Examiners' Report: Final Honour School of Mathematics Part A Trinity Term 2016

October 20, 2016

Part I

A. STATISTICS

- **Numbers and percentages in each class.**
  See Table 1.

Table 1: Numbers in each class

Range	Numbers					Percentages %				
	2016	2015	2014	2013	2012	2016	2015	2014	2013	2012
70–100	50	51	57	49	56	34.97	36.17	36.54	31.21	33.73
60–69	63	59	62	71	78	44.06	41.84	39.74	45.22	46.99
50–59	26	26	31	32	28	18.18	18.44	19.87	20.38	16.87
40–49	3	5	4	4	2	2.1	3.55	2.56	2.55	1.2
30–39	0	0	2	1	2	0	0	1.28	0.64	1.2
0–29	1	0	0	0	0	0.7	0	0	0	0
Total	143	141	156	157	166	100	100	100	100	100

- **Numbers of vivas and effects of vivas on classes of result.**
  Not applicable.

- **Marking of scripts.**
  All scripts were single marked according to a pre-agreed marking scheme which was strictly adhered to. The raw marks for paper A2 are out of 100, and for the other papers out of 50. For details of the extensive checking process, see Part II, Section A.

- **Numbers taking each paper.**
  All 143 candidates are required to offer the core papers A0, A1, A2 and ASO, and five of the optional papers A3-A11. Statistics for these papers are shown in Table 2 on page 2.

Table 2: Numbers taking each paper

Paper	Number of Candidates	Avg RAW	StDev RAW	Avg USM	StDev USM
A0	143	27.7	8.4	65.22	10.61
A1	143	33.03	7.93	67.29	12.13
A2	143	51.53	14.93	66.16	10.37
A3	65	29.94	8.54	65.92	12.89
A4	100	34.56	9.32	67.66	12.1
A5	79	31.49	8.17	63.86	12.81
A6	81	30.58	9.8	65.83	11.22
A7	68	35.68	7.68	65.76	9.21
A8	132	34.9	7.17	68.86	11.26
A9	94	32.28	8.41	66.32	11.6
A10	42	33.88	7.39	65.45	10.99
A11	59	36.46	7.46	66.54	13.18
ASO	143	33.13	8.85	66.58	11.94

B. New examining methods and procedures

This was the third year of the new Part A structure. The core papers AC1 and AC2 were replaced with core papers A1 and A2. The cross-sectional papers AO1 and AO2 were replaced with option papers A3-A11. In addition there is a core cross-sectional paper, ASO, examining the short option courses. From 2015-16 the 3-hour core paper A1 was split into two 1.5-hour papers: A0 Linear Algebra and A1 Differential Equations 1.

From 2015-16 students were able to take 5 or 6 long options. It was anticipated that most students would not wish to take on the extra workload, and that in most years it would be some subset of the first-class students wishing to take 6 long options. In 2016 five candidates were examined in an additional long option.
A student taking 5 long options has each of them counting as a unit's weight towards their overall second year mark. For a student taking 6 long options, their best 4 papers (following the exams) count one unit each and their worst 2 papers count half a unit each. Thus these 6 papers overall still have a weight of 5 units. The results from all 6 papers will appear on the student's exam transcript.
The aim of the above scoring system is to ensure anyone taking on an extra option does not do so lightly (all marks will be reported and all count to some extent) but also that a student will not get a lower overall mark for having taken on the extra workload (the given scoring system was a fair compromise looking at several years' data sets).

C. Changes in examining methods and procedures currently under discussion or contemplated for the future

None.

D. Notice of examination conventions for candidates

The first Notice to Candidates was issued on 17th February 2016 and the second notice on the 3rd May 2016.

These can be found at https://www.maths.ox.ac.uk/members/students/undergraduate-courses/ba-master-mathematics/examinations-assessments/examination-20, and contain details of the examinations and assessments. The course Handbook contains the link to the full examination conventions and all candidates are issued with this at Induction in their first year. All notices and examination conventions are on-line at
https://www.maths.ox.ac.uk/members/students/undergraduate-courses/examinations-assessments/examination-conventions.

Part II

A. General Comments on the Examination

Acknowledgements

The examiners would like to express their gratitude to

- Nia Roderick for her work in supporting the Part A examinations throughout the year, and for her help with various enquiries throughout the year.
- Also Helen Lowe for assistance with information, procedure and other matters.
- Waldemar Schlackow for running the database and the algorithms that generate the final marks, without which the process could not operate.
- Charlotte Turner-Smith and the Academic Administration who helped with marks entry, script checking, and much vital behind-the-scenes work.
- The assessors who set their questions promptly, provided clear model solutions, took care with checking and marking them, and met their deadlines, thus making the examiners' jobs that much easier.
- Several members of the Faculty who agreed to help the committee in the work of checking the papers set by the assessors.
- The internal examiners and assessors would like to thank the external examiners, Martyn Quick (Pure Mathematics) and Warren Smith (Applied Mathematics) for helpful feedback and much hard work throughout the year, and for the important work they did in Oxford in examining scripts and contributing to the decisions of the committee.

Timetable

The examinations began on Monday 13th June at 9.30am and ended on Friday 24th June at 11.00am.

Factors Affecting Performance

A subset of the Examiners attended a pre-board meeting to band the seriousness of each application of Factors Affecting Performance form received from the Proctors' office. The outcome of this meeting was relayed to the Examiners at the final exam board, who gave careful regard to each case, scrutinised the relevant candidates' marks and agreed actions as appropriate. See Section E for further details.

