## 2024 UK PDE NETWORK MEETING 13TH OXBRIDGE PDE CONFERENCE

18 - 22 MARCH 2024



Oxford Centre for Nonlinear P∂E



LONDON MATHEMATICAL SOCIETY EST. 1865



Engineering and Physical Sciences Research Council



## NATIONAL PDE NETWORK MEETING 18 - 21 MARCH

## Monday, 18 March 2024

12:30 - 13:00	Coffee North Mezzanine
13:15 - 13:25	Opening Remarks
13:30 - 14:10	Monica Musso (University of Bath)
14:15 - 14:55	Short Course I-1 Dehua Wang (University of Pittsburgh)
15:00 - 15:30	Coffee break
15:30 - 16:10	Elaine Crooks (University of Swansea)
16:15 - 16:55	Short course II-1 Paolo Secchi (University of Brescia)
17:00	Close of Day 1

## Tuesday, 19 March 2024

08:30 - 09:00	Coffee South Mezzanine
09:00 - 09:30	Matthew Schrecker (University of Bath)
09:45 - 10:25	Short Course I-2 Dehua Wang (University of Pittsburgh)
10:30 - 10:50	Break South Mezzanine
10:50 - 11:30	Mikhail Feldman (University of Wisconsin)
11:35 - 12:15	Short course II-2 Paolo Secchi (University of Brescia)
12:20 - 13:30	Lunch Break
13:30 - 14:10	Myoungjean Bae (Korea Advanced Institute of Science and Technology)
14:15 - 14:55	Short Course I-3 Dehua Wang (University of Pittsburgh)
15:00 - 15:30	Break South Mezzanine
15:30 - 16:10	Wei Xiang (City University Hong Kong)
16:15 - 16:55	Short course II-3 Paolo Secchi (University of Brescia)
17.00	Close of Day 2



## NATIONAL PDE NETWORK MEETING 18 - 21 MARCH

## Wednesday 20 March 2024

08:30 - 09:00	Coffee North Mezzanine
09:00 - 09:40	Siran Li Jiao Tong University, Shanghai
09:45 - 10:25	Josephine Evans (University of Warwick)
10:30 - 10:50	Break South Mezzanine
10:50 - 11:30	Jan Sbierski (University of Edinburgh)
11:35 - 12:15	Qing Han (University of Notre Dame)
12:20 - 13:30	Lunch Break
13:30 - 14:10	Kari Astala (University of Helsinki)
14:15 - 14:55	Ali Taheri (University of Sussex)
15:00 - 15:30	Break North Mezzanine
15:30 - 16:10	Laura Kanzler (CEREMADE, University Paris Dauphine)
16:15 - 16:55	Paul Minter (University of Princeton)
17:00	Close of Day 2

## Thursday, 21 March 2024

10:30-10:50	Coffee
10:50-11:00	Opening Remarks
11:00-11:40	Pierre German (University College London)
11:40-12:15	Vanessa Ryborz (University of Oxford)
12:20 - 13:30	Lunch Break
13:30 - 14:10	Costante Bellettini (University College London)
14:15 - 14:55	Robert J. McCann (University of Toronto)
15:00 - 15:30	Break South Mezzanine
15:30 - 16:10	Xavier Ros-Oton (ICREA University of Barcelona)
16:15 - 16:45	Istvan Kadar (University of Cambridge)
16:50 - 17:30	Jason Lotay (University of Oxford)
17:30	Close of Day 2



## 13TH OXBRIDGE PDE CONFERENCE 18 - 21 MARCH

## Thursday, 21 March 2024

10:30-10:50	Coffee
10:50-11:00	Opening Remarks
11:00-11:40	Pierre German (University College London)
11:40-12:15	Vanessa Ryborz (University of Oxford)
12:20 - 13:30	Lunch Break
13:30 - 14:10	Costante Bellettini (University College London)
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15:00 - 15:30	Break South Mezzanine
15:30 - 16:10	Xavier Ros-Oton (ICREA University of Barcelona)
16:15 - 16:45	Istvan Kadar (University of Cambridge)
16:50 - 17:30	Jason Lotay (University of Oxford)
17:30	Close of Day 2

