Examiners' Report:
Final Honour School of Mathematics Part A
Trinity Term 2019

November 21, 2019

Part I

A. STATISTICS

- **Numbers and percentages in each class.**
  See Table 1.

Table 1: Numbers in each class

Range	Numbers					Percentages %				
	2019	2018	2017	2016	2015	2019	2018	2017	2016	2015
70–100	57	57	57	50	51	35.19	35.62	36.77	34.97	36.17
60–69	71	69	62	63	59	43.83	43.12	40	44.06	41.84
50–59	27	22	31	26	26	16.67	13.75	20	18.18	18.44
40–49	5	9	4	3	5	3.09	5.62	2.58	2.1	3.55
30–39	1	3	1	0	0	0.62	1.88	0.65	0	0
0–29	1	0	0	1	0	0.62	0	0	0.7	0
Total	162	160	155	143	141	100	100	100	100	100

- **Numbers of vivas and effects of vivas on classes of result.**
  Not applicable.
- **Marking of scripts.**
  All scripts were single marked according to a pre-agreed marking scheme which was strictly adhered to. The raw marks for paper A2 are out of 100, and for the other papers out of 50. For details of the extensive checking process, see Part II, Section A.
- **Numbers taking each paper.**
  All 162 candidates are required to offer the core papers A0, A1, A2 and ASO, and five of the optional papers A3-A11. Statistics for these papers are shown in Table 2 on page 2.

Table 2: Numbers taking each paper

Paper	Number of Candidates	Avg RAW	StDev RAW	Avg USM	StDev USM
A0	162	30.44	6.77	64.8	11.51
A1	162	33.84	7.87	66.58	11.54
A2	162	66.43	17.58	66.47	12.08
A3	82	34.65	8.21	67.67	13.3
A4	120	28.34	7.91	65.32	9.66
A5	94	34.91	8.66	67.33	13.28
A6	96	30.91	8.46	66.53	12.64
A7	54	29.57	7.04	64.94	10.31
A8	150	31.98	6.67	65.94	9.54
A9	97	35.01	9.25	66.14	13.32
A10	37	31.43	8.27	64.78	12.24
A11	83	31.1	7.94	64.95	10.74
ASO	161	30.48	9.2	65.91	12.08

B. New examining methods and procedures

There were no changes in 2018–19.

C. Changes in examining methods and procedures currently under discussion or contemplated for the future

None.

D. Notice of examination conventions for candidates

The first notice to candidates was issued on 19th February 2019 and the second notice on the 9th May 2019.

These can be found at https://www.maths.ox.ac.uk/members/students/undergraduate-courses/ba-master-mathematics/examinations-assessments/examination-20, and contain details of the examinations and assessments. The course handbook contains the link to the full examination conventions and all candidates are issued with this at induction in their first year. All notices and examination conventions are on-line at
https://www.maths.ox.ac.uk/members/students/undergraduate-courses/examinations-assessments/examination-conventions.

Part II

A. General Comments on the Examination

Acknowledgements

- Nia Roderick for her work in supporting the Part A examinations throughout the year, and for her help with various enquiries throughout the year.
- Waldemar Schlackow for running the database and the algorithms that generate the final marks, without which the process could not operate.
- Charlotte Turner-Smith for her help and support, together with the Academic Administration Team, with marks entry, script checking, and much vital behind-the-scenes work.
- The assessors who set their questions promptly, provided clear model solutions, took care with checking and marking them, and met their deadlines, thus making the examiners' jobs that much easier.
- Several members of the Faculty who agreed to help the committee in the work of checking the papers set by the assessors.
- The internal examiners and assessors would like to thank the external examiners, Jelena Grbic and Demetrios Papageorgiou, for helpful feedback and much hard work throughout the year, and for the important work they did in Oxford in examining scripts and contributing to the decisions of the committee.

Timetable

The examinations began on Monday 17th June and ended on Friday 28th June.

Mitigating Circumstances Notices to Examiners

A subset of the examiners (the 'Mitigating Circumstances Panel') attended a pre-board meeting to band the seriousness of the individual notices to examiners. The outcome of this meeting was relayed to the Examiners at the final exam board, who gave careful regard to each case, scrutinised the relevant candidates' marks and agreed actions as appropriate. See Section E for further details.

Setting and checking of papers and marks processing

As is usual practice, questions for the core papers A0, A1 and A2, were set by the examiners and also marked by them. The papers A3-A11, as well as each individual question on ASO, were set and marked by the course lecturers/assessors. The setters produced model answers and marking schemes led by instructions from Teaching Committee in order to minimize the need for recalibration.

The internal examiners met in December to consider the questions for Michaelmas Term courses (A0, A1, A2 and A11). The course lecturers for the core papers were invited to comment on the notation used and more generally on the appropriateness of the questions. Corrections and modifications were agreed by the internal examiners and the revised questions were sent to the external examiners.

