Examiners' Report: Final Honour School of Mathematics Part A Trinity Term 2017

November 8, 2017

Part I

A. STATISTICS

- **Numbers and percentages in each class.**
  See Table 1.

Table 1: Numbers in each class

Range	Numbers					Percentages %				
	2017	2016	2015	2014	2013	2017	2016	2015	2014	2013
70–100	57	50	51	57	49	36.77	34.97	36.17	36.54	31.21
60–69	62	63	59	62	71	40	44.06	41.84	39.74	45.22
50–59	31	26	26	31	32	20	18.18	18.44	19.87	20.38
40–49	4	3	5	4	4	2.58	2.1	3.55	2.56	2.55
30–39	1	0	0	2	1	0.65	0	0	1.28	0.64
0–29	0	1	0	0	0	0	0.7	0	0	0
Total	155	143	141	156	157	100	100	100	100	100

- **Numbers of vivas and effects of vivas on classes of result.**
  Not applicable.

- **Marking of scripts.**
  All scripts were single marked according to a pre-agreed marking scheme which was strictly adhered to. The raw marks for paper A2 are out of 100, and for the other papers out of 50. For details of the extensive checking process, see Part II, Section A.

- **Numbers taking each paper.**
  All 155 candidates are required to offer the core papers A0, A1, A2 and ASO, and five of the optional papers A3-A11. Statistics for these papers are shown in Table 2 on page 2.

Table 2: Numbers taking each paper

Paper	Number of Candidates	Avg RAW	StDev RAW	Avg USM	StDev USM
A0	155	29.61	7.94	65.97	11.25
A1	155	34.73	4.6	64.95	7.8
A2	155	68.53	12.78	65.9	9.73
A3	77	29.01	9.01	64.19	13.98
A4	94	35.2	10.1	69.47	15.96
A5	79	32.32	7.78	67.09	11.15
A6	99	28.31	8.99	64.69	12.55
A7	60	30.37	7.24	65.85	10.83
A8	148	35.61	7.93	67.84	12.24
A9	108	31.86	7.46	65.94	10.82
A10	58	33.71	8.11	64.93	11.97
A11	57	22.86	7.36	62.54	10.21
ASO	155	32.61	8.77	67.35	12.81

B. New examining methods and procedures

There were no changes in 2016–17.

C. Changes in examining methods and procedures currently under discussion or contemplated for the future

None.

D. Notice of examination conventions for candidates

The first notice to candidates was issued on 16th February 2017 and the second notice on the 8th May 2017.

These can be found at https://www.maths.ox.ac.uk/members/students/undergraduate-courses/ba-master-mathematics/examinations-assessments/examination-20, and contain details of the examinations and assessments. The course handbook contains the link to the full examination conventions and all candidates are issued with this at induction in their first year. All notices and examination conventions are on-line at
https://www.maths.ox.ac.uk/members/students/undergraduate-courses/examinations-assessments/examination-conventions.

Part II

A. General Comments on the Examination

Acknowledgements

The examiners would like to express their gratitude to

- Nia Roderick for her work in supporting the Part A examinations throughout the year, and for her help with various enquiries throughout the year.
- Also Helen Lowe for assistance with information, procedure and other matters.
- Waldemar Schlackow for running the database and the algorithms that generate the final marks, without which the process could not operate.
- Charlotte Turner-Smith and the Academic Administration Team who helped with marks entry, script checking, and much vital behind-the-scenes work.
- The assessors who set their questions promptly, provided clear model solutions, took care with checking and marking them, and met their deadlines, thus making the examiners' jobs that much easier.
- Several members of the Faculty who agreed to help the committee in the work of checking the papers set by the assessors.
- The internal examiners and assessors would like to thank the external examiners, Martyn Quick (Pure Mathematics) and Warren Smith (Applied Mathematics) for helpful feedback and much hard work throughout the year, and for the important work they did in Oxford in examining scripts and contributing to the decisions of the committee.

Timetable

The examinations began on Monday 12th June and ended on Friday 23rd June.

Factors Affecting Performance

A subset of the Examiners attended a pre-board meeting to band the seriousness of each Factors Affecting Performance application. The outcome of this meeting was relayed to the Examiners at the final exam board, who gave careful regard to each case, scrutinised the relevant candidates' marks and agreed actions as appropriate. See Section E for further details.