## Friday, 22 March 2024

08:30 - 09:00	Coffee South Mezzanine
09:00 - 09:40	Grigalius Taujanskas (University of Cambridge)
09:45 - 10:15	Dominic Wynter (University of Cambridge)
10:20 - 10:50	Samuel Charles (University of Oxford)
10:55 - 11:20	Break South Mezzanine
11:20 - 12:00	Andrea Mondino (University of Oxford)
12:05 - 13:45	Neshan Wickramasekera (University of Cambridge)
12:50 - 13:00	Closing Remarks
13:00	End of Conferences

## Professor Kari Astala

The Burkholder Functional in Elliptic PDE's and Non-linear Elasticity

The Burkholder functional \$B\_p\$ gives a fascinating bridge between martingale inequalities, singular integrals and vector valued calculus of variations. It presents a particularly interesting candidate to test Morrey's conjecture in two dimensions, i.e. whether every rank-one convex integrand or functional in \$\mathbb{R}^{2 \times 2}\$ is quasiconvex.

In this talk, based on joint work with D. Faraco, A. Guerra, A. Koski and J. Kristensen, we study the interaction of  $B_p$  with elliptic PDE's and non-linear elasticity, and obtain e.g. sharp integral bounds via local properties of the functional. In particular, we show that the functional is quasiconvex in the space of Sobolev maps with  $B_p(Df) \log 0$ . This result is also a key to minimization and weak lower semicontinuity properties of several natural energy functionals in non-linear elasticity, where, to avoid cavitation and interpenetration of matter, natural minimisers are Sobolev homeomorphisms.

## Professor Myoungjean Bae

#### Two Types of Sonic Interfaces

In the study of inviscid compressible flows, the Mach number, defined by

M =local speed of a flow/sound speed, plays a significant role in that not only it determines a physical feature of a flow, but also it determines a mathematical feature of the governing equations. Especially when M = 1, it is directly related to the degeneracy of a component in the compressible Euler system (or the Euler-Poisson system). The degeneracy brings a great challenge to establish the well-posedness of a boundary

value problem for a transonic flow containing a sonic interface.

In this talk, I present two examples of sonic interfaces: (1) a sonic arc occurring in a self-similar flow of the compressible Euler system with a pseudo-transonic transition (see [1]);

(2) a sonic interface occurring in an accelerated transonic flow of compressible Euler-Poisson system (see [2]).

The most interesting aspect is that (1) is a weak discontinuity across which all the flow variable are continuous but their derivatives are not, while (2) is a regular interface across which all the flow variable are C^1. And, it leads us to the following question:

What is a general criterion to determine the type of a sonic interface?

This talk is based on collaborations with G.-Q. Chen, B. Duan, M. Feldman, and C. Xie.

[1] Bae, M., Chen, G.-Q., and Feldman, M. Regularity of solutions to regular shock reflection for potential flow. Invent. Math. 175, 3 (2009), 505–543.

[2] Bae, M., Duan, B., and Xie, C. The steady euler-poisson system and accelerating flows with C^1-transonic transitions. preprint

## Professor Costante Bellettini

#### Analysis of Stable Minimal Hypersurfaces: Curvature Estimates and Sheeting

We consider properly immersed two-sided stable minimal hypersurfaces of dimension n. We illustrate the validity of curvature estimates for n \leq 6 (and associated Bernsteintype properties). For n \geq 7 we illustrate sheeting results around "flat points". The proof relies on PDE analysis. The results extend respectively the Schoen-Simon-Yau estimates (obtained for n \leq 5) and the Schoen-Simon sheeting theorem (valid for embeddings).

## Mr Samuel Charles

Global Solutions of the Compressible Euler-Riesz Equations with Large Spherical Initial Data

We are concerned with the global existence of spherically symmetric finite energy solutions of the compressible Euler-Riesz equations (CEREs) for dimensions n > 2, considering both the attractive and repulsive case. Such an equation is incredibly useful in modelling compressible gaseous stars, plasmas, Riesz gases, swarming models and more. As such, the theory of existence is of particular importance. I will give an overview of the derivation and past work on the equation, in particular, considering the recent paper (G. Chen, L. He, Y. Wang, D. Yuan, CPAM, 2023), where solutions of the Euler-Poisson equations (a special case of the Euler-Riesz equations) are approximated by the solutions of the compressible Navier-Stokes-Poisson equations (CNSPEs) using a viscosity method. The difficulty with such a procedure is the possible concentration of the density at the origin. As such, a careful construction of the approximate solutions is required. I will give an overview of our work to generalise the above paper to prove rigorously the existence of global weak solutions to the CEREs.

