people said things like "$EF(E - I) = 0$ so $EF = 0$ or $E - I = 0$"; this reasoning doesn't work. In (b)(ii), some people assumed that E and F could be simultaneously diagonalised; of course if that's true then it is easy to see that $EF = FE$.

Various different correct counterexamples to (b)(iii) and (c) were provided.

Q2: In part (a), some people asserted that T was diagonalisable if and only if $m_T(x)$ could be expressed as a product of linear factors; and some stated that the given matrix A was diagonalisable because $\mathbb{C}$ was algebraically closed. Neither is correct. In (a)(iii), the fact that $x^2 + x + 1$ is reducible in $\mathbb{Z}_3$ was missed by some. The point of part (a) is to find $m_T(x)$ in the various fields given and observe whether it is a product of distinct linear factors; many candidates did this but some did not.

In part (b), common errors included assuming that V_λ had dimension one, and that S and T had the same eigenvectors. Some people used the word "orthonormal", when no inner product had been given. That having been said, this section was well-done on the whole.

Few people spotted how to generalise the argument of (b)(iv) to solve part (c).

Q3: There were many good solutions to this question.

In part (a), parts (ii) and (iii) both involve checking a number of conditions; it is important not to miss any of these out. Plenty of candidates unnecessarily lost marks in this way. There was (perhaps unsurprisingly) quite a bit of confusion about which argument an inner product is linear in. This is really the fault of the profession; but the question was using linearity in the first argument, as I believe the lecture course also did. There was also some confusion about the meaning of positive definiteness; some tried to prove the condition that $\langle u, v\rangle = 0$ implies $u = 0$ or $v = 0$; this is only true in zero- or one-dimensional inner product spaces.

Part (b) in particular was done well. One can do part (iii) by induction on the dimension of V; the appeal to the inductive hypothesis involves the restriction of T to the orthogonal complement of an eigenvector, *not* a map on a quotient space. In part (iv), the question of whether the dimension of V is even or odd is not very important; if the dimension is odd then 0 must be an eigenvalue, but it may be an eigenvalue also if the dimension of V is even.

Q4: (a)(i) On the whole, very good. Many candidates did not know what autonomous meant.
(ii) It was easy to get at least half marks on this question, however, to get full marks needed a better understanding and control of the approximation.
(b) Compound errors were the greatest problem here. Mistakes in calculating the Jacobian caused trouble when calculating the stability, which caused trouble when plotting. Even when all calculations had been done correctly few candidates gave a complete sketch of the phase plane.

Q5: Almost all candidates did this question, and the solutions were very good in general.

Q6: This was the least popular question. The bookwork aspects of this question were in general well done. Some students struggled to apply the bookwork to the specific problem in part (b). Detailed comments are as follows:

- Some candidates failed to consider the sup norm when showing that T maps C_η into C_η.
- A number of candidates failed to correctly deduce that the map was a contraction provided $\eta < 1/(L+1)$ (*i.e.* they used only the condition $|F(x,\mathbf{y})-F(x,\mathbf{z})| \leqslant L\|\mathbf{y}-\mathbf{z}\|$, and did not consider both components of $\|T\mathbf{y} - T\mathbf{z}\|$ when determining the Lipschitz constant.
- In Part (b), the majority of candidates failed to give details of the set S that must be considered to ensure that y is non zero. Some candidates did not even comment on this issue.
- The majority of students did not show that the components of $\mathbf{f}$ were bounded, and hence failed to give all the conditions on η that must be satisfied (*i.e.* $\eta < k/M$).
- No student determined the bounds on the components of $\mathbf{f}$, or the Lipschitz constant, in terms of k.

A2: Metric Spaces and Complex Analysis

Q1: Most candidates completed the bookwork parts (a) and (b) appropriately. In part (c), numerous candidates incorrectly identified $\mathbb{Q} \cap [0,1]$ as compact. Candidates did quite well overall on parts (d) and (e), though a moderate number stumbled by trying to prove these facts abstractly (that is showing there exist such functions without constructing them) rather than simply exhibiting functions (for arbitrary X) that have the indicated properties.

