Infrared surprises in gravity & celestial CFT

2403.13053, 2412.16149, 2412.16142
& work in progress

with **Sangmin Choi** & Alok Laddha

to appear & work in progress

with Sangmin Choi & Ameya Kadhe

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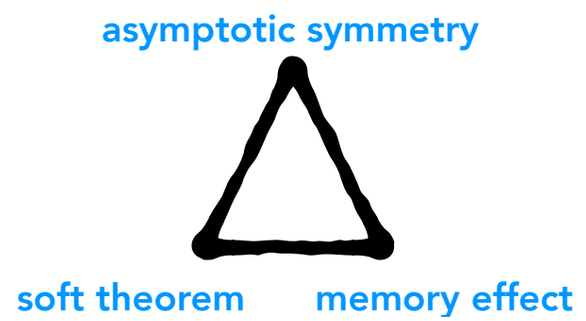
SIMONS FOUNDATION

SIMONS COLLABORATION ON CELESTIAL HOLOGRAPHY

Plan: infrared surprises

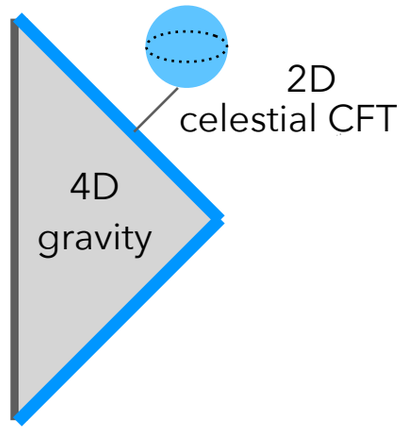
& their connection to ``celestial CFT''

Infrared triangles



Towers of ∞ symmetries

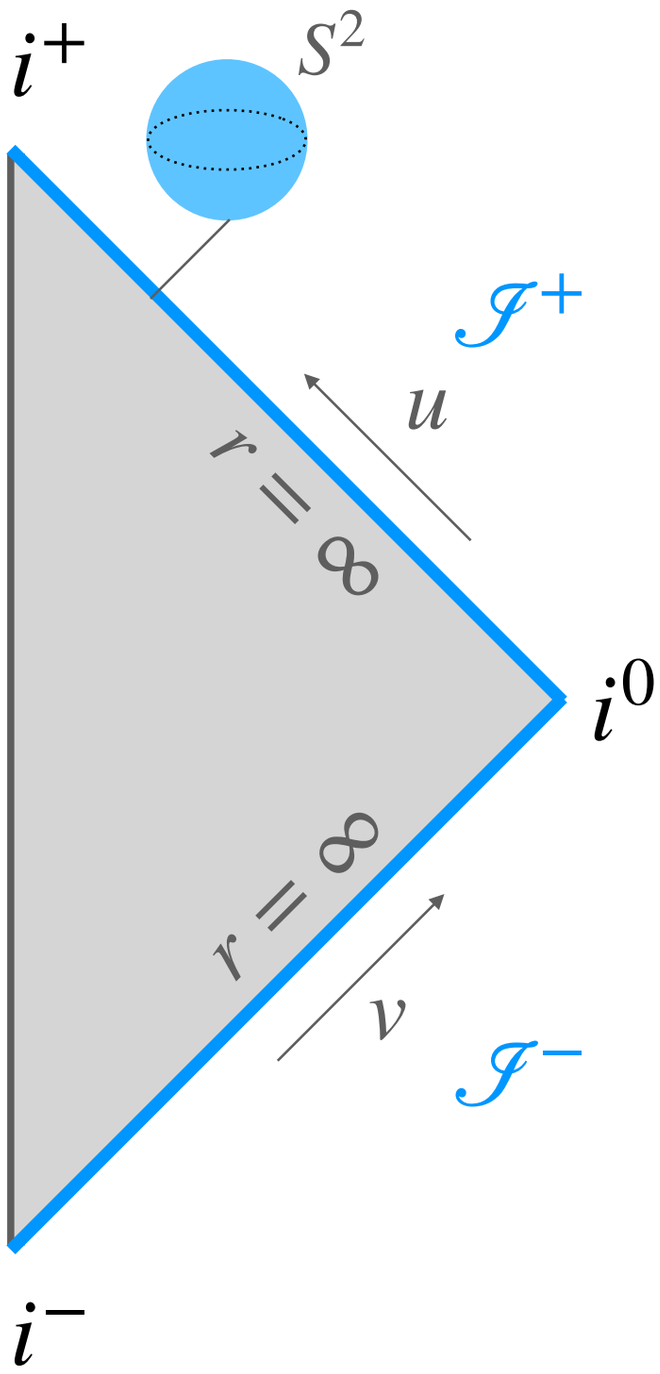
4D = 2D



Long-range effects

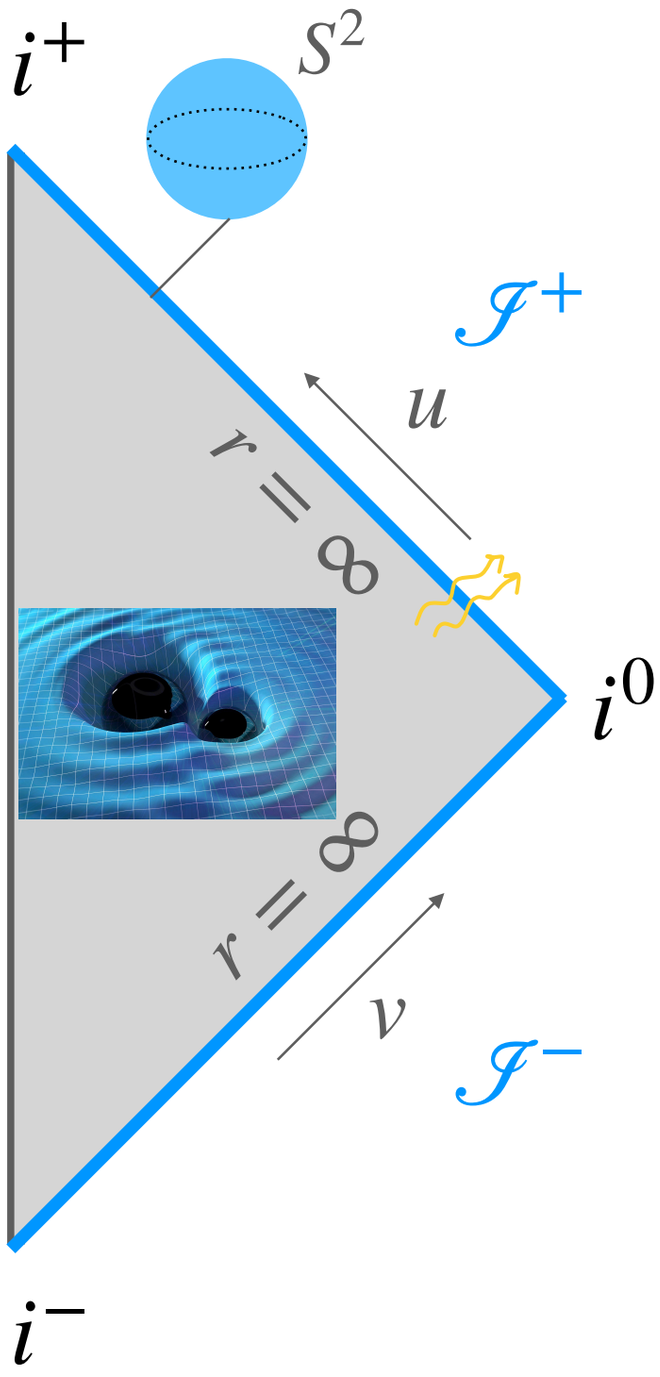
Infrared triangles

Asymptotic symmetries



"Large" gauge/diffeo transformations that
preserve the boundary conditions of the fields,
i.e. their large-distance fall-offs.

Asymptotic symmetries



"Large" gauge/diffeo transformations that preserve the boundary conditions of the fields, i.e. their large-distance fall-offs.

gravity:

mass

$$ds^2 = -(1 + \dots)du^2 - (2 + \dots)dudr$$

angular momentum

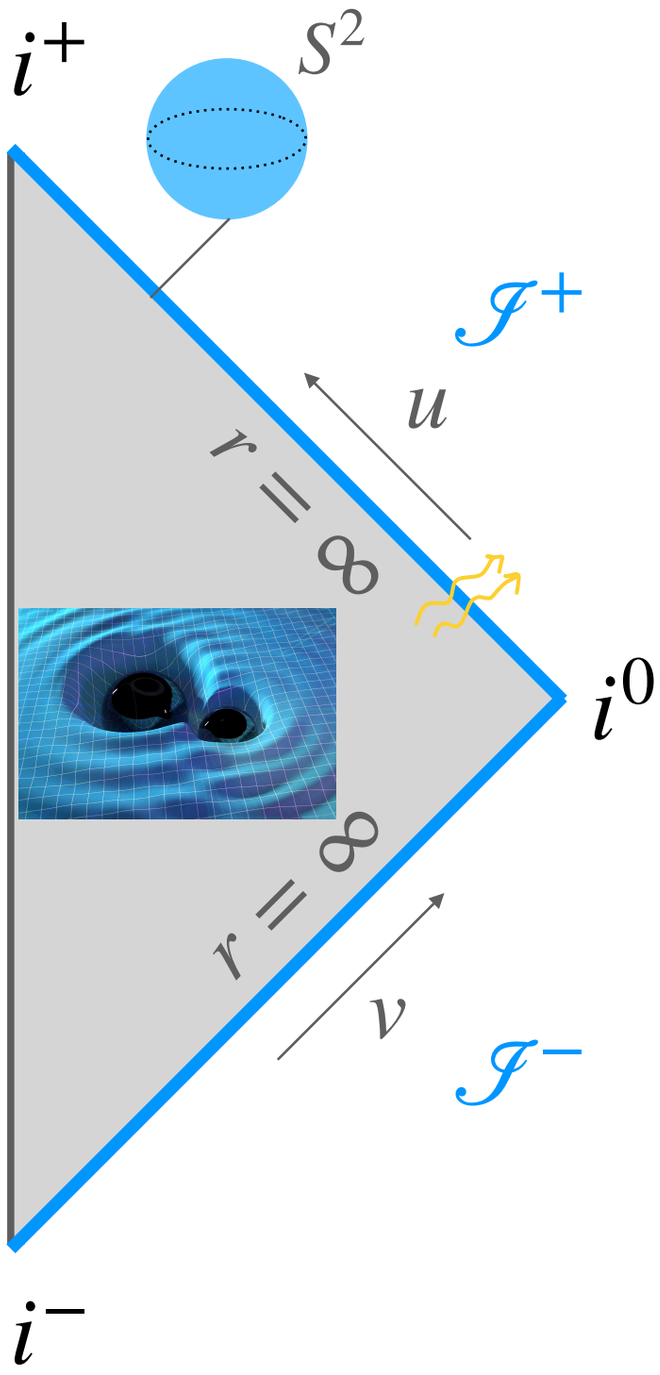
$$+(\dots)dudx^A$$

$$+(r^2\gamma_{AB} + r\,C_{AB} + \dots)dx^A dx^B$$

shear: gravitational waves

⇒ Bondi news $N_{AB} = \partial_u C_{AB}$

Asymptotic symmetries



"Large" gauge/diffeo transformations that preserve the boundary conditions of the fields, i.e. their large-distance fall-offs.

gravity:

Find ξ such that $\mathscr{L}_\xi g_{\mu\nu} \approx "0"$ as $r \to \infty$.

$\updownarrow$

$O(1/r^{\#})$

Unlike gauge redundancies, asymptotic symmetries act non-trivially on the physical data → non-zero charges.

IR triangle

[He,Lysov,Mitra,Strominger'14]

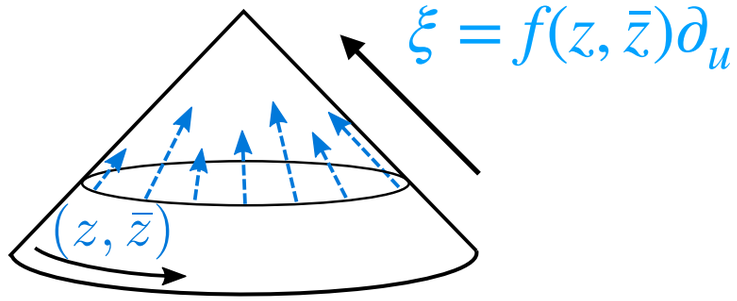
[Strominger,Zhiboedov'14]

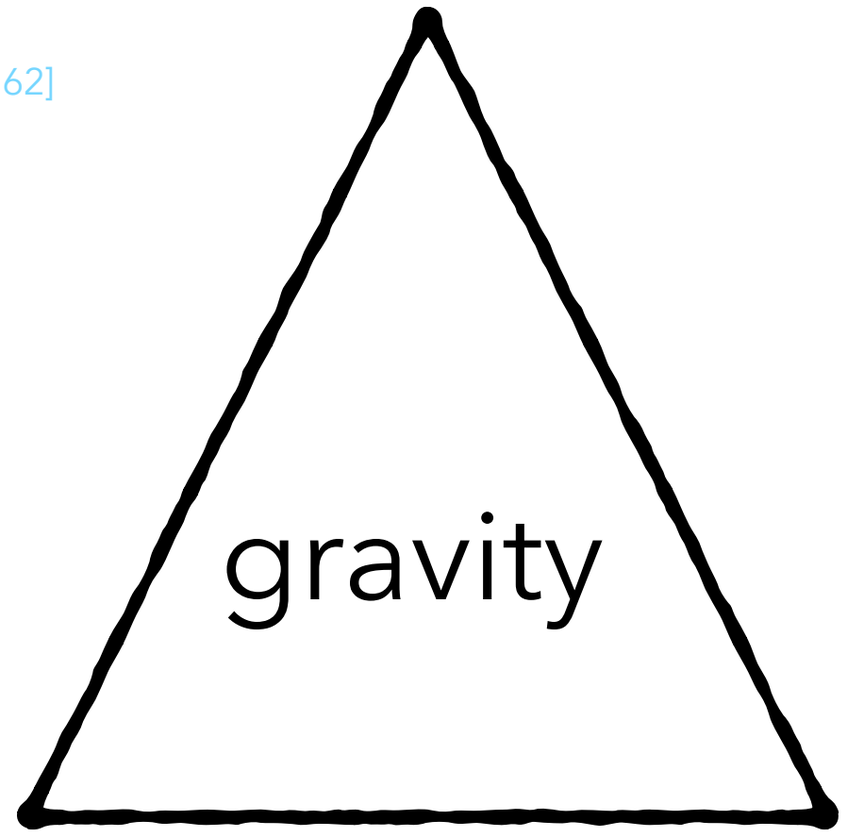
The symmetries of asymptotically flat space are not just Poincaré but an infinite extension!

