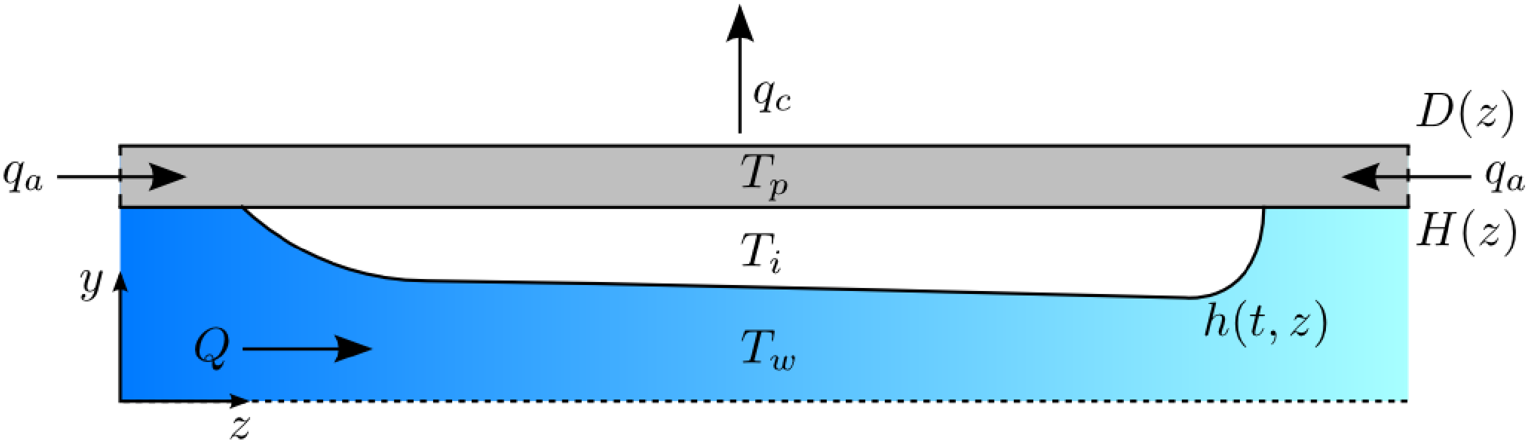
EPSRC Centre for Doctoral Training in
Industrially Focused Mathematical Modelling



Freezing water in a pipe
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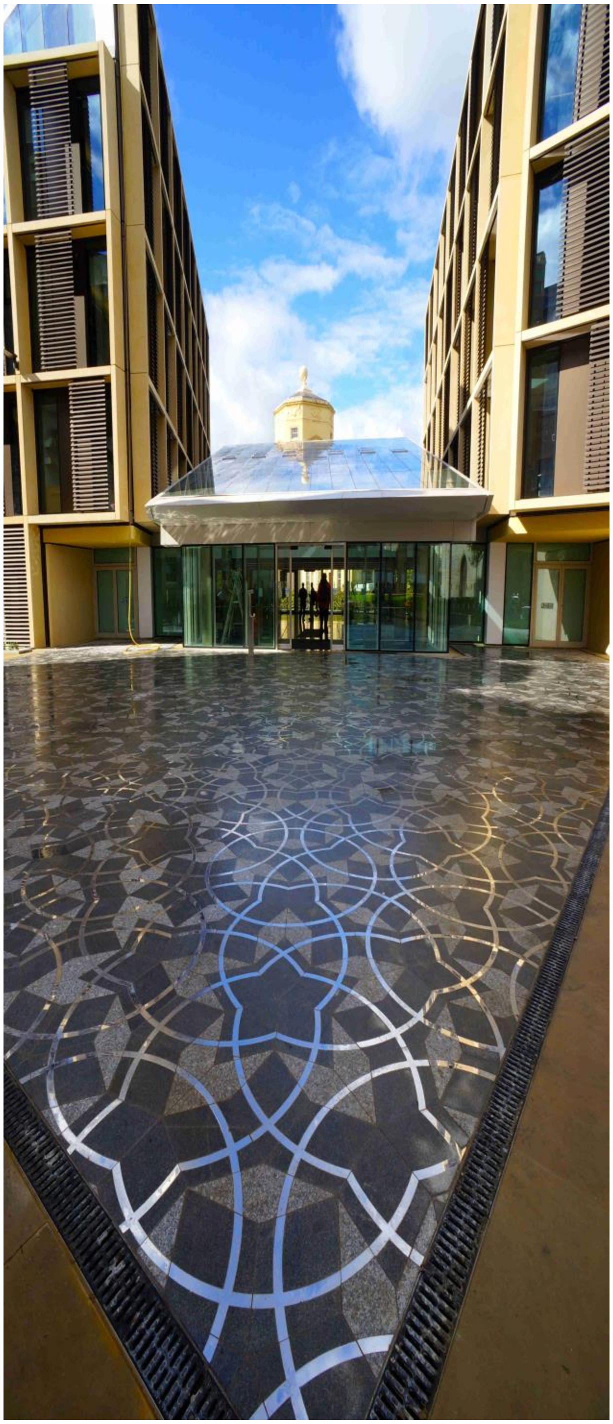