Setting and checking of papers and marks processing

As is usual practice, questions for the core papers A0, A1 and A2, were set by the examiners and also marked by them, with the assistance of an assessor to mark one question on A1. The papers A3-A11, as well as each individual question on ASO, were set and marked by the course lecturers. The setters produced model answers and marking schemes led by instructions from Teaching Committee in order to minimize the need for recalibration.

The internal examiners met in December to consider the questions for Michaelmas Term courses (A0, A1, A2 and A11). The course lecturers for the core papers were invited to comment on the notation used and more generally on the appropriateness of the questions. Corrections and modifications were agreed by the internal examiners and the revised questions were sent to the external examiners.

In a second meeting the internal examiners discussed the comments of the external examiners and made further adjustments before finalising the questions. The same cycle was repeated in Hilary term for the Hilary term long option courses and at the end of Hilary and beginning of Trinity term for the short option courses. *Papers A8 and A9 are prepared by the Department of Statistics and jointly considered in Trinity term.* Before questions were submitted to the Examination Schools, setters were required to sign off a camera-ready copy of their questions.

Examination scripts were collected by the markers from Ewert House or delivered to the Mathematical Institute for collection by the markers and returned there after marking. A team of graduate checkers under the supervision of Charlotte Turner-Smith, Jan Boylan and Hannah Harrison sorted all the scripts for each paper, cross-checking against the mark scheme to spot any unmarked questions or part of questions, addition errors or wrongly recorded marks. Also sub-totals for each part were checked against the marks scheme, noting any incorrect addition. An examiner was present at all times to authorise any required corrections.

Determination of University Standardised Marks

The examiners followed the standard procedure for converting raw marks to University Standardized Marks (USM). The raw marks are totals of marks on each question, the USMs are statements of the quality of marks on a standard scale. The Part A examination is not classified but notionally 70 corresponds to 'first class', 50 to 'second class' and 40 to 'third class'. In order to map the raw marks to USMs in a way that respects the qualitative descriptors of each class the standard procedure has been to use a piecewise linear map. It starts from the assumption that the majority of scripts for a paper will fall in the USM range 57-72, which is just below the II(i)/II(ii) borderline and just above the I/II(i) borderline respectively. In this range the map is taken to have a constant gradient and is determined by the parameters C_1 and C_2, that are the raw marks corresponding to a USM of 72 and 57 respectively. The guidance requires that the examiners should use the entire range of USMs. Our procedure interpolates the map linearly from $(C_1, 72)$ to $(M, 100)$ where M is the maximum possible raw mark. In order to allow for judging the position of the II(i)/III borderline on each paper, which corresponds to a USM of 40, the map is interpolated linearly between $(C_3, 37)$ and $(C_2, 57)$ and then again between $(0, 0)$ and $(C_3, 37)$. It is important that the positions of the corners in the piecewise linear map are not on the class borderlines in order to avoid distortion of the class boundaries. Thus, the conversion is fixed by the choice of the three parameters C_1, C_2 and C_3, the raw marks that are mapped to USM of 72, 57 and 37 respectively.

The examiners chose the values of the parameters as listed in Table 3 guided by the advice from the Teaching Committee and by examining individuals on each paper around the borderlines.

ASO Question 9 When considering the determination of University Standardised Marks for ASO, the examiners felt that question 9 on the paper had not proved to be a fair test of candidate's abilities in comparison to the other questions on this paper. After careful analysis of candidates' performance of question 9 with other questions, and of the distribution of performance on the question, and further noting that the number of attempts on the question was relatively large, the examiners felt confident in agreeing to increase all marks on question 9 by a constant to ensure that all candidates were fairly and equally rewarded for their performance on ASO.

Table 3: Parameter Values

Paper	C1	C2	C3
A0	(36,72)	(21,57)	(11.5,37)
A1	39.2,72)	(30.2,57)	(15.5,37)
A2	(79.2,72)	(55.2,57)	(30.5,37)
A3	(36,72)	(22,57)	(12.06,37)
A4	(40,70)	(23.8,57)	(13.67,37)
A5	(38,72)	(22.5,57)	(13.61,37)
A6	(36.2,72)	(19.7,57)	(11.32,37)
A7	(35.6,72)	(23.6,57)	(12,37)
A8	(40,70)	(26.4,57)	(15.17,37)
A9	(38,72)	(24,57)	(13.79,37)
A10	(40.5,72)	(27,57)	(14,37)
A11	(31,72)	(16.3,57)	(9.36,37)
ASO	(39.2,72)	(22.7,57)	(13.04,37)

Table 4 gives the resulting final rank and percentage of candidates with this overall average USM (or greater).