[Bondi,van der Burg,Metzner'62] [Sachs'62]

BMS group

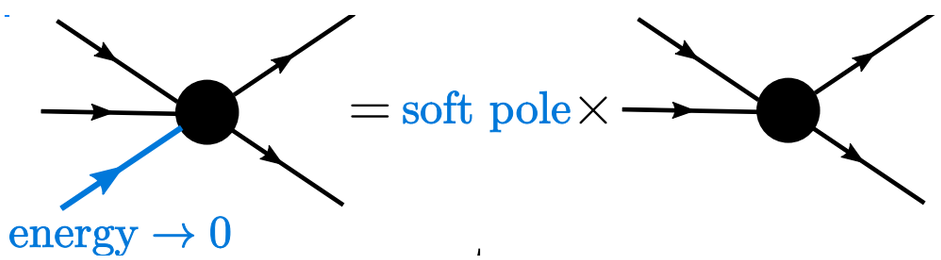
supertranslations





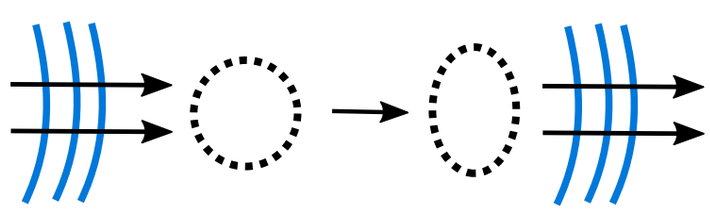
[Weinberg'65]

soft graviton theorem



[Zel'dovich,Polnarev'74 [Braginsky,Thorne'87]

displacement memory effect



IR triangle

on $\mathscr{I}^+$:

supertranslations

superrotations

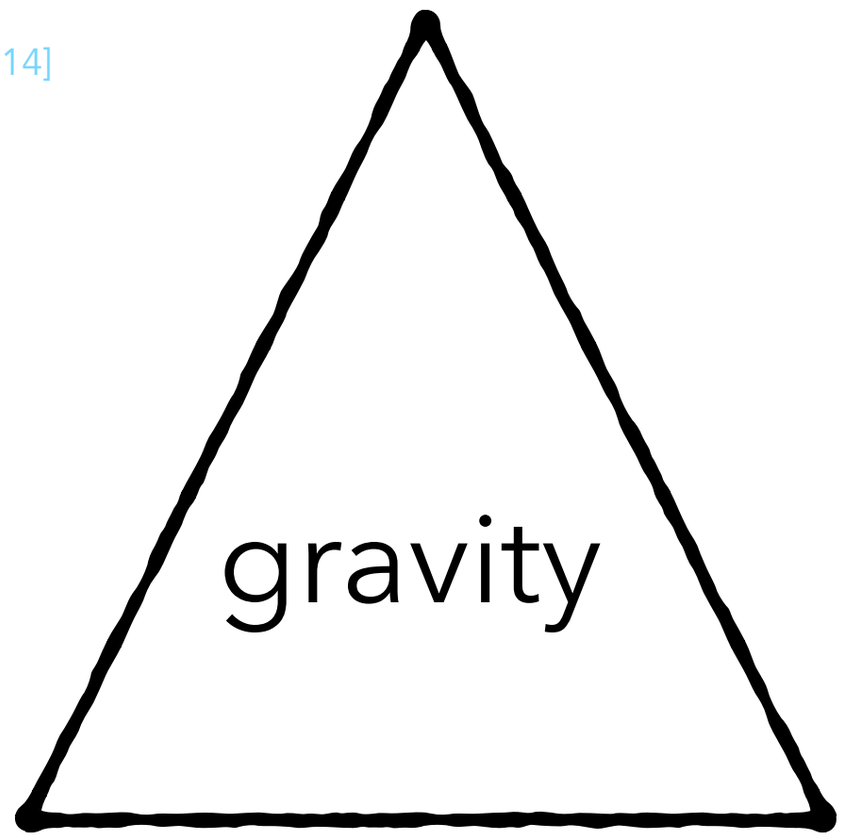
[Barnich,Troessaert'11] [Campiglia,Laddha'14]

extended / generalized BMS group

asymptotic symmetry

$$\xi^u = f \qquad \xi^A = \frac{1}{r} D_A f$$

$$\xi^u = -\frac{1}{2} D_A Y^A \qquad \xi^A = Y^A$$



soft theorem

ω^{-1} leading soft graviton

ω^{0} subleading soft graviton

[Cachazo,Strominger'14]

memory effect

displacement

spin

[Pasterski,Strominger,Zhiboedov'15]

IR triangle

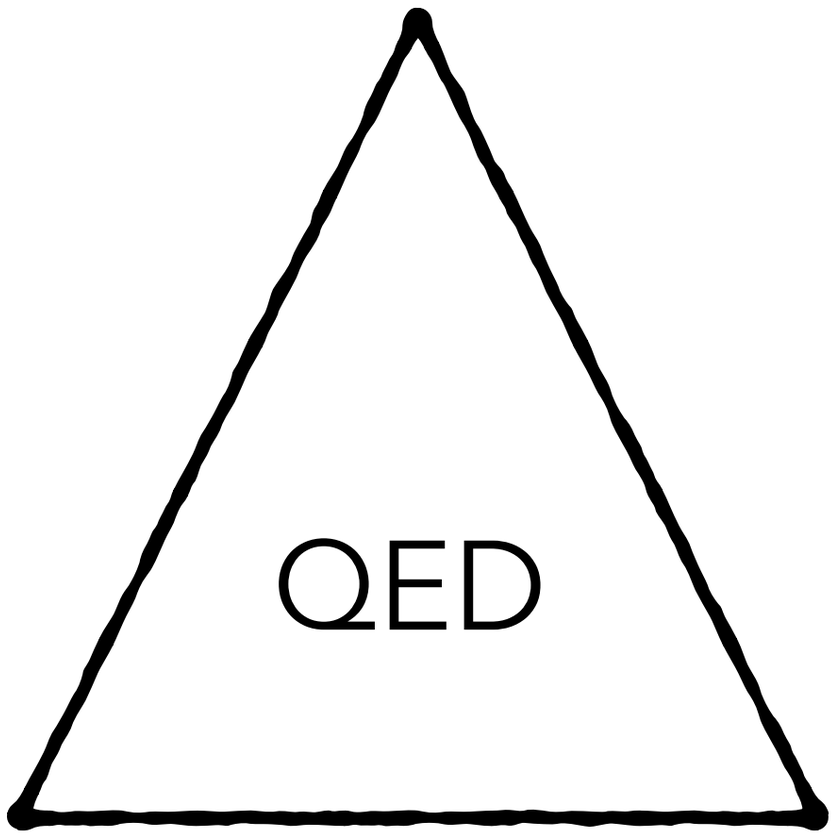
superphaserotation

[He,Mitra,Porfyriadis,Strominger'14]

[Kapec,Pate,Strominger'15]

[Campiglia,Laddha'15]

asymptotic symmetry



soft theorem

ω^{-1} leading soft photon

[Weinberg'65]

memory effect

electromagnetic kick

[Bieri,Garfinkle'13]

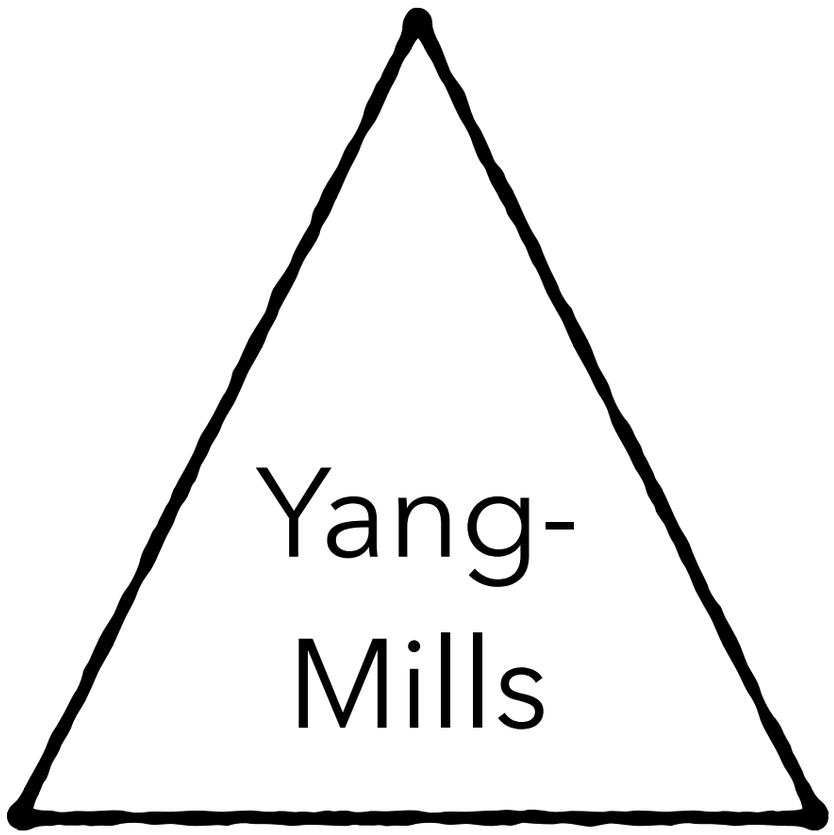
[Pasterski'15]

IR triangle

superphaserotation

[He,Mitra,Strominger'14]

asymptotic symmetry



soft theorem

ω^{-1} leading soft gluon

[Berends,Giele'89]

memory effect

color

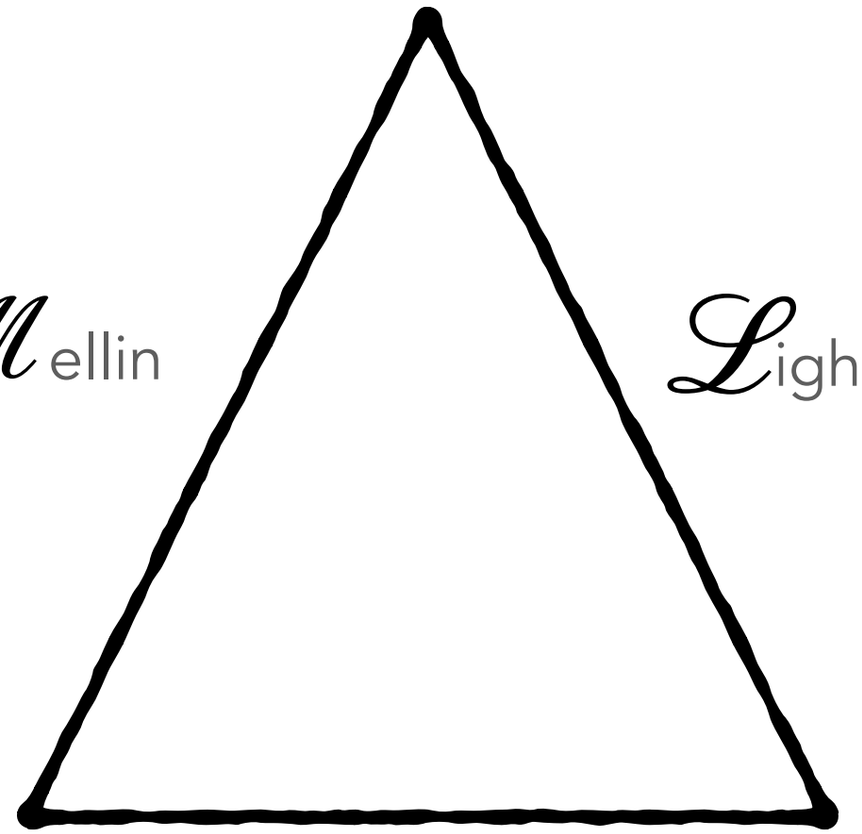
[Pate,Raclariu,Strominger'17]

3 bases for the IR

boost weight

Δ

asymptotic symmetry



$\mathscr{M}$ellin

$$\mathscr{M}(.) = \int_0^\infty d\omega \omega^{\Delta-1}(.)$$

[Pasterski,Shao,Strominger'17]

$\mathscr{L}$ight-ray

$$\mathscr{L}(.) = \int_{-\infty}^{+\infty} du u^{-\Delta}(.)$$

[Pasterski,AP,Trevisani'21]

soft theorem

energy

ω

$\mathscr{F}$ourier

$$\mathscr{F}(.) = \int_{-\infty}^{+\infty} du e^{i\omega u}(.)$$

memory effect

null time

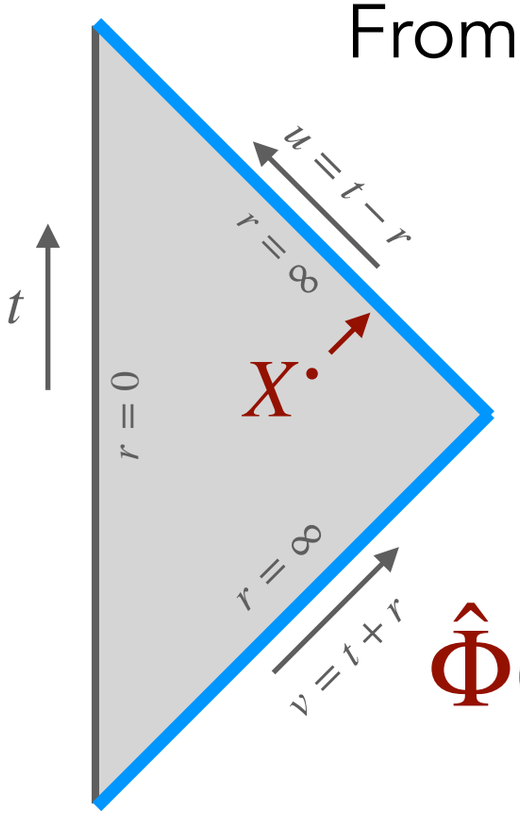
u

4D = 2D

Flat space holography

``Extrapolate" dictionary for flat holography. [Paserski,AP,Trevisani'21]

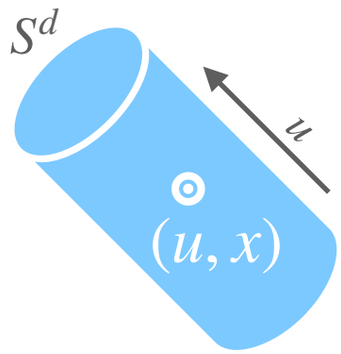
From bulk operators to boundary operators:



$\hat{\Phi}(X)$

large r limit

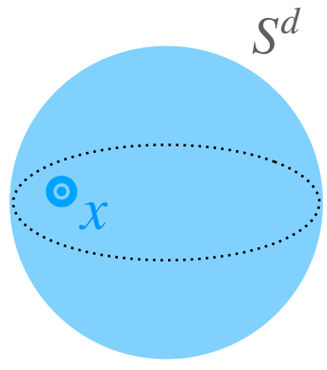
$\lim_{r\to\infty} r^{1-|\ell|}$



$\Phi_\ell(u, x)$

$\mathscr{L}$ight-ray

$\int_{-\infty}^{+\infty} du u^{-\Delta}$



$\mathcal{O}_{\Delta,\ell}(x)$

boost-weight spin

conformal primaries

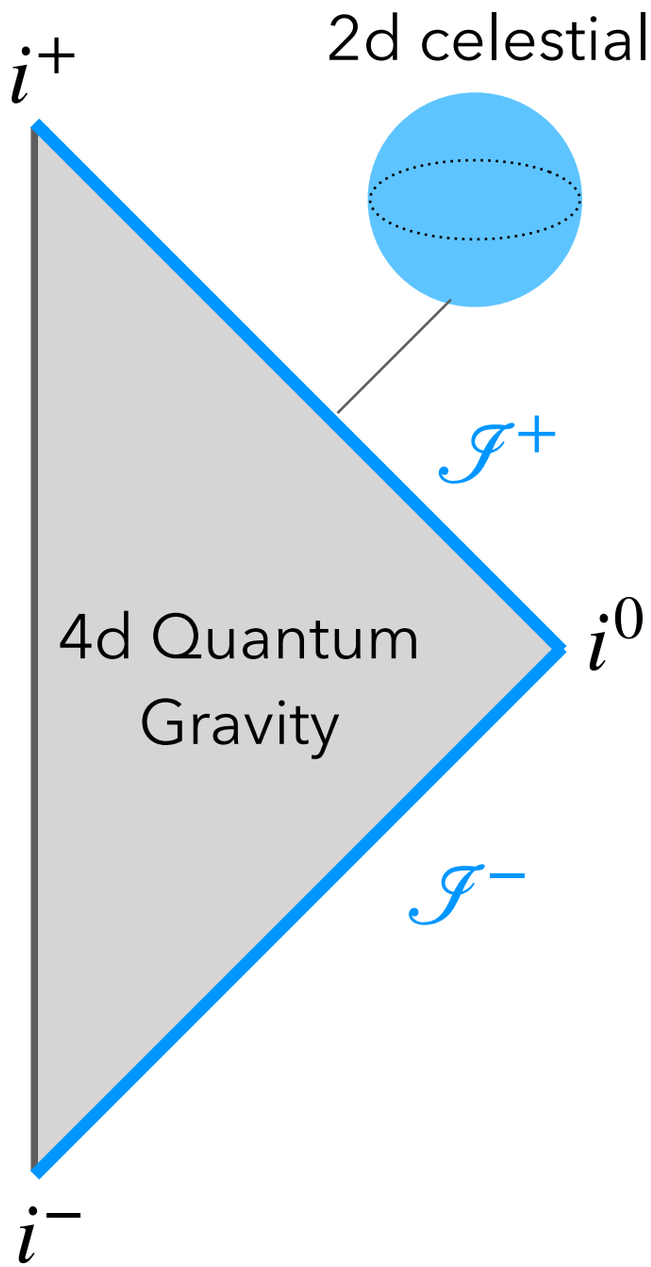
degenerate metric, $c \to 0$ field theory

Carroll weights $(k, \bar{k}) = \frac{1}{2}(1 + \ell, 1 - \ell)$ [Donnay,Herfray, Fiorucci,Ruzziconi'22]

bulk Lorentz acts as boundary global conformal

[Pasterski,Shao'17]

Celestial holography



4d Quantum Gravity

in asymptotically flat spacetimes

=

2d "Celestial CFT"

