UNIVERSITY OF OXFORD

Mathematical Institute

Oxford Mathematics Escape Room

Oxford Mathematics

Introduction

Thank you for participating in the Oxford Mathematics Escape Room. We hope that you enjoyed the evening. This event was made possibly with funding from the Oxford University Public Engagement with Research Seed Fund.

In this booklet, you will find the puzzles, together with solutions and brief notes describing the links to Oxford Mathematics research.

If you enjoyed this, you may wish to attend our Public Lectures. You can sign up by emailing external-relations@maths.ox.ac.uk.

You can also follow us on Facebook (@OxfordMathematics), Twitter (@OxUniMaths) and Instagram (@oxford.mathematics).

Contents

The police briefing

Sometime between 2pm and 3pm, the priceless 400-year-old portrait of Henry Savile was stolen from the office of the Savilian Professor of Geometry. The painting was in the office at 1400, when the Professor left to go to a Research Committee meeting. They left their office door unlocked. At precisely 1405, the Professor's PA heard someone enter the office. The PA heard the person leave again at about 1410.The Professor got back to their office at 1500, and found the painting was missing. The theft was immediately reported to the police. The working theory is that the thief entered the office at 1405.

Nobody has seen the portrait leaving the building, so it is presumably stashed in the building to be retrieved by the thief/thieves overnight.

Forensics found an unexplained partial footprint in the Savilian Professor's office. The working theory is that this belongs to the thief, who discovered their muddy footprints and cleaned their feet at this point. Other muddy footprints have been found in eight other locations in the Andrew Wiles Building. Forensics have discovered that there are 9 different types of muddy footprint.

There is CCTV on the entrance to the building, but the hard drive has malfunctioned and the images have been corrupted. This means that it is not possible to trawl through all the CCTV data.s

By identifying the thief's footprints, it should be possible to work out which places they visited in the Andrew Wiles Building. By finding the shortest path between these places (which the thief presumably took), it will be possible to work out when the thief entered the building, and so to look at just the relevant CCTV images.

There were some mysterious T-shapes found in a corridor by the faculty offices.

The police believe that two people were involved in the theft of this portrait.

Your task is to help the police with their investigation, by uncovering the identity of the two thieves and finding the portrait before it is smuggled out of the building.

Fact sheet

The Savilian Professor left their office at 14:00 to go to a meeting.

The Professor's PA heard someone enter the office at exactly 14:05:00, and leave again about 14:10. They presumed that the Professor had forgotten something and had returned to pick it up.

The theft was reported when the Professor returned at 15:00.

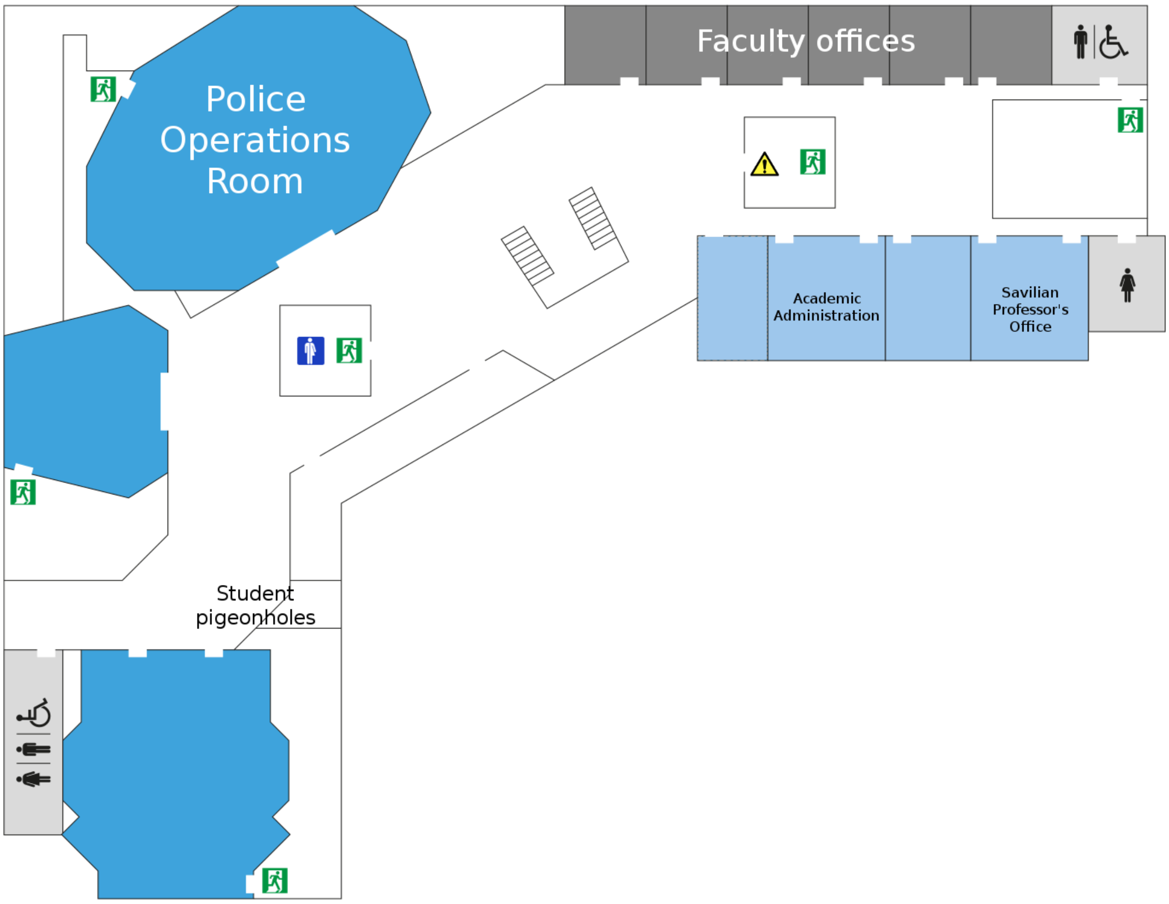
A partially cleaned footprint was discovered in the office. Only two patches of mud remain, and in these the mud has been smudged so the the detail of the print is gone.

