InFoMM
Industrially Focused Mathematical Modelling

Optimisation of filter geometry: modelling the contaminant build-up and efficiency

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When we wash clothes in a washing machine, the clothes release fibres and small microplastic particles. If the water is left to drain freely from the washing machine these fibres and microplastics are released into the wastewater system, and would eventually make their way into rivers and oceans. The environmental implications of this are significant, ranging from microplastics in our drinking water to damaging the ecosystem. To address this problem, a filter is placed in the outflow pipeline of a washing machine. This filter consists of a box with permeable mesh walls that trap the microplastics and fibres (figure 1, left). As contaminant particles build up on the membrane surface, they provide an added resistance to the flow, making it more difficult for water to pass through. Once the filter is full, the user will be notified to replace the filter with a new one.

Beko wish to prolong the filter life and speed up the design phase of a new filter that can contain more contaminant. To tackle this problem, we have developed a mathematical model for simulating the fluid flow in the filter and the resulting distribution of microfibres on the filter surface. Our goal is to optimise Beko's filter design. The problem at hand is one of morphodynamics: we wish to understand the interaction between the unfiltered fluid flow and the resulting build-up and evolution of the contaminants on the surface, which is termed a *cake*. The build-up of this cake alters the geometry within which the unfiltered fluid flows and also affects how difficult it is for the fluid to permeate the cake. This, in turn, changes the flow field, creating a feedback cycle (figure 1, right). To systematically reduce the complexity of this moving-boundary problem, we exploit the fact that the cake builds up much more slowly than the time it takes for fluid to pass through the filter. This allows us to solve a fluid-flow problem for an unchanging cake distribution.

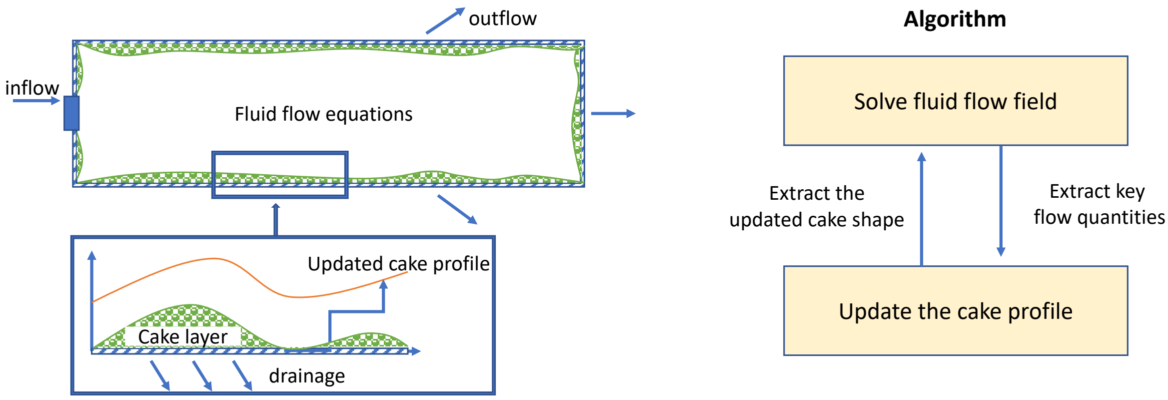

Figure 1 – (Left) A sketch of the filter and the cake layer. (Right) The algorithm used to solve the problem.

Our model is comprised of two components: the flow problem and the evolution of the cake. A key quantity is the reduced Reynolds number, which describes the nature of the unfiltered fluid flow and informs us of the modelling approach we should take. This key quantity is derived from the viscosity, density, and speed of the flow, along with the dimensions of the flow domain. In this problem, the reduced Reynolds number is large, which informs our modelling approach; we use equations seen in modelling inviscid flows such as water flow through a pipe, for example.

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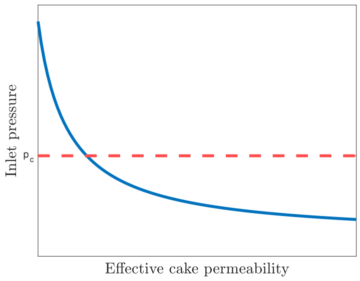
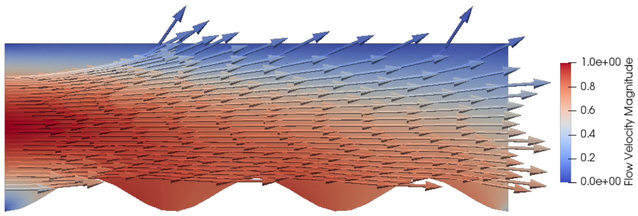
Solving the flow problem allows us to determine how the inlet pressure increases to maintain a constant flow rate of fluid through the filter as the cake layer grows, increasing the wall resistance to the flow. The dashed line in figure 2(a) identifies the critical inlet pressure, p_c, above which the filter efficiency will be compromised. The solid curve in figure 2(a) shows how the inlet pressure varies as we increase the thickness of a uniform cake layer on the filter walls. The numerical results shown in figure 2(a) thus allow us to predict when the inlet pressure will exceed p_c for a given geometry and operating configuration, and the filter might need replacing. We have also experimented with different shapes of cake layers, and compared how these different profiles affect the inlet pressure. The cake evolution exhibits aspects analogous to well-established theories for sand dune formation, which are described by the Exner equation. Insight into an appropriate model for the cake transport has been provided by experiments conducted at Beko, which explore the critical flow rate at which cake movement is initiated. In some cases, the resulting fluid flow problem can be compared to the study of high-speed flow over a flat plate, with the drainage of fluid flowing through the cake providing an interesting and complex extension to this classical model. We can feed results extracted from the fluid flow problem we solve, like those seen in figure 2(b), into our generalised Exner equation to update the cake profile on the boundaries.



(a) The inlet pressure varying with the effective cake permeability. The dashed red line marks the critical inlet pressure.



(b) Flow field through a filter with a sinusoidal bottom boundary. The arrows show the direction of the fluid flow.

Figure 2 – Numerical flow simulation results.

The solver we have developed is easily adaptable for a range of filter shapes and flow speeds. The effect of changing the shape of the filter and the thickness of a cake layer on the flow field can be observed by simple modifications of our code. Thus, our solver provides a framework through which Beko can perform experiments computationally. Key quantities such as the drag on the surface of a cake layer can be extracted from our code, and we have identified how to feed such quantities into our cake-shape equation to observe how the cake builds up and moves over the filter surface in time. In the future, the tool developed during the project can be used to model the cake transport in the filter, and optimise the flow rate and the membrane shape, in order to extend the filter life.

Dr Antonio D'Ammaro, Senior Specialist in Mechanical Technologies at Beko said

> *It was a very ambitious project. The work carried out by Georgia and the team at Oxford University provided insights on the existing filter performance. The resulting tool could allow Beko R&D to rapidly iterate new filter designs.*