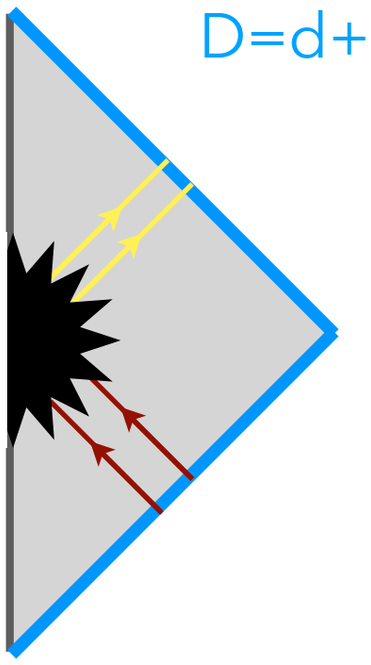
Symmetry & observables

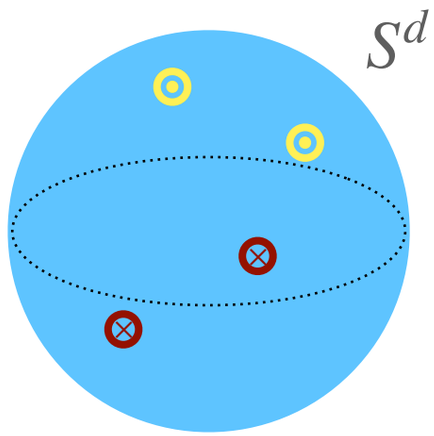
symmetry:

Lorentz group in
D=d+2 dimensions

$\cong$

Euclidean conformal
group in d dimensions

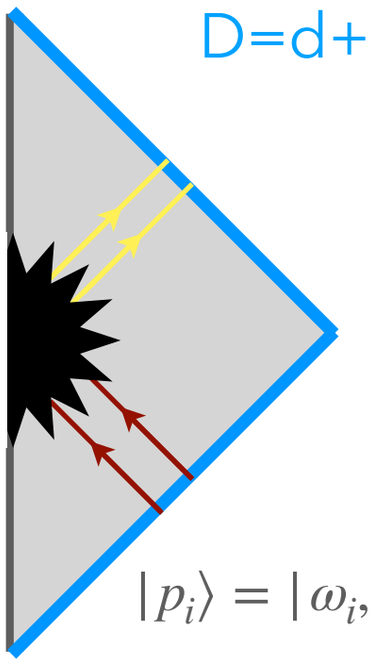




Symmetry & observables

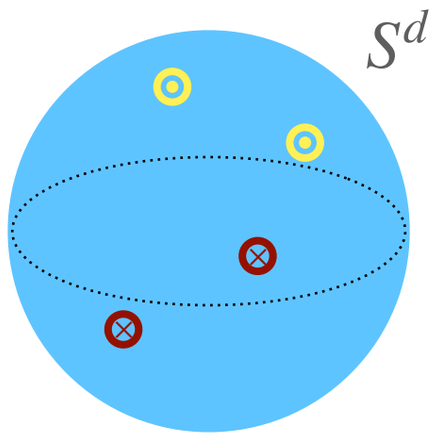
symmetry:

Lorentz group in D=d+2 dimensions $\cong$ Euclidean conformal group in d dimensions



$|p_i\rangle = |\omega_i, x_i\rangle$

energy basis



basic observables in flat space:

S-matrix

Standard amplitudes

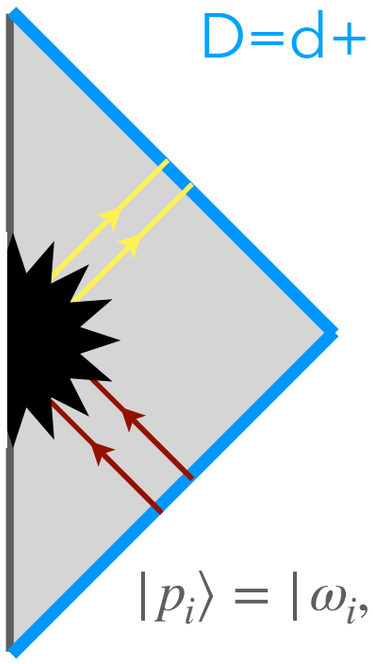
$$\langle p_n^{\text{out}} \dots | \mathcal{S} | \dots p_1^{\text{in}} \rangle$$

translation symmetry

Symmetry & observables

symmetry:

Lorentz group in D=d+2 dimensions $\cong$ Euclidean conformal group in d dimensions



$|p_i\rangle = |\omega_i, x_i\rangle$

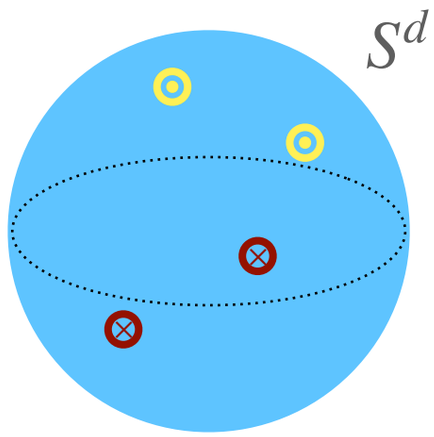
energy basis

basic observables in flat space:

S-matrix

$\mathscr{M}$ellin

$\int_0^\infty d\omega \omega^{\Delta-1}$



$|\Delta_i, x_i\rangle$

boost-weight basis

Standard amplitudes

$\langle p_n^{\text{out}} \dots | \mathcal{S} | \dots p_1^{\text{in}} \rangle$

translation symmetry

Celestial amplitudes

$\langle \mathcal{O}^-_{\Delta_1}(x_1) \dots \mathcal{O}^+_{\Delta_n}(x_n) \rangle$

Lorentz symmetry

$\updownarrow$

global conformal symmetry

[de Boer, Solodukhin'03] [Pasterski,Shao,Strominger'17] [Pasterski,Shao'17]

From global to local conformal

supertranslations,
superrotations, ...

[Barnich,Troessaert'11]

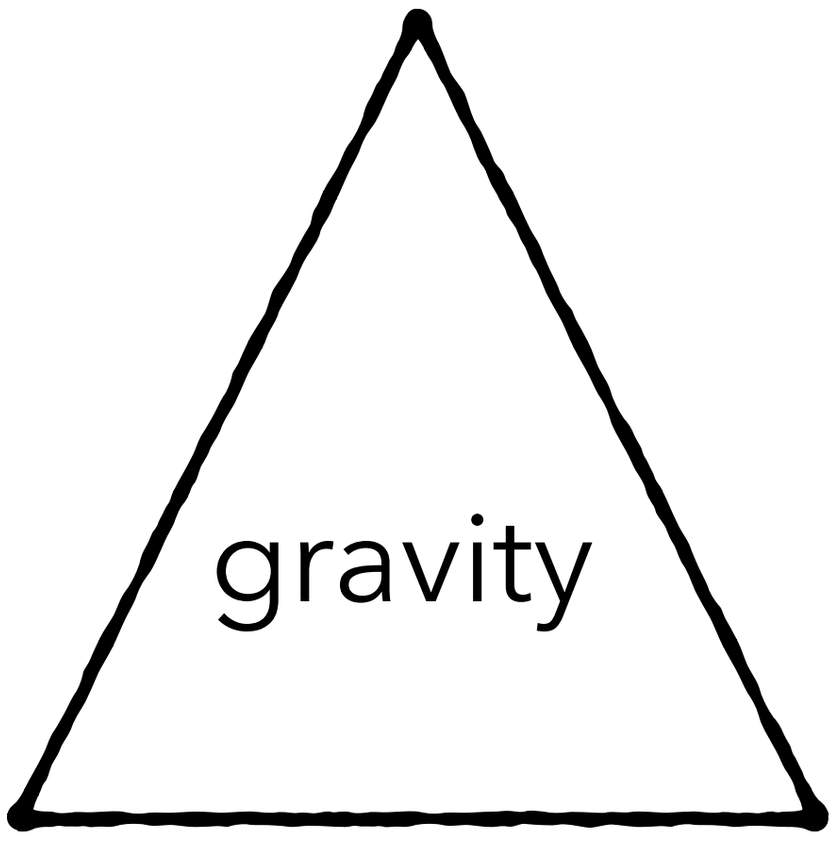
local conformal
symmetry on S^2 !

asymptotic symmetry

on $\mathscr{I}^+$:

$\xi^u = f$ $\qquad$ $\xi^A = \frac{1}{r} D_A f$

$\xi^u = -\frac{1}{2} D_A Y^A$ $\qquad$ $\xi^A = Y^A$



soft theorem

ω^{-1} leading soft graviton,

ω^{0} subleading soft graviton, ...

[Cachazo,Strominger'14]

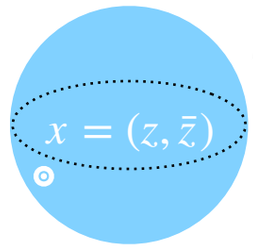
memory effect

displacement,

spin, ...

[Pasterski,Strominger,Zhiboedov'15]

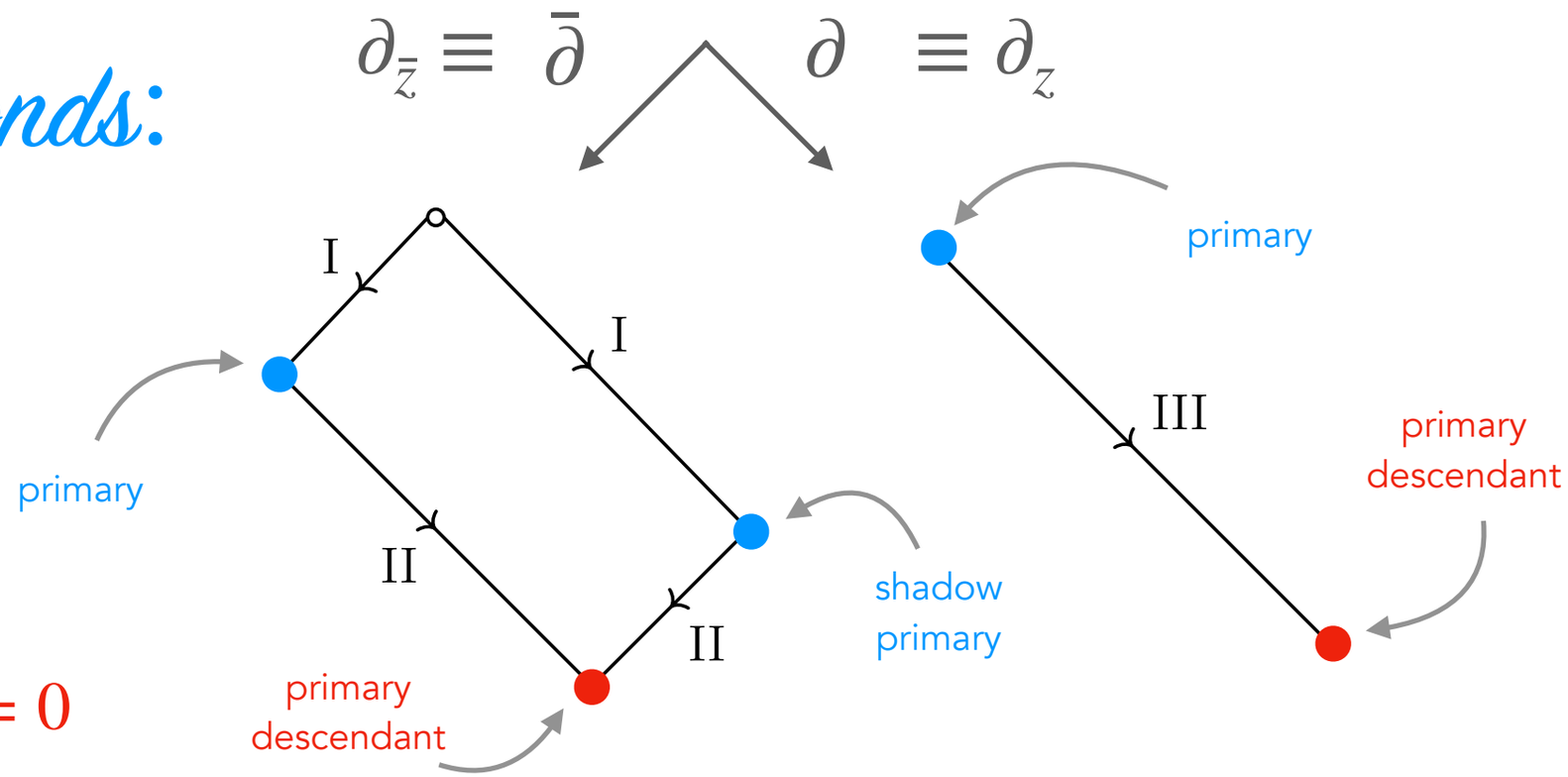
Celestial symmetries



celestial diamonds:

[Pasterski,AP,Trevisani'21]

also: [Banerjee,Pandey,Paul'19]



conservation equation $\bar{\partial}^{\#}\mathcal{O}(z,\bar{z}) = 0$

depends on types I, II, III: spin of descendant >,<,= spin of parent primary

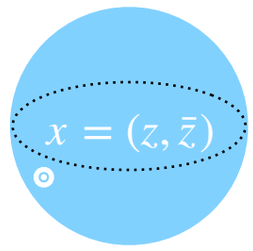
Conformally soft operator = primary

$\downarrow \bar{\partial}^{\#}$

primary descendant = conservation equation for **topological operator**

Noether current: $\mathcal{J} = \sum_{m=0}^{\#-1} (-1)^m \bar{\partial}^m \epsilon(z,\bar{z}) \bar{\partial}^{\#-m-1} \mathcal{O}(z,\bar{z})$

Celestial symmetries



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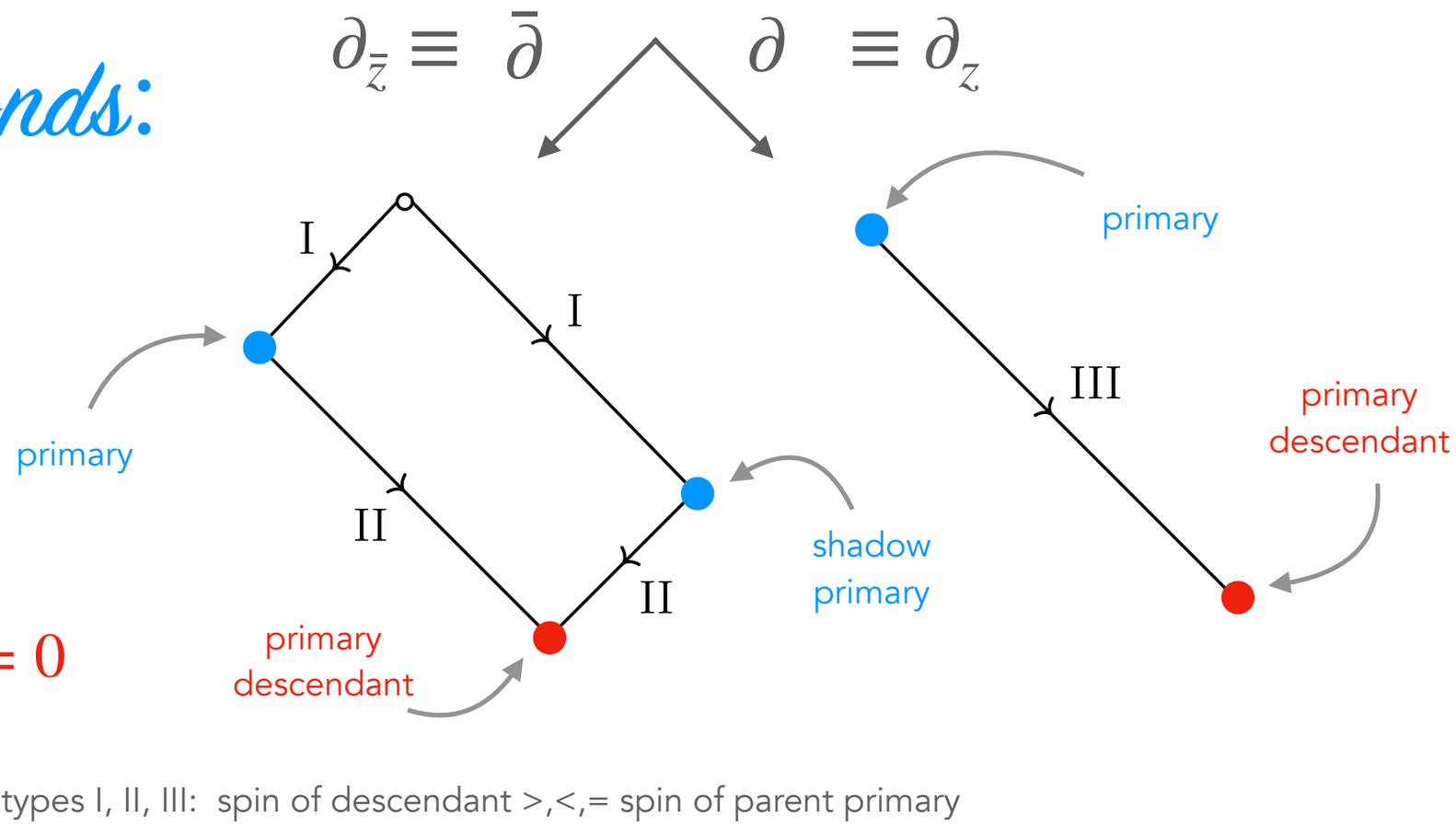


conservation equation $\bar{\partial}^{\#}\mathcal{O}(z,\bar{z}) = 0$

depends on types I, II, III: spin of descendant >,<,= spin of parent primary

QED: $\mathcal{O}_\Delta(x)$ with $\Delta = 1$ generates **superphaserotation** symmetry. [He,Mitra,Strominger'15] [Kapec,Mitra,Raclariu,Strominger'16] [Donnay,AP,Strominger'18] [Donnay,Pasterski,AP'20]

gravity: $\mathcal{O}_\Delta(x)$ with $\Delta = 1,0$ generate **extended BMS** symmetries.

