Examiners' Report:
Final Honour School of Mathematics Part A
Trinity Term 2018

November 5, 2018

Part I

A. STATISTICS

- **Numbers and percentages in each class.**
  See Table 1.

Table 1: Numbers in each class

Range	Numbers					Percentages %				
	2018	2017	2016	2015	2014	2018	2017	2016	2015	2014
70–100	57	57	50	51	57	35.62	36.77	34.97	36.17	36.54
60–69	69	62	63	59	62	43.12	40	44.06	41.84	39.74
50–59	22	31	26	26	31	13.75	20	18.18	18.44	19.87
40–49	9	4	3	5	4	5.62	2.58	2.1	3.55	2.56
30–39	3	1	0	0	2	1.88	0.65	0	0	1.28
0–29	0	0	1	0	0	0	0	0.7	0	0
Total	160	155	143	141	156	100	100	100	100	100

- **Numbers of vivas and effects of vivas on classes of result.**
  Not applicable.

- **Marking of scripts.**
  All scripts were single marked according to a pre-agreed marking scheme which was strictly adhered to. The raw marks for paper A2 are out of 100, and for the other papers out of 50. For details of the extensive checking process, see Part II, Section A.

- **Numbers taking each paper.**
  All 160 candidates are required to offer the core papers A0, A1, A2 and ASO, and five of the optional papers A3-A11. Statistics for these papers are shown in Table 2 on page 2.

Table 2: Numbers taking each paper

Paper	Number of Candidates	Avg RAW	StDev RAW	Avg USM	StDev USM
A0	160	29.49	8.36	66.18	10.57
A1	160	32.91	8.3	66.18	13.17
A2	160	54.56	16.69	65.64	11.97
A3	77	32.9	8.7	64.22	12.32
A4	115	32.82	6.82	66.55	11.01
A5	70	33.5	9.28	65.16	11.28
A6	99	30.4	8.71	64.39	12.35
A7	64	32.33	8.04	64.25	11.77
A8	153	36.03	7.96	66.8	13.16
A9	118	32.88	10.83	66.98	16.44
A10	44	32.45	8.51	62.89	11.82
A11	63	28.11	8.49	67.51	11.42
ASO	160	35.15	8.29	66.05	12.28

B. New examining methods and procedures

There were no changes in 2017–18.

C. Changes in examining methods and procedures currently under discussion or contemplated for the future

None.

D. Notice of examination conventions for candidates

The first notice to candidates was issued on 12th February 2018 and the second notice on the 8th May 2018.

These can be found at https://www.maths.ox.ac.uk/members/students/undergraduate-courses/ba-master-mathematics/examinations-assessments/examination-20, and contain details of the examinations and assessments. The course handbook contains the link to the full examination conventions and all candidates are issued with this at induction in their first year. All notices and examination conventions are on-line at https://www.maths.ox.ac.uk/members/students/undergraduate-courses/examinations-assessments/examination-conventions.

Part II

A. General Comments on the Examination

Acknowledgements

The examiners would like to express their gratitude to

- Nia Roderick for her work in supporting the Part A examinations throughout the year, and for her help with various enquiries throughout the year.
- Also Helen Lowe for assistance with information, procedure and other matters.
- Waldemar Schlackow for running the database and the algorithms that generate the final marks, without which the process could not operate.
- Charlotte Turner-Smith for her support at the initial meetings and her help, together with the Academic Administration Team, with marks entry, script checking, and much vital behind-the-scenes work.
- The assessors who set their questions promptly, provided clear model solutions, took care with checking and marking them, and met their deadlines, thus making the examiners' jobs that much easier.
- Several members of the Faculty who agreed to help the committee in the work of checking the papers set by the assessors.
- The internal examiners and assessors would like to thank the external examiners, Martyn Quick (Pure Mathematics) and Demetrios Papageorgiou (Applied Mathematics) for helpful feedback and much hard work throughout the year, and for the important work they did in Oxford in examining scripts and contributing to the decisions of the committee.

Timetable

The examinations began on Monday 11th June and ended on Friday 22nd June.

Factors Affecting Performance

A subset of the Examiners attended a pre-board meeting to band the seriousness of each Factors Affecting Performance application. The outcome of this meeting was relayed to the Examiners at the final exam board, who gave careful regard to each case, scrutinised the relevant candidates' marks and agreed actions as appropriate. See Section E for further details.

