(a) Model I: $\frac{du}{dt} = f(u)$

Steady state: $\frac{du}{dt} = 0 \Rightarrow f(u^*) = 0.$ $\boxed{1}$

Linear stability: Let $u = u_* + \hat{u}$ where $|\hat{u}| \ll u_*$

Then $\frac{d\hat{u}}{dt} = f(u_* + \hat{u}) = f(u_*) + \hat{u} f'(u_*) + h.o.t.$

To first order, $\frac{d\hat{u}}{dt} = \hat{u} f'(u_*) \Rightarrow \hat{u} = e^{f'(u_*)t}$

$\therefore$ linearly stable if $f'(u_*) < 0.$ $\boxed{2}$

Model II: $N_{t+1} = g(N_t)$

Steady state: $N_* = g(N_*)$ $\boxed{1}$

Linear stability: let $N_t = N_* + n_t$ where $|n_0| \ll N_*$.

Then $\cancel{N_*} + n_{t+1} = g(N_* + n_t) = \cancel{g(N_*)} + n_t g'(N_*) + h.o.t.$

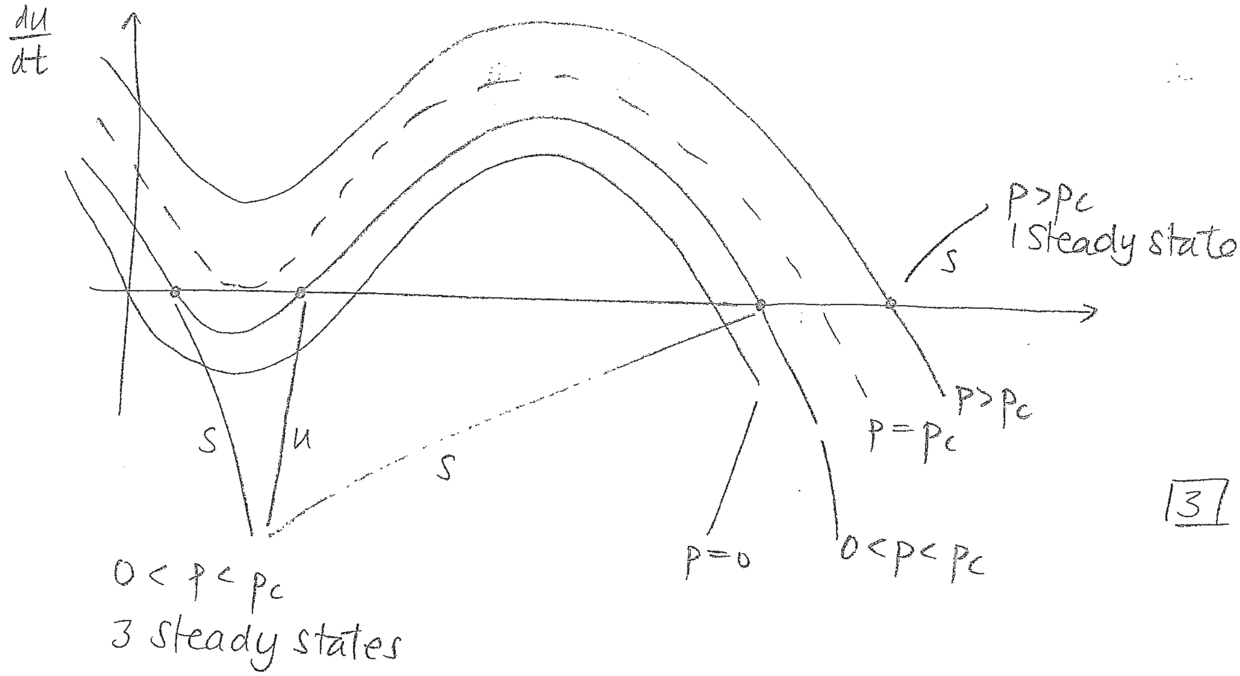
To first order, $n_{t+1} = n_t g'(N_*) \Rightarrow n_t = (g'(N_*))^t n_0$

$\therefore$ linearly stable if $|g'(N_*)| < 1.$ $\boxed{2}$

BOOKWORK – all seen before in lectures TOTAL $\boxed{6}$

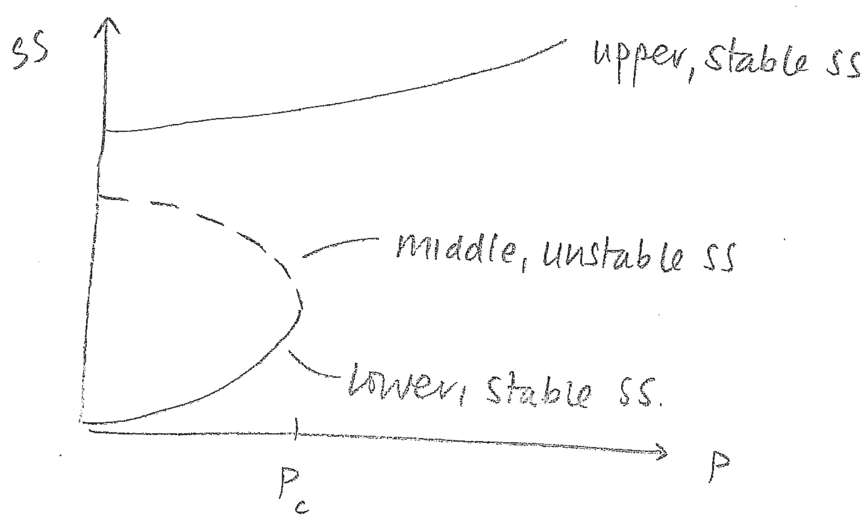
(b) $\frac{du}{dt} = u(u-1)(3-u) + P = f(u)$