$\Delta = 1$: supertranslations; action: $O_{\Delta_k} \to O_{\Delta_k+1}$

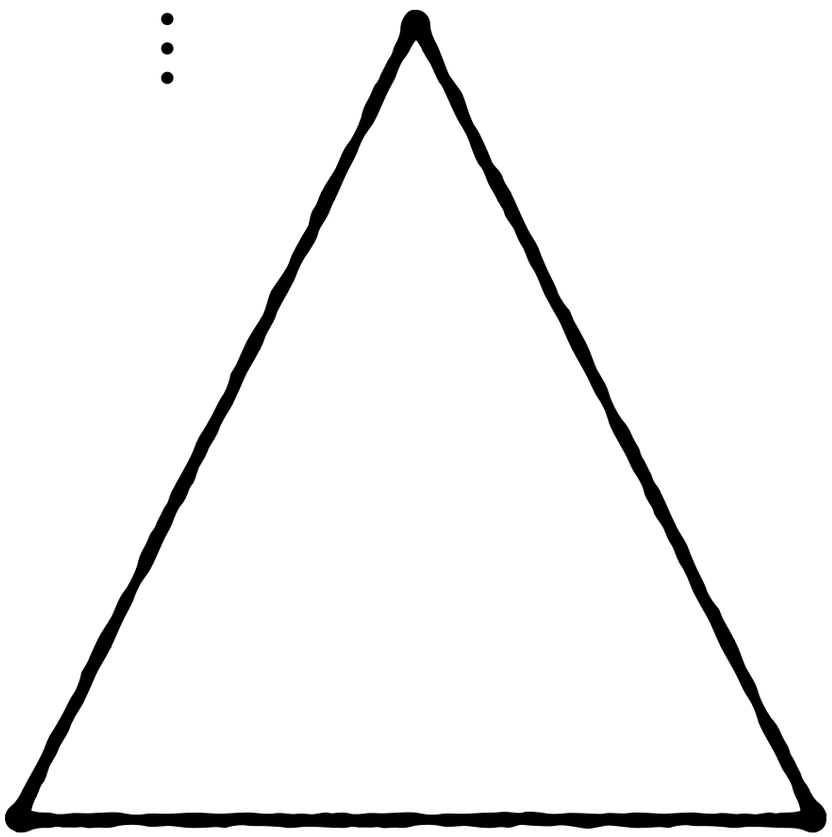
$\Delta = 0$: superrotations: shadow of $\mathcal{O}_{\Delta=0}$ is celestial CFT_2 stress tensor; action: global conformal

Tower of ∞ symmetries

∞ IR triangle

(for projected S-matrix)

asymptotic symmetry



[Hamada,Shiu'18]
[Li,Lin,Zhang'18]

soft theorem

ω^n ⋮

$n = -1,0,1,\ldots$

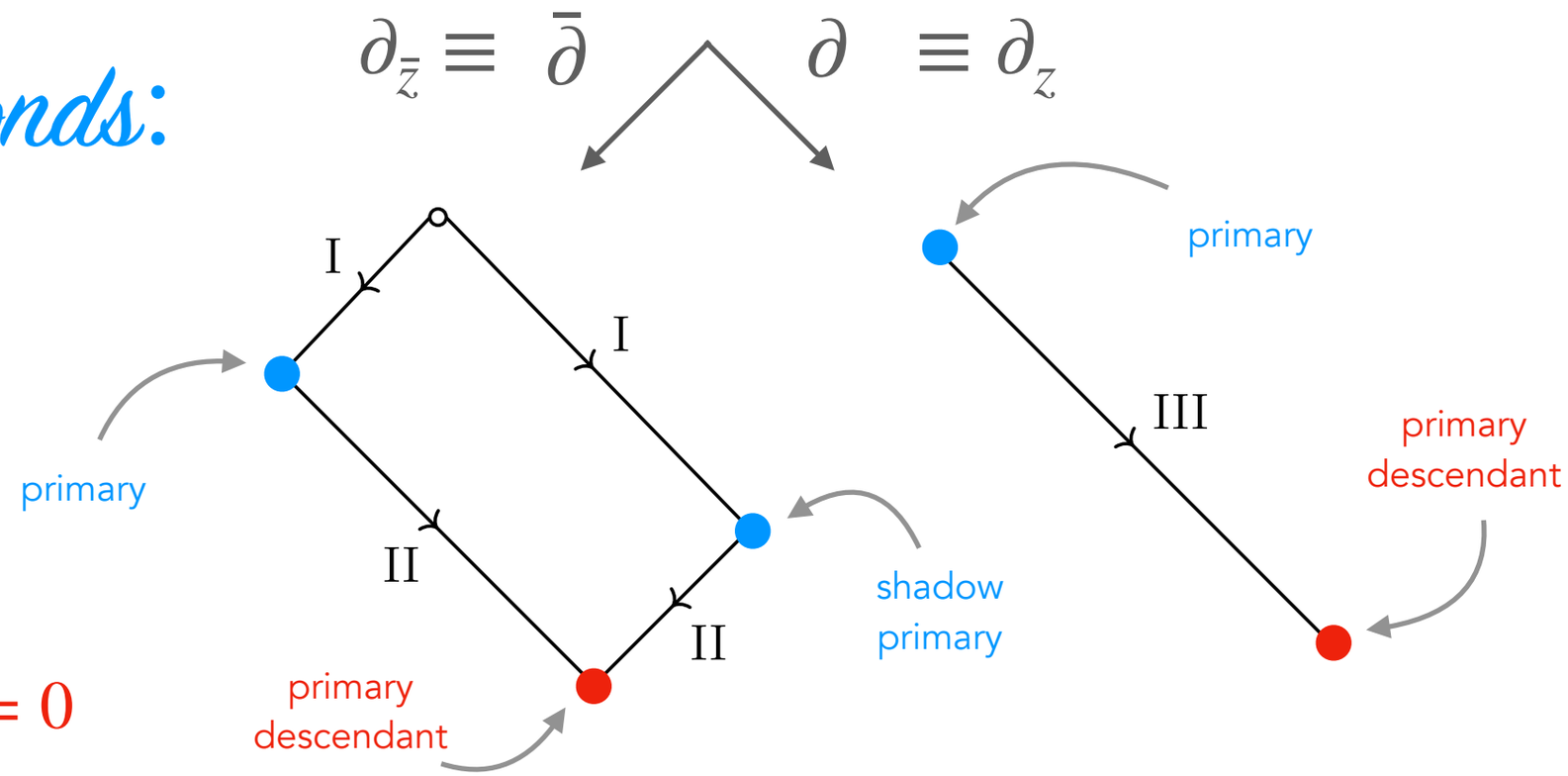
memory effect

⋮

Towers of ∞ symmetries

celestial diamonds:

[Pasterski,AP,Trevisani'21]



conservation equation $\bar{\partial}^{\#}\mathcal{O}(z,\bar{z}) = 0$

depends on types I, II, III: spin of descendant >,<,= spin of parent primary

Classification of all symmetries in celestial $\text{CFT}_{d\geq 2}$.

[Pasterski,AP,Trevisani'21] $d = 2$

[Pano,AP,Trevisani'23] $d > 2$

$d = 2$: $\mathcal{O}_\Delta(x)$ with $\Delta = 1,0,-1,\ldots$ satisfy ∞ dimensional algebra! [Guevara,Himwich,Pate,Strominger'21] [Strominger'21]

Towers of ∞ symmetries

Define a **discrete family** of conformally soft positive-helicity gravitons

$$H^k = \lim_{\varepsilon\to 0} \varepsilon \mathcal{O}_{k+\epsilon,+2} \qquad k = 2,1,0,-1,-2,\ldots$$

supertranslations

superrotation

with weights $(h,\bar{h}) = \left(\frac{k+2}{2}, \frac{k-2}{2}\right)$

and a **consistently-truncated** antiholomorphic mode expansion

$$H^k(z,\bar{z}) = \sum_{n=\frac{k-2}{2}}^{\frac{2-k}{2}} \frac{H_n^k(z)}{\bar{z}^{n+\frac{k-2}{2}}}$$

`wedge condition' = conservation equation for topological operator

$$w_n^p = \frac{1}{\kappa}(p-n-1)!(p+n-1)!H_n^{-2p+4} \qquad p,q = 1,\frac{3}{2},2,\frac{5}{2},\ldots$$

$$[w_m^p, w_n^q] = [m(q-1) - n(p-1)]w_{m+n}^{p+q-2}$$

Arises already in Penrose's twistor construction!

This is the $w_{1+\infty}$ algebra.

[Guevara,Himwich,Pate,Strominger'21]

[Strominger'21]

2D soft actions

A toy model that captures the features of the diamond structure is the higher derivative Gaussian theory with action

[Pasterski,AP,Trevisani'21]

$$S = \int d^2z \left[\partial_z^k \mathcal{O}^s_{\Delta,J} \partial_{\bar{z}}^{\bar{k}} \mathcal{O}^s_{\Delta,J} + \partial_z^k \mathcal{O}^s_{\Delta,-J} \partial_{\bar{z}}^{\bar{k}} \mathcal{O}^s_{\Delta,-J} \right]$$

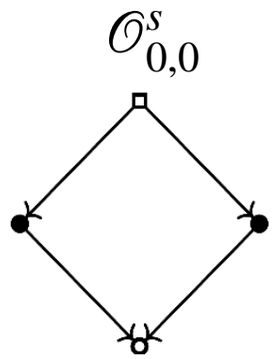
and with conservation equation

$$\partial_z^k \partial_{\bar{z}}^{\bar{k}} \mathcal{O}^s_{\Delta,J} = 0$$

$$\Delta = 1 - \frac{k+\bar{k}}{2} \qquad J = \frac{k-\bar{k}}{2}$$

$$k, \bar{k} \in \mathbb{Z}_>$$

Simplest example:
free boson ($k = \bar{k} = 1$)



QED

Yang-Mills [Cheung,de La Fuente, Sundrum'15] [Magnea'21] [Gonzáles,Rojas'21]

gravity [Nguyen,Salzer'20] [Nguyen'21] [Kalyanapuram'20+'21]

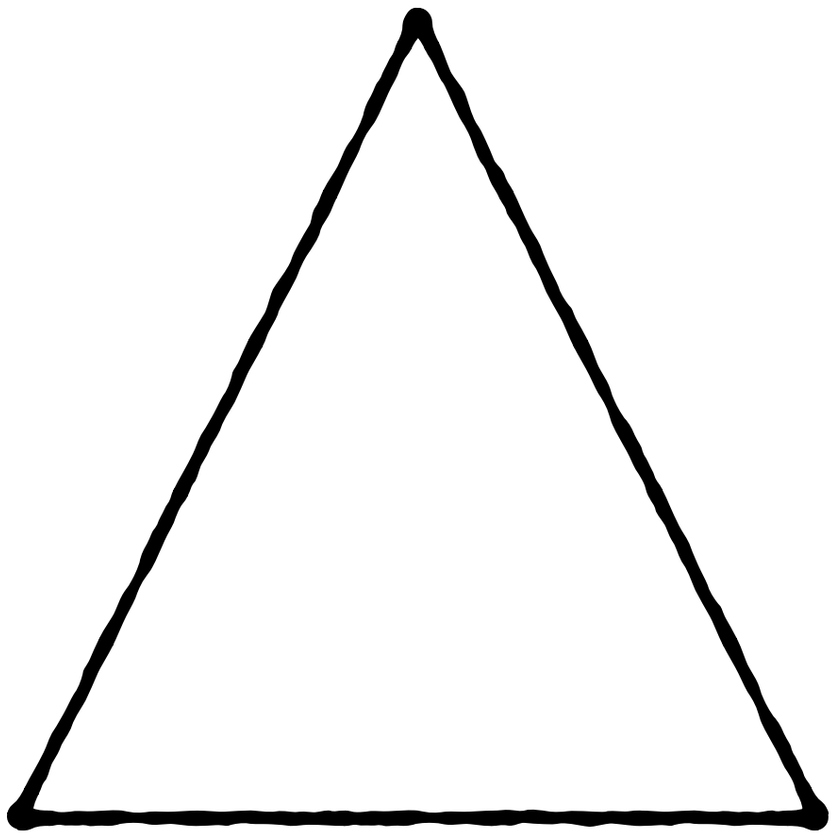
Long-range effects

IR triangle @ tree !

∞-dimensional
symmetry algebra

⊃ local conformal
symmetry on S^2 !

asymptotic symmetry



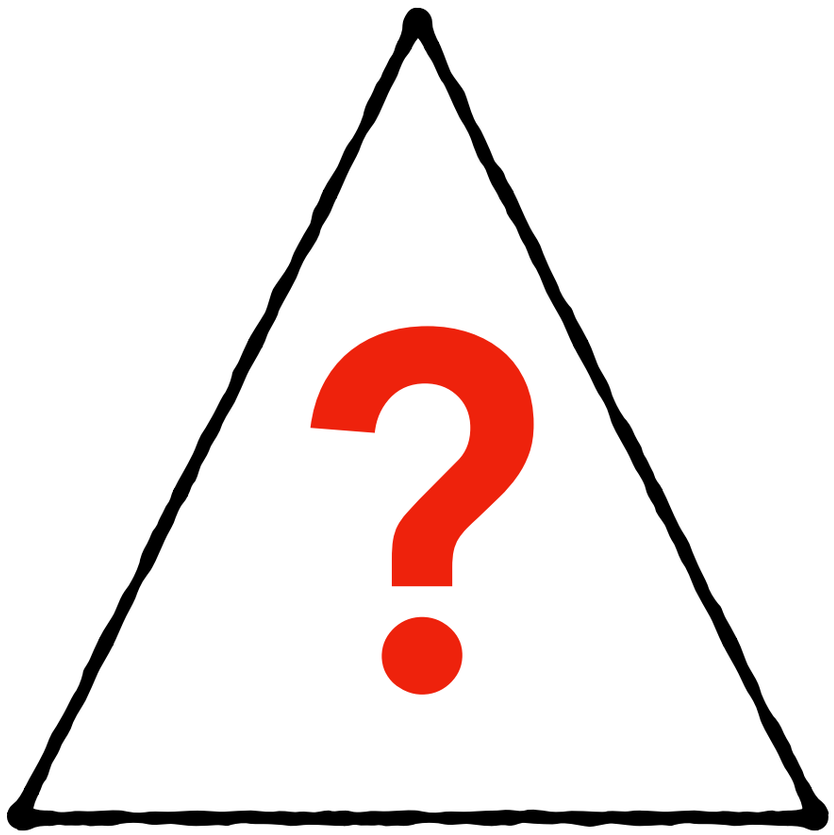
soft theorem **memory effect**

IR triangle @ loop ?