Setting and checking of papers and marks processing

As is usual practice, questions for the core papers A0, A1 and A2, were set by the examiners and also marked by them. The papers A3-A11, as well as each individual question on ASO, were set and marked by the course lecturers. The setters produced model answers and marking schemes led by instructions from Teaching Committee in order to minimize the need for recalibration.

The internal examiners met in December to consider the questions for Michaelmas Term courses (A0, A1, A2 and A11). The course lecturers for the core papers were invited to comment on the notation used and more generally on the appropriateness of the questions. Corrections and modifications were agreed by the internal examiners and the revised questions were sent to the external examiners.

In a second meeting the internal examiners discussed the comments of the external examiners and made further adjustments before finalising the questions. The same cycle was repeated in Hilary term for the Hilary term long option courses and at the end of Hilary and beginning of Trinity term for the short option courses. *Papers A8 and A9 are prepared by the Department of Statistics and jointly considered in Trinity term.* Before questions were submitted to the Examination Schools, setters were required to sign off a camera-ready copy of their questions.

Examination scripts were collected by the markers from Ewert House or delivered to the Mathematical Institute for collection by the markers and returned there after marking. A team of graduate checkers under the supervision of Charlotte Turner-Smith, Nia Roderick and Hannah Harrison sorted all the scripts for each paper, cross-checking against the mark scheme to spot any unmarked questions or part of questions, addition errors or wrongly recorded marks. Also sub-totals for each part were checked against the marks scheme, noting any incorrect addition. An examiner was present at all times to authorise any required corrections.

Determination of University Standardised Marks

The examiners followed the standard procedure for converting raw marks to University Standardized Marks (USM). The raw marks are totals of marks on each question, the USMs are statements of the quality of marks on a standard scale. The Part A examination is not classified but notionally 70 corresponds to 'first class', 50 to 'second class' and 40 to 'third class'. In order to map the raw marks to USMs in a way that respects the qualitative descriptors of each class the standard procedure has been to use a piecewise linear map. It starts from the assumption that the majority of scripts for a paper will fall in the USM range 57-72, which is just below the II(i)/II(ii) borderline and just above the I/II(i) borderline respectively. In this range the map is taken to have a constant gradient and is determined by the parameters C_1 and C_2, that are the raw marks corresponding to a USM of 72 and 57 respectively. The guidance requires that the examiners should use the entire range of USMs. Our procedure interpolates the map linearly from $(C_1, 72)$ to $(M, 100)$ where M is the maximum possible raw mark. In order to allow for judging the position of the II(i)/III borderline on each paper, which corresponds to a USM of 40, the map is interpolated linearly between $(C_3, 37)$ and $(C_2, 57)$ and then again between $(0, 0)$ and $(C_3, 37)$. It is important that the positions of the corners in the piecewise linear map are not on the class borderlines in order to avoid distortion of the class boundaries. Thus, the conversion is fixed by the choice of the three parameters C_1, C_2 and C_3, the raw marks that are mapped to USM of 72, 57 and 37 respectively.

The examiners chose the values of the parameters as listed in Table 3 guided by the advice from the Teaching Committee and by examining individuals on each paper around the borderlines.

Table 3: Parameter Values

Paper	C1	C2	C3
A0	(36.4,72)	(19.4,57)	(10.57,37)
A1	39,72)	(25,57)	(14.36,37)
A2	(67,72)	(37,57)	(21.26,37)
A3	(41.2,72)	(24.7,57)	(14.19,37)
A4	(37.6,70)	(25.6,57)	(14.71,37)
A5	(42.8,72)	(23,57)	(12.52,37)
A6	(38.2,72)	(22.7,57)	(12.47,37)
A7	(40,72)	(25,57)	(14.36,37)
A8	(41.8,72)	(28.3,57)	(16.83,37)
A9	(39,70)	(21.5,57)	(12.35,37)
A10	(41.2,72)	(26.7,57)	(14.19,37)
A11	(33,72)	(18,57)	(10.34,37)
ASO	(42.2,72)	(26.7,57)	(15.34,37)

Table 4 gives the resulting final rank and percentage of candidates with this overall average USM (or greater).