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1. Introduction

Background

Sharp Laboratories of Europe Ltd. (SLE) is working on the new generation of household water purifiers. The technique they are developing is based on the principle that when water freezes, the ice formed is purer than the initial water, because ice rejects the contaminants.

To increase the water efficiency of the purifier we need to maximise the amount of ice formed in the pipe.

The process of water purification under consideration consists of a closed circuit in which contaminated water flows through a freezing chamber. This chamber consists of a series of metal walled channels through which water flows while heat is extracted from its walls. Ice starts forming on the walls, rejecting the contaminants which are carried away by the water flow. Once the ice clogs the freezing chamber, flow ceases and contaminants are no longer dragged away so we have to stop the system. Then, we extract the contaminated water and proceed to melt the ice, obtaining purified water.

The performance of any water purifier can be assessed considering purification efficiency and water efficiency. Purification efficiency is the percentage of contaminants that the system is able to remove. Water efficiency is the percentage of purified water that we can obtain given an initial amount of water. In the freezing purification technique, the amount of pure water is the same as the amount of ice that grows in the system so we want to optimise the system to maximise the amount of ice formed when the freezing chamber clogs.

SLE has performed experiments with this new technique. In the literature there are empirical correlations that describe the evolution of the concentration of contaminant in the ice, which match SLE's experimental results. However, little is known about the factors that determine the distribution and quality of ice in the chamber when it clogs.

In this Miniproject we modelled the process of freezing water in a pipe to understand the physics behind the process and to propose solutions to improve the water efficiency of the system.

Experimental setup

The experimental setup consists of a closed circuit in which water flows through a freezing chamber similar to that sketched in Figure 1. Water flow is represented by large arrows (denoted by Q). It comes into the inlet through the top lid, and flows through the channels towards the outlet from where it is extracted through the top lid again. Heat is extracted from top and bottom surfaces over the channels, in the region coloured in dark blue and represented by small arrows.

In the experiments it is seen that ice grows quite uniformly except for the ends, where it has a rounded shape. It has also been observed that ice does not grow along the entire channel, but there are two regions near the inlet and the outlet where, despite the cooling, ice does not grow. In Figure 2 we can see a numerical recreation of the experiment results in COMSOL

