Delamination and fracture of thin films

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Brittle fracture in thin films

Geyer & Nemat-Nasser (IJSS 1982)
Brittle fracture

Bourdin & Maurini (in preparation)

\[ E(u, K) = \int_{\Omega \setminus K} \frac{1}{2} A(e(u) - \theta I) \cdot (e(u) - \theta I) + G_c H^{n-1}(K) \]

approximated by

\[ E_\ell(u, \alpha) = \int_\Omega \frac{1}{2} (1 - \alpha)^2 A(e(u) - \theta I) \cdot (e(u) - \theta I) + \int_\Omega \left( \frac{9\alpha}{64\ell} + \ell |\nabla \alpha|^2 \right) \]
Thermal shock on a cylinder

Bourdin & Maurini (in preparation)

Numerical results: an overview of 3D results
Cylinder - Cross sections
891,000 elements, 101 time steps. Approx. 6h walltime on 256 cpus (Ranger, TACC)
C. Maurini (UPMC)
Variational fracture mechanics: thermal cracks
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Delamination and fracture of thin films

D. Henao & J.-F. Babadjian, B. Bourdin, A. León, C. Maurini
Thermal cracks

Goehring, Mahadevan & Morris (PNAS 2009)
Thermal cracks

Goehring, Mahadevan & Morris (PNAS 2009)

Delamination and fracture of thin films D.Henao & J.-F.Babadjian, B.Bourdin, A.León, C.Maurini
Cracks in thin films

Krämer et al. (MSE 2008)

Delamination and fracture of thin films
D.Henao & J.-F.Babadjian, B.Bourdin, A.León, C.Maurini
Cracks in thin films

Wu et al. (ASS 2011)
Cracks in thin films

Tsui, McKerrow & Vlassak (JMR 2005)
Delamination

Reis (PNAS 2009)  Airwolf Aerospace
León Baldelli et al. (in preparation)
León Baldelli et al. (in preparation)
Delamination & Fracture

León Baldelli et al. (in preparation)
A 2D model for a thin film bonded on a substrate

Thin film on an elastic substrate with transverse fractures $(\Gamma)$ and debonded regions $(\Delta)$

**Kinematics**

$u$: film membranal displacement

$\varepsilon(u)$: film membranal deformations

**Loadings**

$\varepsilon_0$: inelastic deformations in the film

$u_0$: substrate displacement

**Strain energy of the thin film** (see e.g. Xia and Hutchinson JMPS 2000)

$$P(u, \varepsilon, \varepsilon_0) = \int_{\Omega} \frac{1}{2} A_{\infty} (\varepsilon(u') - \theta I_{2 \times 2}) \cdot (\varepsilon(u') - \theta I_{2 \times 2})$$

$$+ \mathcal{H}^1(J u') + \frac{\mu'}{2} \int_{\Omega \setminus \Delta} u'^2 + \kappa \mathcal{H}^2(\Delta)$$

with

$$A_{\infty} \varepsilon = \frac{2\lambda \mu}{\lambda + 2\mu} \varepsilon_{\alpha\alpha} e_{\beta\beta} + 2\mu e_{\alpha\beta} e_{\alpha\beta}$$
Linear plate theory (e.g. Ciarlet vol. II)

- \( \Omega_\varepsilon = \omega \times (0, \varepsilon) \subset \mathbb{R}^3 \)
- \( \mathbf{v}_\varepsilon = (v_1^\varepsilon, v_2^\varepsilon, v_3^\varepsilon) \in H^1(\Omega_\varepsilon \times (0, \varepsilon)) \)

\[
\frac{1}{2} \int_{\Omega_\varepsilon} 2\mu \left( \begin{array}{ccc}
    e_{11} & e_{12} & e_{13} \\
    e_{21} & e_{22} & e_{23} \\
    e_{31} & e_{32} & e_{33}
\end{array} \right)^2 + \lambda (e_{\alpha\alpha} + e_{33})^2
\]