Table 4: Rank and percentage of candidates with this overall average USM (or greater)

Av USM	Rank	Candidates with this USM or above	%
97	1	1	0.63
91	2	2	1.25
85	3	4	2.50
84	5	5	3.13
83	6	6	3.75
82	7	8	5.00
81	9	10	6.25
80	11	11	6.88
79	12	14	8.75
78	15	16	10.00
77	17	21	13.13
76	22	25	15.63
75	26	28	17.50
74	29	32	20.00
73	33	37	23.13
72	38	42	26.25
71	43	50	31.25
70	51	57	35.63
69	58	65	40.63
68	66	74	46.25
67	75	82	51.25
66	83	89	55.63
65	90	98	61.25
64	99	109	68.13
63	110	114	71.25
62	115	118	73.75
61	119	121	75.63
60	122	126	78.75
59	127	129	80.63
58	130	132	82.50
57	133	136	85.00
56	137	138	86.25
55	139	140	87.50
54	141	143	89.38
53	144	144	90.00
51	145	145	90.63
50	146	148	92.50
49	149	152	95.00
44	153	153	95.63
43	154	154	96.25
42	155	157	98.13
37	158	158	98.75

Table 4: Rank and percentage of candidates with this overall average USM (or greater) [continued]

Av USM	Rank	Candidates with this USM or above	%
33	159	159	99.38
31	160	160	100.00

Recommendations for Next Year's Examiners and Teaching Committee

None

B. Equality and Diversity issues and breakdown of the results by gender

Table 5, page 8 shows percentages of male and female candidates for each class of the degree.

Table 5: Breakdown of results by gender

Class	Number								
	2018			2017			2016		
	Female	Male	Total	Female	Male	Total	Female	Male	Total
70–100	9	48	57	7	50	57	5	45	50
60–69	19	50	69	12	50	62	17	46	63
50–59	8	14	22	12	19	31	12	14	26
40–49	3	6	9	2	2	4	0	3	3
30–39	1	2	3	1	0	1	0	0	0
0–29	0	0	0	0	0	0	0	1	1
Total	40	120	160	34	121	155	34	109	143
Class	Percentage								
	2018			2017			2016		
	Female	Male	Total	Female	Male	Total	Female	Male	Total
70–100	22.5	40	35.62	20.59	41.32	36.77	14.71	41.28	34.97
60–69	47.5	41.67	43.12	35.29	41.32	40	50	42.2	44.06
50–59	20	11.67	13.75	35.29	15.7	20	35.29	12.84	18.18
40–49	7.5	5	5.62	5.88	1.65	2.58	0	2.75	2.1
30–39	2.5	1.67	1.88	2.94	0	0.65	0	0	0
0–29	0	0	0	0	0	0	0	0.92	0.7
Total	100	100	100	100	100	100	100	100	100

C. Detailed numbers on candidates' performance in each part of the exam

Individual question statistics for Mathematics candidates are shown in the tables below.

Paper A0: Linear Algebra

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14.28	14.46	6.24	72	2
Q2	16.03	16.03	4.29	143	0
Q3	12.91	13.20	4.00	105	5

Paper A1: Differential Equations 1

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.34	17.57	5.30	98	3
Q2	15.97	15.94	3.94	137	1
Q3	15.95	15.99	5.32	85	1

Paper A2: Metric Spaces and Complex Analysis

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	12.96	13.32	5.98	109	4
Q2	16.22	16.33	5.43	138	1
Q3	10.79	10.83	3.88	94	1
Q4	12.59	12.78	5.00	137	3
Q5	13.07	13.59	6.34	107	6
Q6	14.82	15.13	4.97	53	2

Paper A3: Rings and Modules

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.87	15.87	5.16	61	0
Q2	16.83	17.04	5.24	68	1
Q3	15.21	16.24	5.15	25	3

Paper A4: Integration

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.96	17.96	3.63	114	0
Q2	14.73	14.81	3.94	110	2
Q3	14.86	16.33	6.64	6	1

Paper A5: Topology

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.97	17.97	4.78	67	0
Q2	14.57	14.57	5.60	46	0
Q3	17.44	17.44	6.31	27	0

Paper A6: Differential Equations 2

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	13.50	13.63	5.21	81	1
Q2	17.30	17.65	5.37	85	2
Q3	11.62	12.69	4.28	32	7

Paper A7: Numerical Analysis

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14.65	14.65	3.97	49	0
Q2	17.69	17.90	4.53	31	1
Q3	16.58	16.58	5.29	48	0