Setting and checking of papers and marks processing

As is usual practice, questions for the core papers A0, A1 and A2, were set by the examiners and also marked by them, with the assistance of an assessor to mark one question on A1. The papers A3-A11, as well as each individual question on ASO, were set and marked by the course lecturers. The setters produced model answers and marking schemes led by instructions from Teaching Committee in order to minimize the need for recalibration.

The internal examiners met in December to consider the questions for Michaelmas Term courses (A0, A1, A2 and A11). The course lecturers for the core papers were invited to comment on the notation used and more generally on the appropriateness of the questions. Corrections and modifications were agreed by the internal examiners and the revised questions were sent to the external examiners.

In a second meeting the internal examiners discussed the comments of the external examiners and made further adjustments before finalising the questions. The same cycle was repeated in Hilary term for the Hilary term long option courses and at the end of Hilary and beginning of Trinity term for the short option courses. *Papers A8 and A9 are prepared by the Department of Statistics and jointly considered in Trinity term.* Before questions were submitted to the Examination Schools, setters were required to sign off a camera-ready copy of their questions. Examination scripts were collected by the markers from Ewert House or delivered to the Mathematical Institute for collection by the markers and returned there after marking. A team of graduate checkers under the supervision of Nia Roderick, Jan Boylan and Hannah Harrison sorted all the scripts for each paper, cross-checking against the mark scheme to spot any unmarked questions or part of questions, addition errors or wrongly recorded marks. Also sub-totals for each part were checked against the marks scheme, noting any incorrect addition. An examiner was present at all times to authorise any required corrections.

Determination of University Standardised Marks

The examiners followed the standard procedure for converting raw marks to University Standardized Marks (USM). The raw marks are totals of marks on each question, the USMs are statements of the quality of marks on a standard scale. The Part A examination is not classified but notionally 70 corresponds to 'first class', 50 to 'second class' and 40 to 'third class'. In order to map the raw marks to USMs in a way that respects the qualitative descriptors of each class the standard procedure has been to use a piecewise linear map. It starts from the assumption that the majority of scripts for a paper will fall in the USM range 57-72, which is just below the II(i)/II(ii) borderline and just above the I/II(i) borderline respectively. In this range the map is taken to have a constant gradient and is determined by the parameters C_1 and C_2, that are the raw marks corresponding to a USM of 72 and 57 respectively. The guidance requires that the examiners should use the entire range of USMs. Our procedure interpolates the map linearly from $(C_1, 72)$ to $(M, 100)$ where M is the maximum possible raw mark. In order to allow for judging the position of the II(i)/III borderline on each paper, which corresponds to a USM of 40, the map is interpolated linearly between $(C_3, 37)$ and $(C_2, 57)$ and then again between $(0, 0)$ and $(C_3, 37)$. It is important that the positions of the corners in the piecewise linear map are not on the class borderlines in order to avoid distortion of the class boundaries. Thus, the conversion is fixed by the choice of the three parameters C_1, C_2 and C_3, the raw marks that are mapped to USM of 72, 57 and 37 respectively.

The examiners chose the values of the parameters as listed in Table 3 guided by the advice from the Teaching Committee and by examining individuals on each paper around the borderlines.

Table 3: Parameter Values

Paper	C1	C2	C3
A0	35.4	17.4	10
A1	39	24	13.8
A2	61.6	34.6	19.9
A3	36	22	12.6
A4	42.5	22.5	12.9
A5	38.8	25.3	14.5
A6	40	18.5	10.1
A7	43.4	25.4	14.6
A8	39	25.5	14.9
A9	39.2	22	13
A10	40.8	26.5	15.7
A11	42.8	29.3	16.8
ASO	41	22.1	12.7

Table 4 gives the resulting final rank and percentage of candidates with this overall average USM (or greater).

Table 4: Rank and percentage of candidates with this overall average USM (or greater)

Av USM	Rank	Candidates with this USM or above	%
94	1	1	0.70
91	2	2	1.40
90	3	3	2.10
86	4	4	2.80
85	5	5	3.50
82	6	8	5.59
81	9	9	6.29
79	10	11	7.69
78	12	13	9.09
77	14	14	9.79
76	15	17	11.89
75	18	19	13.29
74	20	22	15.38
73	23	26	18.18
72	27	32	22.38
71	33	39	27.27
70	40	50	34.97
69	51	55	38.46
68	56	68	47.55
67	69	77	53.85
66	78	87	60.84

65	88	91	63.64
64	92	93	65.03
63	94	100	69.93
62	101	107	74.83
61	108	110	76.92
60	111	113	79.02
59	114	122	85.31
58	123	123	86.01
57	124	125	87.41
56	126	126	88.11
55	127	130	90.91
54	131	133	93.01
53	134	135	94.41
52	136	136	95.10
51	137	139	97.20
48	140	141	98.60
47	142	142	99.30
29	143	143	100.00

Recommendations for Next Year's Examiners and Teaching Committee

The Examiners have no recommendations for Teaching Committee.

B. Equal opportunities issues and breakdown of the results by gender

Table 5, page 8 shows the performances of candidates broken down by gender.

