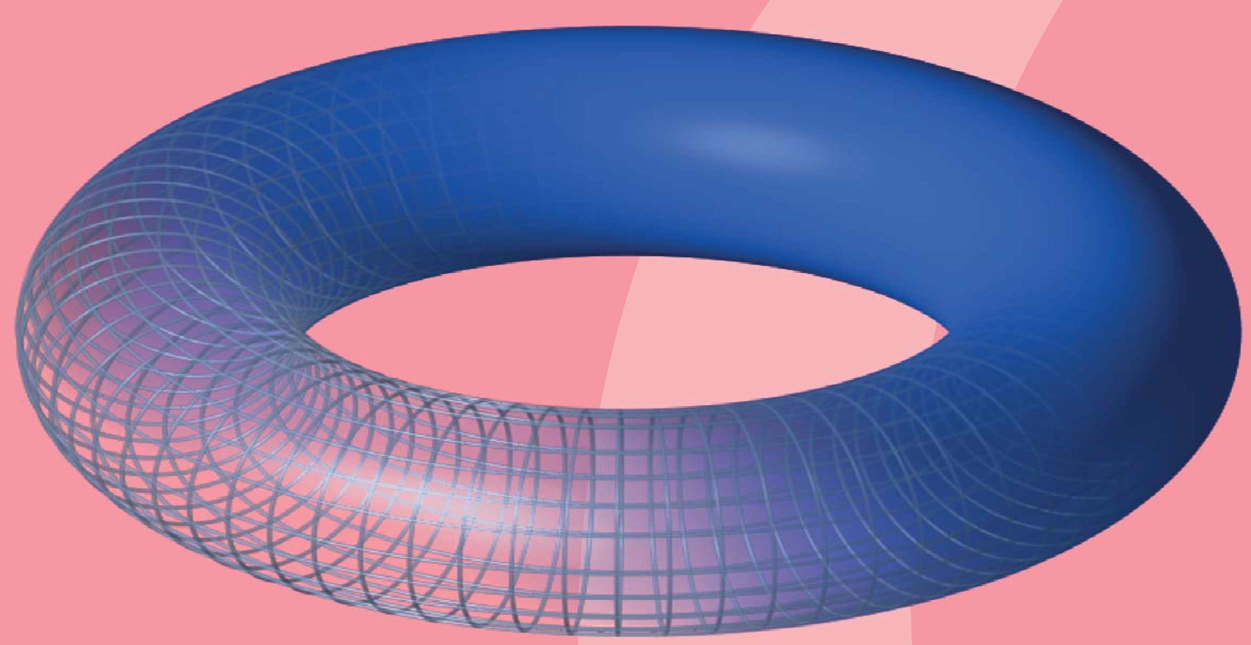
Oxford Mathematics Alphabet

C *is for* Calabi-Yau manifolds

Calabi-Yau manifolds have become a topic of study in both mathematics and physics, dissolving the boundaries between the two subjects.

A manifold is a type of geometrical space where each small region looks like normal Euclidean space. For example, an ant on the surface of the Earth sees its world as flat, rather than the curved surface of the sphere. Calabi-Yau manifolds are complex manifolds, that is, they can be disassembled into patches which look like flat complex space.



What makes Calabi-Yau manifolds so special is that the patches can only be joined together by the complex analogue of a rotation. One solution to this condition in one complex dimension is a torus.