Q2: Most candidates did the bookwork parts (a) and (b) correctly. Most candidates made a good attempt at part (c), though also many errors occurred, such as identifying (iii) as disconnected, or getting confused about the line orientation in (iv) and so identifying it as not path connected; and nearly all candidates were stumped by (vi). Various creative solutions to part (d) appeared, though some candidates didn't quite manage to put together the ingredients (of path-connectedness, the antipodal map, the connectedness of the image, and the intermediate value theorem) that they'd assembled.

Q3: Almost all candidates solved the bookwork parts (a) and (b) correctly. Most candidates also used the linear term factorization to quickly dispatch the first item of part (c), and many simply used direct calculation to establish the second item. Almost no candidates saw how to prove the last item of part (c), with a couple managing by direct computation and an even smaller number by elegant geometric argument.

Q4: Part (a) was well-done in the main, but it was determining the modulus and argument of $1 - e^{i\theta}$ that proved to be the most problematic part of the question. Many could work out that $\left|1 - e^{i\theta}\right| = 2\sin(\theta/2)$ but only a few could work out that $\arg\left(1 - e^{i\theta}\right) = (\pi - \theta)/2$, sometimes by geometric means. Many who could not complete (b) still successfully employed the given expression for $\sqrt{1 - e^{i\theta}}$ in (c) when parametrizing the path integral from (a).

Q5: A popular question with (a) and (b) largely well done, though it was a shame to see many carelessly dropping marks in (a) through inexact or missing hypotheses (e.g. that the contour is simple, closed and positively oriented). It was frustrating to see in (c)(i) little use of the formula from (b)(i); instead of addressing $\sinh z = \pm i \cosh a$ with the previously found formula, many reverted to a basic definition of $\sinh z$ as $(e^z - e^{-z})/2$ or squared the expression found for $\sinh(x + iy)$. Whilst still correct, both led to more intractable algebra. The only singularities of the integrand inside the contour are $\pm a + i\pi/2$; a good number of students who could determine this in (c)(i) still made some progress with (c)(ii) showing that the left and right integrals became negligible, noting that $\sinh(x + \pi i) = -\sinh x$ or showing that any singularities would necessarily be simple poles and finding their residues.

Q6: Part (a) was well-done but many students did not make use of it in addressing (b). Under $z \mapsto 1/z$ the circline $Az\bar{z} + B\bar{z} + \bar{B}z + C = 0$ transforms to $Cz\bar{z} + \bar{B}\bar{z} + Bz + A = 0$ with the former being a circle not through 0, a circle through 0, a line not through 0 or a line through 0 precisely when neither A nor C, just C, just A or both A and C are zero. Instead many students reverted to more standard euclidean equations for circles and lines. The collinearity of P, Q, R in the first diagram was meant to be addressed using basic geometry; if O_1 and O_2 are the circles' centres then O_1QO_2 is a line making the same angle with the parallel lines O_2R and PO_1 and hence RQO_2 and PQO_1 are equal angles. Many unsuccessfuly tried to use function $1/z$ for this part. Instead $1/z$ transforms the second diagram to the first if any of the four points of tangency are taken as the origin.

ASO: Short Options

Q1. Number Theory:

The bookwork parts of the question were done very well. Those who knew Pollard's p-1 method were able to perform the straightforward calculations required for the factorisation in 1b. However, very few made a successful attempt at 1c.

Q2. Algebra 3:
There were a number of good attempts, showing a strong and coherent understanding of semi-direct products. Some errors were made in showing that $x^i y^j x^r y^s = x^{i+4^j r} y^{j+s}$ but then, in (b), showing the defining and verifying the group structure of a semi-direct product was very well done. Many noted that $G \cong C_9 \rtimes C_3$ using $\varphi(y)(x) = x^4$ or noted that G is an internal semi-direct product of $\langle x \rangle \cong C_9$ and $\langle y \rangle \cong C_3$. For part (c), as G is not abelian and $\langle x \rangle$ is normal, then $\{e\} \neq G' \leqslant \langle x \rangle$ and as $\langle x \rangle$ is abelian then $G'' = \{e\}$ and so the derived length is two. Finally G and H are not isomorphic as every non-trivial element of H has order 3 whilst x has order 9.