## Professor Elaine Crooks

Travelling Waves and Minimality Exchange in Smectic C\* Liquid Crystals

We consider minimality conditions for the speed of monotone travelling waves in a model of a sample of smectic C\* liquid crystal subject to a constant electric field, dealing with both isotropic and anisotropic cases. Such conditions are important in understanding switching properties of a liquid crystal, and our focus is on understanding how the presence of anisotropy can affect the speed and nature of switching. Through a study of travelling-wave solutions of a quasilinear parabolic equation, we obtain an estimate of the influence of anisotropy on the minimal speed, and sufficient conditions for linear and non-linear minimal speed selection mechanisms to hold in different parameter regimes. We also discuss sufficient conditions for so-called `minimality exchange' in a general class of parameter-dependent monostable reaction-diffusion equations with explicit travelling-wave solutions, when the minimal wave speed switches from the linearly determined value to the speed of the explicitly-determined front as a parameter changes. This is joint work with Michael Grinfeld and Geoff McKay (Strathclyde).

## Dr Josephine Evans

#### Non-equilibrium Steady States in a BGK Model for Dilute Gases

This is based on a joint work with Angeliki Menegaki. I will discuss a model for heat transfer in gasses and the existence and dynamic stability of non-equilibrium steady states. I will talk about this in the context of hypocoercivity and explain the additional challenges in showing results about non-equilibrium steady states in general as well as in our particular case.

## Professor Mikhail Feldman

Nonlinear Equations of Mixed Type in Two-dimensional Riemann Problems Involving Transonic Shocks

We discuss several two-dimensional Riemann problems in the framework of potential flow equation and isentropic Euler system. Examples include regular shock reflection, Prandtl reflection, and four-shocks Riemann problem. Solutions of these problems are self-similar. Potential flow equation and isentropic Euler system for self-similar solutions are of mixed elliptic-hyperbolic type in the selfsimilar coordinates. We first review recent results on existence, regularity and properties of global self-similar solutions involving transonic shocks for the 2D Riemann problems described above in the framework of potential flow equation. A well-known open problem is to extend these results to compressible Euler system, i.e. to understand the effects of vorticity. We show that for the isentropic Euler system, solutions have low regularity, specifically velocity and density in the subsonic (elliptic) region do not belong to the Sobolev space \$H^1\$ in selfsimilar coordinates. We further discuss well-posedness of the transport equation for vorticity in the resulting low regularity setting,.

## Professor Pierre Germain

#### Asymptotic Stability of Solitons in 1D Dispersive Problems

A soliton is asymptotically stable if, for small perturbations, the solution decomposes into soliton + (decaying) radiation as time goes to infinity. I will present results on the asymptotic stability of solitons of mKdV, NLS, as well as the Phi4 model (in which case the soliton is the 'kink'). A key idea is to take advantage of nonlinear resonances. This is based on articles with Charles Collot, Fabio Pusateri, and Frederic Rousset.

## Professor Qing Han

#### The Isometric Immersion of Surfaces with Finite Total Curvature

In this talk, we discuss the smooth isometric immersion of a complete simply connected surface with a negative Gauss curvature in the three-dimensional Euclidean space. For a surface with a finite total Gauss curvature and appropriate oscillations of the Gauss curvature, we prove the global existence of a smooth solution to the Gauss-Codazzi system and thus establish a global smooth isometric limmersion of the surface into the three-dimensional Euclidean space. Based on a crucial observation that some linear combinations of the Riemann invariants decay faster than others, we reformulate the Gauss-Codazzi system as a symmetric hyperbolic system with a partial damping. Such a damping effect and an energy approach permit us to derive global decay estimates and meanwhile control the non-integrable coefficients of nonlinear terms.