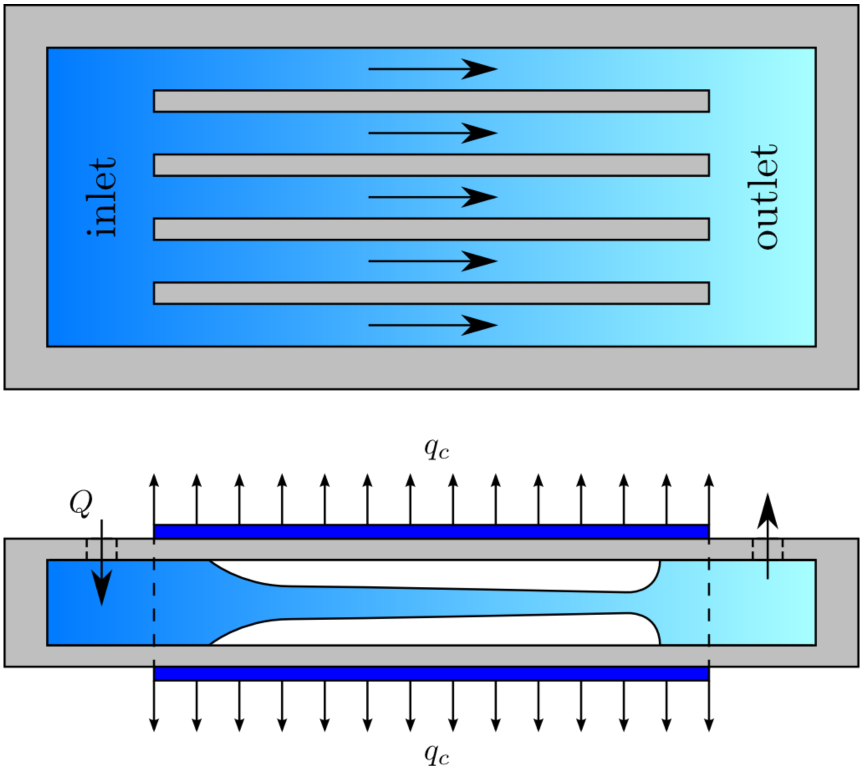

Figure 1: Sketch of the freezing chamber. Flow represented by large arrows and heat extraction represented by small arrows (uniform in the whole dark blue region).

Figure 2: COMSOL simulation performed previously by SLE to replicate the experimental observations. We see ice (white) and water (black) in the freezing chamber where water flows left to right.

2. Freezing water in a pipe

Mathematical model

The channels are long and narrow, allowing simplified models to be developed that more clearly reveal the physics at play.

In this Miniproject we explored the physics behind ice growth, so that once we know the phenomena that govern the process we can suggest improvements to the system. The models and results discussed in this report are for a simplified two-dimensional model, but the key ideas and the conclusions hold as well in the three-dimensional.

We first consider a model in which we include water and ice, leaving the metal of the freezing chamber outside of the model. Plots of the ice growth predicted by this simplified model are shown in Figure 3. As we can see, this model fails to reflect the rounded shape that happens at both ends of the ice layer in the experiments.