In a second meeting the internal examiners discussed the comments of the external examiners and made further adjustments before finalising the questions. The same cycle was repeated in Hilary term for the Hilary term long option courses and at the end of Hilary and beginning of Trinity term for the short option courses. *Papers A8 and A9 are prepared by the Department of Statistics and jointly considered in Trinity term.* Before questions were submitted to the Examination Schools, setters were required to sign off a camera-ready copy of their questions.

Examination scripts were collected by the markers from Exam Schools or delivered to the Mathematical Institute for collection by the markers and returned there after marking. A team of graduate checkers under the supervision of Charlotte Turner-Smith and Nia Roderick sorted all the scripts for each paper, cross-checking against the mark scheme to spot any unmarked questions or part of questions, addition errors or wrongly recorded marks. Also sub-totals for each part were checked against the marks scheme, noting any incorrect addition. An examiner was present at all times to authorise any required corrections.

Determination of University Standardised Marks

The examiners followed the standard procedure for converting raw marks to University Standardized Marks (USM). The raw marks are totals of marks on each question, the USMs are statements of the quality of marks on a standard scale. The Part A examination is not classified but notionally 70 corresponds to 'first class', 50 to 'second class' and 40 to 'third class'. In order to map the raw marks to USMs in a way that respects the qualitative descriptors of each class the standard procedure has been to use a piecewise linear map. It starts from the assumption that the majority of scripts for a paper will fall in the USM range 57-72, which is just below the II(i)/II(ii) borderline and just above the I/II(i) borderline respectively. In this range the map is taken to have a constant gradient and is determined by the parameters C_1 and C_2, that are the raw marks corresponding to a USM of 72 and 57 respectively. The guidance requires that the examiners should use the entire range of USMs. Our procedure interpolates the map linearly from $(C_1, 72)$ to $(M, 100)$ where M is the maximum possible raw mark. In order to allow for judging the position of the II(i)/III borderline on each paper, which corresponds to a USM of 40, the map is interpolated linearly between $(C_3, 37)$ and $(C_2, 57)$ and then again between $(0, 0)$ and $(C_3, 37)$. It is important that the positions of the corners in the piecewise linear map are not on the class borderlines in order to avoid distortion of the class boundaries. Thus, the conversion is fixed by the choice of the three parameters C_1, C_2 and C_3, the raw marks that are mapped to USM of 72, 57 and 37 respectively.

The examiners chose the values of the parameters as listed in Table 3 guided by the advice from the Teaching Committee and by examining individuals on each paper around the borderlines.

Table 4 gives the resulting final rank and percentage of candidates with this overall average USM (or greater).

Table 3: Parameter Values

Paper	C1	C2	C3
A0	(35.4,72)	(24.9,57)	(14.3,37)
A1	(40.5,72)	(24.7,57)	(14.19,37)
A2	(81.8,72)	(45.8,57)	(26.31,37)
A3	(40,72)	(26,57)	(14.94,37)
A4	(36.4,72)	(18.4,57)	(10.57,37)
A5	(41,70)	(25.1,57)	(16,37)
A6	(37,72)	(22,57)	(12.64,37)
A7	(35.8,72)	(22.3,57)	(15,37)
A8	(37.8,72)	(24.3,57)	(13.96,37)
A9	(42,70)	(24.8,57)	(14.25,37)
A10	(39.2,72)	(22.7,57)	(16.5,37)
A11	(39.4,72)	(21.4,57)	(16.3,37)
ASO	(38.6,72)	(19.1,57)	(10.97,37)

Table 4: Rank and percentage of candidates with this overall average USM (or greater)

Av USM	Rank	Candidates with this USM or above	%
94	1	1	0.62
90	2	2	1.23
86	3	4	2.47
85	5	5	3.09
84	6	6	3.70
83	7	7	4.32
82	8	8	4.94
81	9	9	5.56
80	10	10	6.17
79	11	11	6.79
78	12	15	9.26
77	16	19	11.73
76	20	26	16.05
75	27	28	17.28
74	29	31	19.14
73	32	36	22.22
72	37	41	25.31
71	42	50	30.86
70	51	57	35.19
69	58	62	38.27
68	63	70	43.21
67	71	79	48.77
66	80	86	53.09
65	87	91	56.17
64	92	100	61.73

Table 4: Rank and percentage of candidates with this overall average USM (or greater) [continued]

Av USM	Rank	Candidates with this USM or above	%
63	101	111	68.52
62	112	117	72.22
61	118	121	74.69
60	122	128	79.01
59	129	135	83.33
58	136	138	85.19
57	139	146	90.12
55	147	148	91.36
53	149	152	93.83
52	153	153	94.44
51	154	154	95.06
50	155	155	95.68
48	156	156	96.30
47	157	157	96.91
44	158	158	97.53
43	159	160	98.77
36	161	161	99.38
26	162	162	100.00

Recommendations for Next Year's Examiners and Teaching Committee

Cross sectional paper: The examiners noted that the ASO paper remains difficult to scale yet different standards of questions remain an issue. The Examiners recommend that Teaching Committee monitor which courses are consistently hard or have low numbers of question attempts.