#### Mr Istvan Kadar

#### A Scattering Theory Construction of Dynamical Solitons

In the past 20 years the mathematical study of dynamical properties of black holes has greatly accelerated. Indeed, the first mathematical description of a dynamical black hole is from 2013. Many of the works focus on symmetry reduced settings, or toy problems that ought to capture important phenomenology of the Einstein equations. Outside a few exceptions, these works are focused on the solution in a neighbourhood of a single black hole, resulting in almost spherically symmetric setting in the far region. In this talk, I will present a model problem that I think captures some, albeit not all, important features and difficulties of studying multi black hole solutions in asymptotically flat spacetimes and report on progress in the understanding of this model. In particular, I will present the construction of a solution to the energy critical wave equation in a neighbourhood of timelike infinity that has a prescribed radiation field through null infinity.

## Dr Laura Kanzler

Quantitative Fluid Approximation for Heavy Tailed Kinetic Equations with Several Invariants

In past works it has been demonstrated that using an appropriate rescaling, linear kinetic equations with heavy tailed equilibria give rise to a scalar fractional diffusion equation. In this talk an extension of this is presented, where the linear kinetic equations under consideration, not only conserves mass, but also momentum and energy. In the limit, fractional diffusion equations are obtained for the energy and the mass, while the equation for the momentum is trivial. The methods of proof presented rely on spectral analysis combined with energy estimates. It is constructive and provides explicit convergence rates.

## Professor Siran Li

# The Isometric Immersions Problem: From Perspectives of PDE, Geometry, and Physics

We report our recent work on a classical problem in differential geometry: isometric immersions and/or embeddings of Riemannian and semi-Riemannian manifolds. The underlying PDE is the system of Gauss--Codazzi--Ricci equations. Existence of isometric immersions is studied under various curvature conditions, via elliptic and hyperbolic PDE techniques. Weak continuity of isometric immersions is investigated with the help of the theory of compensated compactness. Connections to other problems in mathematical physics, including fluid dynamics, harmonic maps, and nonlinear elasticity, will be discussed.

#### Professor Jason Lotay

#### Translators in Lagrangian Mean Curvature Flow

Lagrangian mean curvature flow is a geometric evolution equation that is potentially a powerful tool in geometry and topology. A key issue in the flow is the understanding of formation of singularities. There are many possible singularity models, but a very special case is given by selfsimilar solutions that simply translate along the flow. I will describe joint work with F. Schulze and G. Szekelyhidi that shows that in certain cases we can guarantee that the singularity model has to be a translator: this is the first such classification result beyond the setting of flows of curves.

## Professor Robert J McCann

#### Lipschitz Free Boundaries in the Monopolist's Problem

The principal-agent problem is an important paradigm in economic theory for studying the value of private information; the nonlinear pricing problem faced by a monopolist is one example; others include optimal taxation and auction design. For multidimensional spaces of consumers (i.e. agents) and products, Rochet and Chone (1998) reformulated this problem to a concave maximization over the set of convex functions, by assuming agent preferences combine bilinearity in the product and agent parameters with a (quasi)linear sensitivity to prices. This optimization corresponds mathematically to a convexity-constrained obstacle problem. The solution is divided into multiple regions, according to the rank of the Hessian of the optimizer.

We show the free boundary separating the highest rank regions to be locally Lipschitz. Combining our techniques with those of Rochet and Chone allows us to confirm conjectured aspects of the solution to their square example and gives the first analytical description of an overlooked market segment.

Based on work-in-progress with Cale Rankin (University of Toronto) and Kelvin Shuangjian Zhang (Fudan University).

## Professor Andrea Mondino

A Sharp Isoperimetric-type Inequality for Lorentzian Spaces Satisfying Time-like Ricci Lower Bounds

In the seminar, I will discuss recent joint work with Fabio Cavalletti, where we establish a sharp and rigid isoperimetric-type inequality in Lorentzian signature under the assumption of Ricci curvature bounded below in the time-like directions. The inequality is proved in the high generality of Lorentzian pre-length spaces satisfying timelike Ricci curvature lower bounds in a synthetic sense via optimal transport. The results are new already for smooth Lorentzian manifolds. Applications include an upper bound on the area of Cauchy hypersurfaces inside the interior of a black hole (original already in Schwarzschild) and an upper bound on the area of Cauchy hypersurfaces in cosmological space-times.

## Professor Paul Minter

#### Stable Minimal Hypersurfaces in R^5

The stable Bernstein problem asks whether a complete, two-sided, immersed stable minimal hypersurface in R^n is necessarily flat. The original PDE version of this question for entire functions satisfying the minimal surface equation inspired some of the early developments of geometric measure theory. In this talk, I will discuss joint work with Otis Chodosh, Chao Li, and Doug Stryker which resolves this question in R^5. Our proof uses ideas inspired from the study of uniformly positive curvature quantities in geometry, for instance bi-Ricci curvature and mu-bubbles.

