

Lecture proposal

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Term: TT2021 / MT2021

Topic: "Optimal Transport and lower curvature bounds"

Aimed at: students with some background on Optimal Transport and Riemannian Geometry.

Outline: The aim of this lecture is to give an introduction to the interplay between Optimal Transport and lower bounds on the Ricci curvature on Riemannian manifolds.

I will mainly focus on the Lagrangian side of the theory. Starting from the Euclidean picture I will discuss the equivalence between geodesic convexity of the entropy over the Wasserstein space and non negativity of the Ricci curvature.

Then I will briefly mention the dual Eulerian perspective on the topic, based on the study of the heat equation and on Bochner's inequality.

Some hints to the developments of the theory of metric measure spaces with lower curvature bounds of the last twenty years will be given in the end of the lecture.

Relevant references:

- D. Cordero-Erausquin, R.-J. McCann, M. Schmuckenschläger: *A Riemannian interpolation inequality à la Borell, Brascamp and Lieb*. *Invent. Math.* **146** (2001), no. 2, 219–257.
- M.-K. Von Renesse. K.-T. Sturm: *Transport inequalities, gradient estimates, entropy, and Ricci curvature*. *Comm. Pure Appl. Math.* **58** (2005), no. 7, 923–940.
- C. Villani: *Optimal transport. Old and new*. Grundlehren der Mathematischen Wissenschaften [Fundamental Principles of Mathematical Sciences], **338**. Springer-Verlag, Berlin, 2009.

Further readings:

- K.-T. Sturm: *On the geometry of metric measure spaces. I.-II*, *Acta Math.* **196** (2006), no. 1, 65–131.
- C. Villani: *Synthetic theory of Ricci curvature bounds*. *Jpn. J. Math.* **11** (2016), no. 2, 219–263.
- L. Ambrosio: *Calculus, heat flow and curvature-dimension bounds in metric measure spaces*. *Proceedings of the International Congress of Mathematicians—Rio de Janeiro 2018*.