A number of other footprints have been discovered at seven places around the building. A careful analysis reveals there are 9 different types of print.

The working hypothesis is that one of these prints belongs to the thief. They entered the building, ran a number of errands, and then went to the Savilian Professor's office, arriving there at 14:05:00. Before leaving the Professor's office the thief realised they were traipsing mud around, and tried to clean the prints in the office, and their shoes (perhaps they even removed their shoes).

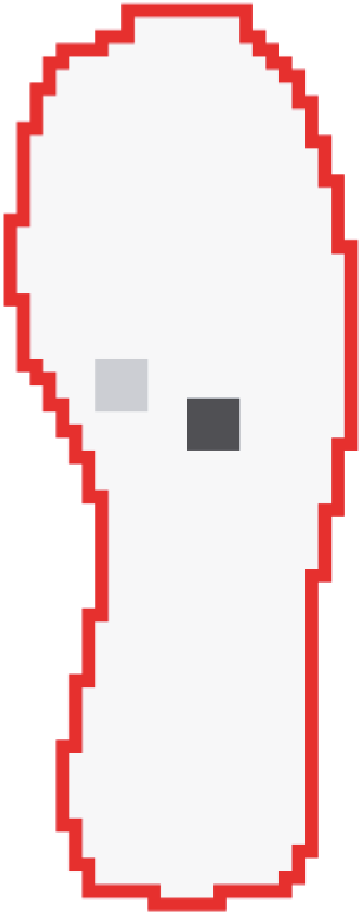
There is CCTV footage available for the front entrance of the building, but the hard drive has been corrupted so that the images are only partial and it takes some time to retrive an image from the system. There is therefore no way we can go through all the footage.

If we can work out which places the thief visited between entering the building at 1 and arriving at the Savilian Professor's office 9, and we can work out which route they took (the working hypothesis is that they would take the most efficient route), then we can identify what time they entered the building, and therefore which CCTV images we should focus on.


Faculty offices
Police Operations Room
Academic Administration
Savilian Professor's Office
Student pigeonholes

The Operations Room:

The muddy footprints



The footprint evidence found in the Savilian Professor's room.
The mud in each square has run, so that all we know is the average intensity for those squares in the original print.