Paper A8: Probability

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.51	17.63	4.68	144	1
Q2	14.17	14.81	5.39	32	3
Q3	19.23	19.23	3.78	130	0

Paper A9: Statistics

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	13.56	14.00	6.78	53	2
Q2	16.75	17.04	5.88	99	2
Q3	17.14	17.27	5.89	84	1

Paper A10: Fluids and Waves

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14.00	15.07	6.63	27	3
Q2	17.12	17.45	4.89	42	1
Q3	14.65	15.16	5.31	19	1

Paper A11: Quantum Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	10.75	10.75	4.40	51	0
Q2	16.58	16.58	4.63	50	0
Q3	15.76	15.76	4.51	25	0

Paper ASO: Short Options

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	21.14	21.14	4.03	63	0
Q2	18.07	18.07	3.52	29	0
Q3	18.00	18.00	4.21	8	0
Q4	15.59	16.31	4.58	16	1
Q5	18.67	19.01	4.83	83	2
Q6	15.61	16.11	5.38	55	2
Q7	11.50	11.50	4.56	24	0
Q8	11.33	11.33	3.06	3	0
Q9	15.10	15.10	5.75	39	0

Part A 2018: Comments on sections and on individual questions

The following comments were submitted by the assessors.

Core Papers

A0: Algebra 1

Question 1. Popular question with many good solutions.

Question 2. This was the most popular question with good results. The most challenging part was (c)(ii), proving that a positive A must be self-adjoint. Many candidates wrote that this follows immediately from $\langle v, Av\rangle = \langle A^*v, v\rangle$ for all v but on its own this is not enough to deduce that $A = A^*$, in fact it is not true in real inner product spaces. The correct solutions proceeded by substituting $v+w$ and $v+iw$ for v in the above equation and solving for $\langle Av, w\rangle$.

Question 3. A common difficulty was in (a)(iii), where many candidates tried to argue with dimension or dual basis which only works if $\dim V$ is finite.

Part (b)(ii) was the hardest with few correct solutions. Perhaps the shortest proof was to consider the values of $R(e_i')(e_j)$ and show that sometimes there is no T such that $R = T'$, for example if $R(e_i')(e_j) = 1$ for all i, j.

A1: Differential Equations 1

Question 1. A number of candidates attempted creating strange (and often incorrect!) piecewise solutions in part a)ii (evidently following the pattern of previous years, but that was not needed here), one could simply use $y \equiv 1$ and the separating variables solution. Full marks in part b) required careful/clear reasoning on maximising h via $Mh \leq k$.

Question 2. This was attempted by most candidates, but marks were not very high on average. Many candidates failed to note in part a that applying linear stability analysis to a linear system gives exact (not approximated) information. In part c, very few saw the 'trick': there was no need to compute the solution explicitly or even a Jacobian, one only needed to recognise that trajectories are straight lines through the origin and thus never intersect in the positive quadrant.

Question 3. The most common mistake in this problem was applying the Maximum principle in part c)ii. This is neither applicable in this problem (the Max Principle as given in lectures was not for the form of system appearing here), nor necessary. The quickest approach was to consider the differential equation (or inequality rather) satisfied by $I(t) = \int_0^1 \frac{1}{2}w^2\,dx$, from which some straightforward logic gives that $I(t) \equiv 0$.

A2: Metric Spaces and Complex Analysis

Question 1. This question explored the properties of the space of continuous functions. The first part of 1a was straight forward and generally done well though only few students realised that the metric may not be defined if Y is not bounded. The second part of 1a was rather difficult for the students as it was genuinely new, and only few students were able to prove this. Partial credits were given for clear statements what needed to be proven, realisation of how the compactness of K and openess of U may play a role. Only a couple of candidates considered the distance function from the image of K to the complement of U. Part b was more straight forward though not easy.

Question 2. Part 2a is standard book work and was generally done quite well. Part 2b proved more challenging with some candidates trying to prove the opposite of what is true in either case. The third part should have been straight forward and not difficult. However, for the first part of 2c few candidates realised that one needed to choose a paths between the two points under consideration and integrate along that, while in the second part marks were lost because candidates neither checked that the function took X to X or that X is complete.