Table 5: Breakdown of results by gender

Range	Total		Male		Female	
	Number	%	Number	%	Number	%
70–100	50	34.97	45	41.28	5	14.71
60–69	63	44.06	46	42.2	17	50
50–59	26	18.18	14	12.84	12	35.29
40–49	3	2.1	3	2.75	0	0
30–39	0	0	0	0	0	0
0–29	1	0.7	1	0.92	0	0
Total	143	100	109	100	34	100

C. Detailed numbers on candidates' performance in each part of the exam

Individual question statistics for Mathematics candidates are shown in the tables below.

Paper A0: Linear Algebra

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	13.65	13.88	5.50	130	3
Q2	14.00	14.28	4.86	100	2
Q3	10.77	13.02	6.47	56	13

Paper A1: Differential Equations 1

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.05	17.50	5.67	127	5
Q2	16.26	16.44	4.16	88	2
Q3	12.86	14.83	5.34	71	24

Paper A2: Metric Spaces and Complex Analysis

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	11.31	11.68	5.22	80	4
Q2	10.30	10.77	5.64	106	9
Q3	15.06	15.06	4.60	109	0
Q4	10.71	10.84	4.34	108	4
Q5	16.56	16.94	5.82	110	4
Q6	9.88	10.46	5.25	59	6

Paper A3: Rings and Modules

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.02	16.02	4.85	61	0
Q2	15.56	15.68	4.48	38	1
Q3	11.75	12.03	5.80	31	1

Paper A4: Integration

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.69	18.75	3.29	63	1
Q2	17.33	17.56	5.81	72	1
Q3	15.03	15.55	7.02	65	3

Paper A5: Topology

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.38	15.48	3.87	71	1
Q2	16.94	16.94	5.42	53	0
Q3	14.40	14.44	4.07	34	1

Paper A6: Differential Equations 2

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14.96	15.09	5.89	44	1
Q2	14.30	14.41	5.16	63	1
Q3	16.25	16.45	5.73	55	1

Paper A7: Numerical Analysis

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14.97	15.59	5.66	27	2
Q2	17.82	17.87	4.10	60	1
Q3	19.04	19.04	4.21	49	0

Paper A8: Probability

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.30	18.43	4.75	118	1
Q2	16.11	16.61	4.79	87	4
Q3	15.97	16.73	4.87	59	5

Paper A9: Statistics

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	10.12	10.86	4.17	14	3
Q2	16.14	16.14	4.23	91	0
Q3	17.02	17.02	4.94	83	0

Paper A10: Waves and Fluids

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	19.37	19.37	3.36	38	0
Q2	15.12	15.12	4.51	42	0
Q3	13.00	13.00	5.60	4	0

Paper A11: Quantum Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14.37	14.67	3.29	48	3
Q2	20.76	21.00	4.18	54	1
Q3	19.00	19.56	6.58	16	1

Paper ASO: Short Options

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	13.79	14.33	5.61	48	4
Q2	17.80	17.80	4.13	15	0
Q3	16.77	16.77	5.59	13	0
Q4	19.31	19.31	4.37	13	0
Q5	18.84	18.84	4.41	86	0
Q6	18.39	18.39	4.16	38	0
Q7	11.97	11.97	5.18	34	0
Q8	15.67	15.67	2.08	3	0
Q9	15.46	15.46	4.25	35	0

D. Part A 2016: Comments on sections and on individual questions

The following comments were submitted by the assessors.

Core Papers

A0: Algebra 1

In all three questions, many candidates omitted parts of multi-part solutions. For example, if the question asked for a map to be proved to be a bijection, some candidates would forget to prove that it was one-to-one.

Q1:

The point of parts (a) and (b) is to prove the result of part (b)(iii): that a linear transformation is upper-triangularisable if and only if its minimum polynomial is a product of linear factors. Performance even on the bookwork (which should have been an easy way to earn marks) was highly variable. In part (b), some candidates gave the proof that any linear transformation whose minimum polynomial is a product of linear factors has a Jordan Normal Form; this is correct but the proof is rather more difficult than the proof laid out in the question.

In part (c) there were many errors of calculation, and there was generally no sign that candidates who had made errors had tried to check their answers. It is easy to check whether λ is an eigenvalue of A: just see whether $A - \lambda I$ has rank less than three. One can also check whether the three eigenvalues one has obtained add up to the trace of the matrix (which is very easy to calculate). Indeed, the trace of A is $3 + \alpha$; if one spots that 1 is an eigenvalue (since $A - I$ is clearly singular), and if one has no time to do the calculations, one could try 2 and α and see if they work.

Q2:

Some candidates did the bookwork in this question very poorly. Those that knew their bookwork, however, did well in part (a) on the whole.

The vital point in part (b), of course, is that the vector space V is infinite dimensional so certain theorems from the course do not apply. Many students spotted this and a number did the entirety of part (b), including spotting how to show that the map Φ is not onto. Some, however, tried to apply the rank-nullity formula, and other theorems that were not applicable.

Q3:

The proof for part (a) is a variation on the proof that a self-adjoint map is diagonalisable, and many candidates completed it successfully.

In part (b), the second part of (i) is trivial—non-diagonalisable linear transformations exist, so not every sum of diagonalisable linear transformations is diagonalisable—but many did not spot this. Part (iii) proved to be quite testing; the point of course is to carefully work out what the individual Jordan blocks look like.