Footprint number 3

Footprint number 2

Footprint number 1

Footprint number 6

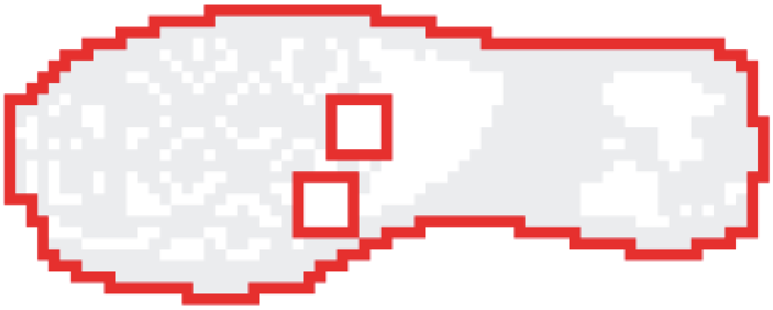
Footprint number 5

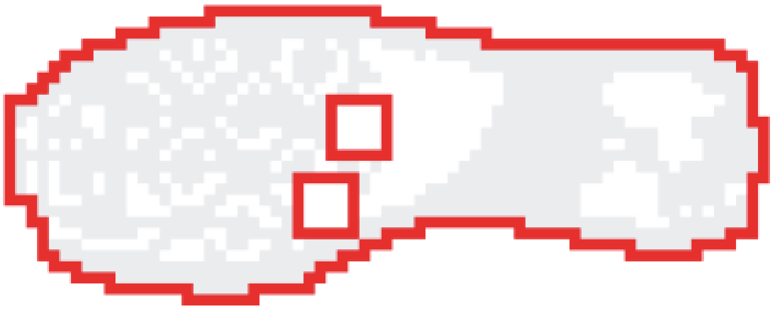
Footprint number 4

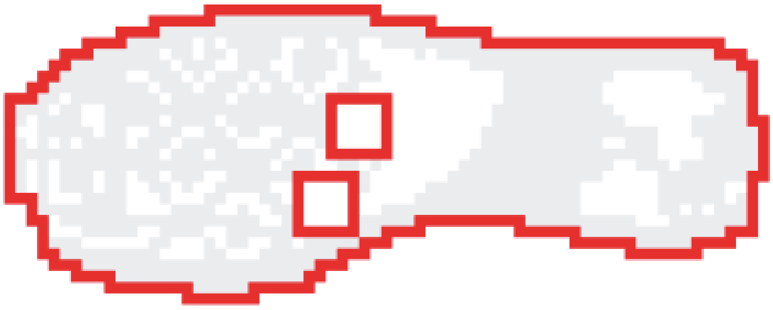
Footprint number 9

Footprint number 8

Footprint number 7





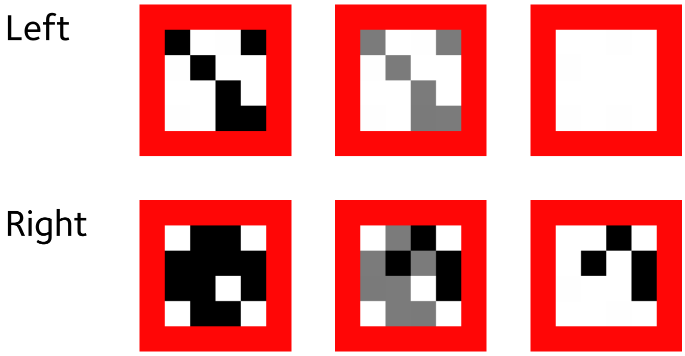


Intensity gauge

The muddy footprints: Solution

Footprint 8 belongs to the thief.

There were two squares of mud remaining of the footprint found in the Savilian Professor's office. Looking at these two regions in the nine footprints found by Forensics, there are three different possibilities for each square.



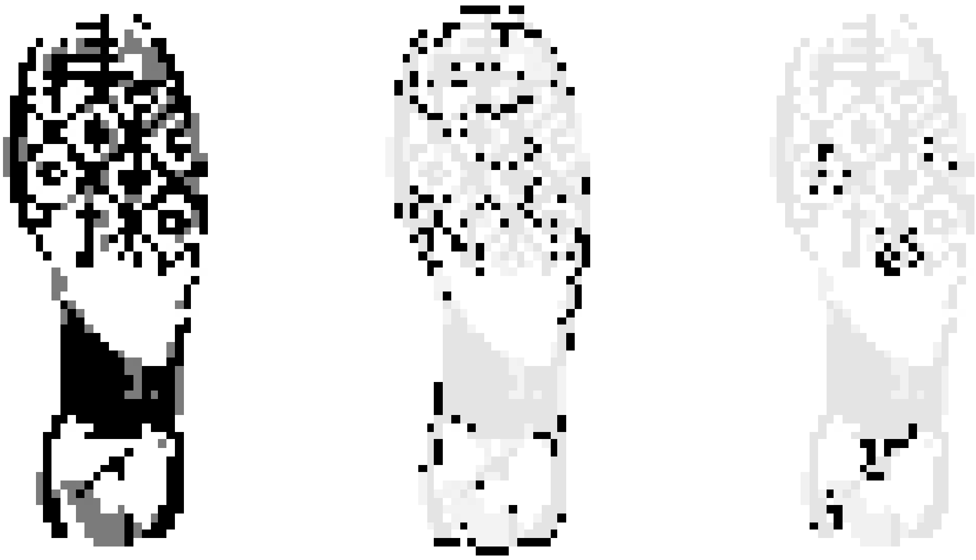
It's possible to see by eye now that for the suspect's footprint, we need the middle of the three boxes on the left, and the first box on the right. These correspond to footprint 8.