Q3. Projective Geometry:
This question was about the cross-ratio. There were 8 attempts. The bookwork and more routine parts of the questions ((a),(b) and (d)) were mostly done well. Candidates mostly knew the definitions and could do the routine calculations. Part (c) was found more challenging though two or three candidates gave good answers here. No-one quite got the geometric interpretation of harmonically separated points at the end of (e).

Q4. Introduction to Manifolds:
This question was about using Lagrange multipliers to find critical points of functions on submanifolds of $\mathbb{R}^n$. There were 22 attempts.
The bookwork in part (a) was generally well done, although some candidates were a little careless with some details. Part (b), though also bookwork, was less well done; most people knew the Lagrange multiplier condition but only a few really gave a good explanation of the underlying geometry.
Most candidates were able to use Lagrange multipliers to find the maximum in (c). However some candidates were a bit careless about checking that the constraint equation defined a manifold, about compactness, and about behaviour at the endpoints of the curve in the positive quadrant.
Part (d), about using the result of (c) to derive the Hölder inequality, was found more difficult, but quite a few candidates were able to do this successfully.

Q5. Integral Transforms:
A popular question, largely well done, with most scripts showing fluent command of the transform and its properties. However most candidates arrived at $\overline{J_0}(p) = A/\sqrt{1+p^2}$ but did not correctly conclude that $A = 1$ from noting $\overline{J_0'}(\infty) = 0$. Most marks lost were due to omitting statements of the standard results used – definition and properties of the convolution, sifting property of the delta function, injectivity of the Laplce transform. etc..

Q6. Calculus of Variations:
This was a popular question, with over 70 entries. On the whole it was well done, with more than half of the entries achieving over 20 marks. Virtually everyone could derive the Beltrami identity satisfactorily, and almost all saw its relevance to the problem. The difficulty for the weaker candidates lay in solving the resulting first-order ODE, something that ought to have been familiar from the first term of Prelims onwards. Too much time and effort was wasted over this. In the later part of the question, almost all could write down the natural boundary condition as a formula, but rather few used the simplicity of the circles to go directly to statements of the solutions.

Q7. Graph Theory:
Overall there was a good spread of marks, with some excellent answers. In part (a)(iii), those who used induction on the number of vertices of T did well, but many tried induction on K which does not work easily. In parts (b) (ii) and (iii), many seemed to look for more difficulties than were there, and thus found them!

Q8. Special Relativity:
There were only six attempts, even fewer than last year. The definition of a Lorentz transformation was correctly stated by virtually all, but demonstration of the required 'if and only if' proposition in the 2-dimensional case was less assured, even though this question had been addressed on the first worksheet. The definition of rapidity was well understood, and so was the scenario of the 'moving walkway' problem, but even the most successful candidate made rather heavy weather of taking the limit so as to obtain the required result.

Option Papers

A3: Algebra 2

Q1: This question was attempted by most students. Parts (a) and (b) were generally done well, while part (c) was probably the most demanding on the paper, with only a few students solving it completely. On the other hand, quite a few students realized how to utilize the previous parts to solve part (d).

Q2: This was the best-answered, but least popular question. If candidates had come to terms with the notion of a module they usually acquitted themselves very well on almost all parts of the question. A variety of different methods were used in the final part, which was nice to see.

Q3: Many students made a slip of some sort establishing the basic properties of content for nonzero polynomials in $\mathbb{Q}[t]$, but overall candidates did well on the question. (This is perhaps because Gauss's Lemma is more subtle than it appears?) Many students provided good solutions to the parts establishing that the prime ideal P is principal. In the final part, while most correctly produced an example of a non-principal prime ideal, quite a few people did not carefully check both properties.

A4: Integration

Q1: Parts (a) and part (b) are routine bookwork, most attempts being able to answer these parts more or less fully and to obtain good marks. It is a bit disappointing that a few students couldn't apply the additivity of the Lebesgue measure to a concrete case and to deduce the uniform continuity, and thus lost a big chunk of marks for part (c).