Core paper checkers: The examiners also recommend that lecturers of the core papers should be the official checkers for papers A0, A1 and A2.

Electronic marksheets: The examiners suggested it would be useful to receive excel files to complete the marksheets electronically, as is done at other institutions. This would ease the workload for markers and administrative staff, make the calculation of the checksum unnecessary and enable initial calculation of paper statistics by assessors for their reports. It is understood that university policy on secure transmission would need to be adhered to.

B. Equality and Diversity issues and breakdown of the results by gender

Table 5, page 7 shows percentages of male and female candidates for each class of the degree.

Table 5: Breakdown of results by gender

Class	Number								
	2019			2018			2017		
	Female	Male	Total	Female	Male	Total	Female	Male	Total
70–100	12	45	57	9	48	57	7	50	57
60–69	28	43	71	19	50	69	12	50	62
50–59	12	15	27	8	14	22	12	19	31
40–49	2	3	5	3	6	9	2	2	4
30–39	0	1	1	1	2	3	1	0	1
0–29	0	1	1	0	0	0	0	0	0
Total	54	108	162	40	120	160	34	121	155
Class	Percentage								
	2019			2018			2017		
	Female	Male	Total	Female	Male	Total	Female	Male	Total
70–100	22.22	41.67	35.19	22.5	40	35.62	20.59	41.32	36.77
60–69	51.85	39.81	43.83	47.5	41.67	43.12	35.29	41.32	40
50–59	22.22	13.89	16.67	20	11.67	13.75	35.29	15.7	20
40–49	3.7	2.78	3.09	7.5	5	5.62	5.88	1.65	2.58
30–39	0	0.93	0.62	2.5	1.67	1.88	2.94	0	0.65
0–29	0	0.93	0.62	0	0	0	0	0	0
Total	100	100	100	100	100	100	100	100	100

C. Detailed numbers on candidates' performance in each part of the exam

Individual question statistics for Mathematics candidates are shown in the tables below.

Paper A0: Linear Algebra

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	13.60	14.45	4.66	71	7
Q2	14.04	14.19	4.24	122	4
Q3	16.52	16.60	3.45	131	1

Paper A1: Differential Equations 1

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.42	16.48	5.43	116	1
Q2	12.79	13.85	5.63	60	6
Q3	18.51	18.51	3.69	148	0

Paper A2: Metric Spaces and Complex Analysis

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.35	17.35	4.40	118	0
Q2	17.64	17.82	5.41	147	2
Q3	16.39	16.39	4.95	139	0
Q4	15.09	15.38	6.37	74	2
Q5	16.37	16.71	5.81	103	3
Q6	13.29	14.74	8.41	65	8

Paper A3: Rings and Modules

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.13	15.17	4.24	75	1
Q2	18.50	18.76	5.06	49	1
Q3	19.20	19.60	4.79	40	1

Paper A4: Integration

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.20	16.70	4.42	47	2
Q2	13.70	13.79	4.92	98	1
Q3	12.99	13.32	4.94	95	4

Paper A5: Topology

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.69	17.69	4.67	90	0
Q2	17.82	17.82	4.71	77	0
Q3	14.45	15.14	5.66	21	1

Paper A6: Differential Equations 2

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.44	17.44	4.28	95	0
Q2	13.49	13.62	5.62	76	1
Q3	11.71	13.10	6.54	21	3

Paper A7: Numerical Analysis

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14.88	14.88	2.88	40	0
Q2	17.10	17.39	5.14	28	1
Q3	12.88	12.88	3.49	40	0

Paper A8: Probability

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.30	17.72	4.37	121	4
Q2	12.21	12.93	4.24	69	7
Q3	15.74	16.01	3.64	110	3

Paper A9: Statistics

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.64	16.09	7.31	32	1
Q2	15.58	15.58	4.63	77	0
Q3	19.39	19.78	5.00	85	3

Paper A10: Fluids and Waves

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.17	16.90	4.31	21	2
Q2	12.97	13.52	4.52	27	2
Q3	16.85	17.04	5.93	26	1

Paper A11: Quantum Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14.37	14.64	4.69	55	2
Q2	16.95	16.95	4.79	80	0
Q3	13.56	13.55	3.98	31	1

Paper ASO: Short Options

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	9.73	10.43	6.83	58	5
Q2	15.38	15.60	5.13	47	1
Q3	21.13	21.13	2.47	8	0
Q4	18.00	18.00	4.73	7	0
Q5	16.44	16.44	3.86	79	0
Q6	15.49	15.65	5.05	54	1
Q7	13.06	13.06	4.68	17	0
Q8	-	-	-	3	0
Q9	17.82	17.82	4.12	49	0

D. Comments on papers and on individual questions

The following comments were submitted by the assessors.