- \( v_3^\varepsilon(x_1, x_2, \varepsilon x_3) = \varepsilon u_3^\varepsilon(x_1, x_2, x_3) \)
- \( v_\alpha^\varepsilon(x_1, x_2, \varepsilon x_3) = \varepsilon^2 u_\alpha^\varepsilon(x_1, x_2, x_3) \)

\[
\Gamma - \lim = \frac{1}{2} \int_{\Omega} \frac{2\lambda \mu}{\lambda + 2\mu} e_{\alpha\alpha}(\mathbf{u})e_{\beta\beta}(\mathbf{u}) + 2\mu e_{\alpha\beta}(\mathbf{u})e_{\alpha\beta}(\mathbf{u}),
\]

\[ e_{\alpha 3}(\mathbf{u}) = e_{33}(\mathbf{u}) = 0. \]
Dimension reduction

- Le Dret & Raoult, JMPA '95: *nonlinear membrane model*
- Fonseca & Francfort, JRAM '98: *3D-2D in optimal design*
- Bhattacharya & James, JMPS '99: *martensitic thin films*
- Braides, Fonseca & Francfort, IUMJ '00: *inhomogeneous thin films*
- Mora & Scardia, JDE '12: *convergence of equilibria of physical plates*
- ...
Brittle thin films

Braides & Fonseca (AMP 2001),
Bouchitté, Fonseca, Leoni & Mascarenhas (ARMA 2002):

\[ \Omega_\varepsilon = \omega \times (0, \varepsilon), \ u \in GSBV_p(\Omega; \mathbb{R}^3), \]

\[ J_\varepsilon = \int_\Omega W\left( \nabla_\alpha u \left| \frac{1}{\varepsilon} \nabla_3 u \right. \right) + \int_{J_u} \vartheta \left( u^+ - u^-, \nu_\alpha(u), \frac{1}{\varepsilon} \nu_3(u) \right) d\mathcal{H}^2 \]

\[ \vartheta \text{ symmetric, positively 1-homogeneous, Lipschitz, linear at infinity} \]

\[ |F|^p \leq W(F) \leq C(1 + |F|^p), \text{ continuous} \]

\( \Gamma \)-converges to

\[ \int_{\omega} Q\bar{W}(\nabla_\alpha u) + \int_{J_u \cap \omega} R\bar{\vartheta}(u^+ - u^-, \nu_\alpha(u)) d\mathcal{H}^1 \]

\[ u \in SBV_p(\Omega; \mathbb{R}^3) : \nabla_3 u = 0, \nu_3(u) = 0. \]
Brittle thin films

- **Giacomini CVPDE ’05:**
  *Ambrosio-Tortorelli for the quasistatic evolution*

- **Babadjian CVPDE ’06:**
  *Convergence of the quasistatic evolutions*
Bhattacharya, Fonseca & Francfort (ARMA 2002):

\[ \Omega_\varepsilon = \omega \times (-\varepsilon s, \varepsilon h), \ u \in W^{1,p}(\Omega^+; \mathbb{R}^3) \cap W^{1,p}(\Omega^-; \mathbb{R}^3), \]

\[ \int_{\Omega^+} W^+ \left( \nabla \alpha u \, \frac{1}{\varepsilon} \nabla_3 u \right) + \int_{\Omega^-} W^- \left( \nabla \alpha u \, \frac{1}{\varepsilon} \nabla_3 u \right) + \varepsilon^{\alpha-1} \int_\omega \| [u] \|_\gamma \]

\[ \frac{1}{C} |F|^p - C \leq W(F) \leq C(1 + |F|^p), \text{ continuous} \]

\[ \Gamma \text{-converges to } \int_\omega hQ \overline{W}^+ + sQ \overline{W}^- \text{ (small debonding with small energy, independent oscillations in each layer; in the limit } u \text{ is continuous and independent of } x_3). \]
Cohesive interfacial energy $\int_{\omega} |[u]|^\gamma$

- **Ansini AA ’04:** *nonlinear Neumann sieve*
- **Ansini, Babadjian & Zeppieri M3AS ’07:** *multiscale Neumann sieve*
  - Ansini-Braides JMPA ’02, AAP ’01; Attouch-Picard RSMUPT ’87; Conca JMPA 1985, 1987; Damlamian RDMUPT ’85; Del Vecchio AMPA ’87; Murat ’85; Sanchez-Palencia ’80, ’81, ’85
- **Roubíček, Scardia, Zanini CMT ’09:** $\int_{\Gamma_c} k z [u]^2_{\Gamma_c} d\mathcal{H}^{n-1}$
- **Freddi, Paroni, Roubíček & Zanini ZAMM ’11:** transverse fracture as a form of delamination
1D delamination and debonding

León Baldelli, Bourdin, Marigo & Maurini CMT ’12:

$$\int_{\omega \setminus \Gamma} \frac{1}{2} (u'(x) - t)^2 \, dx + \int_{\Omega \setminus \Gamma} \frac{1}{2} u(x)^2 \, dx + \#(\Gamma) + \gamma \mathcal{H}^1(\Delta)$$

Fig. 17
Coupled experiment #2: transverse fracture and debonding. Three transverse fractures and debonding.