Question 3. This question was of a more conceptual nature than the other two. Part (b) proved to be a bit harder than anticipated, and several candidates showed difficulties with the proper definition of branch cuts. Furthermore, a small but still surprising fraction of students showed difficulties with the chain-rule for two variables, in the last bit of part (a) of the question. Finally, several candidates only tried part (a), which lowered considerably the total mark. Candidates found Q3 slightly harder than the other two on Complex Analysis (but the difference was less than two marks).

Question 4. The question worked quite well, although part (c) proved to be slightly easier than anticipated. In part (b) a dramatically high number of students wrote $1/30 + 1/30 = 1/60$.

Question 5. The question worked quite well. Part (b) was slightly harder than anticipated and several students had difficulties defining/finding the appropriate contour. This was somehow mitigated by part (c) being easier than anticipated.

Question 6. Only relatively few candidates chose this problem. Part 6a proved - not surprisingly - conceptually difficult. Many candidates just did not understand what there was to prove and quite a few simply left this part out. Only a handful of the candidates demonstrated that they had properly understood the relation of the $PGL(2\mathbb{C})$ and the group of Möbius transformations. The second part of the question was more straight forward with most candidates able to articulate well why the constructed map in 6b(iii) could not be a Möbius transformation.

Long Options

A3: Rings and Modules

Question 1. A popular question with quite a range in the quality of solutions. In (a)(ii) most appreciated that $\mathbb{Z}_{340} \cong \mathbb{Z}_4 \oplus \mathbb{Z}_5 \oplus \mathbb{Z}_{17}$ but quite a few scripts overcomplicated the question, and instead of factorizing the cubic as $(x+1)(x^2+1)$ sought to make use of $(x-1)(1+x+x^2+x^3) = x^4-1$. In all there were 18 distinct solutions and the primes were chosen so that $x^2 = -1 = 4$ or 16 was straightforward to solve. However some scripts did not make use of $\mathbb{Z}_5$ and $\mathbb{Z}_{17}$ being fields and instead wasted time determining the cubic for all possible values of x.

(b) was largely well done but most attempts at (c) were incomplete. Some care is needed showing any ring isomorphism would fix the rationals, and then some further explanation needed as to why it can't be that just one of $\sqrt{a}$ and $\sqrt{b}$ is irrational. Finally then it amounts to showing that there are no square roots of a in $\mathbb{Q}\left[\sqrt{b}\right]$.

Question 2. Part (a) was largely well done with most candidates knowing to consider the ideal generated by a and b. Some few scripts assumed the harder result that PIDs are UFDs; this doesn't aid in answering the question and such solutions typically received little or no credit.

In (b) $\mathbb{F}_8$ is a field with 8 elements as $\mathbb{Z}_2[x]$ is a PID, x^3+x+1 is irreducible over $\mathbb{Z}_2$ and so generates a maximal ideal, and by the division algorithm each coset has a representative of degree 2 or less. Over $\mathbb{F}_8$ we find that y^3+y+1 has roots x^2, x, x^2+x (to find the third consider the coefficient of y^2). Over $\mathbb{F}_8$ we see that y^8-y splits as 8 linear factors but over $\mathbb{Z}_2$

$$y^8 - y = y(y-1)(y^3+y+1)(y^3+y^2+1),$$

the remaining elements $x^2+1, x+1, x^2+x+1$ being roots of the second irreducible cubic factor.

Question 3. This was the least popular question, though still reasonably popular and a comparable average. In (a) most candidates were aware of what was involved in determining that there are 9 abelian groups of order 216 but quite a few dropped marks by not being careful enough counting up the possibilities.

In (b) if $A = PBP^{-1}$ then the map $\mathbf{v} \mapsto P\mathbf{v}$ is a module isomorphism from M_A to M_B and a similar argument can be made for the converse. Some candidates did make progress making use of the uniqueness of rational canonical form but this was much more involved than was expected.

In (c) all three modules are isomorphic. B is in fact the rational canonical form of A or (i) and (ii) can be seen to be isomorphic to $\mathbb{R}[x]/\langle x^2-2x+2\rangle$. A basis for the third (as a real vector space) is $\{1, i\}$ and multiplication by $(1+i)$ is then seen to be given by multiplication by A noting

$$(1+i)\,(a+ib) = (a-b) + i(a+b).$$

The most disappointing scripts were those which had the minimal polynomials of A or B (or both) being of degree greater than 2, something which should have been clearly impossible from A0 Linear Algebra.