#### Professor Monica Musso

#### Leapfrogging of Vortex Rings for Incompressible Euler Equations

We consider the Euler equations for incompressible fluids in 3-dimensions. A classical question that goes back to Helmholtz is to describe the evolution of vorticities with a high concentration around a curve. The work of Da Rios in 1906 states that such a curve must evolve by the so-called "binormal curvature flow". The existence of true solutions whose vorticity is concentrated near a given curve that evolves by this law is a long-standing open question that has only been answered for the special case of a circle traveling with constant speed along its axis, the thin vortex rings. In this talk I will discuss the construction of helical filaments, associated with a translating-rotating helix, and of two vortex rings interacting with each other, the so-called leapfrogging. These results are in collaboration with J. Davila, M. del Pino, and J. Wei.

#### Professor Xavier Ros-Oton

#### The Singular Set in the Stefan Problem

The Stefan problem, dating back to the XIXth century, is probably the most classical and important free boundary problem. The regularity of free boundaries in the Stefan problem was developed in the groundbreaking paper (Caffarelli, Acta Math. 1977). The main result therein establishes that the free boundary is \$C^\infty\$ in space and time, outside a certain set of singular points. The fine understanding of singularities is of central importance in a number of areas related to nonlinear PDEs and Geometric Analysis. In particular, a major question in such a context is to establish estimates for the size of the singular set. The goal of this talk is to present some recent results in this direction for the Stefan problem. This is a joint work with A. Figalli and J. Serra (J. Amer. Math. Soc. 2024).

## Ms Vanessa Ryborz

#### On the Equivalence of Distributional and Synthetic Ricci Curvature Lower Bounds

For a weighted Riemannian manifold, the Ricci curvature tensor depends on up to second derivatives of the metric tensor and the weight function. If the metric and the weight are of regularity below \$C^2\$, one can use the distribution theory on manifolds to generalise the Ricci curvature tensor and the notion of lower Ricci curvature bounds. Using optimal transport, Sturm and Lott- Villani introduced synthetic notions of lower Ricci curvature bounds on metric measure spaces, that do not even require a manifold structure. On smooth manifolds, this definition agrees with classical (pointwise) lower Ricci curvature bounds.

In this talk, I will present a joint work with Andrea Mondino where we prove the equivalence of distributional and synthetic Ricci curvature lower bounds for a weighted Riemannian manifold with a continuous metric that admits an \$L^2\_{\rm loc}\$-connection and a \$C^0\cap W^{1,2}\_{loc}\$-weight. The regularity is sharp, in the sense that these are the minimal assumptions where both approaches are well defined.

## Dr Jan Sbierski

#### The Interior of Black Holes: Instability and Inextendibility

In this talk I will review the problem of understanding the interior of dynamical rotating black holes. In particular I will present some recent work that establishes an instability for the linearised Einstein equations in the interior of Kerr black holes and I will also discuss how to capture the strength of singularities that form in the black hole interior by investigating in which regularity class the solution is inextendible.

## Dr Matthew Schrecker

#### Gravitational Landau Damping

In the 1960s, Lynden-Bell, studying the dynamics of galaxies around steady states of the gravitational Vlasov-Poisson equation, described a phenomenon he called "violent relaxation," a convergence to equilibrium through phase mixing analogous in some respects to Landau damping in plasma physics. In this talk, I will discuss recent work on this gravitational Landau damping for the linearised Vlasov-Poisson equation. In particular, I will discuss the critical role played by the interaction of the vacuum boundary for the kinetic Vlasov equation with the elliptic regularisation of the Poisson equation in distinguishing damping from oscillatory behaviour in the perturbations. This is based on joint work with Mahir Hadzic, Gerhard Rein, and Christopher Straub.

#### Professor Paolo Secchi

#### Characteristic Boundary Value Problems and Magneto-Hydrodynamics

The aim of the course is to provide an introduction to the theory of initial boundary value problems for Friedrichs symmetrizable systems, with particular interest for the applications to the equations of ideal Magneto-Hydrodynamics (MHD).