Core Papers

A0: Algebra 1

Question 1. This was the least popular question, with few correct solutions to (b). Part (a)(iii) was completed by very few candidates despite allowing several possible solutions, eg. choose any B which does not preserve an eigenline of T.

Part (b)(i) was completed successfully by most candidates which attempted the question. In part (b)(ii) few chose to consider matrix A with $m_A = (x+1)^2$, which is the only conjugacy class of non-squares in $SL(2, C)$.

There were surprisingly few correct answers to (b)(iii) despite the simple argument that the squaring map is not injective and hence cannot be surjective on a finite set.

Question 2. This was a popular question with good answers overall.

There were many attempts of (b)(i) with mixed success. Candidates who tried to prove $\mathrm{Im}(T') = \ker(T)^0$ often run into difficulty in one inculsion, moreover this does not help in (ii) since V is infinite dimensional. A few candidates found the easier argument that $\ker T' = (\mathrm{Im} T)^0$ which immediately also gives the equality for part (ii) since it only needs that $\dim W$ is finite.

Question 3. This was a popular questions with good answers.

In (c)(i) many candidates proved that $\ker N = \ker N^*$ but struggled with the second equality and did not observe that $\ker N^* = (\mathrm{Im} N)^{\perp}$.

A common mistake in (c)(ii) was to deduce that $NN^* = N^*N$ immediately from $\langle NN^*v, v\rangle = \langle N^*Nv, v\rangle$ for all $v \in V$ (whereas the correct solution either linearizes by replacing v by $v+w$ and $v+iw$, or applies the spectral theorem to the self-adjoint $NN^* - N^*N$.)

A1: Differential Equations 1

Question 1. This question was attempted by most candidates, and to a generally good standard. The conceptual idea in part (a) was grasped by most candidates, but full marks required a clear definition of continuous dependence on data and care with norms. There were a number of different ways to obtain the integral equation in part b; a common problem was commenting on how to extend the uniqueness to all x. In (c), deriving the bound on M_n followed the same ideas as seen in lectures, but the difference in the integral limits meant that an $n!$ term does not appear in the denominator. A number of candidates seemed to have expected the $n!$ and so put it there without deriving it and then tried to prove an incorrect formula by induction! A geometric series emerges in part (iv) but requires being careful with the first two iterations and spotting the pattern.

Question 2. This question was the least commonly attempted. It also had the lowest average mark though several perfect marks were attained as well. Common mistakes included conceptual errors about the Cauchy data and not understanding the equivalence of characteristics intersecting and the Jacobian vanishing. Full marks in part (a)(iv) required commenting that the Jacobian is never 0 (as $\beta = 1$ is in the region $0 \leq \beta \leq 1$ found in part (iii)) and thus the solution is well defined; thus the domain of definition is the region contained by the characteristics passing through the end points of the boundary curve.

Part (b) (i) and (ii) was a straightforward computation though some care was required in inverting from (r, s, t) to (x, y, z). Full marks in part (iii) required commenting on the Jacobian and translating the boundary condition $s \geq 1$ into a well-defined region of (x, y, z) space.

Question 3. This question was attempted by most candidates. Full marks in part (a)(ii) required clear arguments about what a solution trajectory means, i.e. that if at any time $x^2 + y^2 = 4$ then this will be true for all time, e.g. by showing that $\frac{d}{dt}(x^2 + y^2)$ will equal zero. In part (iv) many candidates solved explicitly for $r(t)$ and $\theta(t)$ to show that $r = 2$ is a limit cycle. This was perfectly fine, but a much quicker solution could be obtained by simply considering the sign of $\dot{r}$ as a function of r.

Part (b) did not cause too much trouble for most candidates. Full marks required (at least brief) mention of why the eigenvalues of $M = \left(\begin{smallmatrix} 0 & 1 \\ f_u & f_v \end{smallmatrix}\right)$ are relevant to the question of stability. Algebraic manipulations on the eigenvalues to produce the various types of equilibrium points were mostly well done.

A2: Metric Spaces and Complex Analysis

Question 1. The first part of question 1, where students had to compare notions of convergence for different metrics, was messy.

Question 2. Overall, the students performed a little bit less well in Question 2 compared to Question 1. The very last part (the last two points) of Question 2 created some confusion: the question asked to find a bijection between $[0, 1]$ and S^1 just after the students had shown that no homeomorphism existed. I have the feeling that this question was worded in a way that made even the good students perform poorly.

Question 3.

This is a popular question, almost every candidate attempted this question with reasonable good marks (about 15 or more). The first two parts are basically book-work, but still a few candidates cannot state Taylor's expansion and integral formula correctly with omission of conditions. Various different definitions for removable singularities caused some confusion for some candidates. Many candidates were unable to argue correctly for part (c)(iv).