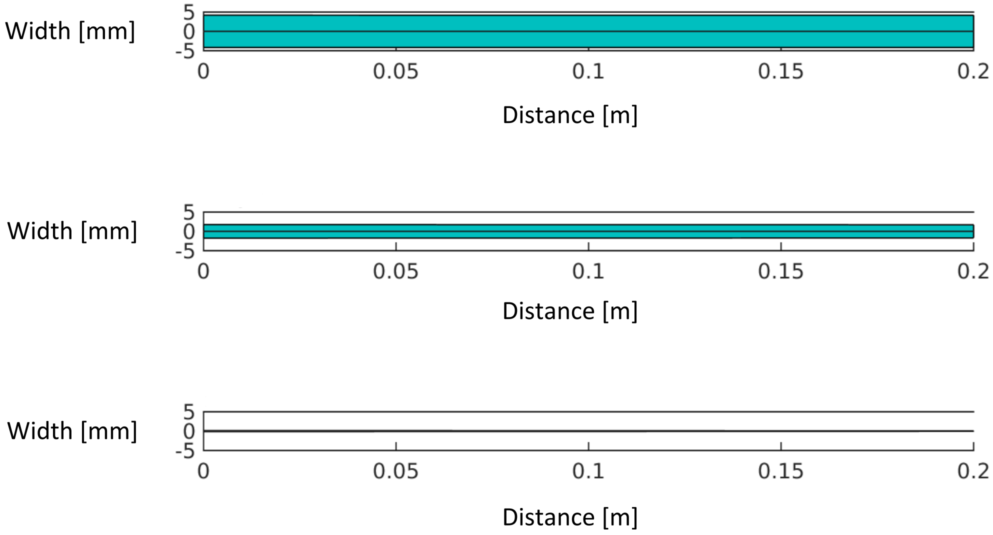

Figure 3: Shape of the ice predicted for the simple model at different times. Ice shown in white, water in blue.

Therefore, we expanded the model to include the conduction of heat through the metal plate. In particular, we allow for a possible heat flux from the inlet and the outlet flowing through the metal towards the freezing region, as shown in Figure 4.

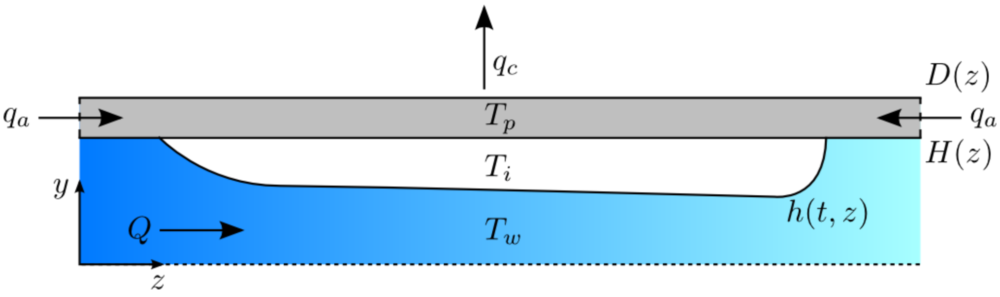

Figure 4: Diagram of the model with all the elements that take part in it. We model the heat flux flowing in from the ends of the system as $\mathbf{q_a}$.

The governing equations of the model are heat conservation in the metal, ice and water with interface conditions to connect the phases. The most important of these conditions is the Stefan condition, which describes the movement of the ice-water interface. We expand the equations to model appropriately the regions near the ends in which ice does not grow so heat exchange is direct from water to metal.

Figure 5 shows a numerical solution for the evolution of the ice shape. Notice that now the model describes the rounded shape near the edges of the ice that we saw in the experiments.

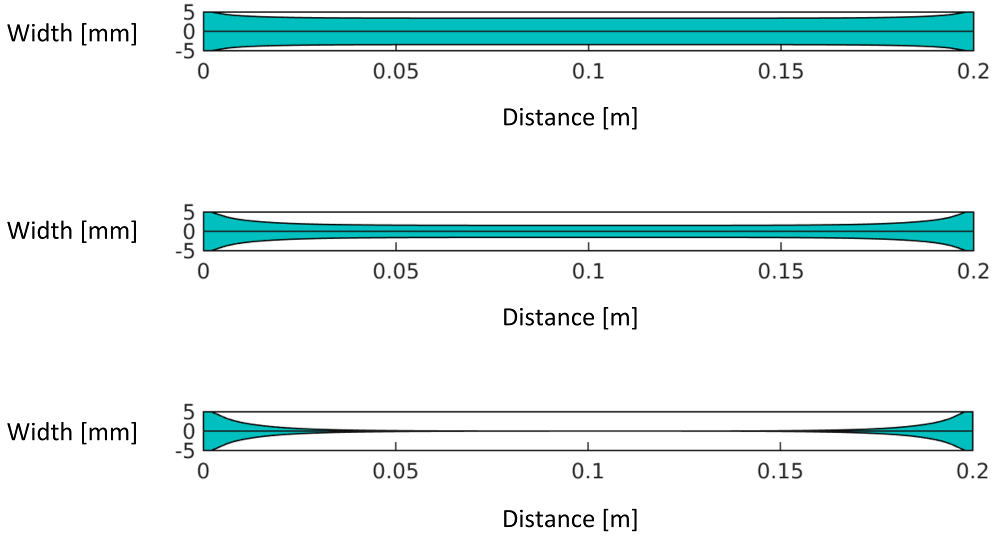

Figure 5: Shape of the ice predicted for the extended model at different times. Ice shown in white, water in blue. Observe that now the model describes the rounded shape at the ends of the ice layer as seen in the experiments.