A4: Integration

As expected, Q.3 on L^p-spaces was attempted by few candidates. In order to allow for many candidates effectively having little choice, Q.1 and Q.2 included considerable fairly standard material, but they also had parts which required something more imaginative than routine manipulations. The outcome was a mark distribution where very low marks, and extremely high marks, were very scarce, and little scaling was required (at least in the opinion of the marker). The great majority of the candidates showed a good understanding of the underlying material.

Question 1. Most candidates got a high proportion of the first 20 marks, usually dropping a few marks. Hardly any candidates quoted the Substitution Theorem for Lebesgue integration in (b)(iv) although some stated it in full for (c)(i); consequently most arguments for (b)(iv) went by a more laborious route. There were a fair number of attempts at (c)(ii) by various methods which could have worked but were often not carried through. One common error was to regard the equality of $\int_{\mathbb{R}} e^{-x^2}\,dx$ and $\int_{\mathbb{R}} e^{-(x-i/2)^2}\,dx$ as being an instance of the Substitution Theorem instead of an argument via Cauchy's Theorem.

Question 2. As expected, (a) and (b) were answered well in most cases. Part (c)(i) suffered from the usual tendency for students to use the continuous-parameter version of DCT with a dominating function which depended on the parameter y, sometimes explicitly and sometimes hidden by describing as a "constant" a quantity which had been constant in (b)(ii) where y was constant but which depended on y. Nevertheless there were a good number whose dominating function was independent of y at least locally, but some arguments failed to cover continuity at $y = 0$. Both (c)(ii) and (c)(iii) allowed a variety of approaches, some of which worked better than others. Hardly any candidates thought of checking whether $\int_0^\infty \frac{\partial df}{\partial dy}(x,0)\,dx$ existed before setting off on more complicated calculations. The non-existence of that integral would have given a big clue that F is not differentiable at 0, and it could have earned several marks.

Question 3. Very few attempts.

A5: Topology

Question 1.

Virtually all candidates attempted this question.

Part a was generally well done, but some candidates used the characterization of connectedness via functions without justifying it and a point was taken off. The second part was generally well done.

Part b (i) was done by most candidates. In part b (ii) most candidates realized that they should use 1 (a) but several did not choose the right sets and were given only partial credit.

In part c many students had the right intuition that $\mathbb{R}$ is not homeomorphic to a product as $\mathbb{R}$ minus a point is disconnected, however very few gave a complete argument showing that $X \times Y \setminus \{a\}$ is connected if $X.Y$ are connected.

Question 2.

Part a was generally well done. The proofs that compactness implies sequential compactness were not always complete, often the construction of a subsequence converging to a limit point was missing and marks were taken off for this. Some candidates failed to see that sequential compactness was useful for the last part and usually failed to give a correct proof.

In part b the proof of compactness of $\hat{X}$ was generally correct but many candidates claimed that $\mathcal{T} \subset \mathcal{T}'$ implies that X is a subspace of X' which is incorrect.

Part c was more challenging. A fair number of candidates showed that the compactification of $\mathbb{R}$ is sequentially compact either identifying it with S^1 or using the definition. Some did see that the compactification of $\mathbb{Q}$ is not Hausdorff but only a handful realized that it is sequentially compact. Several claimed incorrectly that it is not as $\mathbb{Q}$ is not.

Question 3.

Part a. Frequently the proof that the torus is a quotient space of the square was done 'by picture' and marks were taken off for this as a formal proof was possible and easy to do. The triangulation part was done by most candidates and the statement of the classification theorem was generally well done even though sometimes there were a omissions and a mark was taken off.

Part b was generally well done.

Part c was more challenging but quite a few candidates gave a complete proof. Some gave incomplete arguments either showing that the space is a surface without identifying the surface or giving some reasons why it should be N_2 without a proper justification. Partial credit was given in these cases.

A6: Differential Equations 2

Question 1. This question was attempted by most candidates. Some easy marks were missed in part b by not saying anything about boundary conditions satisfied by the eigenfunctions or Green's function. Part c was 'hit or miss', those that saw how to approach the problem (which had two different possible routes) tended to get full marks. In part d, the delta function showing up in the operator caused issue for a number of candidates, but simply required writing a solution in $x < 0$ and $x > 0$ with a jump condition in $y'(x)$ across $x = 0$.