Question 4.

This is a less popular question but still many candidates attempt the question. Very few candidates realized that the set in part (a)(i) is the interaction of a sequence of closed sets so that it is also closed, while most who attempted the question produced good proof for the openness of the set. The book work about identity theorem received good solutions while few candidates proved it by using the first part of (a). Most candidates got most marks for part (b) though few who attempted can justify their answers to the last bit part of (b) which are unfortunate. Many candidates were unable to prove the standard facts about essential singularities by using Laurent's expansion.

Question 5.

A good number of candidates attempted this question, most candidates among them were able to give complete answers for part (a) which is mainly book-work. No candidate realized the real integral can be worked out just by using Cauchy's formula. For part (b), most who attempted were able to set up a contour and specify a holomorphic branch, but many candidates could not do estimates to justify the calculations by using the residue theorem. The most pain mistakes spotted were inequalities between complex numbers. Part (c) should follow the residue theorem, but amusingly only few candidates were able to factorize out the principal part of f'/f by using the form of f about the zero a suggested in the question, and were able to figure out the residues of $\varphi f'/f$.

Question 6.

This is a question about conformal manppings which is not so popular as others but still many candidates attempted. Many candidates who attempted didn't know the standard method to produce Mobius transformations mappings standard simply domain like half plane to the unit disk, and therefore very few marks could claim. For those who understand the properties of Mobius transformations and other few conformal mappings can thus justify their credits.

Long Options

A3: Rings and Modules

Question 1.

This was easily the most popular question with parts (a) and (b) well done in the main. Some argued in (b) along the lines of I maximal implies R/I is a field implies R/I is an integral domain implies I is prime, and such arguments gained full credit if each implication was proved. A more direct route is as follows: if $ab \in I$, $a \notin I$ and I is maximal then $\langle a \rangle + I = R$ so that $ra + i = 1$ for some $r \in R$ and $i \in I$ and so $b = rab + bi \in I$.

Part (c) proved more elusive and frequently scripts simply claimed $\mathbb{Z}[x]/I \cong \mathbb{Z}[x]$ implies $I = \{0\}$ which, whilst true, is not at all obvious. Rather for a homomorphism ϕ to be onto, it's necessary that $\phi(x)$ has degree 1 and it can be further shown that $\phi(x) = \pm x + c$ for surjectivity. These maps are then invertible and so isomorphisms. Quite a few incorrect scripts implicitly or explicitly said $\mathbb{Z}[x]$ is a PID or ED, which it isn't, or made use of the division algorithm, which doesn't exist.

Ideals for parts (i) and (ii) do exist, with many selecting $I = \langle 3, x\rangle$ and $I = \langle x^2 - 1\rangle$ as possibilities. No ideal I exists such that $\mathbb{Z}[x]/I \cong \mathbb{Z}[x, y]$. This would mean there were surjective homomorphisms

$$\mathbb{Z}[x] \to \mathbb{Z}[x, y] \to \mathbb{Z}[x],$$

the second map setting $y = 0$ and then by (c) these maps would in fact be isomorphisms which is not the case.

Question 2.

A well done question with many gaining high or full marks. In (b)(iii) it was necessary to find a rational 2×2 matrix which has minimal polynomial $x^2 - 2$, and similarly in (c)(ii) it was necessary to find a 2×2 matrix with entries $\mathbb{F}_4$ with a quadratic minimal polynomial that is irreducible over $\mathbb{F}_4$. In (d) many appreciated finding a C with minimal polynomial $m_\alpha(x)$ would resolve the question, and most further recalled that the companion matrix of $m_\alpha(x)$ is such a matrix.

Question 3.

The bookwork of part (a) was generally well done. The expected method for (b) was to find the characteristic polynomial of A and note that A has 3 distinct eigenvalues $-1, 1, 3$ and so a basis of eigenvectors $\mathbf{v}_{-1}, \mathbf{v}_1, \mathbf{v}_3$ (which need not be explicitly determined). Then $M = \langle \mathbf{v}_{-1}\rangle \oplus \langle \mathbf{v}_1\rangle \oplus \langle \mathbf{v}_3\rangle$ is the desired decomposition into submodules.

For (c) note that B has no real eigenvalues. Consequently $BP\mathbf{v}_\lambda = PA\mathbf{v}_\lambda = \lambda\mathbf{v}_\lambda$ means that $P\mathbf{v}_\lambda = \mathbf{0}$ and so $P = 0$ as $\mathbf{v}_{-1}, \mathbf{v}_1, \mathbf{v}_3$ are a basis.

A4: Integration

The overall marks were quite low, partly because the questions were rather long and they had very tough riders. In addition, a large number of candidates did not realise that $|f|$ must have a finite integral if f is to be (Lebesgue) integrable, and this depressed marks on Q.2 and Q.3.