There were some good answers to part (c), but there was also a certain amount of confusion; some candidates implicitly assumed that the transformation was self-adjoint. The point of course is to find a vector which is provably an eigenvector of both T and T^*. Once we know that T is diagonalisable, it is then clear that T and T^* have the same eigenvectors, but this is not clear at the outset and the existence of a common eigenvector needs to be proved.

A1: Differential Equations 1

Q1:

Overall, the standard of answers showed a decreasing trend from parts (a) to (e).

- Part (a): This part was solved by most students. In the definition of autonomous system, some replies were ambiguous about what is meant by dependence on time.
- Part (b): The most common error was a wrong classification of the critical point which, in some cases, was classified twice (for example both as a spiral and as a centre).
- Part (c): Common errors were identifying $x = 0$ as a nullcline when $|x| > \alpha$ and drawing orthogonal trajectories on the line $y = -x$.
- Part (d): A common issue was dependence on the result of point b) and maintenance of overall coherence.
- Part (e): This part was often not addressed and sketches were frequently ambiguous.

Q2:

- Part (a): This was very well done.
- Part (b): This was generally well done. Common errors included inaccurate statement of Gronwall's inequality, and failing to include that the result must hold for all $x \in [a-h, a+h]$.
- Part (c)(i) was well done. Part (c)(ii): Many candidates did not deduce that $k < 1$. Additionally, many candidates failed to give the correct bound on f, and hence gave an incorrect relationship between h and k.
- Part (c)(iii): Only one candidate spotted that the correct equation to solve now is $dy/dx = \sqrt{x(y^2-1)}$. Many candidates did not spot that $y = 1$ is now a solution. Only a handful of candidates were able to show that there were infinitely many solutions. The majority of candidates were able to say why the uniqueness result is not contradicted.

Q3:

- Part (a) was mostly well done, although some candidates confused characteristics with the data curve.
- Part (b): some candidates failed to show the characteristic projections were circles centre the origin (or it took a lot of algebra to get to this point). Some candidates solved the characteristic equations to show that $z \exp x^2$ is constant on the characteristic curves, instead of differentiating along the characteristic curve.
- Part (c): This was generally well done.
- Part (d): Very few candidates were able to determine the appropriate form of s for each data segment as a function of $x$and y.

A2: Metric Spaces and Complex Analysis

Q1:
(a) Essentially all candidates got these definitions.
(b) Most candidates correctly gave this proof from the course, though there were a number of erroneous arguments involving arbitrarily chosen open covers from which a finite subcover gave no information.
(c.i,c.ii) Most candidates correctly identified these spaces, though often by using explicit convergence computations rather than a succinct comparison to known metric spaces. (c.iii) Most candidates correctly identified this space, though some didn't check that the limit function is again bounded. (c.iv) Many candidates correctly noted the Cantor set is closed therefore complete; those who thought of the Cantor set in terms of ternary expansion tended to erroneously conclude the opposite.
(d.i) A decent number of candidates saw a simple sequence without a convergent subsequence; some took the more laborious but correct route of constructing an open cover without a finite subcover. (d.ii) Though some correctly saw the answer here (HC is sequentially compact and sequentially compact metric spaces are compact), only a few candidates supplied a proof (that HC is sequentially compact).
Q2:
(a.i) Essentially all candidates got these definitions. (a.ii) There were a surprising range of mistakes in this part—for instance, some candidates simply proved the contraction mapping theorem (which is not what was asked); others proved the contraction mapping theorem for the contraction f^n (which could be assumed from (a.i)) and then erroneously leaped to the desired conclusion. A healthy handful of candidates did see the simple solution that for a the fixed point of f^n, we have $f(a) = f(f^n(a)) = f^n(f(a))$, so $f(a)$ is a fixed point of f^n and is therefore a, as required.
(b) Many candidates went through Cauchy gymnastics, with varying levels of success, rather than using the hint to observe that the function $d(x, f(x))$ must attain its minimum, which if not zero leads to a contradiction and if zero provides the desired fixed point.
(c.i) Most candidates provided clear examples here. (c.ii) Very many candidates gave functions which were not Lipschitz but also were not defined everywhere on $[0, 1]$. (c.iii) Many candidates correctly identified this function. (c.iv) Many candidates correctly identified this function, with some simply saying it isn't a contraction because it isn't an endomorphism (following the letter of the course definition) and many proving it isn't a contraction even in the more general sense that doesn't insist on an endomorphism.
(d) Many candidates were stumped by this final part, but a healthy number gave excellent answers showing a clear intuitive and computational grasp of the concepts.
Q3:
The first part of the question was entirely book-work and was generally done well. Points were lost when it was not clear in the statement of Morera's Theorem that the integral had to evaluate to zero for *all* curves. [10 marks]
In the second part students found it surprisingly difficult to prove that $\overline{g(\bar{z})}$ is holomorphic. The most popular approach was to express the derivative as a limit and work from there. Others showed that the Cauchy-Riemann equations had to hold. My favorite method (and most within the spirit of the question) was to use Morera's theorem also for this part. For the last task of the question, many realised that the trick was to write the contour integral for a triangle into a contour integral for a closed curve above and below the real line. Few