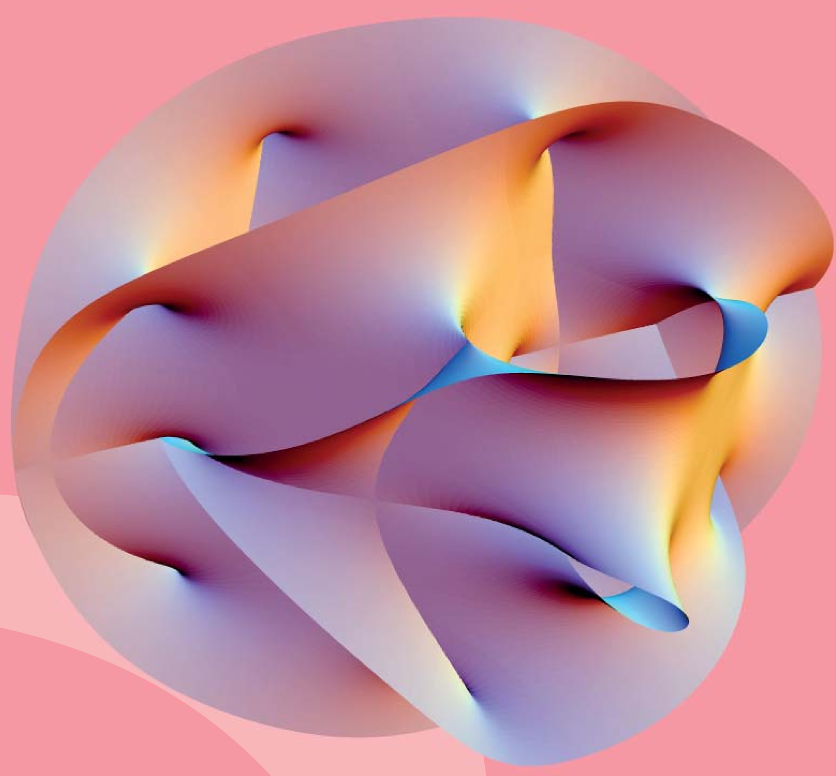
In any dimension, Calabi-Yau manifolds can be described as solutions to algebraic equations. We can describe an n-dimensional sphere as the real solution to

$$x_1^2 + \ldots + x_{n+1}^2 = r^2.$$

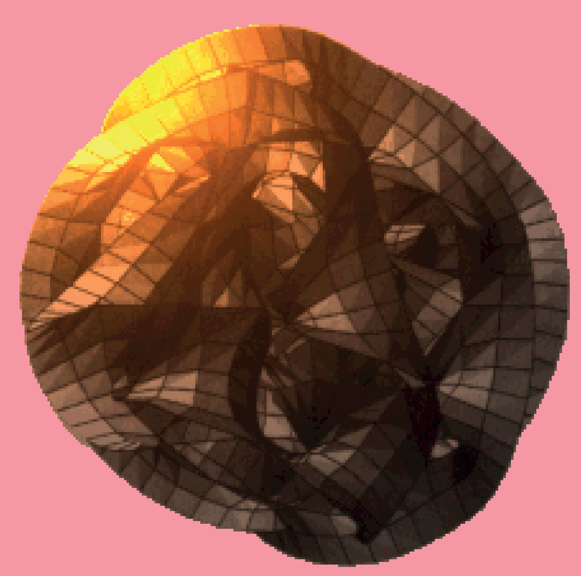
Likewise the famous complex three-dimensional Calabi-Yau manifold called the 'quintic' is given by the (complex) solutions to

$$x_1^5 + x_2^5 + x_3^5 + x_4^5 + \psi x_1 x_2 x_3 x_4 = 1$$

for a parameter ψ.



Proving a conjecture of Eugenio Calabi, Shing-Tung Yau has shown that Calabi-Yau manifolds have a property which is very interesting to physics. Einstein's equations show that spacetime curves according to the distribution of energy and momentum. But what if space is all empty? By Yau's theorem, not only is flat space a solution but so are Calabi-Yau manifolds.



Every example of a Calabi-Yau threefold gives rise to a different universe with different sets of elementary particles and particle interactions. This makes the question of classification tremendously interesting. However, to date it is not even clear whether there are finitely many. So far, physicists and mathematicians have constructed roughly half a billion examples.

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Top image: Calabi-Yau quintic by Lunch
Bottom image : Calabi-Yau manifold by Nina Hernitschek