Question 1. This question on σ-algebras was probably unexpected and it was not very popular, but it worked well. Many candidates had difficulties with (c)(i) and (c)(iv), while (c)(v) was a very tough rider and very few candidates saw that every subset of $[0, 1]$ is the image of a null set.

Question 2. It was disppointing that very many candidates thought that the statement in (a)(i) is true, even though it had been emphasised in lectures that Lebesgue integrability is not preserved under integration by parts. The answers in (b) varied greatly in levels of rigour, and were marked accordingly (for example, applying MCT for Series directly to a series with changing signs was penalised). A number of candidates invoked Abel's continuity theorem in (b)(iii), which is a good move, but only a handful gave a logically correct argument (by contradiction).

Question 3. Many candidates failed to appreciate that $|f|$ must have a finite integral if f is to be (Lebesgue) integrable, and this severely affected parts (b) and (d). It was not easy to award partial marks and this made the overall marks quite low. Part (d) was very tough: even amongst those who did consider $|\tilde{h}|$, very few got to the correct solution.

A5: Topology

Question 1. Almost all candidates attempted this question. Part (a) was well done. Several candidates did not manage to show that if $p_X \circ f, p_y \circ f$ are continuous then f is continuous. Sometimes no justification was given for the continuity of paths in the proof of path connectedness of $X \times Y$ and a mark was taken off-as one could simply invoke part ii.

In part bi several candidates defined the function from $[0, 1]$ to Y only on $0, 1$ and not on the whole interval and a mark was taken off. Part bii was treated in various ways either by the definition or by showing that the set is homeomorphic to a closed interval. A common mistake was to assert that if the image of a set under a continuous map is compact then the set is compact. Marks were taken off when the proof of the homeomorphism was not complete. Several candidates managed to complete part biii. Some tried to express B as a union of intervals and this didn't help.

Question 2. Many candidates attempted this question. Part a i: Some students gave a wrong definition for connectedness for subsets and several who gave the right definition used the wrong one when proving that the image of a connected set is connected.

Part a ii. Most candidates got full marks but a few stated that the image of an open set under p is open.

Part bi. Several candidates gave incomplete proofs that $p(K)$ is closed missing one of the two cases.

Part bii was generally well done. A few students however claimed that the space is Hausdorff. Part biii was generally well done. Sometimes in the last part the definition of the open saturated set was wrong but partial credit was given as the approach was sound.

Question 3. Only 26 students attempted this question. Some candidates apparently forgot to define the link in part ai. In the proof of aii some students asserted that a simplicial complex of dimension n embeds in $\mathbb{R}^n$ and so failed to give a correct proof for the Hausdorff part.

Part aiii proved to be quite challenging-many candidates had the right idea but did not manage to give a formal proof so they received only partial credit.

Part bi was generally well done but some candidates gave an unnecessarily lengthy argument showing that $|K|$ is a polygon with a cssi.

Part bii was quite challenging and only one candidate gave a full proof. Other candidates received partial credit for realizing that they had to complete the simplicial complex to a closed surface.

A6: Differential Equations 2

Question 1.

This question was attempted by almost everyone. Parts (b) and (c) contained bookwork and standard techniques which allowed most candidates to accumulate good marks. Parts (a) and (d) were less familiar and were only convincingly answered by the strongest candidates. In part (a), many tried to assume a particular form of $\mathcal{L}$, rather than differentiating with respect to m. In parts (b) and (c), many candidates made heavy weather of solving simple constant-coefficient ODEs. In part (d), several candidates fallaciously assumed the existence of a solution to the homogeneous ODE that satisfies the given inhomogeneous boundary conditions.

Question 2.

This question was quite popular but proved difficult for many candidates. In part (a), there were very many algebraic errors in classifying the critical points and calculating the indicial equations. Very few candidates spotted the symmetry between the points at $x = \pm 1$, and several lost marks by just omitting the analysis of the point at infinity. Part (b) was generally done better, although most did not give sufficient consideration to the parity of λ. There were not many answers to part (c) that convincingly explained the (lack of) boundary conditions.

Question 3.

This question was not popular and generally not well done. In part (a), very few were able to explain (e.g. via monotonicity) why the given equation has a unique solution. Many made the algebra much more complicated by pointlessly multiplying through by the denominator on the left-hand side. In part (b), most were able to find the governing equations for y_0 and y_1. Quite a few stated the required solvability condition, but did not make full use of the hint to calculate the given equation for A. Part (c) was generally done quite well.

A7: Numerical Analysis

Question 1.

The lowest mark for this question was 6, and the highest was 23 (out of 25). Parts a) – c) of this problem were standard arguments from the earlier part of the course, but some candidates lost points due to inaccuracies or leaving out crucial detail. In Part b) many candidates used an ansatz based on a degree 2 polynomial with general coefficients rather than the explicit form of the Lagrange interpolation polynomial for the given nodes, which made the calculations unnecessarily long and fraught with opportunities to make mistakes. Part d) drew on the later part of the course and required using the roots of the given polynomial as nodes and careful integration to obtain the correct weights.