(a) Energy plot shows an evolution consisting in three subsequent transverse fractures and debonding. Boundary layer effects appear during the very first phase of debonding, as the linear growth of the debonding energy in the very first debonding phase indicates,

(b) plot of the displacement and damage fields, the film exhibits three transverse cracks at the end of the loading phase,

(c) plot of the displacement and debonded domain (shaded in light gray). The latter is not symmetric for each part of the bar since the total energy is insensitive to the arrangement of the debonded domain is uniquely determined as a function of the loading. Equivalently, debonding may appear only at the ends of the domain. Moreover, this is a property is true of all local minima of the energy.

The modeling of transverse cracks requires us to formulate problem in terms of global minimization, as customary in the variational approach to fracture mechanics with a Griffith-type surface energy. We showed that transverse cracks are equally spaced and lead to periodic solutions. This behavior was only postulated in previous studies. The coupling of transverse fracture and debonding produces an interesting and rich behavior even in the 1D setting. Through analytical results and phase diagrams, we unveiled the dependence of the key qualitative properties of the solutions on the two non-dimensional parameters of the model. In the numerical part, we proposed a finite elements implementation of a regularized model. Our numerical approach is used to illustrate key properties of the model identified in the analysis section. A natural extension of this work is to tackle the 2D case, which is known to lead to intriguing complex crack and debonding patterns, like fracture networks, parallel crack arrays with stop-and-go phenomena, or spirals. The extension of the numerical model to 2D is straightforward. It is under active development, and the analytical results presented here will be used for its verification. The rigorous derivation of the thin-film model with fracture and debonding, starting from a 3D variational model with Griffith surface energy is pending. Preliminary $\Gamma$-convergence results are available and will be reported in a forthcoming paper. Our final aim is to compare the analytical and numerical results.
León Baldelli, Bourdin, Marigo & Maurini CMT ’12:

1D delamination and debonding

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Fig. 7

Energy curves \( \hat{E}_t^n \) with possible debonding and transverse fracture for \( \gamma = 2.2 \), \( L = 6 \). Each curve is for a specific number \( n - 1 \) of transverse fractures, corresponding to the value at the intersection with the axis \( t = 0 \). The solid continuous lines indicate the energy is obtained for a state without debonding, the dashed line for a state with debonding.

Fig. 8

The optimal displacement fields for the states A, B, C of Fig. 7. Debonding regions are indicated with a dashed line.

Fig. 9

Key properties of the solution of the static problem of a film of dimensionless length \( L \) and relative debonding toughness \( \gamma \) (see Proposition 4).

- Plot of \( \bar{n} \) as a function of \( L \) and \( \gamma \), \( \bar{n} \) being the minimum number of pieces into which the film is split by transverse fractures when debonding appear.
- Plot of \( t^* \) as a function of the relative debonding toughness \( \gamma \), \( t^* \) being the critical load beyond which the optimal solution is that of a debonded film without transverse fractures (\( t^* \) is independent of the dimensionless length \( L \) of the film).
Multi-layer asymptotic analysis

- \( \Omega = \omega \times (0, 1), \Omega' = \omega \times (-1, 0); \omega \subset \mathbb{R}^2 \)

- **Vanishing Young’s modulus** in the bonding layer

\[
(\lambda_\varepsilon, \mu_\varepsilon) = \begin{cases} 
(\lambda, \mu) & \text{in } \Omega \\
\varepsilon^2(\lambda', \mu') & \text{in } \Omega'
\end{cases}
\]

- Rescaled energies

\[
J_\varepsilon(u, \Omega) = \frac{1}{2} \int_{\Omega} 2\mu \left| \begin{pmatrix}
e_{11} & e_{12} & \varepsilon^{-1}e_{13} \\
e_{21} & e_{22} & \varepsilon^{-1}e_{13} \\
\varepsilon^{-1}e_{31} & \varepsilon^{-1}e_{32} & \varepsilon^{-2}e_{33}
\end{pmatrix} \right|^2 + \lambda(e_{\alpha\alpha} + \varepsilon^{-1}e_{33})^2
\]

\[
J_\varepsilon(u, \Omega') = \frac{1}{2} \int_{\Omega'} 2\mu \left| \begin{pmatrix}
\varepsilon e_{11} & \varepsilon e_{12} & e_{13} \\
\varepsilon e_{21} & \varepsilon e_{22} & e_{13} \\
e_{31} & e_{32} & \varepsilon^{-1}e_{33}
\end{pmatrix} \right|^2 + \lambda(e_{\alpha\alpha} + \varepsilon^{-1}e_{33})^2
\]