Table 4: Rank and percentage of candidates with this overall average USM (or greater)

Av USM	Rank	Candidates with this USM or above	%
90	1	1	0.65
86	2	3	1.94
85	4	5	3.23
84	6	6	3.87
83	7	7	4.52
82	8	8	5.16
81	9	10	6.45
80	11	12	7.74
79	13	14	9.03
78	15	16	10.32
77	17	20	12.90
76	21	22	14.19
75	23	24	15.48
74	25	30	19.35
73	31	35	22.58
72	36	42	27.10
71	43	51	32.90
70	52	57	36.77
69	58	61	39.35
68	62	66	42.58
67	67	76	49.03
66	77	83	53.55
65	84	87	56.13
64	88	94	60.65
63	95	101	65.16
62	102	108	69.68
61	109	117	75.48
60	118	119	76.77
59	120	128	82.58
58	129	130	83.87
57	131	131	84.52
56	132	139	89.68
55	140	141	90.97
54	142	143	92.26
53	144	146	94.19
52	147	148	95.48
51	149	149	96.13
50	150	150	96.77
48	151	152	98.06
43	153	153	98.71
41	154	154	99.35
35	155	155	100.00

Recommendations for Next Year's Examiners and Teaching Committee

B. Equality and Diversity issues and breakdown of the results by gender

Table 5, page 8 shows percentages of male and female candidates for each class of the degree.

Table 5: Breakdown of results by gender

Class	Number								
	2017			2016			2015		
	Female	Male	Total	Female	Male	Total	Female	Male	Total
70–100	7	50	57	5	45	50	10	41	51
60–69	12	50	62	17	46	63	17	42	59
50–59	12	19	31	12	14	26	10	16	26
40–49	2	2	4	0	3	3	3	2	5
30–39	1	0	1	0	0	0	0	0	0
0–29	0	0	0	0	1	1	0	0	0
Total	34	121	155	34	109	143	40	101	141
Class	Percentage								
	2017			2016			2015		
	Female	Male	Total	Female	Male	Total	Female	Male	Total
70–100	20.59	41.32	36.77	14.71	41.28	34.97	25	40.59	36.17
60–69	35.29	41.32	40	50	42.2	44.06	42.5	41.58	41.84
50–59	35.29	15.7	20	35.29	12.84	18.18	25	15.84	18.44
40–49	5.88	1.65	2.58	0	2.75	2.1	7.5	1.98	3.55
30–39	2.94	0	0.65	0	0	0	0	0	0
0–29	0	0	0	0	0.92	0.7	0	0	0
Total	100	100	100	100	100	100	100	100	100

C. Detailed numbers on candidates' performance in each part of the exam

Individual question statistics for Mathematics candidates are shown in the tables below.

Paper A0: Linear Algebra

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	12.44	12.96	4.35	68	7
Q2	15.11	15.56	4.91	131	5
Q3	14.69	15.04	5.53	111	4

Paper A1: Differential Equations 1

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	19.97	20.09	2.48	149	1
Q2	16.21	16.22	3.86	72	1
Q3	13.59	13.73	3.88	89	3

Paper A2: Metric Spaces and Complex Analysis

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.32	17.33	3.73	132	1
Q2	17.67	17.71	4.49	134	1
Q3	14.68	15.03	4.52	68	4
Q4	13.20	14.08	5.02	39	6
Q5	18.35	18.46	3.69	141	1
Q6	16.97	17.35	5.05	103	4

Paper A3: Rings and Modules

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.51	15.98	5.60	58	3
Q2	12.07	12.09	4.65	65	2
Q3	16.47	16.81	4.97	31	1

Paper A4: Integration

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	19.71	19.71	5.01	76	0
Q2	16.95	17.07	6.02	58	1
Q3	15.05	15.20	5.99	54	1

Paper A5: Topology

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.82	16.82	4.72	78	0
Q2	14.99	15.15	4.05	68	1
Q3	16.38	17.58	6.56	12	1

Paper A6: Differential Equations 2

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	12.54	13.16	5.42	62	7
Q2	15.36	15.56	4.87	73	2
Q3	12.74	13.51	5.60	63	6

Paper A7: Numerical Analysis

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.39	15.39	4.41	56	0
Q2	14.74	14.74	3.89	57	0
Q3	17.14	17.14	4.63	7	0

Paper A8: Probability

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.23	17.98	5.78	95	7
Q2	18.17	18.32	4.28	140	2
Q3	15.95	16.34	4.63	61	3