∞-dimensional
symmetry algebra

⊃ local conformal
symmetry on S^2 ?

asymptotic symmetry



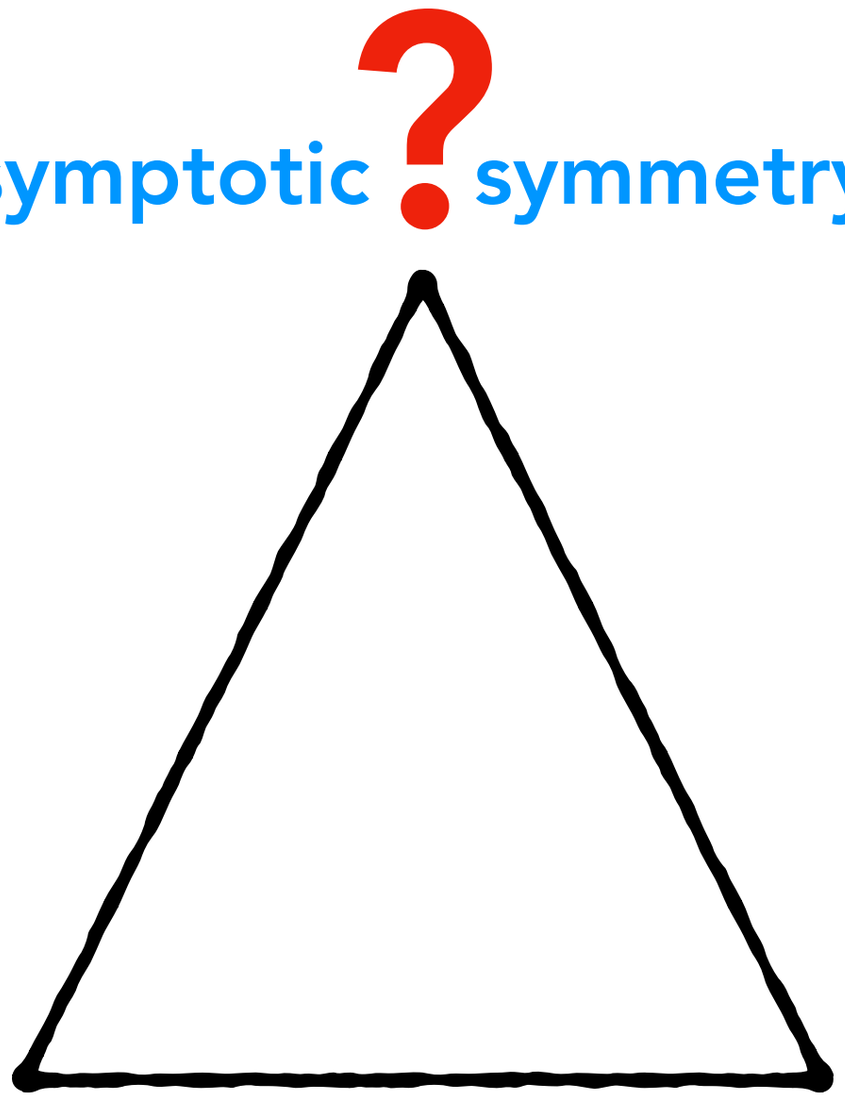
soft theorem **memory effect**

IR triangle @ long-range IR effects

classical and quantum

∞-dimensional symmetry algebra

⊃ local conformal symmetry on S^2 ?



logarithmic soft theorem

[Laddha,Sen'18]
[Sahoo,Sen'18]
[Saha,Sahoo,Sen'19]

tail memory effect

[Sahoo'20]
[Sahoo,Sen'21]
[Sahoo,Krishna'23]

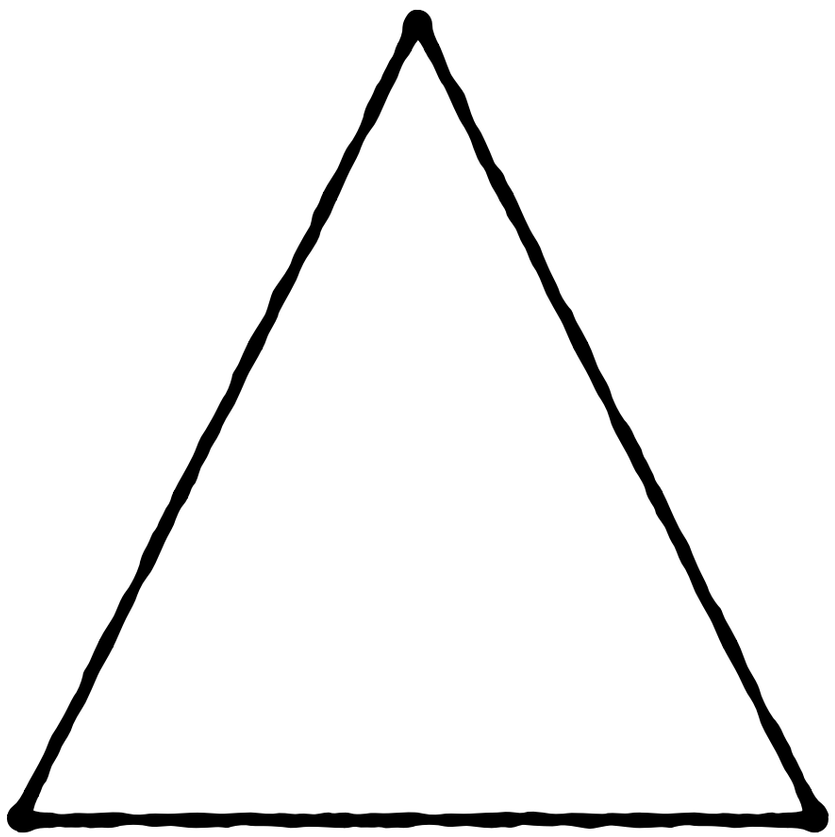
IR triangle @ long-range IR effects

classical and quantum

∞-dimensional symmetry algebra

⊃ local conformal symmetry on S^2 ?

asymptotic ? symmetry



logs render ambiguous all subleading tree-level soft theorems

↑

logarithmic soft theorem

tail memory effect

ω^{-1} leading soft

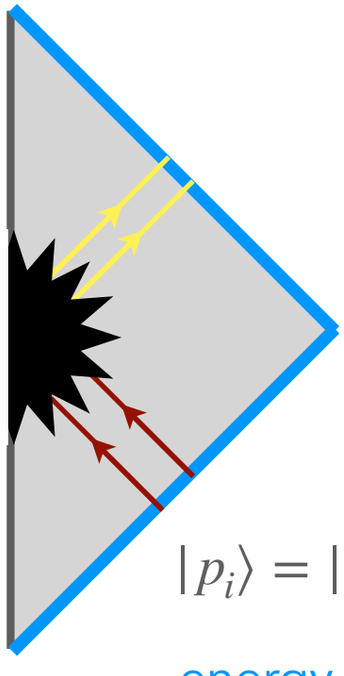
$\log \omega$ subleading soft

⋮

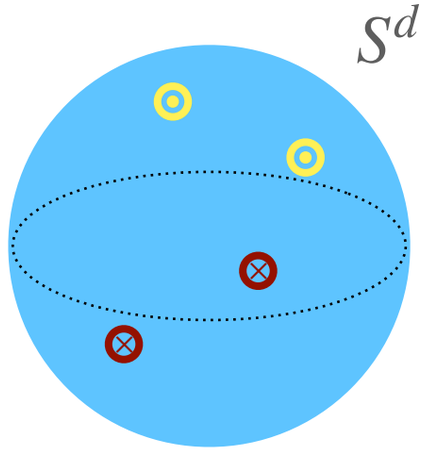
[Laddha,Sen'18]
[Sahoo,Sen'18]
[Saha,Sahoo,Sen'19]

[Sahoo'20]
[Sahoo,Sen'21]
[Sahoo,Krishna'23]

Soft tower



S-matrix



$|p_i\rangle = |\omega_i, x_i\rangle$

energy basis

$\xrightarrow[\int_0^\infty d\omega\omega^{\Delta-1}]{\mathscr{M}\text{ellin}}$

$|\Delta_i, x_i\rangle$

boost-weight basis

$\langle p_n^{\text{out}} \dots | \mathcal{S} | \dots p_1^{\text{in}} \rangle$

translation symmetry

$\langle \mathcal{O}^-_{\Delta_1}(x_1) \dots \mathcal{O}^+_{\Delta_n}(x_n) \rangle$

Lorentz symmetry

energetically soft expansion:

$\frac{1}{\omega}, 1, \omega, \dots$	**tree-level**	$\Delta = 1, 0, -1, \dots$ simple poles
$\log\omega, \dots$	**loops**	$\Delta = 0, \dots$ higher poles

``conformally soft" poles:

Same as tree-level subleading symmetry operators!

Power-law soft theorems

Tree-level amplitudes admit a soft expansion:

[Low'58] [Weinberg'65] [Cachazo,Strominger'14]
[Hamada,Shiu'18] [Li,Lin,Zhang'18]

hard momenta helicity

$$\mathscr{M}_{N+1}(p_1,\ldots,p_N;(\omega,q,\ell)) = \sum_{n=-1}^{\infty} \omega^n S_n(\{p_i\},(q,\ell))\mathscr{M}_N(p_1,\ldots,p_N) + \ldots$$

soft momentum

$$p^\mu = \omega q^\mu(z,\bar{z})$$

⊃ non-universal *

$n=-1$	S_{-1}	Weinberg (leading) soft factor	tree exact & universal
$n=0$	S_0	subleading tree soft factor	
$n>0$	$S_{n>0}$	sub^{n+1} leading tree soft factors	

* see [Elvang,Jones,Naculich'16] for classification in EFT of massless particles

Logarithmic Soft Theorems

Long-range effects yield **novel soft theorems**:

[Sahoo,Sen'18]
[Saha,Sahoo,Sen'19]

hard momenta helicity

$$\mathscr{M}_{N+1}(p_1,\ldots,p_N;(\omega,q,\ell)) = \sum_{n=-1}^{\infty} \omega^n(\ln\omega)^{n+1}\, S_n^{(\ln\omega)}(\{p_i\},(q,\ell))\,\mathscr{M}_N(p_1,\ldots,p_N) + \ldots$$

soft momentum

$$p^\mu = \omega q^\mu(z,\bar{z})$$

⊃ non-universal
$\sim \omega^n(\ln\omega)^{m\neq n+1}$

$n=-1$	$S_{-1}^{(\ln\omega)} \equiv S_{-1}$	Weinberg (leading) soft factor	tree exact & universal
$n=0$	$S_0^{(\ln\omega)} \neq S_0$	leading log soft factor	1-loop exact & universal
$n>0$	$S_{n>0}^{(\ln\omega)} \neq S_{n>0}$	sub^n leading log soft factor	

Is the **universality** of the loop exact logarithmic soft theorems a consequence of **asymptotic symmetries of the S-matrix** ?

Conservation laws

To establish a symmetry interpretation for a soft theorem from *first principles*: for asymptotic symmetry transformations δ compute charges $Q^{\pm}$ from the symplectic structure

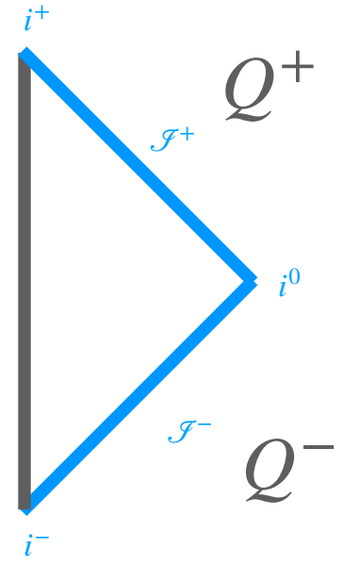
$$\Omega_{i^{\pm} \cup \mathscr{I}^{\pm}}(\delta, \delta') = \delta' Q^{\pm}$$

in the **covariant phase space formalism** and show that the

charge conservation law

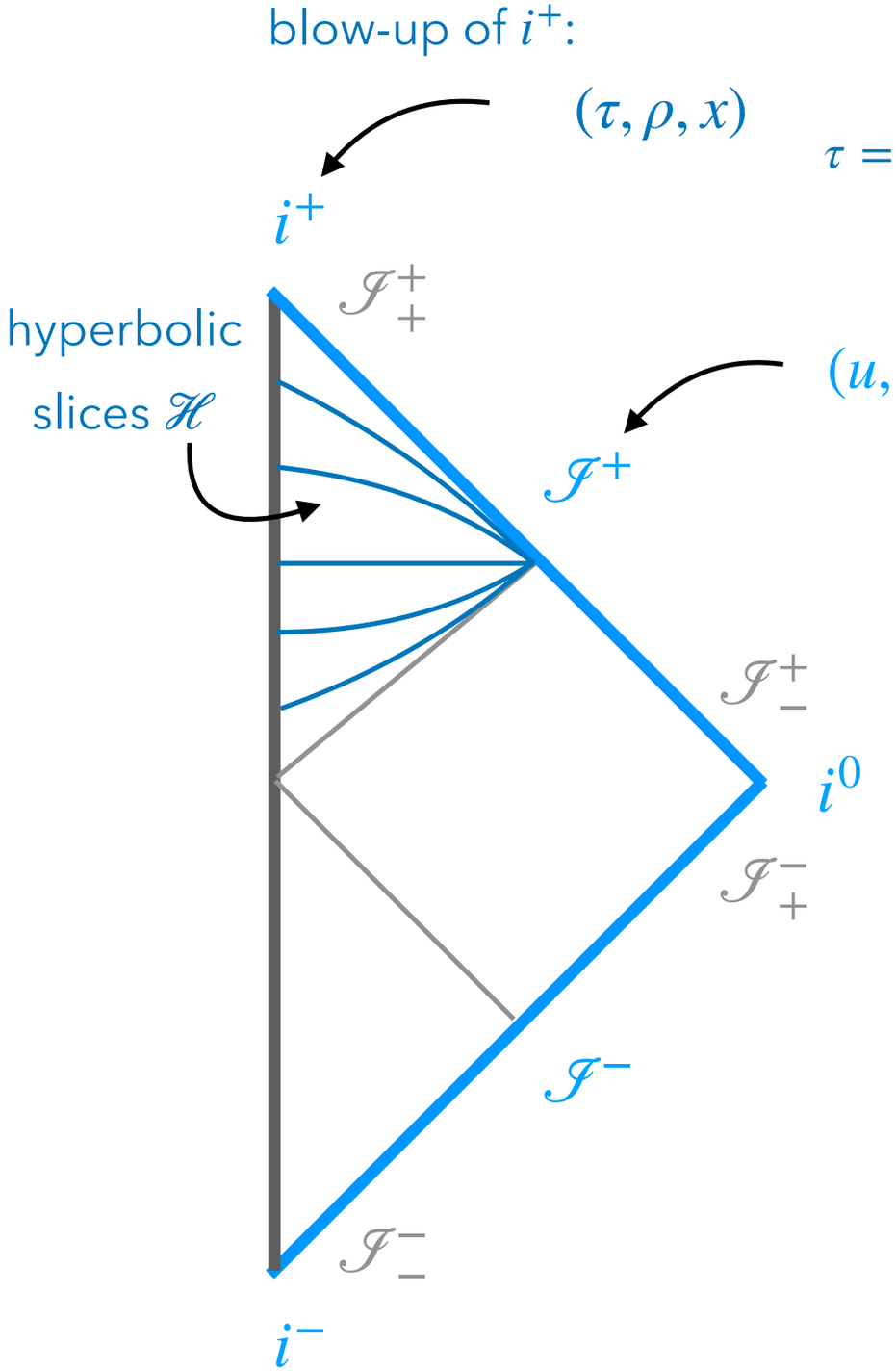
Upon identifying the fields and symmetry parameter antipodally:

$$Q^{+} = Q^{-}$$



corresponds to the **soft theorem**.