noted that some extra work was needed to allow part of the contour to be on the real line. [12 marks]
(145 attempts: 19 scripts with < 10 marks, 27 with ≥ 20 marks)
Q4:
The first part was very easy and was generally done well. [6 marks]
Many students were able to use the first part of the question to derive that the absolute value had to be constant on the boundary of any circle (and hence on the boundary of any region using the deformation theorem). Very few however where able to prove the hint itself that a holomorphic function with constant absolute value has to be constant, and too many confused themselves ($|z| = \alpha$ does not imply $z = \pm\alpha$ when working over the complex numbers!). A few good and even perfect scores showed that this was not an impossible question. But generally more help was needed. [12 marks]
The trick here was to consider $f(z)/z$. Those who saw this generally received full marks. [7 marks]
(142 attempts: 78 scripts with < 10 marks, 7 with ≥ 20 marks)
Q5:
This is a standard though not easy contour integration question. The first part was bookwork. [5 marks]
The second part asked for a non-trivial residue computation. The easiest was to expand the trigonometric functions in a Taylor series and compute the coefficient of z^{-1}. The algebra could get messy but a surprising number of students managed to persevere and produce the answer (convincingly with out bluffing). [6 marks]
For the third part too often points were lost because the imaginary poles were missed. But generally students understand how to do this type of question. [15 marks]
(153 attempts: 29 scripts with < 10 marks, 54 with ≥ 20 marks)
Q6:
(a.i) A surprising number of candidates missed that the assumption of simple connectivity is essential to the theorem, and a disturbing number of candidates seemed not to have encountered the Riemann Mapping Theorem at all. (a.ii) Most candidates appropriately applied Liouville's Theorem.
(b) This challenging part expected candidates to understand or see that angle- and sense-preserving maps satisfy the Cauchy-Riemann equations (and from there apply Goursat's theorem).
(c) Almost all candidates got parts (i), (iii), and (iv), but almost all candidates erroneously claimed for (ii) that the image was the whole interior of the unit disc (and many were led astray in part (d) by this mistake).
(d) Many candidates made a good effort on this part, in the first part applying either a square root or a log (followed by standard transformations), and in the second part, a Mobius transformation, eg $1/(z-2)$ (followed by standard transformations).

ASO: Short Options

Q1. Number Theory:
The bookwork parts of the question were done very well. Very few made a successful attempt at 1e.

Q2. Group Theory:

The question was largely done well, although no-one scored full marks. Parts (a) and (b) were done well in the main although some few answers included S_3 in the composition series of A_4. Many did (c)(i) by doing a case-by-case anaysis of the entries of an invertible 2×2 matrix over $\mathbb{Z}_3$. Whilst commonly successful, this is time consuming and it is easier to note the first column must be non-zero and the second column independent of the first. Many made (c)(ii) unnecessarily lengthy by carefully verifying the subgroup test and normality for $\mathrm{SL}(2,3)$ and listing some of its elements, when it was intended just to show det is a homomorphism and take N for its kernel. Finally in (c)(iii), no script quite appreciated the full details of the question being asked. Many correctly showed that $\alpha^3 = I_2, \beta^2 = -I_2, (\alpha\beta)^3 = I_2$ but failed to conclude that this means $\pm\alpha \mapsto \alpha, \pm\beta \to \beta$ induces a well-defined homomorphism from $N/\{\pm I_2\}$ to A_4. Some further argument (say considering orders of elements and/or subgroups) is then needed to show an isomorphism is in fact induced.

Q3. Projective Geometry:
This question was about conics, and explored some of the theory of polarity.
There were 15 attempts, of which 5 answers were in the 18-25 range, 5 in the 13-17 range and 5 in the below 13 range. The question produced a good spread of marks, separating out the stronger from the weaker students.
The bookwork and more routine parts of the question ((a),(b) and (c)) were mostly done well, except that several candidates failed to appreciate in part (b) the important point that one needed to use nonsingularity of the conic to ensure the quadratic equation was nontrivial.
As expected, parts (d) and (e) proved more challenging, but some candidates did very well and came up with neat arguments for the final section of part (e).

Q4. Introduction to Manifolds:
This question was about derivatives, the inverse function theorem and the distinction between local and global diffeomorphisms.
There were 15 attempts (including two physics candidates). Eleven were in the 18-25 range and two in the 13-17 range.
The bookwork was generally well done, although some candidates forgot that the C^1 condition is needed for the inverse function theorem.
Part (c) created unexpected problems for some candidates. Some were a bit too sketchy in their proof that invertibility was an open condition (I was looking for a recognition that det was a polynomial map, hence continuous). More surprisingly, some candidates were unsure about the matrix algebra needed to calculate the derivative of the inversion map. Several forgot that when inverting a matrix product the order had to be reversed.
Part (d), on the exponential map, was generally well done, although most candidates were a bit sketchy in their proof that the remainder term was $o(h)$.
Part (e) was generally very well done–most candidates got the idea that matrices in the image of exp would have a square root.

Q5. Integral Transforms:
The question was popular and largely well done. In (b)(ii) quite a few scripts took k to be a positive integer, rather than a general positive real number, and so gave the answer in terms of a factorial rather than the gamma function. The expected method in (c) was to write down $\overline{\mathrm{erf}}(p)$ as a double integral and swapping the order of integration. Quite a few scripts cleverly avoided this by either considering $\mathrm{erf}(x)$ as a convolution or integrating

$\int_0^\infty \mathrm{erf}(x)e^{-px}\,\mathrm{d}x$ by parts.