Paper A9: Statistics

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14.55	14.85	3.73	71	4
Q2	16.76	16.90	5.09	78	2
Q3	15.87	16.20	3.87	66	3

Paper A10: Waves and Fluids

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.35	18.35	3.92	57	0
Q2	15.64	16.85	6.97	33	3
Q3	12.83	13.58	4.36	26	3

Paper A11: Quantum Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	11.19	11.19	3.46	54	0
Q2	11.31	11.47	5.29	53	1
Q3	13.00	13.00	3.56	7	0

Paper ASO: Short Options

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.59	18.59	4.23	64	0
Q2	20.66	20.66	2.97	32	0
Q3	19.00	19.00	5.73	8	0
Q4	14.79	14.79	6.53	14	0
Q5	16.12	16.39	5.44	83	2
Q6	15.82	16.06	5.42	48	1
Q7	11.22	12.47	4.88	19	4
Q8	11.00	11.00	4.06	4	1
Q9	11.60	11.68	2.84	37	3

Part A 2017: Comments on sections and on individual questions

The following comments were submitted by the assessors.

Core Papers

A0: Algebra 1

Question 1. The earlier parts of this question were generally done well and there were some good attempts at the later parts also. No one part of the question appeared much harder than all of the others, judging by the solutions I marked.

There were a few common errors of mathematics, such as asserting that since the minimum polynomial of a particular transformation was monic and divided the characteristic polynomial, the two must be equal; and there were some common errors of exam technique. Many people included superfluous material, such as proofs of results the question explicitly said could be assumed; and a very few candidates fell into the trap of excessive formality: of phrasing parts of the solution in terms of quotient modules over various rings, and losing sight of what the question was actually asking for.

Question 2. There were many good attempts at this question. In part (a)(i), many people, in defining the dual space, said what the underlying set was but did not say what the vector space operations (addition and scalar multiplication) were; so they defined it as a set but not as a vector space. In part (b)(i), in the proof that U' is naturally isomorphic to V'/U°, quite a few people said (either explicitly or imiplicitly) that if $U \leq V$, then U' is a subspace of V'. This is not the case; in fact the opposite is true: U' is a quotient of V' (which is after all what the question asked one to prove). Many candidates did this part of the question by appealing (correctly) to the First Isomorphism Theorem applied to the restriction map from V' to U'. For the later parts of the question, an instruction to "deduce" something indicated that only methods of proof using facts proved immediately beforehand could be considered to be valid solutions.

Several people noted in their solutions to **question 3**. that the question had to do with Sturm-Liouville theory, and more candidates must have noticed the fact. The point of the question, of course, is to see how (this instance of) the theory can be established within core Linear Algebra.

Again there were many good solutions, and some solutions using correct methods unanticipated by the examiner. There were one or two common errors. In the proof in part (a)(ii), that every self-adjoint linear transformation on a complex vector space has an orthonormal basis of eigenvectors, some people forgot to prove that there are any eigenvectors at all; this is precisely the point at which it matters that we are dealing with a complex vector space. In (b)(ii), for the proof that T is self-adjoint, the slickest method (though not, it seems, the only one) is to use integration by parts. But it is not at all enough simply to observe that all eigenvalues of T are real; there are plenty of non-self-adjoint operators with this property. There were many different ways of answering (b)(iv) correctly and many different correct solutions were given. The trick, of course, is that in part (iv), there is no mention of a space V_n, so if one is "gluing together" bases for different V_n, one must make sure that they harmonise correctly.

A1: Differential Equations 1

- **Question 1** This was a popular question, and generally done well. In parts (a)(i) and (a)(iii) nearly all candidates gave the correct definitions. In part (a)(ii) some lost marks due to incomplete explanations of linearisation or how eigenvalues relate to stability. In part (b)(i) common mistakes included missing that one of the critical points was not valid for $k > 2$, and making small errors in solving a quadratic to find the eigenvalues. (ii) The phase plane sketches were mostly good, with a detail to watch being the direction of trajectories through the nullclines. (iii) This part was found difficult, or not attempted, with very few correct answers. Many misunderstood the hint and how it related to Bendixson-Dulac Theorem (eg. that $a_1, \dots, a_4$ should be greater than zero).

- **Question 2** This question was well done in general. Parts (a)(i) and (a)(ii) were answered well. Many candidates missed out key steps in the argument for part (a)(iii). In part (b)(ii) some candidates failed to restrict $k < 1$. Some candidates failed to deduce that there is a Lipschitz constant. Most candidates were able to find the solution, though some solutions given were *increasing* functions of x and hence incorrect. In part (b)(iii) some candidates failed to spot that $y = -1$ is a solution, and when finding the second solution some candidates gave a function that was an *increasing* function of x. Very few candidates were able to correctly answer (b)(iv).