Structure at ∞



Euclidean AdS_3 coordinates

$$\tau = \sqrt{t^2 - r^2} \qquad \rho = \frac{r}{\sqrt{t^2 - r^2}}$$

retarded Bondi coordinates

$$u = t - r$$

$$\Omega_{i^+ \cup \mathscr{I}^+}(\delta, \delta') = \delta' Q^+$$

limit $\tau \to \infty$ at fixed ρ — limit $r \to \infty$ at fixed u

$$Q^+_H + Q^+_S = Q^+$$

hard charge — soft charge

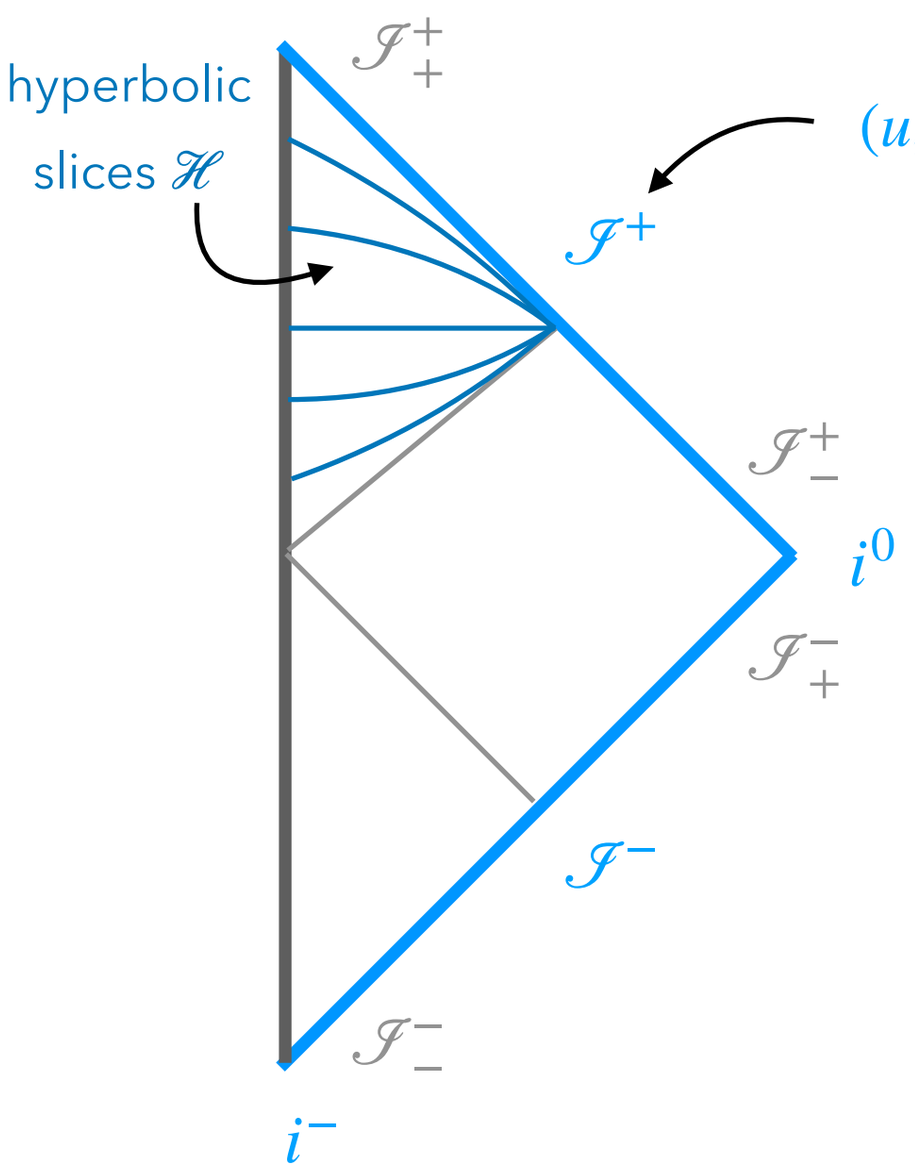
Structure at ∞

blow-up of i^+:

(τ, ρ, x)

Euclidean AdS_3 coordinates

$\tau = \sqrt{t^2 - r^2}$ $\quad \rho = \frac{r}{\sqrt{t^2 - r^2}}$



(u, r, x)

retarded Bondi coordinates

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limit $\tau \to \infty$ at fixed ρ $\quad$ limit $r \to \infty$ at fixed u

$$Q^+_H + Q^+_S = Q^+$$

hard charge $\quad$ soft charge

in soft theorem

action on hard matter $\quad$ soft insertion

Soft graviton factor

[Weinberg'65]

[Cachazo,Strominger'14]

Leading soft factor $\sim \frac{1}{\omega}$:

$$S_{-1} = \frac{\kappa}{2} \sum_{i=1}^{N} \frac{\varepsilon_{\mu\nu} p_i^\mu p_i^\nu}{q \cdot p_i}$$

p_i^μ... hard momenta

$p^\mu = \omega q^\mu$... soft momentum

$\varepsilon^{\mu\nu}$...soft graviton polarization

Subleading soft factor $\sim \omega^0$:

$$S_0 = -\frac{i\kappa}{2} \sum_{i=1}^{N} \frac{\varepsilon_{\mu\nu} p_i^\mu q_\lambda J_i^{\lambda\nu}}{q \cdot p_i}$$

ambiguous if long-range IR effects

Soft graviton factor

[Weinberg'65]
[Sahoo,Sen'18]
[Saha,Sahoo,Sen'19]

Leading soft factor $\sim \frac{1}{\omega}$:

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p_i^μ... hard momenta
$p^\mu = \omega q^\mu$... soft momentum
$\varepsilon^{\mu\nu}$...soft graviton polarization

Subleading soft factor $\sim \ln \omega$:

$$S_0^{(\ln\omega)} = S_{0,\text{classical}}^{(\ln\omega)} + S_{0,\text{quantum}}^{(\ln\omega)}$$

classical: late time gravitational radiation from particle acceleration via long-range gravitational interaction

$$S_{0,\text{classical}}^{(\ln\omega)} = \frac{i(\frac{\kappa}{2})^3}{8\pi} \sum_{i=1}^{N} \frac{\varepsilon_{\mu\nu} p_i^\nu q_\rho}{q \cdot p_i} \sum_{j\neq i, \eta_i\eta_j=1} \frac{(p_i \cdot p_j)\left[p_i^\mu p_j^\rho - p_j^\mu p_i^\rho\right]\left[2(p_j \cdot p_j)^2 - 3p_i^2 p_j^2\right]}{\left[(p_i \cdot p_j)^2 - p_i^2 p_j^2\right]^{3/2}}$$

quantum: $\omega_{\text{soft}} \ll \omega_{\text{loop}} \ll \omega_{\text{hard}}$

(1-loop)

$$S_{0,\text{quantum}}^{(\ln\omega)} = -\frac{(\frac{\kappa}{2})^3}{16\pi^2} \sum_{i=1}^{N} \frac{\varepsilon_{\mu\rho} p_i^\rho q_\nu}{q \cdot p_i} \left(p_i^\mu \partial_{p_i}^\nu - p_i^\nu \partial_{p_i}^\mu\right) \sum_{j\neq i} \frac{2(p_i \cdot p_j)^2 - p_i^2 p_j^2}{\sqrt{(p_i \cdot p_j)^2 - p_i^2 p_j^2}} \ln\left(\frac{p_i \cdot p_j + \sqrt{(p_i \cdot p_j)^2 - p_i^2 p_j^2}}{p_i \cdot p_j - \sqrt{(p_i \cdot p_j)^2 - p_i^2 p_j^2}}\right)$$

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+ drag

quantum: $\omega_{\text{soft}} \ll \omega_{\text{loop}} \ll \omega_{\text{hard}}$ $\omega_{\text{IR}} \ll \omega_{\text{loop}} \ll \omega_{\text{soft}}$

(1-loop)

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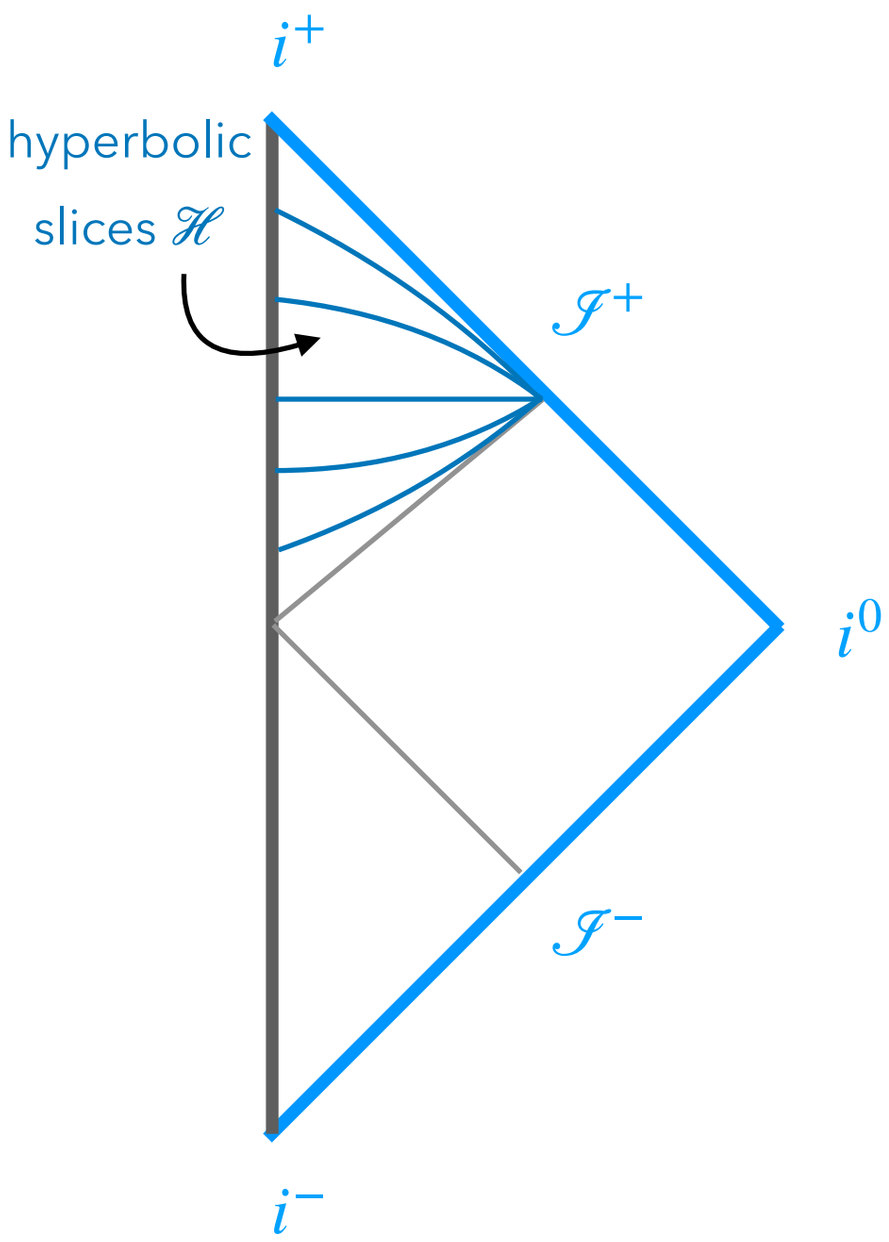
Symmetry @ $i^+ \cup \mathscr{I}^+$

$x \in S^2$

$y^\alpha = (\rho, x) \in \mathscr{H}$



Superrotation across $i^+ \cup \mathscr{I}^+$

$\mathscr{I}^+$: $Y^A(x)$

$$\delta_Y \gamma_{AB} = 2D_{(A}Y_{B)} - D\cdot Y \gamma_{AB}$$

$$\delta_Y C_{AB} = \left[\mathscr{L}_Y - \tfrac{1}{2} D\cdot Y(1 - u\partial_u)\right] C_{AB} + u\left[D_{(A}(D^2+1)Y_{B)} - D_A D_B D\cdot Y - \tfrac{1}{2}\gamma_{AB}(D^2+4)D\cdot Y\right]$$

i^+ : $\bar{Y}^\alpha(y)$ $\qquad \delta\varphi = \bar{Y}^\alpha \partial_\alpha \varphi$

vector Green's function

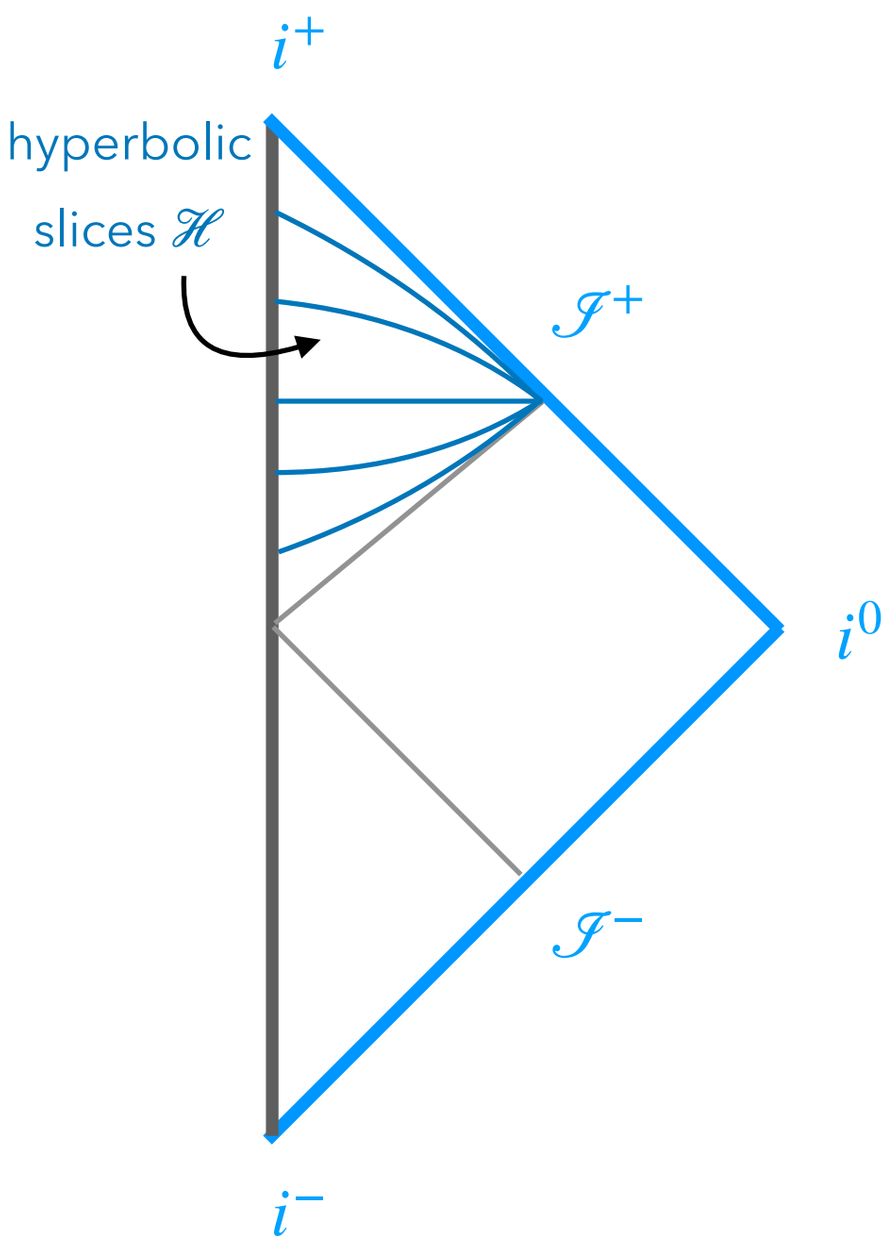
$$i^+ \cup \mathscr{I}^+ : \quad \bar{Y}^\alpha(y) = \int_{S^2} d^2x\, G^\alpha_A(y;x) Y^A(x)$$

$\Rightarrow$ Superrotation vector field extends smoothly across $i^+ \cup \mathscr{I}^+$.

Phase space @ $i^+ \cup \mathscr{I}^+$

$x \in S^2$

$y^\alpha = (\rho, x) \in \mathscr{H}$



Asymptotic phase space on $i^+ \cup \mathscr{I}^+$

$$i^+ : h_{\tau\tau}(\tau, y) \overset{\tau\to\infty}{=} \frac{1}{\tau} h_{\tau\tau}(y) + \dots$$

'Coulombic' mode sourced by matter stress tensor

$$\varphi(\tau, y) = \frac{\sqrt{m}}{2(2\pi)^{3/2}} \sum_{n=0}^{\infty} \frac{e^{-im\tau}}{\tau^{\frac{3}{2}+n}} \left(\overset{\ln}{b}_n(y) \ln\tau + b_n(y) \right) + \text{c.c.} + \dots$$

$\overset{\ln}{b}_n$ and b_{n+1} for $n \geq 0$ fixed by $b \equiv b_0$

$$\mathscr{I}^+ : \; C_{AB}(u, x) \overset{u\to\pm\infty}{=} C^{(0),+}_{AB}(x) + \frac{1}{u} C^{(1),+}_{AB}(x) + \dots$$

$= \lim_{r\to\infty} \frac{1}{r} h_{AB}(u, r, x)$ — displacement memory — tail

$\Rightarrow$ Tails and logs from long-range interactions.