Question 2.

The marks on this question ranged from 7 to 24 (out of 25). This question seemed slightly easier than the other two, perhaps because candidates felt at home with its linear algebra flavour. Part b) contained novel material in the form of an easy equation, which was correctly established by all candidates who attempted it, followed by instructions to construct a counterexample to this equation to prove that the L1 norm is not an inner product norm. Surprisingly few candidates managed to do this correctly, the main problem seeming to be that they hadn't properly understood the definition of the L1 norm. Part c) was book work and led to Part d), which could be solved in two ways, one directly using the recursion result of Part c), and one longer, more tedious, approach based on computing the degree 2 Laguerre polynomial explictly before expressing it in terms of the the two given polynomials to find the recursion parameters. All but one candidate chose the long route of the second approach.

Question 3.

Marks ranged from 2 to 19 (out of 25). This problem had an algorithmic flavour. The book work parts were solved well by most candidates, except the considerations about flop counts, which caused confusion. Some candidates stated the correct order of the flop count but gave no details about the multiplicative constant, which was an important missing detail since we were comparing two different algorithms of the same order of complexity. Part d) required to show that applying the symmetric QR algorithm to a tridiagonalised matrix is of lower complexity order than applying it to the original matrix. Many candiates overlooked important details such as that the Householder matrices for tridiagonalisation are constructed differently to Part b) or arguing that the symmetrix QR algorithm leaves the tridiagonal structure invariant.

A8: Probability

See Mathematics and Statistics report.

A9: Statistics

See Mathematics and Statistics report.

A10: Fluids and Waves

Question 1.

Question 1 was answered by 24 candidates. Parts 1(a)-(b) were well answered. No candidate was able to determine the correct pressure field in 1(c), although the majority were able to find the streamfunction, given in the question.

Question 2.

Question 2 was answered by 29 candidates. Many candidates were unable to derive the complex potential for the source from the integral expression for Qi in 2(a)(i), despite it being in the lecture notes. 2(a)(ii) was well answered. 2(b) was well answered, with most candidates verifying that the given map was indeed a conformal map into the right half plane. Only a couple of candidates were able to obtain the correct expression for the complex potential for the dipole in 2(c).

Question 3.

Question 3 was answered by 28 candidates and was the best answered question of the three. The main errors came in 3(b), where some candidates forgot to incorporate the fluid velocities U_j $(j = 1, 2)$ into the boundary conditions. However everyone knew the methods for deriving both the dynamic and kinematic boundary conditions, as well as determining the dispersion relation for both ω and c. Candidates who omitted the U_j were able to use the quadratic equation for ω, given on the question paper, to determine the correct stability conditions.

A11: Quantum Theory

Question 1.

This question was attempted by 62 students. I think the level was adequate. The average mark was slightly above 14/25. As a small remark: in the proposed solutions for part a it was stated j=0. As noted by many students, this is only true if the x-dependent part of the stationary state is real.

Question 2.

This question was attempted by almost all students, and the average mark was slightly below 17/25. I felt the level was adequate. The grades in this question were slightly higher than expected. The reason for this was that the students responded very well to bookwork and similar material involving commutators.

Question 3.

This question was attempted by 33 students. Less than the other two questions, but being the third question, this is reasonable. The average mark was slightly above 13/25, and hence a bit lower than the other two questions, but again, this being the third question many students may have had less time to devote to this one.

Short Options

ASO: Q1. Number Theory

The questions consisted of several parts, some easy and standard, and others hard. Part (d) was found particularly hard by candidates although there were 3 complete solutions. Part (b) could be done in several different ways, including by more-or-less quoting a result from lectures (provided this was done correctly). The intended solution to (f) was to work modulo 9, but several candidates observed that one may also work modulo 8.

ASO: Q2. Group Theory

(59 attempts) The question turned out to be challenging with relatively few scripts achieving more than 20 marks. This was probably mainly due to the fact that candidates had to know several aspects of the course well to score highly. Though the Sylow Theorems were the central theme, group actions and derived series made an appearance as well.

Essentially all candidates reproduced the definitions in part (a) but many struggled to prove that a group of order p^2 is abelian. This part of the question was not directly related to the Sylow theorems. The hint was not always taken up. Most students solved part (i) of part (b). The case analysis was more difficult for part (ii).

ASO: Q3. Projective Geometry

9 solutions were handed in. The level of the solutions was high; the average score was 20.33 (81%). Several students had essentially complete answers, losing points only on accuracy (forgetting for example to give a geometric interpretation of the span).