3. Improving the efficiency

When SLE set this Miniproject, one of the objectives they were interested in was how to optimise the shape of the pipe to maximise the amount of ice formed before clogging. However, with the model we developed we find that the maximum improvement in efficiency they could achieve by modifying the shape was around 2% in the best scenario, and there was a stronger dependence on other parameters in the system such as the inlet or the coolant temperature. Having established the impact of the conduction in the metal plates we conducted a brief study of the system in which the heat flux through the sides of the plate was set equal to zero. We found that in that case, which sets the theoretical limit of the efficiency, the efficiency could be increased by more than 10%, reaching values over 99%. This points towards insulation as a key feature of the experimental design.

Insulation

We saw in the previous section that the heat flux flowing into the freezing region increases the temperature near both ends and reduces the efficiency of the system. We suggest that the efficiency could be increased by installing some insulation that prevents the unwanted heat flux into the metal.

The effect will depend on the configuration of the insulator we set up, so in this section we discuss the case in which a thin layer of insulator is placed in the inlet and the outlet both on the top and bottom of the metal plates as seen in Figure 6.

Water efficiency is defined as:

$$\frac{\text{volume of ice}}{\text{total volume of channel}}$$

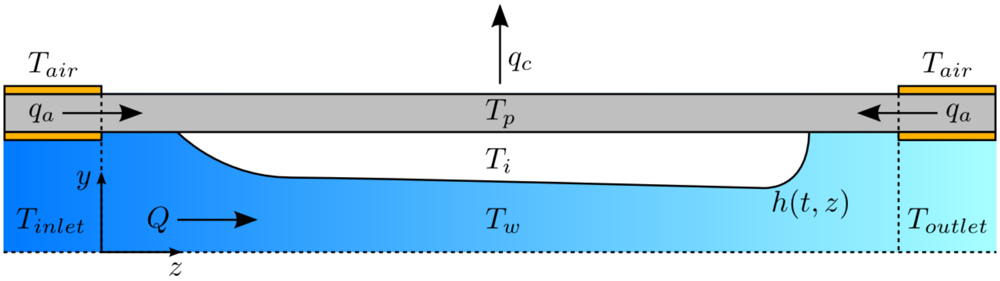

Figure 6: Diagram of the model with the insulator set up. We consider insulation at both inlet and outlet with water and with the air.

For given insulator properties (thickness and conductivity of the insulator) we solve the model to find the water efficiency. We first saw that it is especially important to insulate properly the surface in contact with air, and a layer 3 cm thick of plastic foamed insulator was considered to have the desired effect. The efficiency will depend mainly on the inlet temperature and we found that for an inlet temperature of 2°C the insulation required is 6.5 mm of plastic foam insulator, whereas for an inlet temperature of 1°C 2.7 mm of the same insulator would be enough to achieve the same efficiency. For any of the temperatures in the range studied a layer of 2.5 mm is enough to push efficiency over 90%. These results are shown in Figure 7, in which there are plots of efficiency versus thickness of insulator at inlet and outlet for different inlet water temperatures.