Q6. Calculus of Variations:
This question was very well done and where candidates struggled was not so much on material from the course, but due to a lack of "mathematical street smarts". For example, some candidates struggled to integrate (b) (iii); most candidates did not correctly derive the expression for y in (c)(i) by recognising that $cosh(A) = cosh(-A)$; some candidates could not sketch the curve $y = sinh(x)$ (in (c)(ii)).
Some candidates thought that the force, rather than the energy, was to be minimised [(b)(ii)], while some people just wrote down a function J to be minimised without saying that it was potential energy.
Question (c)(iii) was misleading because the positive solution is required for a minimum but the question focussed on extremals. I was therefore generous in the marking of this part. Only one candidate properly explained why the zero solution was not appropriate (it leads to y being infinite).

Q7. Graph Theory:
Most candidates were able to solve (a).
In (b), common mistakes were adding edges not in G, or claiming that there is an edge of G between any two components of a subforest.
Most marks were lost in (c), particularly for assuming that X=E(G), despite the explicit instruction not to do this.

Q8. Special Relativity:
The question was found hard by the candidates and attracted few attempts.

Q9. Modelling in Mathematical Biology:
Part (a). This was all bookwork and on the whole well done. Part (b). This part was generally well answered. Many candidates lost marks for not e.g. fully justifying why steady states were linearly stable or unstable. In addition, many didn't state that there was no hysteresis loop. Part (c). This part was less well answered. Many candidates lost marks in deriving the required results by algebraic rather than graphical means. It was generally helpful to write $S = S(N^*)$ in part (ii).

Option Papers

A3: Algebra 2

Q1:
This question was generally well answered. Students found a couple of ways of proving that elements of $1 + N$ were invertible. Checking it was a subgroup of the group of units using the subgroup test, as a number of students did, is actually clumsier than a direct check. The final part proved somewhat trickier, with many students incorrectly asserting that in order for an element of $\mathbb{Z}/n\mathbb{Z}$ to be nilpotent, it prime factors had to coincide with those of n.
Q2:
This was the second most popular question on the paper. The first part was solved by most students without issue. There were two approaches to the second part – finding an

isomorphism with a quotient of $\mathbb{Q}[t]$ or finding a $\mathbb{Q}$-basis. In either case candidates had difficulties supplying all the technical details. Part *c*), although similar to questions about polynomials having integer roots was found more challenging. While clearing denominators is the simplest approach, it is also possible to adapt the notion of content for polynomials, as some candidates did. For the final part a number of candidates attempted to prove the wrong assertion (i.e. a positive answer) by emulating the proof that the Gaussian integers are an ED. Arithmetic errors sometimes allowed them to complete the proof. Of those who attempted to prove the correct assertion most modified a method used in the problem sets.
Q3:
The bookwork in this question was well answered, though many people failed to precisely state the uniqueness part of the canonical form theorem (which is perhaps understandable as the course does not prove it). Some students lost time trying to prove statements which are false in part *b*), and quite a number of candidates made elementary arithmetic errors in part *c*) leading them to incorrect conclusions. On the other hand, many students correctly used either the canonical form or primary decomposition theorem to correctly calculate the number of isomorphism classes of abelian groups of order 675.

A4: Integration

Q1:
Most candidates did quite well on this question, and collected most of the marks on the book work part (a). Most candidates have no difficulty to justify the limit in part b(ii) by using MCT, but very few are able to justify the limit in (b)(iii) and thus lose most of the marks assigned for this part.
Q2:
Those who attempted this question were able to claim some marks on part (a), but not many of them are able to carry out the computations required satisfactory. A few candidates quoted the MCT wrongly, and a significant number of students are unable to write down the function $f(x)$ into a series then apply MCT series version, though the identity asked to prove suggested in this direction. Part (b) is a standard question for differentiating under integrals, most of those who attempted are able to justify the integrability and to write down the formulas for the derivatives of F, but complete solutions are rare.
Q3:
Most candidates who attempted are able to state Fubini and Tonelli theorems correctly. For part (a), it is surprising that a significant number of candidates are unable to work out the repeated integrals involving simple power functions, and thus are unable to identify the range of α by using Tonelli theorem correctly. Those who attempted part (b) understand, even though a few forget to say it, that one should use the Tonelli theorem to show the integrability, but again surprisingly, most of them could not compute simple repeated integrals, and thus lose many marks on this part.

A5: Topology

Q1:
Parts 1.a and 1.b were attempted by almost all candidates, and were well answered by a large majority. A number of candidates wrote the definition of a connected topological space correctly, but when reformulating it for a subset of a topological space made mistakes. The

question 1.b.iii was also less well answered, some candidates arguing either that unions of connected components are also connected, or that the open and closed set A must be connected.

In part 1.c, question (i) was generally well answered, as well as the first part of (ii), while the question whether path-connectedness is inherited by the closure was answered correctly by a surprisingly small number of candidates, considering that it referred to an example that has been discussed in both the Topology and in the Metric Spaces course.

Question 1.c.iii was more demanding, there were few attempts to answer it, and even fewer that would discuss thoroughly both the first and the second part of this question.

Q2:

Parts 2.a and 2.c were done by a large number of candidates, and almost all answered correctly.

In part 2.b.i the same imperfect understanding of the subspace topology became apparent, either one or the other of the two equivalent statements was incorrect, assuming that the sets were open (respectively closed) in the space X and contained in the subspace K either in the definition or in the proof of one of the implications.

Part 2.b.ii was answered correctly by almost all the candidates that attempted it, in part 2.b.iii a large number of candidates used the Heine-Borel theorem, which was not what was required.