Gravity: hard and soft charges

[Choi,Laddha,AP'24]

$$\Omega_{i^+ \cup \mathscr{I}^+} = \Omega_{i^+} + \Omega_{\mathscr{I}^+}$$

$$Q = Q_H + Q_S$$

hard charge soft charge

$$\Omega_{i^\pm \cup \mathscr{I}^\pm}(\delta, \delta_Y) = \delta Q_\pm$$

superrotation

Gravity: hard and soft charges

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diverges logarithmically as $\tau \to \infty, u \to \infty$

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Gravity: hard and soft charges

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diverges logarithmically as $\tau \to \infty, u \to \infty$

regulate via late-time cutoff Λ^{-1}

$$\Omega_{i^+ \cup \mathscr{I}^+} = \Omega_{i^+} + \Omega_{\mathscr{I}^+}$$

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hard charge soft charge

$$\Omega_{i^\pm\cup\mathscr{I}^\pm}(\delta\,, \delta_Y) = \delta Q_\pm$$

superrotation

Regularized Noether charge:

$$Q^\Lambda = \ln\Lambda^{-1}\left(Q_H^{(\ln)} + Q_S^{(\ln)}\right) + \left(Q_H^{(0)} + Q_S^{(0)}\right) + \dots$$

$\overset{1}{h}_{\tau\tau}$ 'Coulombic'

$$Q^{(\ln)}_{H,+}[\bar{Y}] = \int_{i^+} d^3y\, \bar{Y}^\alpha\, \overset{3,\ln}{T}{}_{\tau\alpha}$$

leading interacting stress tensor

$$Q^{(0)}_{H,+}[\bar{Y}] = \int_{i^+} d^3y\, \bar{Y}^\alpha \overset{3}{T}_{\tau\alpha}$$

free stress tensor

$$Q_S^{(\ln)}[Y] = -\frac{2}{\kappa}\int_{\mathscr{I}^+} du\, d^2x\, D_z^3 Y^z\, \partial_u(u^2\partial_u C^{zz}) + \text{c.c.}$$

log soft projector

$$Q_S^{(0)}[Y] = -\frac{2}{\kappa}\int_{\mathscr{I}^+} du\, d^2x\, D_z^3 Y^z\, u\partial_u C^{zz} + \text{c.c.}$$

D_A ... covariant derivative on S^2

sub tree soft projector

$Q^{(0)}$ charge

$$Q^{\Lambda} = \ln \Lambda^{-1} \left(Q_H^{(\ln)} + Q_S^{(\ln)} \right) + \left(Q_H^{(0)} + Q_S^{(0)} \right) + \dots$$

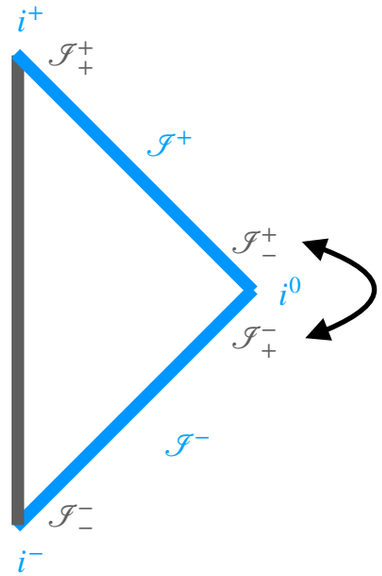
$$\equiv Q^{(0)}$$

Conservation law:

[Campiglia,Laddha'15]

$$Q_+^{(0)} = Q_-^{(0)}$$

Upon identifying the fields and gauge parameter antipodally:



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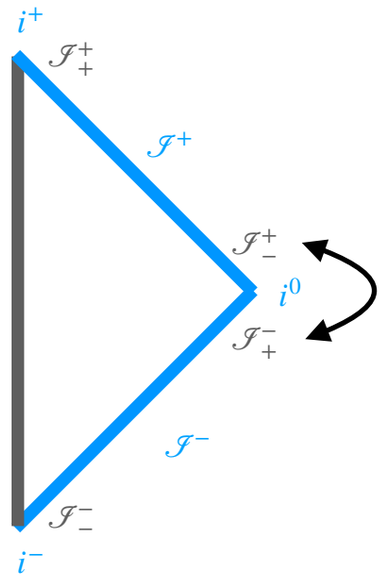
judicious choice of **superrotation** $Y^A(x)$

Upon identifying the fields and gauge parameter antipodally:

[Kapec,Lysov,Pasterski,Strominger'14]



Tree-level subleading soft **graviton** theorem:

$$\mathscr{M}_{N+1} = \left(\omega^{-1} S_{-1} + \omega^0 S_0 \right) \mathscr{M}_N + \dots$$

tree-level soft expansion

[Cachazo,Strominger'14]

$Q^{(0)}$ charge

$$Q^{\Lambda} = \ln \Lambda^{-1} \left(Q_H^{(\ln)} + Q_S^{(\ln)} \right) + \left(Q_H^{(0)} + Q_S^{(0)} \right) + \dots$$

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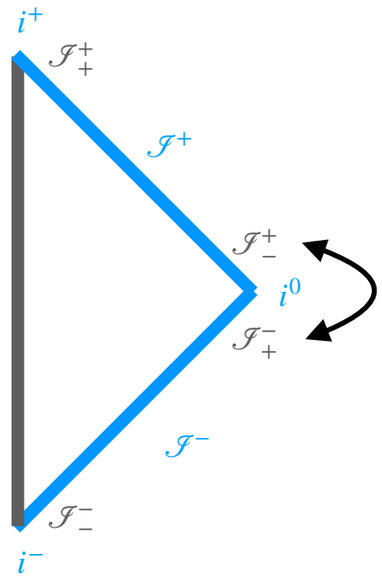


Tree-level subleading soft **graviton** theorem:

$$\mathscr{M}_{N+1} = \left(\omega^{-1} S_{-1} + \omega^0 S_0 \right) \mathscr{M}_N + \dots$$

tree-level soft expansion

[Cachazo,Strominger'14]

Recall: IR effects render subleading soft theorem at tree-level ambiguous.

$Q^{(\ln)}$ charge

$$Q^{\Lambda} = \ln \Lambda^{-1} \left(Q_H^{(\ln)} + Q_S^{(\ln)} \right) + \left(Q_H^{(0)} + Q_S^{(0)} \right) + \dots$$

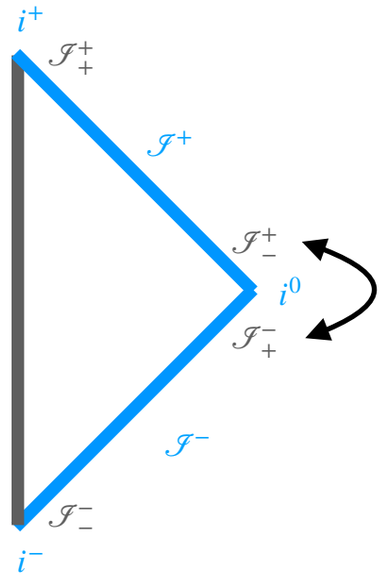
$$\equiv Q^{(\ln)}$$

Conservation law:

[Choi,Laddha,AP'24]

$$Q_+^{(\ln)} = Q_-^{(\ln)}$$

Upon identifying the fields and gauge parameter antipodally:



This is **exact** in the **coupling** κ !

$Q^{(\ln)}$ charge

$$Q^{\Lambda} = \ln \Lambda^{-1} \left(Q_H^{(\ln)} + Q_S^{(\ln)} \right) + \left(Q_H^{(0)} + Q_S^{(0)} \right) + \dots$$

$$\equiv Q^{(\ln)}$$

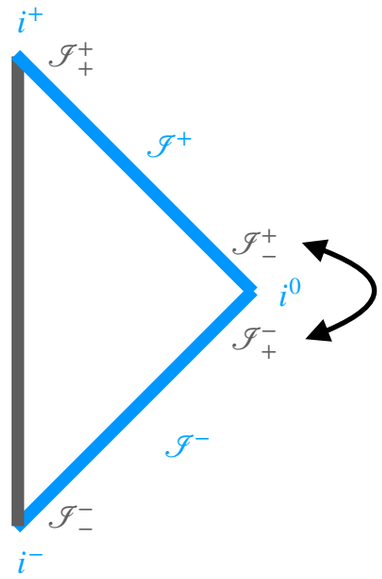
Conservation law:

[Choi,Laddha,AP'24]

$$Q_+^{(\ln)} = Q_-^{(\ln)}$$

judicious choice of
superrotation $Y^A(x)$

Upon identifying the fields and gauge parameter antipodally:



Logarithmic soft **graviton** theorem:

$$\mathscr{M}_{N+1} = \left(\omega^{-1} S_{-1} + \omega^0 \ln \omega \, S_0^{(\ln \omega)} \right) \mathscr{M}_N + \dots$$

log soft expansion

[Sahoo,Sen'18] [Saha,Sahoo,Sen'19]

This establishes the symmetry interpretation of the classical logarithmic soft graviton theorem.* [Choi,Laddha,AP'24]

* up to drag terms

Gravity is a drag

Due to the spacetime curvature caused by the matter the soft graviton experiences a gravitational drag at late times:

$$\Delta_{\text{drag}} S^{(\ln\omega)}_{0,\text{classical}} = -\frac{i}{4\pi}\log\omega \sum_{j,\eta_j=-1} (q\cdot p_j)\underbrace{\frac{\kappa}{2}\sum_{i=1}^{N}\frac{\varepsilon_{\mu\nu}p_i^\mu p_i^\nu}{q\cdot p_i}}$$

[Saha,Sahoo,Sen'19]

leading soft factor S_{-1}

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[Saha,Sahoo,Sen'19]

The resulting time delay to reach a detector at distance r can be captured by defining the retarded time at the detector:

$$u = t - r + \log r \times f_{\text{drag}}$$

effect of the long range gravitational force on the gravitational wave as it travels from the scattering center to the detector

$$f_{\text{drag}} = 2G\sum_{j=1}^{N} q\cdot p_j$$

$$q^\mu = (1, \frac{\vec{x}}{r})$$

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How does the drag show up in the charge?

Gravity is a drag

The time-delay results in a logarithmically divergent phase which effectively shifts the shear:

u^0 term in $u \to \pm\infty$ expansion: depends on matter stress tensor at i^+

$$C_{AB}(u,x) \to C_{AB}(u,x) - \frac{\kappa}{2}\ln r \overset{0}{h}_{rr}(x)\partial_u C_{AB}(u,x)$$

sQED: [Bhatkar'19]

This results in an additional contribution to the log soft charge:

$$Q^{\text{drag}}_{S,+}[Y] = \ln\Lambda^{-1}\left[-\frac{1}{2\kappa}\int_{\mathscr{I}^+} du d^2x\, D_z^3 Y^z \overset{0}{h}_{rr}\underbrace{\partial_u C^{zz}}\right]$$

leading soft projector

action on Fock state

judicious choice of **superrotation** $Y^A(x)$

This yields the drag term in the soft theorem:

$$S_{-1} = \frac{\kappa}{2}\sum_{i=1}^{N}\frac{\varepsilon_{\mu\nu}p_i^\mu p_i^\nu}{q\cdot p_i}$$

leading soft factor

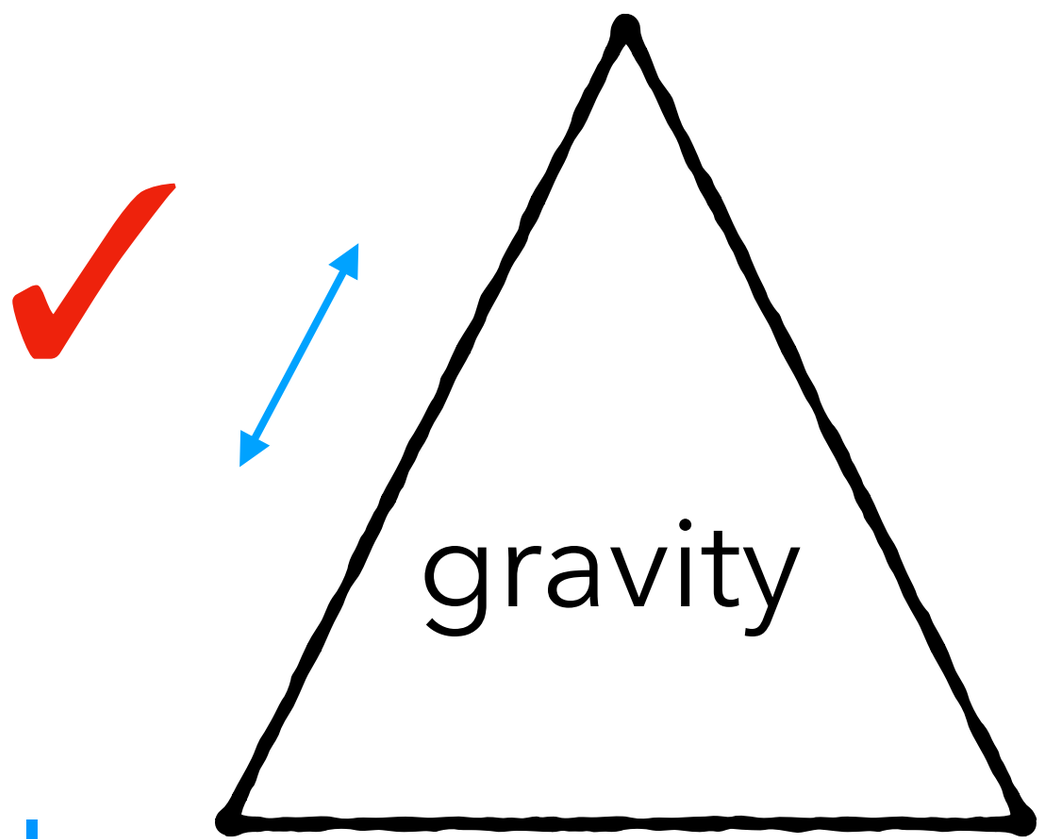
$$\Delta_{\text{drag}} S^{(\ln\omega)}_{0,\text{classical}} = -\frac{i}{4\pi}\log\omega\sum_{j,\eta_j=-1}(q\cdot p_j)\,S_{-1}$$

[Choi,Laddha,AP'24]

Classical superrotation IR triangle

[Choi,Laddha,AP'24]

superrotation



classical log soft theorem

[Laddha,Sen'18]
[Sahoo,Sen'18]
[Saha,Sahoo,Sen'19]

Memory and its tail

Shear @ late times: $C_{AB}(u,x) \overset{u\to+\infty}{=} C^{(0),+}_{AB}(x) + \frac{1}{u} C^{(1),+}_{AB}(x) + \dots$

linear displacement memory sourced by matter field:

$$\Delta C^{(0)}_{AB} = C^{(0),+}_{AB} - C^{(0),-}_{AB}$$

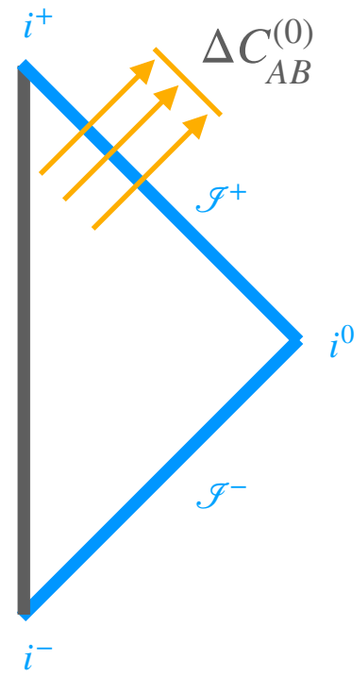
free

matter stress tensor

$$C^{(0),\pm}_{AB} = -\frac{\kappa^2}{8\pi}\int_{i^\pm} d^3y \frac{(\partial_A q\cdot\mathcal{Y})(\partial_B q\cdot\mathcal{Y}) + \frac{1}{2}\gamma_{AB}}{q\cdot\mathcal{Y}} {}^{3}T^{\text{matt}}_{\tau\tau}(y)$$

$$x^\mu = \tau\mathcal{Y}(y)$$

unit vector in Minkowski



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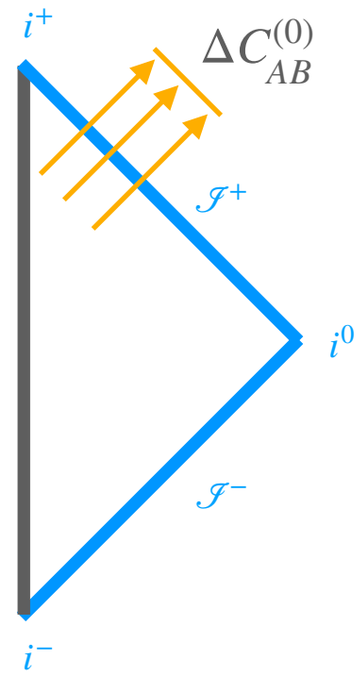
matter stress tensor

$$\mathscr{P}^\alpha(y) = \overset{3}{T}{}^{\rm matt}_{\tau\tau}(y)\mathscr{Y}^\alpha$$

linear momentum density of massive scalar

$$x^\mu = \tau\mathscr{Y}(y)$$

unit vector in Minkowski



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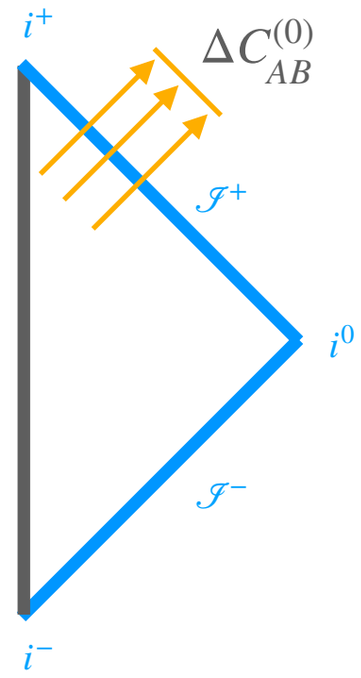