ASO: Q4. Introduction to Manifolds

8 solutions were handed in. The average score was 18.13 (72%). There were very few complete solutions, with many students realising the importance of the space of skew matrices to the question but not getting their arguments fully watertight.

ASO: Q5. Integral Transforms

Part (a) was generally very well answered, with most candidates picking up most marks on the bookwork (some struggled with the definition of a distribution). The greatest cause of lost marks here was in (a)(ii) in which many candidates did not note that $x\phi$ was a test function, a fact on which most solutions relied. Many candidates struggled with (b), with very few students correctly identifying or justifying the presence of a jump in y of -1 at $x = 0$ and many claiming that y had to be continuous. Taking the Fourier Transform and solving the resulting algebraic equation was generally well done, but several candidates struggled from this stage to calculate the inverse Fourier Transform via contour integration. Among those who did this successfully, several candidates did not notice that their solution was valid for only half of the real line and as a result overlooked the jump in the solution. A few candidates used their knowledge of the Fourier Transform of $e^{-|x|}$ to reach the correct answer without contour integration. (c) was generally well done, although some candidates struggled to accurately calculate the Laplace Transform for xf' in (c)(ii). Several candidates made sign errors in solving the resulting ODE, but most who did solve it were able to successfully complete the remainder of the question.

ASO: Q6. Calculus of Variations

- (a) Very well done.
- (b) Very, very few convincing answers for this.
- (c) First part – reasonably well done, but some candidates struggled to show $y = n\frac{dx}{d\psi}$

  (i) For $n = -\frac{1}{2}$, it follows from $y^n cos\psi = constant$ that y is a constant multiplied by $cos^2\psi$, so that the result follows from the double angle formula. However, a substantial number of students did the following: $\frac{dx}{d\psi} = \frac{y}{n} = -2K\ \cos^2\psi$, where K is a constant. Hence $\frac{dx}{d\psi} = -K(1 +\ \cos(2\psi)$. Then they integrated to find x, then differentiated x to find y from $y = n\frac{dx}{d\psi}$. While this is correct, it is not the most efficient way to answer the question.

  (ii) Some good attempts.

  (iii) Here, the boundary conditions imply that $\cosh(\frac{1-\mathrm{K}_1}{\mathrm{K}}) = \cosh(\frac{-1-\mathrm{K}_1}{\mathrm{K}}) = \alpha$. Many candidates then simply stated that $K_1 = 0$ to give the answer. However, as an examiner, it is not clear if they did this because they know the properties of cosh, or if they did it because it gave the correct answer.

ASO: Q7. Graph Theory

Most candidates were able to recall the definitions and bookwork asked for in part (a), but not so many recalled the bookwork in part (b). Part (c) caused great difficulties: very few candidates could prove optimality in (ii) or give a valid example for (iii).

ASO: Q8. Special Relativity

Part A: The students answered well to this question showing that they learned basic definitions and properties of the Lorentz group.

Part B: The students did not perform very well when trying to answer this question. The style of the question and the type of reasoning involved is probably a little different from the rest of their math classes. However, we looked at some examples very close to this question in class.

Part C: This question required more original thinking and the students performed relatively well.

ASO: Q9. Modelling in Mathematical Biology

Part A. About a quarter of candidates received partial marks because they did not read the entire question (most did not answer both the interpretation and dimensions). A few errors with dimensions of quantities.

Part B. Most candidates got this right, the ones with partial marks was because there was no constant in the solution.

Part C. A quarter of candidates received partial marks because they did not notice that N is constant, S and I do not depend on R, and that they can write R in terms of the first two equations.

Part D. Nearly everyone received full marks.

Part E. Only a few candidates received full marks for this question.
Nearly all candidates could solve the steady-states, but most missed a mark on the constraint for the second steady-state to be biologically plausible.
Nearly everyone could calculate the Jacobian, but many made mistakes on determining stability from it.
Only a few candidates received full marks for calculating the reproductive rate and determining whether there can be an epidemic.

E. Comments on performance of identifiable individuals

This section has been redacted from the public report.

F. Names of members of the Board of Examiners

- **Examiners:**
  Prof. Fernando Alday (chair)
  Dr Neil. Laws
  Prof. Derek Moulton
  Prof. Nikolay Nikolov
  Prof. Zhongmin Qian
  Prof Jelena Grbic (External Examiner)
  Prof Demetrios Papageorgiou (External Examiner)
- **Assessors:**
  Prof. Charles Batty
  Dr Joshua Bull
  Dr Richard Earl

Prof. Ben Green
Prof Heather Harrington
Prof. Raphael Hauser
Prof. Peter Howell
Prof. Sam Howison
Prof. Peter Keevash
Prof. Philip Maini
Dr Carlo Meneghelli
Prof. Irene Moroz
Prof. Panos Papazoglou
Prof. Balazs Szendroi
Prof. Ulrike Tillmann
Prof. Andy Wathen
Dr Matthias Winkel