- \( u \in H^1(\Omega \cup \Omega'), u(\cdot, -1) = 0 \)
Multi-layer asymptotic analysis

- Energies:

\[
J_\varepsilon(u, \Omega) = \frac{1}{2} \int_\Omega 2\mu \left| \begin{pmatrix} e_{11} & e_{12} & \varepsilon^{-1}e_{13} \\ e_{21} & e_{22} & \varepsilon^{-1}e_{13} \\ \varepsilon^{-1}e_{31} & \varepsilon^{-1}e_{32} & \varepsilon^{-2}e_{33} \end{pmatrix} \right|^2 + \lambda (e_{\alpha\alpha} + \varepsilon^{-1}e_{33})^2
\]

\[
J_\varepsilon(u, \Omega') = \frac{1}{2} \int_{\Omega'} 2\mu \left| \begin{pmatrix} \varepsilon e_{11} & \varepsilon e_{12} & e_{13} \\ \varepsilon e_{21} & \varepsilon e_{22} & e_{13} \\ e_{31} & e_{32} & \varepsilon^{-1}e_{33} \end{pmatrix} \right|^2 + \lambda (e_{\alpha\alpha} + \varepsilon^{-1}e_{33})^2
\]

- \( u \in H^1(\Omega \cup \Omega') \), \( u(\cdot, -1) = 0 \), then
Multi-layer asymptotic analysis

▶ Energies:

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J_\varepsilon(u, \Omega) = \frac{1}{2} \int_\Omega 2\mu \left| \begin{pmatrix}
    e_{11} & e_{12} & \varepsilon^{-1} e_{13} \\
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    \varepsilon e_{11} & \varepsilon e_{12} & e_{13} \\
    \varepsilon e_{21} & \varepsilon e_{22} & e_{13} \\
    e_{31} & e_{32} & \varepsilon^{-1} e_{33}
\end{pmatrix} \right|^2 + \lambda (e_{\alpha\alpha} + \varepsilon^{-1} e_{33})^2
\]

▶ \(u \in H^1(\Omega \cup \Omega'), u(\cdot, -1) = 0\), then

▶ Theorem

\[ J_\varepsilon(u, \Omega) + J_\varepsilon(u, \Omega') \Gamma\text{-converges to} \]

\[
\frac{1}{2} \int_\omega \left[ \frac{2\lambda \mu}{\lambda + 2\mu} e_{\alpha\alpha} e_{\beta\beta} + 2\mu e_{\alpha\beta} e_{\alpha\beta} \right] + \frac{\mu'}{2} \int_\omega u_\alpha u_\alpha
\]
For \( u \in SBV(\Omega \cup \Omega' \cup \Omega''; \mathbb{R}^3) \), \( u = w \) a.e. in \( \Omega'' \), define

\[
F_\varepsilon(u) = \int_\Omega (|\nabla' u - A_0|^2 + \varepsilon^{-2} |\partial_3 u|^2) \, dx + \int_{\Omega'} (\varepsilon^2 |\nabla' u|^2 + |\partial_3 u|^2) \, dx \\
+ \int_{\Omega \cap J_u} \left| (\nu_u)' \frac{1}{\varepsilon} (\nu_u)_3 \right| \, d\mathcal{H}^2 + \int_{\Omega' \cap J_u} |(\varepsilon (\nu_u)', (\nu_u)_3)| \, d\mathcal{H}^2.
\]

**Theorem**

\( F_\varepsilon \) converges to

\[
\int_\omega |\nabla' u - A_0|^2 \, dx' + \int_{\{|u-w| \leq 1\}} |u-w|^2 \, dx' \\
+ \mathcal{H}^1(J_u) + \mathcal{H}^2(\{|u-w| > 1\}),
\]

for \( u \in SBV(\omega, \mathbb{R}^3) \) and \( \max_\alpha \|u_\alpha\|_{L^\infty} \leq M \).