i) $\frac{du}{dt}$



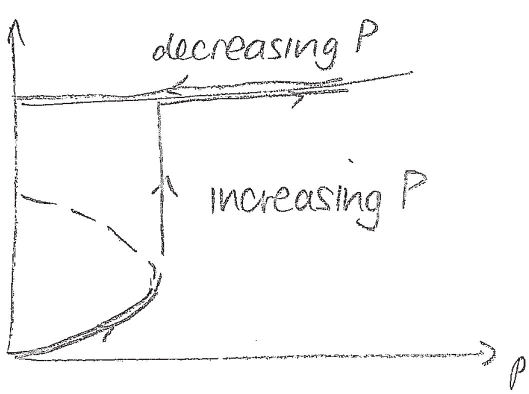
$\boxed{3}$

ii) ss



$\boxed{3}$

(iii)



No hysteresis as system can't get back to lower steady state

$\boxed{3}$

SIMILAR – similar examples covered in lectures / problem sheets

TOTAL $\boxed{9}$

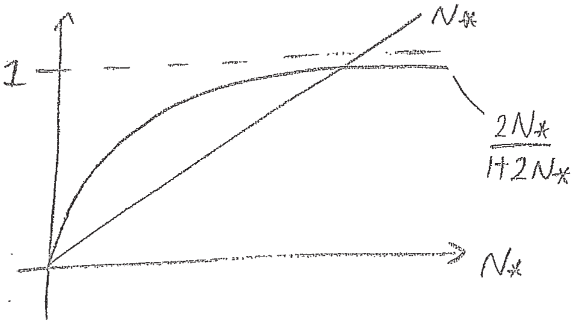
(c) $$N_{t+1} = \frac{2N_t^2}{(N_t+s)(1+2N_t)}$$

(i) $$N_* = \frac{2N_*^2}{(N_*+s)(1+2N_*)}, \quad N_* \neq 0 \Rightarrow (N_*+s)(1+2N_*) = 2N_*$$

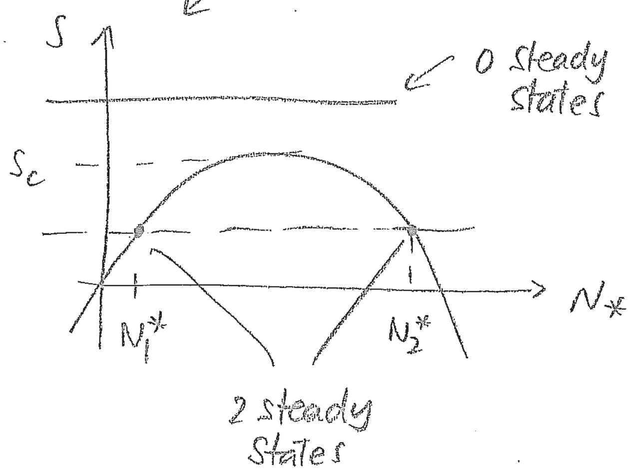
$\boxed{2}$

(ii) Rearrange to write $s = s(N^*)$

$$s = \frac{2N_*}{1+2N_*} - N_*$$



use this to sketch $s(N_*)$



$$\frac{ds}{dN_*} = 0 \Rightarrow \frac{2(1+2N_*) - 4N_*}{(1+2N_*)^2} - 1$$

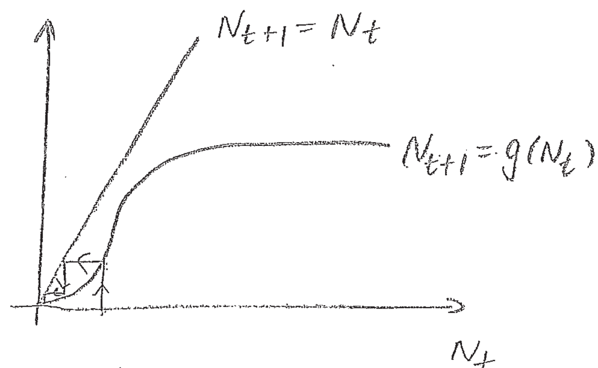
$$\Rightarrow 2 = (1+2N_*)^2$$

$$N_* = \frac{\sqrt{2}-1}{2}$$

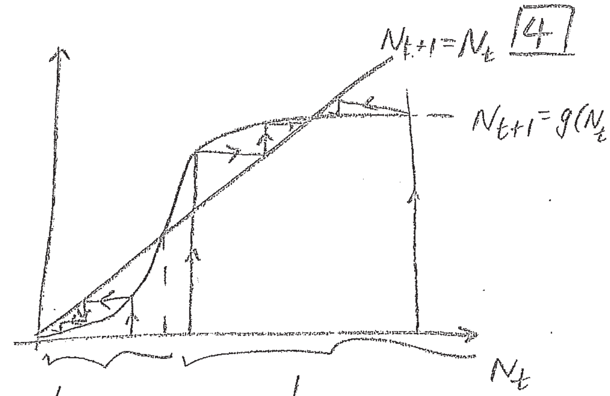
$$\therefore s_c = s\left(\frac{\sqrt{2}-1}{2}\right) = \frac{(\sqrt{2}-1)^2}{2}$$

$\boxed{4}$

(iii)



extinction for all values of N_0



extinction if $N_0 < N_1^*$

$N_t \to N_2^*$ if $N_0 > N_1^*$

$\boxed{4}$

(i) SIMILAR

(ii) NEW RIDER

(iii) SIMILAR / NEW RIDER

TOTAL $\boxed{10}$