Q3:

In part 3.a a surprisingly large number of candidates attempting it did not know how to reformulate the Hausdorff property in the quotient as a condition in the initial space, and this affected the correctness of their answers to the following questions.

In part 3.b most answers have been correct, the agreeable surprise being that some candidates produced original examples displaying a good intuition.

In part b.ii the most frequent mistake was to deduce that every continuous map f defines an injective continuous map of the quotient, while this is clearly not the case if X is itself Hausdorff and f is a constant map.

Many candidates attempted the standard question 3.c.i, a small number only attempted 3.c.ii, even though it was based on well known features of the cofinite topology and it did not require elaborate arguments.

A6: Differential Equations 2

Q1:

Part a) required fully deriving the Wronskian condition. In b) ii), a number of candidates neglected to include the non-homogeneous boundary condition in solvability. A variety of approaches were shown in iv), quickest was to apply across the equation.

Q2:

Full marks on a) ii) required justifying the domain $(-\infty, \infty)$ by considering boundary terms appearing in integration by parts. Many candidates lost marks in b) i) by not checking $x = \infty$ as a singular point.

Q3:

This question had the highest average mark. Some candidates did not recognize the need to expand in half powers of $\mathcal{E}$ in b). A common and costly mistake in c) was not expanding $p(x)$ in the inner solution to get an easy exponential solution at leading order.

A7: Numerical Analysis

This seems to have been a reasonably successful paper with quite a wide range of marks including several high scores.

Q1:

was the least attempted and least well done of the questions though there were still several high scores. A considerable number of candidates had difficulty in deriving the quadrature rule for the non-standard interpolation problems addressed. Too many spent more time than necessary calculating rather than thinking: having just proved a uniqueness theorem, simply noting this and that the data given in part (b) clearly fits $-x$ rather than using the general Lagrange form could have saved some time.

Q2:

was attempted by nearly all candidates. The QR and LU parts were reasonably well done, but several candidates misinterpreted what Gershgorin's Theorem actually says.

Q3:

many candidates spent far too long on the first part especially calculating integrals which were obviously zero because of simple symmetry/antisymmetry (even/odd) considerations. A pleasing number worked out the unseen and rather non-standard final part on matrix best approximation.

A8: Probability

Q1:

This was a very popular question, and many students scored high marks. It was interesting how many students were unable to give a precise definition of convergence in distribution. Part (b) was generally well done, although part (iv) tested the students' understanding of the central limit theorem, and a number of students struggled here. Part (c) proved to be quite challenging, and tested the students' understanding of convergence in distribution and also their skills in first year analysis. Most students were able to follow the hint to get started, but only the strongest students were able to produce complete or near-complete solutions.

Q2:

The students typically performed very well on part (a) and parts (b)(i) and (b)(iii), showing a decent understanding of the theory of Markov chains. Questions on stationary distributions are common in Part A Probability exams, but the students still found part (b)(ii) challenging. Indeed, in order to score a high mark the students were required to perform a quite demanding calculation. Part (iv) can actually be solved using a really quite simple argument, although only the very best students noticed this - many wasted time on calculations that had no hope of succeeding.

Q3:

This was the least popular of the three questions. The material in (a) is straightforward and didn't pose problems for well-prepared candidates (in part (ii), the fact that T_1 and $T_2 - T_1$ are independent is a key point). In (b)(i), many answers were unconvincing; the key point is that the independent increments property of the Poisson process gives that $N_{n+1} - N_n$ is independent of $(N_t, t \leq n)$ and hence that $X_{n+1} - X_n$ is independent of $X_1, X_2, \ldots, X_n$. Part (iii) is a variation on the proof of recurrence for a 1D simple symmetric random walk, and many candidates did this well. Part (iv) eludedmost candidates. Some tried to calculate the mean return time directly, which was not successful. The key point is that if the chain were positive recurrent, it would have a stationary distribution; then there are various ways to show that this cannot hold; one is to observe that $p_{00}^{(n)} \to 0$ (from part (iii)) so the stationary distribution would have to put weight 0 at 0, but this is quickly seen to be impossible.

A9: Statistics

Q1:
This question was not popular. Part (a) is standard material from lectures that was done well. The sketches asked for in (b) were part of a question on a problem sheet. However this part of the question was not done well, few candidates answered thoroughly and most lost marks. In (c)(i) it is essential to say that $\overline{X}$ and Q_X are independent, as well as giving their marginal distributions. The quality of answers to (c)(ii) and (c)(iii) was pretty varied, even though they are relatively small variations on standard results. Most candidates struggled with (d), though there were a few nice answers based on the asymptotic χ^2 distribution of the likelihood ratio statistic.

Q2:
This question was popular. Parts (a) and (b) were done well and marks were generally high. However very few candidates did (c) well and many did not explain why their sketch took the form it did, even though the question explicitly asked this. It is possible to sketch the two power functions in (c) without any detailed calculation of the form of these functions. However many candidates seemed to have little intuition about the situation and had power functions that were decreasing when they should have been increasing, etc. Some realised that $w_b(\lambda)$ must be below $w_a(\lambda)$ for $\lambda > \lambda_0$ as the test in (a) is UMP, but few spotted that $w_b(\lambda)$ increases as λ moves away from λ_0 in either direction.

Q3:
A popular and high-scoring question. Parts (a) and (b) were usually done well. There were some excellent answers to (c), but by far the most common error was not realising that to find the marginal posterior distribution for ψ the joint posterior must be integrated over μ.