Question 2. This question was attempted by the most candidates, and average marks were the highest for this question. It was a computational question more than conceptual. Some candidates evidently spent too much time making the calculations longer than necessary, and such inefficiency probably took away valuable time from the other questions.

Question 3. This question was tried by the fewest number of candidates, and had the lowest average mark. It was a conceptual question, and could be answered with very little computation. Some candidates spent far more time than needed answering part bi – the question did not require to solve explicitly for y_1, all that was needed was to see from the structure that y_1 would have terms $x \cos x$ and/or $x \sin x$ to see that y_1 is growing unbounded with increasing x and thus is of the same size as y_0. Part bii and part c could both be answered by observing the necessary orthogonality of y_0^3 (in part bii) and $Y_0^3 + 2Y_{0_s}$ (in part c) to $\cos x$ and $\sin x$.

A7: Numerical Analysis

This appears to have been a reasonable exam with candidates scoring a range of marks including several very high ones.

Question 1. on LU factorization was attempted by a large majority of candidates. Scores were typically in the middle range with just a few candidates able to correctly identify a matrix similar to A when pivoting is employed.

Question 2. on the Power Method and QR Algorithms was not so popular, but high scores were achieved in a good proportion of the attempts. Only one candidate mentioned the singularity in $\frac{1}{x-\mu}$ and the consequent effect of significantly separating the eigenvalues near to μ.

Question 3. on orthogonal polynomials and best approximation attracted a large number of attempts and a wide range of marks.

A8: Probability

See Mathematics and Statistics report.

A9: Statistics

See Mathematics and Statistics report.

A10: Fluids and Waves

Question 1: 1(a)-(c) was well answered.
1(d) No one was able to show that there is a closed dividing streamline, by computing the gradients at x=0 and at the two stagnation points. Some candidates were able to derive the equation for the dividing streamline. The sketch of the flow sometimes omitted the two stagnation points as well as the dividing streamline.

Question 2: was the most popular question, answered by nearly all candidates. 2(a) was well answered.
2(b) very few candidates got the correct answer using Cauchy's Residue Theorem.
2(c) only a few candidates realised that the vortex can move up or down, depending on the relative sizes of a and d.

Question 3: 3(a) was well answered.
3(b) Many of the candidates had problems deriving the expression for $\eta(x, t)$. Some tried to convert the x and t variation into a travelling wave, rather than the standing wave given by

pressure $p(x,t)$.
3(c) Very few candidates realised the wave was a standing wave, forced by the pressure.

A11: Quantum Theory

Question 1: Most candidates attempted this even though many of them were not prepared at all for the first 5 marks on what was intended to be a standard bookwork proof. Almost all could sail through the separation-of-variables argument for the energy eigenstates, but many found the later calculations too difficult — even though they were only integrals of a type well-studied in first-year work. The last part was, as expected, found too demanding by all but a few candidates.

Question 2: Popular. Even the weaker candidates were well-prepared in the use of the $a_{\pm}$ operators, gaining a good tally of marks. The questions on measurement were generally answered with confidence and good intuitive sense, but careful explanations of 'collapse' were often lacking.

Question 3: Less popular, but those who did attempt it were generally good at parts (a) and (b), the series solution method being confidently handled. One point that no candidate addressed was that the matching of powers in the series solution must including a matching of the very first term, which shows that a_1 is undetermined and so yields the existence of a non-trivial solution. Apart from this point, many candidates scored a good run of marks on part (b). Part (c), however, was found very hard indeed. Almost all the candidates expected the work to follow on from part (b) and did not see that it meant going back to the original 3-dimensional equation. Only a few candidates made any headway.

Short Options

ASO: Q1. Number Theory

The question turned out to be rather easy, with a large number of candidates scoring close to full marks. This may also be due to the fact that the question was subdivided into three quite separate parts.

Although there were many perfect or almost perfect solutions, there were also quite a few nonsensical calculations – a particularly large number of candidates thought that 8 divides 36.

ASO: Q2. Group Theory

Part 2a was straight forward and generally done well showing that candidates had a good understanding of the Jordan-Hölder Theorem. The second part however proved surprisingly difficult. Finding a good N proved tricky and few candidates were able to show that the nilpotent subgroup was solvable (few simply appealed to the general fact that p-groups are solvable).