- **Question 3** This question was the harder out of question 2 and 3 with many candidates struggling to complete the question. Part (a) was well done. While part (b) was generally well done, some students failed to solve the correct characteristic equations. Common slips in determining the canonical equation included incorrectly computing u_{xx} and u_{yy} in terms of the characteristic variables. Part (c) presented difficulties to many candidates. Very few candidates were able to sketch the region where the solution is uniquely determined. Only a handful of candidates were able to correctly answer part (d).

A2: Metric Spaces and Complex Analysis

Question 1. Nearly all candidates attempted this question. It was generally well done. Marks were lost when proving the triangle inequality in part (b)(i) – some candidates getting rather entangled in the algebra. The only part that proved challenging was (b)(iii). Relatively few hesitated to take the maximum of all the N_i for each coordinate sequence, and fewer knew to first control the 'tail' of the summation before controlling the first finite terms. So the final four points were rarely earned.

Question 2. Again nearly all candidates attempted this question and most got solid marks. As to be expected, some had trouble when it came to the visualisation of the geometry. On the other hand, it was interesting to see that some candidates had trouble defining connectedness and path connectedness, yet were able to identify the connected components in the example.

Question 6. About 2/3 of the candidates handed in scripts for this question. But there were a number of attempts from candidates who were not prepared and not able to go very far. Part (a) seemed more challenging than planned, with the majority of students jumping into the algebra in part (a)(ii). Finding the Möbius transformation in (b)(i) proved difficult for most, while the last part was generally answered in a rush.

Long Options

A3: Rings and Modules

QUESTION 1: The various parts of (b) were largely intended to allow students to show their knowledge of bookwork and canonical examples. So (iii) was an opportunity to show finite integral domains are fields and (v) was an opportunity to show finite fields have a power of a prime elements. On many occasions, several successfully, candidates took a more first-principles approach, for example showing that there is no integral domain of order 6 to complete (iii) and then addressing (v) simultaneously. There were many good answers to this question, but also frustratingly many that showed patchy appreciation of the material: e.g. carefully presented bookwork or other inventive approach to do (iii), only to claim $\mathbb{Z}_2[i]$ or $\mathbb{Z}_2 \times \mathbb{Z}_2$ are fields with four elements, or to waste time in (v) discounting S_3 as a possible additive group of a ring of order 6. Likewise many did not quickly discount (ii) as impossible because, by Lagrange's theorem, a ring of order 5 cannot include an element of additive order 2.

QUESTION 2: This question was popular but less well done than the first question. Retrospectively, it would probably have been fairer, or made the average mark more comparable, to explain that $|a + b\omega|^2 = a^2 - ab + b^2$ should be used as the norm in $\mathbb{Z}[\omega]$, but given that this is a generalization of the norm in the Gaussian integers, and given that said expression appears in the next part, it had been hoped that these would be steers enough. In any case (c) was not well done and few made it to the very end. Relatively few also took time to sketch the lattice $\mathbb{Z}[\omega]$ in the Argand diagram and approach the problem geometrically (again a generalization of showing the Gaussian integers have a division algorithm). For the very last part it was important to appreciate that $n^2 + 3$ factorizes in $\mathbb{Z}[\omega]$ and so p is not prime in that ring.

QUESTION 3: A less popular question but well done in the main by those who attempted it. In (c) a mark was commonly lost for not treating the special case $a = 0$ separately. It had been intended that m_U and c_U could be quickly determined from m_T and c_T which could be read off the given matrix; some approached the question in this manner whilst others calculated c_U directly. It had also been intended for the very last part to note T^{-1} is a linear combination of T and I, and so apply the previous part; no-one did this but many still successfully completed the question.

A4: Integration

Question 1. This is the most popular question. Most of the candidates were able to prove the sub-additivity of the out measure, but yet most of them forget to consider the easy case that the sum of out measures is infinity, which requires to argue differently. Well several candidates were unable to state the definition of Lebesgue measurable sets and therefore unable to show that null sets are measurable. For the last part of the question, most of candidates were able to state DCT correctly, but could not spot a simple control function to justify the limit procedure.