Q2: Part (a) is a standard example of integrability, with many attempts doing well, though a few candidates applied the comparison theorem directly to functions which change signs. Most of those attempted were able to supply substantial arguments to answer part (b), but a few students did not realize that a control function when applying DCT should not depend on the parameter. Part (c) is the challenging part of this question. Many attempts failed to produce a correct control function, though almost all those attempts understood that one should apply a convergence theorem to justify the limit under integration.

Q3: Part (a) is routine and is a slight modification of an example in the lecture notes. Most attempts did well on this part. Part b) proved tricky, although it follows a simple application of Tonelli's theorem and change of variables (either using change of variables for the double integral, or do change of variables for a repeated integral). Many students who attempted this question knew one should use Tonelli's theorem to prove the integrability, then apply Fubini's theorem to conclude the inequality, but many candidates couldn't implement this idea fully.

A5: Topology

Q1: Parts (a)(i), (a)(ii) were generally well done. However a few candidates did not define connected subsets and others did not use this definition correctly in part (a)(ii). Part (a)(iii) was generally well done.
Part (b)(i) was done by most candidates but the proof given was often longer than necessary. Part (b)(ii) was generally well done but there were careless errors in the proof of both directions-in particular often candidates took two points $(x_1, y_1), (x_2, y_2)$ and assumed that both $x_1 \neq x_2$ and $y_1 \neq y_2$ thus not covering all cases.
Part (b)(iii) was more challenging but many candidates had the right idea how to approach this. Some tried to argue using path connectedness which of course was not possible as the spaces were not assumed to be path connected.
Several candidates answered either the first or the second part of (b)(iv) and a few answered both.

Q2: Many candidates attempted this. Part (a) was generally well done. Part (c)(i) was done well by most candidates. In part (c)(ii) several candidates argued just by a picture and did not get any marks for this as they were supposed to give a formal proof using part (b). Candidates gave a range of different arguments for (c)(iii) sometimes reducing it to (c)(ii) or arguing for compactness by definition. Even though these approaches were longer when they managed to give a complete proof they got full credit for their answers. Part (c)(iv) was well done but some candidates mistakenly asserted that the topology in the second case is the indiscrete topology. Finally part (c)(v) was quite demanding and only one candidate gave a complete proof.

Q3: Part (a)(i) was generally well done but sometimes the description of the topological realization was not very precise. Part (a)(ii) was well done but some candidates tried to map the simplicial complex to a simplex of maximal dimension which was incorrect. Part (a)(iii) was well done even though some candidates forgot the Hausdorff condition. Part (a)(iv) was answered by many candidates but some only gave a representation as a quotient space and sometimes the simplicial complex structure was incorrect.
In part (b)(i) most candidates had an intuitive understanding and quite a few gave a precise definition. The classification theorem was generally stated correctly. In part (b)(iii) some candidates assumed that the surfaces were described by 4 letter words in the classification theorem without proof. Quite a few candidates approached this correctly and got either full marks for complete solutions or partial marks when they missed some case.

A6: Differential Equations 2

Q1: Was attempted by nearly all candidates. Most received full marks on (a) and (b), although some spent more time than needed on (b). Few people saw the way to construct the Green's function in (c).

Q2: Produced widely varying attempts. Common mistakes were: not forming the general solution using Frobenius machinery in (b), spending time unnecessarily computing coefficients in (b), and not paying close attention to the form of polynomial in (c).

Q3: Was attempted by about half the candidates. A few were hurt by early algebra mistakes in the asymptotic root finding. The final part caused the most trouble (c)(ii), with few people recognising the structure $Ly_{11} = f(y_1(x); \lambda_1)$ in order to use FAT.

A7: Numerical Analysis

Q1: Was reasonably well done. In (b) one should be careful to note the fixed sign when using the Integral MVT. For the "not a contradiction" bit, many simply said "well its only a bound", missing that the error in the formula is actually attained for some ξ (the "cannot be improved" bit). Part (d) was an application of Richardson extrapolation.

Q2: Was reasonably well done. In part (a), the presence of the italicized word "unique" does not free one from the burden of existence. Many people were able to re-use their result in (b) for (c)(i) by noting that q interpolates f' on the η_i. A few noted that ξ was an unknown (and perhaps non-differentiable) function of x.