ASO: Q3. Projective Geometry

Part (a) was well done. In (b)(ii) there were some scripts which didn't appreciate the importance of the conic being non-empty. Non-singular real conics are projectively equivalent to one of $\varepsilon_0 x_0^2 + \varepsilon_1 x_1^2 + \varepsilon_2^2 x_2^2 = 0$ where $\varepsilon_i = \pm 1$. But if the ε_i all have the same sign then the conic is empty, so that non-empty, non-singular conics are equivalent to $x_0^2 = x_1^2 + x_2^2$. In (c) most did not note that the second conic is singular; its equation can be rewritten as $(x + y + 1)(x + 2y - 1) = 0$.

ASO: Q4. Introduction to Manifolds

As anticipated the bookwork part of question (a) went well, but the execution of the Lagrange multiplier method in question (b) seems to have caused some problems. However, most of the candidates that tried managed to get a good part of the available marks.

ASO: Q5. Integral Transforms

The question was largely well done with many perfect or high-scoring solutions. Most scripts appreciated how part (c) relied on (b)(iii). Unfortunately many of the marks lost on other scripts were the result of carelessly not quoting transforms of standard functions or standard properties of the Laplace transform; the question makes clear that these need not be proved but some nod to these standard results or brief marginal comment was at least expected, rather than their apparent use without any explanation in a calculation. Some few candidates had clearly not revised the material on distributions, though this often did not stop them completing the remainder of the question.

ASO: Q6. Calculus of Variations

In the very first part, many candidates did not state that the variation function had to be sufficiently smooth.

Virtually all candidates struggled with some of the subtle analysis required in b(ii). Less than 5 were able to make a convincing argument as to how the inclusion of the constraint led to determining the last unknown constant of integration.

Unfortunately the wording of b(iii) confused some candidates (perhaps it would have been better to say "total kinetic energy") but there was only 1 mark for the calculation of the KE and I was very lenient.

Overall, the answers were solid.

ASO: Q7. Graph Theory

Most candidates were able to recall the definitions and bookwork asked for in part (a).

In (b), the most common error was assuming that a cover must be contained within one part of the bipartite graph.

Many candidates were able to guess the correct answers for part (c), but only one candidate was able to give the proofs.

ASO: Q8. Special Relativity

Part (a) was a bookwork definition of time-like and light-like trajectories and of proper time along a trajectory and a demonstration of Lorentz invariance of the proper time. Several candidates instead gave the criteria for the displacement vector of a given point on a trajectory from the origin being time-like or light-like. Additionally, factors of the speed of light (c) were frequently omitted from the definition of the proper time. Lorentz invariance of the proper time was generally well treated.

Part (b) required computing velocity four-vectors. This done fairly well in general, though a crucial absolute value was occasionally omitted from the denominator that led to nonsensical results down the line. The second part was a classic time-dilation effect, and most candidates set up the calculation reasonably well. However, in many instances, no sanity check was applied to the final answer. The final parts involved the relativistic Doppler formula, which could be cited or re-derived, but in most instances the derivation went awry.

The required sketch could be produced without an exact knowledge of the relativistic Doppler formula, using only that the frequency should diverge when the light source approaches at the speed of light and goes to zero when the light source recedes at the speed of light. This went largely un-answered, however.

ASO: Q9. Modelling in Mathematical Biology

The early parts of the question were answered relatively well, though most candidates forgot about the initial condition in (b). In (c) many candidates did not spot that there was a condition on β for the non-zero steady state to be feasible. Many candidates struggled to sketch the phase plane in (d), and very few correctly identified the conditions in (e).

E. Comments on performance of identifiable individuals

Removed from public version of the report.

F. Names of members of the Board of Examiners

- **Examiners:**
  Prof Fernando Alday (chair)
  Dr Neil. Laws
  Prof. Derek Moulton
  Prof. Nikolay Nikolov
  Prof. Ulrike Tillmann

Dr Martyn Quick (External Examiner)
Prof Demetrios Papageorgiou (External Examiner)

- **Assessors:**
  Prof. Ruth Baker
  Prof. Charles Batty
  Prof. Christopher Beem
  Dr Richard Earl
  Prof. Ben Green
  Prof. Andrew Hodges
  Prof. Peter Keevash
  Prof. Jan Kristensen
  Prof. Philip Maini
  Prof. Lionel Mason
  Prof. Irene Moroz
  Prof. Derek Moulton
  Prof. Panos Papazoglou
  Prof. Andy Wathen
  Dr Matthias Winkel