Question 2. More than half of the candidates attempted this question, which is quite standard. Some candidates could not state the change formula for Lebesgue integrals correctly, though were able to apply the formula to complete part (a)(ii). A few candidates forget to consider the absolute value of the function f and to consider the integral of the function itself which actually does not exist. For part (b), few candidates stated MCT and Fatou's lemma incorrectly (forgetting to mention that the functions are non-negative or measurable etc.). Part (b) (ii) appeared as part of exam question several times in the past, most of the candidates did well, but some candidates were unable to justify the exchange of order in taking sum and taking integration, and while a few candidates calculated the integral in part (b) (iii) without justification (by using part (b)).

Question 3. Those who attempted this question did quite well on part (b) of the question, while many candidates were unable to prove the integrability of the function in part (a) (ii), which requires some estimates together with the comparison argument. For part (iii), a few candidates even could not work out the derivative of the function with respect to the parameter t, and got a wrong answer for this simple function. Some though very few of the candidates were able to justify the procedure of taking derivative under Lebesgue integration, and most of those attempted could not find a simple control function.

A5: Topology

Question 1. Virtually all candidates attempted this question.

Part a was generally well done, a few candidates managed to do only one direction of the equivalence in a.ii.

The first part of question b was generally well done. Around half of the candidates did not manage to give a counterexample but the ones who did explained it well and received full marks for this.

Few students did not define connected correctly and many did not manage to answer the second part of part c.

Many students answered correctly the first part of part d and several managed to answer the full question often assuming the result of part c.

Question 2. Most of the candidates attempted this question.

Parts a.i, a.ii were generally well done. Some candidates had difficulties with a.ii even though it was bookwork.

In part b.i many condidates defined the quotient space as a set without giving the topology and received no marks. The candidates struggled with the second part of this: many answered a different question expressing S^1 as a quotient of I and several used a map $S^1 \to I$ which was not continuous giving a wrong answer.

Part b.ii was generally well done.

In part c.i many candidates said that the topology on $Z \times Z$ is the co-finite topology which is false. However most deduced correctly that $Z \times \mathbb{R}, Z \times Z$ are not homeomorphic. Several candidates answered at least partly the question about K, quite a few showed that it is not compact in $Z \times \mathbb{R}$ considering the projection map to $\mathbb{R}$. They had more difficulties when they considered it as a subset of $Z \times Z$ as they were not clear about the product topology. Several candidates attempted to identify W up to homeomorphism but few succeeded.

Only a couple of candidates managed to answer c.ii.

Question 3. Only 14 students attempted this question.

Even though few students attempted this several who did received quite high marks.

Part a.i was generally well done.

In part a.ii some candidates gave an incomplete statement of the classification theorem and received partial marks.

Many candidates did part a.iii justifying their solutions only by a diagram- and received full marks as this was acceptable.

Many candidates managed to do the first part of a.iv but only one answered correctly the second part.

Many candidates attempted part b arguing using diagrams but several miscalculated saying for example that the surface is the projective plane or the torus rather than the Klein bottle. Candidates that made these mistakes received partial credit when they used the right method.

A6: Differential Equations 2

Q1 Most candidates did very well with the first part of the question, but struggled with the second part, in particular how to deal with the delta in the operator. Successful candidates saw that this should be treated by defining two different regions (or three regions when defining the Green's function) with appropriate jump conditions. A common mistake was attempting to plug in x=1 and incorrectly writing $\delta(x-1) = 1$ when $x = 1$. No!

Q2 Full marks in aii) required noting the requirement that y and $y'(x)$ remain bounded at the boundary, which effectively replaces the boundary condition. In constructing the series solution in biii), a number of candidates attempted the "decomposed solution" route; this could potentially work, but given that the question specifically requested the full solution as a series – not a series plus another term – this route would require expressing the extra function in the form of a series. Only one or two candidates properly saw this.

Q3 The biggest issue with this question was not seeing the short route, which likely led a lot of exam time being spent on unnecessary calculations. In particular: aii), once it is determined that for $c = -1$ no contradiction is reached, this also means that the coefficient b_2 is free, and can be set to 0, thus giving the explicit solution almost immediately. Also, in bii), a number of candidates spent time trying to compute an adjoint operator, which is not needed, rather the question required seeing the structure $Ly_i = -y_{i-1}$ at each order and thus showing that $Ly = 0$ has only the solution $y = 0$ to apply Fredholm Alternative.

A7: Numerical Analysis

Question 1 on interpolation and quadrature was attempted by almost all candidates and marks ranged from low to very high with most achieving reasonable scores. In part (a), several candidates attempted to calculate the interpolating polynomial for the given data using the Lagrange form; unsurprisingly none succeeded. Few related the answers they calculated in the last part to the degree of accuracy of Simpson's rule.