We first analyse different kinds of boundary conditions and present the main results about the well-posedness. In the case of the characteristic boundary, we discuss the possible loss of regularity in the normal direction to the boundary and the use of suitable anisotropic Sobolev spaces in MHD.

Finally, we give a short introduction to the Kreiss-Lopatinskii approach and discuss a simple boundary value problem for the wave equation that may admit estimates with a loss of derivatives from the data.

## Professor Ali Taheri

Gradient Estimates on the Witten Laplacian, Curvature Conditions and Liouville Theorems on Smooth Metric Measure Spaces

In this talk I will present gradient estimates of elliptic types (specifically, of Hamilton and Souplet-Zhang types) for a class of nonlinear parabolic equations on smooth metric measure spaces. The Laplace-Beltrami operator gives its place to the Witten Laplacian and this context the Riemannian metric and potential are taken to evolve with time (a geometric flow). The estimates are established under different curvature conditions and lower bounds on the Bakry-Emery Ricci tensor and are then used to prove a number of important results such as Harnack inequalities, spectral bounds, sharp Logarithmic Sobolev inequalities (LSI) and general Liouville and global constancy results. If time allows, I will present applications of the above to the (super) Perelman-Ricci flow, heat entropy formula and ancient/eternal solutions.

#### Dr Grigalius Taujanskas

#### Low Regularity Wave Maps on the Einstein Cylinder via Peter—Weyl Theory

Nonlinear wave equations of wave maps type are typically expected to be well-posed for initial data just above scaling critical regularity. For wave maps on Minkowski space, this is by now well-understood as a consequence of works of Klainerman–Machedon, Tataru, Tao, and others, and effectively relies on sharp null form estimates which exploit the special "null" structure of the nonlinearities. In Fourier space, these estimates capture cancellations between parallel propagating waves. I will introduce a new approach to obtain a wide range of analogous estimates on the Einstein cylinder, where traditional Fourier theory is unavailable, using instead the Lie group structure of SU(2), an emergent periodicity of the conformal wave equation on the Einstein cylinder, and Peter--Weyl theory. The estimates we obtain hold for a slightly different set of exponents than in flat space, including edge cases which are forbidden in flat space, however with an arbitrarily small loss which we fundamentally trace down to the non-commutativity of SU(2). Time permitting, I will outline how the estimates may be used to show almost optimal well-posedness of wave maps equations on the Einstein cylinder.

#### Professor Dehua Wang

Euler Equations and Mixed-Type Problems in Gas Dynamics and Geometry

In this short course, we will discuss the Euler equations and applications in gas dynamics and geometry. First, the basic theory of Euler equations and mixed-type problems will be reviewed. Then we will present the results on the transonic flows past obstacles, transonic flows in the fluid dynamic formulation of isometric embeddings, and the transonic flows in nozzles. We will discuss global solutions and stability obtained through various techniques and approaches. The short course consists of three parts and is accessible to PhD students and young researchers.

## Professor Neshan Wickramasekera

#### Analysis of Singularities of Area Minimising Currents

In monumental work dating back to the early 1980's, Almgren established that the Hausdorff dimension of the singular set of an \$n\$-dimensional area minimising rectifiable current of a Riemannian manifold of dimension \$n + k\$ is no larger than \$n-2\$. This bound is sharp when the codimension  $k \ge 2$  (whereas in codimension 1, the sharp bound is \$n-7\$, which had been established in stages over the decade 1960-1970 by De Giorgi, Federer, Fleming, Almgren and Simons). In codimension \$\geq 2\$, the presence of branch point singularities, i.e. singular points where one tangent cone is an \$n\$-dimensional plane (with integer multiplicity 2 or larger), makes the problem more difficult and subtle than in codimension 1. In Almgren's approach (later presented with some technical streamlining and more accessibility by De Lellis--Spadaro), the lack of an estimate giving decay of the current towards a unique tangent plane at branch points is a major contributing factor to the exceeding intricacy of the proof. Almgren's work left open the question of uniqueness of the

tangent cones entirely, as well as other central questions concerning the local structure of the singular set and the asymptotic behaviour of the current on approach to the singular set.