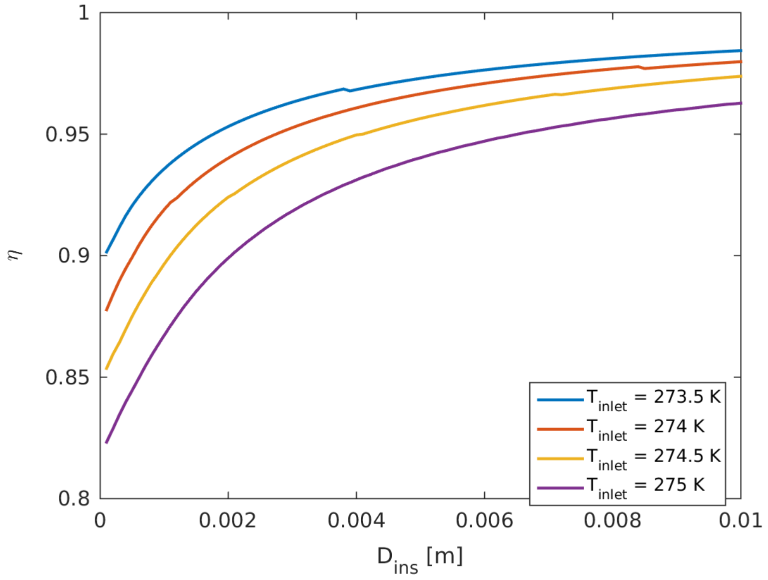

Figure 7: Efficiency versus thickness of insulator at inlet and outlet for different inlet temperatures. We consider the insulator as plastic foamed insulator.

Multichannel approach

As seen in Figure 1, the freezing chamber is formed by various parallel channels with their ends connected to the same inlet and outlet. We considered how the flow through the system is distributed between the individual channels. We derived a model for the flow and showed that its behaviour is analogous to an electrical circuit. The flow acts as a current and every channel presents a resistance to this current. The more ice in one channel, the higher the resistance. Therefore, as all channels are connected at the inlet and the outlet we can study the flow to each channel as if it was a circuit with parallel resistances with a source of current. The study of the multichannel system can be important to determine the effect of one channel over the rest and thus how this can affect the total efficiency of the system.

Extending the model

Some next steps that could be taken to extend the model are the following:

- Include a mushy layer to describe the compositional effects neglected in this model and necessary to study more concentrated liquids.
- Consider other configurations of the insulator and use models to determine their effectiveness.
- Consider a three-dimensional model to more accurately describe the shape of the freezing chamber.
- Study the potential for flow instability in the multichannel freezing chamber.

4. Discussion, Conclusions and Recommendations

After modelling and analysing the process of freezing water in a pipe, the main conclusion we can extract is that the principal phenomenon that causes the rounded shape observed in the experiments is heat conduction in the metal plate which allows heat to flow into the freezing region from the inlet and outlet water chambers at the ends.

Insulation of inlet and outlet showed to be the key factor to improve the efficiency of the system.

For this reason, we have focused on the potential for insulation of the system to reduce this effect and improve the efficiency of the system. The simulations of a perfectly insulated model show that by insulating properly we can achieve an improvement greater than 10% which means having efficiency over 99%. We also found that with a layer of 3 cm of plastic foamed insulator on the outer surface and a layer of 5 mm of the same insulator on the inner inlet and outlet surfaces we can achieve efficiency over 95% for most of the cases. Therefore, it seems reasonable to follow this direction and work further on the insulation to improve the efficiency of the system.

5. Potential Impact

The short term impact of this project is a change in the approach to the problem, as the experiments and simulations will now focus on the insulation of the system. As Phil Roberts, Senior Researcher in SLE, commented: "This Miniproject showed us that we were asking the wrong question: we should focus on the insulation of the system rather than on its shape. Mathematical modelling helped us to understand which physical phenomena were important in our system and gave us ideas on how to improve its efficiency". From a medium term perspective, the project is now stronger, and therefore more likely to turn into a commercial product.

In a long term view, this Miniproject will help to design a purification device with much higher water efficiency in order to achieve a good purification efficiency the device may need to realise various freezing/melting cycles, so any improvement in water efficiency has a great effect on the overall efficiency. Finally, if the device is successful and efficient, this Miniproject will have helped to consolidate a new generation of water purifiers.