Question 2 was also attempted by the vast majority of candidates. In (a) the descriptions of Gaussian elimination varied considerably in their accuracy. In (b) almost all failed to provide a correct proof that $A = LDL^T$. The last part was quite well done by use of a single Householder reflection or (perhaps more easily) by a Givens rotation.

Question 3 was attempted by only a handful of candidates, but scores were quite high for those that did.

A8: Probability

See Mathematics and Statistics report.

A9: Statistics

See Mathematics and Statistics report.

A10: Fluids and Waves

Question 1: Nearly all candidates attempted this question, with an average of 20/25. All parts were attempted reasonably well.

Question 2: Next most popular question. Not as well answered, with a mean of 15/25. Quite a few candidates didn't observe in b(iii) that the region between the large circle and $C(t)$ contained no singularities, validating a holomorphic Laurent expansion, and failing to transform to circular variables. Some candidates got the wrong integrand in the derivation of Kelvin's Circulation Theorem, and forgot the single-valuedness.

Question 3: Least popular question, with a mean of 13/25. The basic definitions in (a) caused a few problems, especially the mathematical equation for the group speed. Part b(i) and (ii) was well answered, apart from the bottom boundary condition. Quite a few candidates omitted to incorporate the time variation of h. Very few candidates derived the expression for b, and only a couple attempted part (c).

A11: Quantum Theory

Q.1: This was attempted by all but one candidate. Most were well-prepared for the derivation of the energy eigenstates for this elementary system, though proofs in (a)(ii) were often patchy.

In part (b), many candidates noticed that the given function $\psi(x, 0)$ could be put in the form $(\frac{1}{2}\psi_2 +$ a second term), but only a few actually showed that this second term contained no component of ψ_2. None of those who did noticed that this fact could be deduced simply from looking at its parity. Many candidates were defeated by the expectation value calculations. A common mistake, dooming a number of efforts, lay in forgetting to take the complex conjugate where required in evaluating matrix elements.

Q.2: This was attempted by all but 2 candidates. Part (a) was particularly well prepared for, and many candidates gave a commendable account of the logic of the ladder argument. A good proportion of candidates saw that an induction proof would work for the first segment of part (b). But very few candidates were able to attempt the main application in part (b), which essentially required an explicit evaluation of ψ_2.

Q.3: Only 9 candidates attempted this question. They were generally well prepared for the commutation relations. Candidates did not seem to notice that the question did not require rehearsing the whole of the ladder argument, and that some simpler statements would have sufficed. The standard result in (b)(i) was generally done satisfactorily but the remaining parts were poorly attempted.

Short Options

ASO: Q1. Number Theory

There was one question, but it was divided into four small subquestions on different topics. As a result the marks were generally quite high, as candidates could get half marks for only having understood half the course. However, the question did seem to test understanding reasonably well. A few specific points:

(a) Quite a few candidates - probably even the majority of candidates - did not seem to notice that 111 is composite. This cost them most of the 4 points available for the rider, though I did sometimes give one sympathy mark for an otherwise correct computation.

(b) A surprising number of candidates think that $1 + \cdots + n = n(n-1)/2$. Otherwise, there were a lot of good solutions to this question.

(c) It turned out that there was an almost trivial solution to the last part, namely $n = 2017 \times 2 = 4034$. This was not spotted by me or the checker, and in fact only by a small percentage of the candidates (perhaps around 10). Quite a large number of other solutions were found (possibly even all of them, I didn't check).

(d) This was done quite well, even though the checker thought the unseen part might be a little tricky.

ASO: Q2. Group Theory

The question was generally answered well. Essentially all understood the Sylow Theorems and could apply them. Most were also able to argue that a group of order 70 had to be a semi-direct product and that it therefore was determined by a group homomorphisms from the 2-Sylow subgroup to the automorphisms of the normal cyclic subgroup. Some struggled to clearly argue that there were only four such homomorphis. Many forgot to show that the listed groups are actually non-isomorphic.

ASO: Q3. Projective Geometry

Question 3 was attempted by 9 candidates, for an average of over 18 marks. The main source of problems was that several candidates mis-remembered the statement of Desargues' theorem, and then somehow convinced themselves that the false statement holds true in the example.

ASO: Q4. Introduction to Manifolds

Question 4 was attempted by 15 candidates, for an average of about 15.5 marks. The bookwork was done well, but several candidates struggled to use the definition from (a)(ii) to do (b)(ii). Also a surprising number of candidates appear never to have seen a hyperboloid.