The talk (based on recent joint work with Brian Krummel) will describe a new approach to this problem that leads to progress on these questions, which is based on a combination of relatively elementary parts of Almgren's program and new ideas. In particular, the work introduces and uses an intrinsically defined frequency function for the current relative a plane (the planar frequency function) which is approximately monotone when decay towards the plane holds, and which takes correct values (i.e. \$\leq 1\$) on cones. The new approach first establishes decay estimates at \$H^{n-2}\$ a.e. point, which in combination with the planar frequency function lead to a considerably simpler proof of the dimension bound on the singular set, and then to a finer description of the singular set. The key overall outcomes are: (i) the tangent cone at \${\mathcal  $H^{n-2}$  a.e. point is unique; (ii) the singular set is locally the finite union of pieces each of which is locally compact and locally (n-2)-rectifiable (with locally finite measure), and (iii) the current at  ${\rm H}^{n-2}$  a.e. branch point has (in addition to a unique tangent plane) a unique tangent plane) a unique non-zero higher order blow-up either relative to the tangent plane (if the rate of decay towards the tangent plane is less than quadratic in the scale) or relative to a center manifold (if the rate of decay towards the tangent plane is quadratic or higher). [In contemporaneous work, De Lellis, Minter and Skorobogatova have developed a different method which takes the full extent of Almgren's program as a starting point and provides conclusion (i) as well as part of conclusion (ii), namely, countable \$(n-2)\$-rectifiability of the singular set.]

#### Mr Dominic Wynter

#### Shock Profiles for the Non-Cutoff Boltzmann Equation

We construct kinetic shock profiles for the Boltzmann equation with long range interactions, in the hard potential regime. We prove existence and uniqueness up to translation of solutions close to Navier-Stokes traveling waves, using stability estimates for the Boltzmann equation and for traveling fluid waves, and construct solutions by a Galerkin approximation.

#### Professor Wei Xiang

#### Normal Shock with Large Swirl Velocity in a Finite Cylinder

In this talk, we introduce our recent work on the existence and unique location of the three-dimensional normal shock with large vorticity for axisymmetric full Euler equations in a cylinder, with appropriate boundary conditions at the entrance of the nozzle and the receiver pressure at the exit of the nozzle. It is the first mathematical result on the threedimensional transonic shock with large vorticity.



## Parking

Parking is not available at the Institute. Here is a link to our travel page <u>https://www.maths.ox.ac.uk/about-us/travel-</u> <u>maps</u>

Vehicle access is available for drop-off and collection of materials and equipment. Please provide the name of the driver, vehicle model and registration in advance.

## Mobility

There are two disabled parking spaces located adjacent to the northern entrance to the Andrew Wiles building. Please contact the event organisers in advance if you require a space.

For those who cannot manage stairs, pass the through the large glass doors nearest reception and access the lifts to go to the mezzanine (-1) level. The reception staff can release the door lock and/or escort occasional visitors. Regular building users will have card access. The doors are not powered.

## Hearing loop

There is an induction hearing loop system installed in all mezzanine floor teaching rooms and lecture theatres and one on the reception desk. The hearing loop provides a signal that is picked up by the hearing aid when it is set to 'T' (Teslacoil) setting. Those lecturing should be aware that they need to stand close to the lectern or use the lapel microphone in order to ensure that their voice is picked up by the hearing loop system. If you have any difficulties with the hearing loop system please make the organisers aware.

#### Wifi

Network connection when attending a meeting does not require any local IT account. Network connection can be obtain via Eduroam or The Cloud wifi services in the building.

## Toilets

There are large male and female toilet facilities at either end of the mezzanine floor. There are also gender-neutral accessibility toilets near these male/female toilets as well as gender-neutral toilets accessible to all just inside core 2 and 3 (i.e. in the big grey 'columns' on either side of the cafe area which straddle all floors and also house the lifts).

## Catering

The Café is situated on the Mezzanine floor and is open from 8:30 – 16:15 Monday to Friday. The café serves a wide variety of food including:

Breakfast 8:30 – 10:30: cereals, yoghurts, Danish pastries, fresh fruit, traditional English breakfast.

Lunch 12:00 – 14:00: daily hot choices, hot deli sandwich, jacket potatoes, fresh soup and a range of sandwiches, wraps and salads. The daily menu for the cafe is displayed in the mezzanine and on the kitchen noticeboards and as an online weekly menus.

The café is open to all members of the University. The open and welcoming nature of the building means members of the public may also enter and are welcome.

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