A10: Waves and Fluids

Q1:
All but 4 candidates attempted this question. Many could not write down the Euler equations at all. Common mistakes were writing down equations that mixed scalar and vector terms, or omitting the pressure gradient completely. Some wrote down the compressible Euler equations, while others just wrote down a momentum equation and omitted $\nabla \cdot \mathbf{u} = 0$.

Parts (b) and (c) were largely done well. The most common error was not verifying that the velocity field described an incompressible flow by showing that $\nabla \cdot \mathbf{u} = 0$. Few candidates commented explicitly that they should treat X, Y, Z as constants when differentiating to follow a fluid element. Two candidates used the $\mathbf{u} \cdot \nabla \boldsymbol{\omega} - \boldsymbol{\omega} \cdot \nabla \mathbf{u}$ form of the vorticity equation, which is much quicker to verify.

Part (d) caused a lot of difficulty. Many candidates separated the Ω terms from the α terms, but few identified the former as a line vortex and the latter as a stagnation point flow. Several candidates decomposed the latter into a flow in the xy plane and a flow along z, neither of which was incompressible. No candidate mentioned vortex stretching as being responsible for the growth in the Ω terms over time. Many candidates confused sink flows with vortices, or wrote that the alpha flow was uniform through confusing a velocity field with a potential.

Q2:
Every candidate attempted this question. Part (a) was done surprisingly poorly, having been set twice previously (in 2008 and 2015) and on a problem sheet. A few candidates simply wrote out the proof of Blasius' theorem for the force, ignoring the moment asked for in the question. Many others had difficulty arriving at an expression for the moment of the force exerted by the pressure. Several candidates used the Bernoulli theorem for rotational flow, then relied on the boundary being a streamline, rather than using the Bernoulli theorem for irrotational flow. This is rather clumsy, although not incorrect.
Parts (b) and (c) were mostly found straightforward, though several candidates used the conformal maps the wrong way round. They found the moment on the wrong shape, which one candidate went on to do correctly. Only two candidates drew a diagram to indicate the branches of $\sqrt{\zeta^2 - 4c^2}$, and roughly half explicitly used $z \sim \zeta$ at large distances to justify the flow being a uniform stream at infinity. Many did not explain why they chose the g_+ solution for the inverse Joukowski map.
Candidates were expected to transform the integral in ζ back to an integral in z around $|z| = a$ using the chain rule, as covered in lectures, and evaluate the residues at $z = 0$ and $z = \pm c$. Some managed to do all this correctly. It was worth using the $g_+g_- = c^2$ relation between the two inverse Joukowski maps to simplify the complex potential before differentiating, though two candidates then lost a factor of a^2/c^2. One could also evaluate the integral by finding the coefficient of $1/z$ in a Laurent expansion valid in an annulus containing $|z| = a$, as in the Complex Analysis lecture notes, which noone tried. Some candidates tried to directly evaluate the integral round the ellipse in ζ using Cauchy's formula, but they all ignored the branch cut along $[-2c, 2c]$.
Several candidates asserted that the moment aligns the ellipse with its semimajor axis parallel to the flow (as seems intuitive, but is incorrect) rather than with the semimajor axis perpendicular to the flow (broadside on).
Q3:
This question received only 4 attempts, though it is very similar to a problem sheet question. Most attempts provided insufficient justification for the end results supplied in the question. For example, it was common to just drop the pressure, since it led to the required result, instead of stating that the pressure is constant on a free surface.
Only one candidate produced a qualitatively correct sketch, of functions supplied in the question, to show that the free surface rises monotonically away from the symmetry axis with a continuous gradient at $r = a$.

A11: Quantum Theory

Q1:
This was a straightforward 'particle-in-a-box' problem and the candidates were expected to immediately set V=0 in the Schrödinger equation. In the event, most did but 8 of 54 attempts showed some confusion about what V should be. The average on this question was lower than I expected chiefly because parts (b)(ii) and (iii) were not well done. Whatever confusion there was about V in part (a) was not reflected in lower scores in that part.
Q2:
This was a straightforward question on the algebraic parts of quantum theory and was generally well-done. The main challenge was in choosing suitable observables A to solve part (b)(ii) given the guidance of part (b)(i) and this spread out the candidates.

Q3:
A straightforward angular momentum problem, this drew fewer attempts than the others but it was well done.

E. Comments on performance of identifiable individuals

Removed from public version of the report.

F. Names of members of the Board of Examiners

- **Examiners:**
  Dr Robin Knight (Chair)
  Prof. Christopher Douglas
  Dr Neil. Laws
  Prof. Ulrike Tillmann
  Prof. Sarah Waters
  Dr Martyn Quick (External Examiner)
  Dr Warren Smith (External Examiner)

- **Assessors:**
  Dr Jennifer Balakrishnan
  Prof. Ruth Baker
  Prof. Andrew Dancer
  Prof. Paul Dellar
  Prof. Cornelia Drutu
  Dr Richard Earl
  Dr Adam Gal
  Prof. Peter Keevash
  Prof. James Martin
  Prof. Lionel Mason
  Prof. Kevin McGerty
  Prof. Derek Moulton
  Dr Daniele Muraro
  Prof. Philip Maini
  Prof. Zhongmin Qian
  Prof. Paul Tod
  Prof. Andy Wathen