Q3: Had lots of high-scoring answers, and perhaps easier than intended or everyone learned this material really well. Common mistakes: J(2,n) in principle changes the entire 2nd and nth row whereas many assumed it was more local, e.g. in (d). Part (b) was generally well done (this was intended as "new" material), and the most common mistake was not explaining that $Q^{-1}b$ can be computed as $Q^T b$, and furthermore, using n^2 flops.

A8: Probability

There were plenty of good scripts, although few that were 100% convincing throughout. Questions 1 and 2 were much more popular than Question 3 (although I don't think Q3 is any harder, and of those who attempted it many were quite successful).

Q1: Everything up to (c)(i) is fairly standard, with some easyish marks. The asymptotics in (c)(ii) caused a few more problems. Even if for some reason you can't get the convergence of the mgfs to work out fully, there is plenty of partial credit available for stating the continuity theorem (those who did this accurately were certainly in the minority), for observing that the limit in distribution should be standard normal, and for observing that hence the sequence of mgfs should converge to the one obtained in (b)(iii).

Q2: The essentials of the transformation to polar coordinates caused little difficulty. For the joint distribution of Θ and R^2, a good answer would be something like: Θ has uniform distribution on $(0, 2\pi)$, R^2 has exponential(1/2) distribution (or equivalently χ_2^2 distribution if you prefer), and the two are independent (the last point is particularly important).

(a)(ii) caused a lot of problems. It's immensely helpful to start off with a good diagram; most straightforwardly, draw the region $Y > c|X|$ in the XY-plane, but also a graph showing the functions $\sin\theta$ and $c|\cos\theta|$ on the interval $[0, 2\pi]$ works well. In either case, it becomes clear that the region we want corresponds to an interval of Θ, and the calculation is not hard. Most candidates however tried to proceed by symbolic manipulation alone, and most came unstuck (the interaction of inequalities and inverse trig functions is easy to get wrong).
In (b) the key observation is that the positive XY-quadrant does NOT map to the whole of the positive UV-quadrant, but instead to the set $0 < U < V < 2U$. If you miss this in (i), then (ii) and (iii) are likely to give you impossible answers, and should act as some kind of sanity check. But plenty of candidates wrote something like a "uniform distribution on $[0, \infty)$", or on $[u, \infty)$, in part (ii), without noticing that something was amiss.

Q3: Parts (a) and (b) were mostly done quite successfully (although, as would be expected, writing out a completely grammatical argument involving almost sure convergence was the exception rather than the rule). In (b)(iii), very few candidates were willing to state clearly the result involving minimal non-negative solutions to systems of recurrence relations along with their boundary conditions, and without this the argument will not be complete. A few tried to short-cut this by invoking $h_k \to \infty$ as $k \to \infty$ as an extra boundary condition, but of course this is essentially what we are trying to prove. Part (c) is less familiar, but the hint in (c)(ii) is reasonably generous, and there were plenty of convincing solutions.

A9: Statistics

Q1: Part (a) was done very well. In (b)(i) most candidates had no trouble showing the X_i are identically distributed with cdf F, but most said nothing about the X_i being independent. There were varying degrees of success with (b)(ii), most candidates could make at least a reasonable attempt. For the two distributions in (c), it is necessary to determine the variance of each, and to find the value of the pdf at the median in each case. The median (and mean) of each does not require a calculation, neither does evaluating the pdf at median, but many mistakes were made. Finding the variance of the second distribution is more difficult, but can quickly be related to the variance of an exponential distribution – only a few candidates got (close to) the correct answer. Some candidates answered the last bit without giving the required reasons.

Q2: Most candidates did (a) well, but a full answer needs to either specify what the critical region is or say how to calculate the p-value – a few candidates gave neither of these. In (b)(ii), "what would you conclude?" invites saying more than reject/don't reject – from the information given the p-value is only a bit away from 0.05, so it is possible to interpret this, and this should be included in a full answer. There were some good answers to (c), but many candidates made errors doing the required maximisations – we maximise over β under the null, and over β and θ under the alternative. The resulting likelihood ratio depends on neither β nor θ – some answers depended on β, others on θ.