ASO: Q5. Integral Transforms

Parts (a) and (b) were in the main well done, with students showing a good operational knowledge of the Fourier transform and many being able to adapt it to find the integral in (a)(iv). The last part, relating Laplace's equation in the quadrant to that in the half-plane was less well done. Even when it was appreciated that $z \mapsto z^2$ conformally maps the quadrant onto the half-plane, few were then able to deduce that $g(x, y) = f(x^2 - y^2, 2xy)$.

ASO: Q6. Calculus of Variations

There were 60 attempts at this question and the average score was 15.5 out of 25 so, overall, candidates did reasonably well on this paper. Where many candidates lost marks was in not going into sufficient detail in proofs, leaving out crucial steps. This happened in a number of parts of the question:

- (a) (iii) At one point the proof requires an integration by parts. Many candidates skipped that and simply wrote down the answer of this operation that, while correct, lost a mark because it was not clear if they were doing this by memory or if they really understood it. [Note that more than one candidate thought that the integral of a product was the product of the integrals]
- (b) (ii) Having found one extremal, many candidates then wrote down the sum as a solution without justifying that this is valid because the ODE is linear.
- (b) (iii) Many candidates simply wrote down that

$$\int_0^\pi [\Sigma_{n=1}^{n=\infty} a_n sin(n\pi x)]^2 dx = \int_0^\pi \Sigma_{n=1}^{n=\infty} [a_n sin(n\pi x)]^2 dx.$$

  This is true, but has to be proved, by appealing to the orthogonality property proved in (a)(iii). Some candidates simply quoted

$$\int_0^\pi sin(m\pi x) sin(n\pi x) dx = 0 \quad n \neq m,$$

  but, again, this needed to be proved.

No one got the very last part of the question correct, as this required them to notice that any admissible function could be written as a sum of extremals. I was very lenient in marking this and gave full marks if they simply considered the sum.

ASO: Q7. Graph Theory

Most candidates were able to recall the definitions asked for in part (a) although a common confusion was using “the” where “a” was required.

In (b), most candidates did not realise that an example requires negative weights.

Part (c) was found quite straightforward by those who attempted it.

Very few candidates made much progress on (d): most answers claimed to prove false statements (particularly that T contains all w-shortest vu-paths) or were not relevant to the question (no marks were given for Dijkstra’s algorithm); only one candidate gave a complete solution.

ASO: Q8. Special Relativity

Part (a) was a bookwork definition of four-momentum. Most candidates answered reasonably well, though there were a number of dropped Lorentz factors and incorrectly written spatial momenta for the massless particle.

Part (b, i) was a conservation of relativistic momentum problem. Most candidates identified this fact, but few successfully set p the equations without making mistakes, and not gone safely all the way to the supplied solution.

Parts (b, ii) and (b, iii) was a fairly simple time-dilation/length-contraction exercise. It could be solved using the supplied answer to part (b, i), but did not receive many attempts.

ASO: Q9. Modelling in Mathematical Biology

All candidates struggled with this question; many did not seem to have a grasp of essential concepts from the course. In (a), many could not write down appropriate biological interpretations of the various terms in the model, and very few linked the presence of repeated outbreaks to the steady state being a centre. In (b), many candidates did not properly non-dimensionalise the model, and all candidates struggled with the final part (sketching the right-hand side of the expression given in the question gives the result in a single step).

E. Comments on performance of identifiable individuals

Removed from public version of the report.

F. Names of members of the Board of Examiners

- **Examiners:**
  Dr Robin Knight
  Dr Neil. Laws
  Prof. Alex Scott
  Prof. Ulrike Tillmann
  Prof. Sarah Waters (Chair)
  Dr Martyn Quick (External Examiner)
  Dr Warren Smith (External Examiner)

- **Assessors:**
  Prof. Ruth Baker
  Prof. Christopher Beem
  Dr Mike Chen
  Dr Janet Dyson
  Dr Richard Earl
  Prof. Ben Green
  Prof. Andrew Hodges
  Prof. Peter Keevash
  Prof. Philip Maini
  Prof. James Martin
  Prof. Lionel Mason
  Prof. Irene Moroz
  Prof. Derek Moulton
  Prof. Panos Papazoglou
  Prof. Zhongmin Qian
  Prof. James Sparks
  Prof. Endre Suli
  Prof. Balazs Szendroi
  Prof. Andy Wathen