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linear momentum density of massive scalar

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unit vector in Minkowski

tail to the memory
sourced by matter field:

$$\Delta C^{(1)}_{AB} = C^{(1),+}_{AB} - C^{(1),-}_{AB}$$

long-range interaction

$$C^{(1),\pm}_{AB} = -\frac{\kappa^2}{8\pi}(\partial_A q^\mu)(\partial_B q^\nu)\int_{i^\pm} d^3y\Bigg[\frac{(q\cdot\mathscr{Y})\mathscr{D}^\alpha(\mathscr{Y}_\mu\mathscr{Y}_\nu) - (\mathscr{Y}_\mu\mathscr{Y}_\nu + \frac{1}{2}\eta_{\mu\nu})\mathscr{D}^\alpha(q\cdot\mathscr{Y})}{q\cdot\mathscr{Y}}\overset{3,\ln}{T}{}^{\rm matt}_{\tau\alpha}$$

matter stress tensor

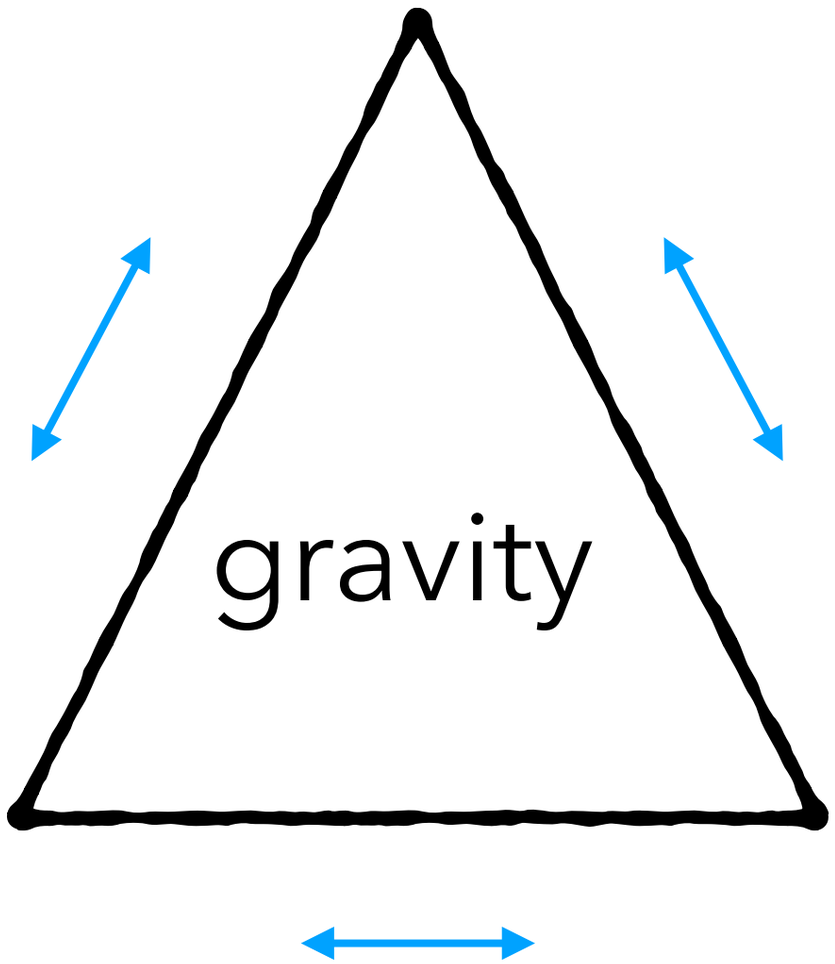
$$-\left((\mathscr{Y}_\mu\mathscr{Y}_\nu + \frac{1}{2}\eta_{\mu\nu})k^{\alpha\beta} + (\mathscr{D}^\alpha\mathscr{Y}_\mu)(\mathscr{D}^\beta\mathscr{Y}_\nu) - \frac{1}{2}\eta_{\mu\nu}(\mathscr{D}^\alpha\mathscr{Y}_\sigma)(\mathscr{D}^\beta\mathscr{Y}^\sigma)\right)\overset{2}{T}{}^h_{\alpha\beta}\Bigg]$$

gravity stress tensor

Classical superrotation IR triangle

[Choi,Laddha,AP'24]

superrotation



classical log soft theorem

[Laddha,Sen'18]
[Sahoo,Sen'18]
[Saha,Sahoo,Sen'19]

tail to the memory effect

[Saha,Sahoo,Sen'19]
[Choi,Laddha,AP'24]

particles

fields ✓

Log symmetry for scalar QED

Gauge field $\mathscr{A}_\mu$ + minimally coupled complex massive scalar field ϕ

$$\delta\mathscr{A}_\mu = \partial_\mu \epsilon^{\mathscr{I}^+} \qquad\qquad \delta\phi = ie\epsilon^{i^+}\phi$$

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divergent superphaserotation:

$$r\epsilon(x) + \frac{u}{2}(2 + D^2)\epsilon(x) + \ldots \qquad \tau\bar{\epsilon}(y) + \ldots$$

$$\bar{\epsilon}(y) = \int_{S^2} d^2\hat{x}\, G(y;x)\epsilon(x)$$

$$r\epsilon(x) + \ldots \qquad \tau\rho = r$$

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linear and log divergence as $\tau \to \infty, u \to \infty$

$$\longrightarrow \quad \Omega_{i^+ \cup \mathscr{I}^+} = \Omega_{i^+} + \Omega_{\mathscr{I}^+}$$

subtract linear & regulate log via late-time cutoff Λ^{-1}

$$Q^\Lambda = \ln \Lambda^{-1} \left(Q_H^{(\ln)} + Q_S^{(\ln)} \right) + \left(Q_H^{(0)} + Q_S^{(0)} \right) + \ldots$$

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[Campiglia,Laddha'19]

$$Q_+^{(\ln)} = Q_-^{(\ln)} \qquad Q_+^{(0)} = Q_-^{(0)}$$

We show $Q_+ = Q_-$ is **exact in coupling** !

[Choi,Laddha,AP'24]

matches logarithmic soft photon theorem

[Sahoo,Sen'18] [Saha,Sahoo,Sen'19]

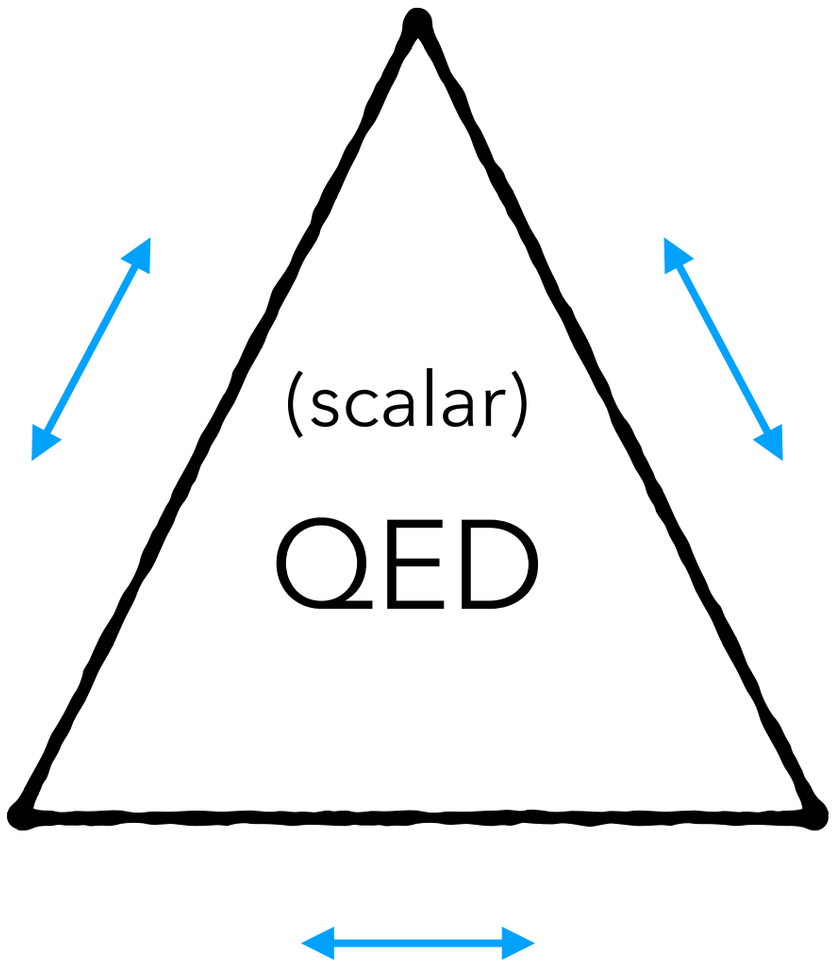
matches subleading tree-level soft photon theorem

[Low'58]

Classical super*phase*rotation IR triangle

[Choi,Laddha,AP'24]

superphaserotation



classical log soft theorem

[Laddha,Sen'18]
[Sahoo,Sen'18]
[Saha,Sahoo,Sen'19]

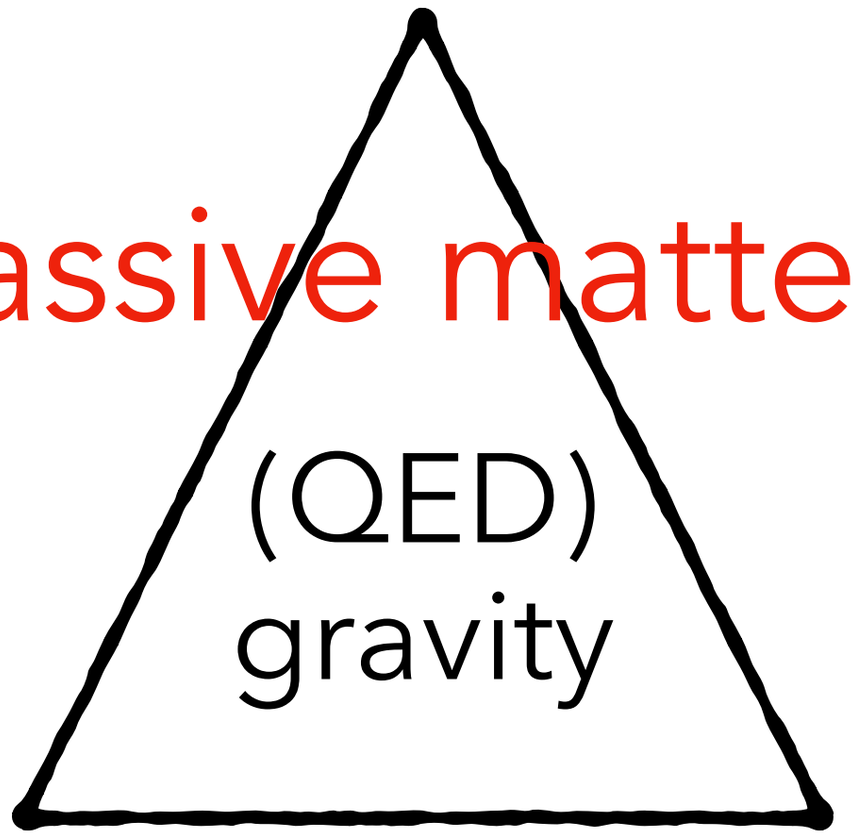
tail to the memory effect

[Saha,Sahoo,Sen'19] particles
[Choi,Laddha,AP'24] fields

Classical super(phase)rotation IR triangle

[Choi,Laddha,AP'24]

super(phase)rotation



classical log soft theorem

[Laddha,Sen'18]
[Sahoo,Sen'18]
[Saha,Sahoo,Sen'19]

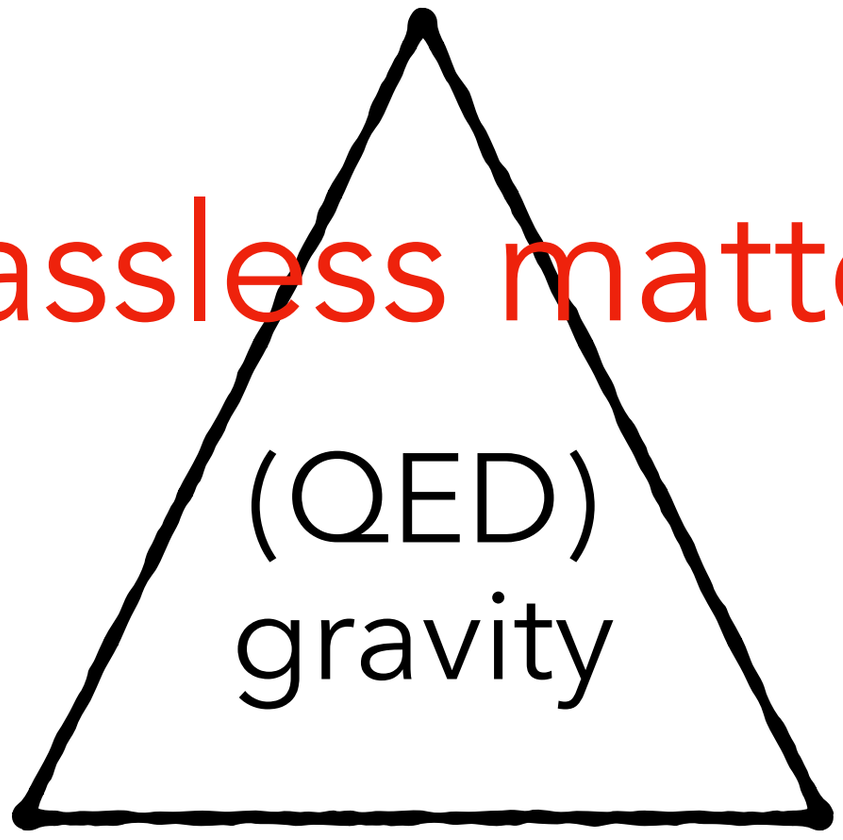
tail to the memory effect

[Saha,Sahoo,Sen'19] particles
[Choi,Laddha,AP'24] fields

Classical super(phase)rotation IR triangle

[Choi,Kadhe,AP - to appear]

super(phase)rotation



(more subtle)

classical log soft theorem

tail to the memory effect

Summary

Noether charge from long-range IR effects:

$$Q^{\Lambda} = \ln \Lambda^{-1} \left(Q_H^{(\ln)} + Q_S^{(\ln)} \right) + \left(Q_H^{(0)} + Q_S^{(0)} \right) + \dots$$

physical IR scale

- super (phase) rotation symmetry ✓
- associated charge Q^{Λ} from first principles & exact in the coupling !
- regulator Λ^{-1} arises from the relevant infrared scale: large $|\tau|$ & $|u|$
- Λ can be removed in the end since $Q_{\pm}^{(\ln)}$ are finite
- log charge conservation law = log soft theorem ✓

exact in **coupling** !

Long-range interactions lead to new conservation laws!

Infrared surprises

Infrared triangles

Complete?

4D = 2D

What are the axioms of celestial CFT?

Exact celestial duals? [Costello,Paquette'22] [Costello,Paquette,Sharma'22]

Towers of ∞ symmetries

Symmetries of what theories?

How powerful constraints?

Long-range effects

Beyond gravity & QED ? Beyond leading log ?

see also [Donnay,Nguyen,Ruzziconi'22], [Agrawal,Donnay,Nguyen,Ruzziconi'23]

Quantum log soft factor?

$\log(u)$? [Campiglia,Laddha'19]

logarithmic CFT? [Bissi,Donnay,Valsesia'24]

Thank you!