Q3: This question was slightly less popular than Qs 1 and 2, but only marginally so. At the end of (a), some candidates gave the posterior odds instead of the posterior probability that was asked for. Not surprisingly there were some algebraic mistakes in (b)(i), and hence in (ii) and (iii), but errors in (i) didn't prevent candidates from being able to attempt (ii)/(iii).

A10: Waves and Fluids

Q1: This was the least popular, and the least well done. Some candidates tried to obtain the path-independence of the integral expression for the streamfunction ψ from $\nabla \times u = 0$ (as for ϕ) rather than from $\nabla \cdot u = 0$. Similarly, some candidates tried to find the potential flow condition in part (b) from $\nabla \cdot u = 0$, which is automatically satisfied when u is expressed using a streamfunction, rather than from $\nabla \times u = 0$. In part (c) only a few candidates realised that they needed to show that $\tilde{\Psi}$ satisfies the potential flow condition from part (b) if Ψ does. In part (d), no-one could find the streamfunction for uniform flow in spherical coordinates by integrating the given expressions for u_r and u_θ from part (b).

Q2: All candidates attempted this question. Most candidates either ignored the moment (Prelims Dynamics) or wrote down an integral involving $x.dx$ without any mention of cross products or perpendicular distances. The standard proof of the Blasius force theorem was reproduced correctly by almost all candidates, though a surprising number did not even attempt a modified proof for the moment. Almost all candidates found the correct complex potential and velocity from the method of images in part (c). For part (d), many candidates obtained correct solutions for the force and moment, though a few just calculated residues rather than closing the contour and estimating the contribution from a large semicircle. Some incorrectly claimed that the contribution to the moment integral from a large semicircle went to zero. It is easier to spot that the integrand is an odd function of x, and the advice to consider symmetric intervals $[-R, R]$ avoids any issues with convergence.

Q3: This was the second most popular question. Almost all candidates could derive the mass and momentum conservation equations from Reynolds' transport theorem, though a few confused material and space-fixed volumes. No-one could linearise these equations and eliminate one variable to derive the linear wave equation, though many candidates went on to derive the correct linearised boundary condition on ϕ at $x = 0$ in part (c). Most candidates asserted that $\phi \to 0$ as $x \to \infty$, though in general ϕ is only bounded. For part (d), many candidates wrote down $\nabla^2\phi = 0$, ignoring the wave equation for ϕ displayed in part (b). Quite a few candidates made good attempts at separable solutions for ϕ, and one candidate produced a near-complete solution. The solution for $\omega > \omega_c$ is a right-going wave travelling away from $x = 0$, from a group velocity argument, so it is helpful to consider a separable solution proportional to $\sin(kx - \omega t)$.

A11: Quantum Theory

Q1: A question on stationary states with a finite square well, this was popular and high-scoring. The parts done least well were the explanation why one could restrict to eigenstates of parity and the graphical solution at the end.

Q2: A question on finding the energy levels of the harmonic oscillator algebraically, on the course for the first time this year, I think this was the most popular question. The parts done least well were the explanation of finding E_n, i.e. the actual raising and lowering part, and the significance of a nondegenerate ground state.

Q3: A question on series solutions for the hydrogen-like atom, this was the least popular. The parts done least well were justifying the choice of solution of the indicial equation, and justifying the requirement for f to be polynomial.

Overall, averages marks were high so I adhered closely to the mark scheme.

F. Comments on performance of identifiable individuals

Removed from the public version of the report.

G. Names of members of the Board of Examiners

- **Examiners:**
  Dr R. Earl (Chair)
  Prof. C. Douglas
  Dr R. Knight
  Dr N. Laws
  Prof. S. Waters
  Dr M. Wildon (External Examiner)
  Dr W. Smith (External Examiner)

- **Assessors:**
  Dr J. Balakrishnan
  Prof. A. Dancer
  Prof. P. Dellar
  Dr A. Hodges
  Prof. C Macdonald
  Prof. J. Martin
  Prof. C. McDiarmid
  Prof. K. McGerty
  Prof. D. Moulton
  Prof. P. Papazoglou
  Prof. Z. Qian
  Prof. P. Tod
  Dr T. Woolley