The muddy footprints: Explanation and discussion

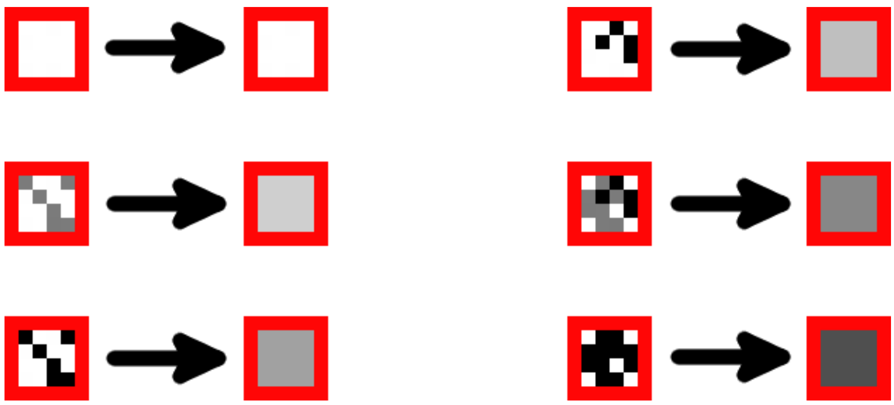
We could check that it's footprint 8 with the help of the intensity gauge. The grey colour in the suspect's footprint is halfway between black and white. In the left box, there are always 10 small white squares, and then six small squares that are black, grey or white. It's possible to assign numerical values to these to measure the intensity. For example, if black has intensity 1, grey intensity 0.5 and white 0, then averaging over the sixteen squares would give $6/16 \approx 0.4$ on the left, $(6 \times 0.5)/16 \approx 0.2$ in the middle, or 0 on the right. Using the intensity gauge shows that the suspect's footprint has intensity 0.2.

We can do something similar for the right box. Here there are 5 white squares, 4 black squares, and 7 that are black, grey or white. These correspond to intensities $11/16 \approx 0.7$, $7.5/16 \approx 0.5$, and $4/16 = 0.25$. Using the intensity gauge shows that the suspect's footprint has intensity 0.7.

To generate the footprints, we created a “base” footprint (shown here on the left), and two “perturbation footprints” (centre and right). Starting from each base footprint, we added a white, grey or black version of each of the two perturbation footprints (adding a white version doesn’t change anything). This gave the nine footprints.



For the suspect’s footprint, we found the intensity of the smudged square (a number between 0 and 1), and converted this to greyscale. Because the two chosen smudged squares overlap with the perturbation footprints, this is enough information to find whether the white, grey or black version of each perturbation footprint was added.



The intensity scale is useful to overcome optical illusions where the shade of a colour appears different to the human eye depending on the surrounding colours. It gives a precise way to measure the level of grey.

This problem is related to the mathematical area of compressed sensing. This is important in a range of practical applications, for example in medical diagnostic imaging.

One way to store a black-and-white digital photograph is to record the grey level of each pixel in the image. (This is the RAW data format.) This gives a high-fidelity image, but it takes a lot of memory to store so much information.

Another way to store the image is to build up an approximation to it as a sum of multiples of special “basis” images. Often the number of basis images that have to be added to get a good approximation of the original image is much smaller than the number of pixels, so this involves storing less information. If this is the case, the representation of the image is called “sparse”.

For our puzzle, we took a baseline image and added multiples of two basis images, so that just two numbers were needed to record each image. We restricted these two numbers each to be 0, 0.5 or 1, so that we generated 9 images in total.

In real image compression, the basis images have to work for any possible starting image you want to compress, so they are much more generic than the footprints that we chose. It might take 40,000 basis images to reproduce the image sufficiently accurately.

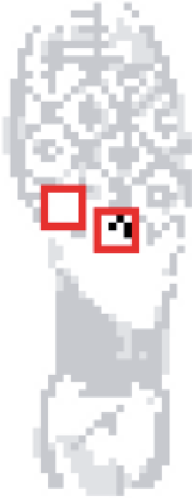
When you take a digital photo, the camera first records the greyscale for each pixel. It then compresses the image by calculated the required multiples (coefficients) of the basis images. These coefficients are then stored on the memory card – between them they capture all the information needed for a program (that knows the basis images) to reconstruct the original image.

The idea of compressed sensing is to bypass the first step. Why do all the work of calculating the greyscale of 4 million pixels, if you are only going to store 40 thousand coefficients? Compressed sensing calculates the coefficients directly. Because there are far fewer coefficients than there are pixels, this can be achieved in a fraction of the time. This is important for applications. For example, in medical imaging this can reduce the time a patient has to spend in an MRI scanner, and minimises the blurring of images caused by the patient moving.

In our puzzle, we were able to identify the two coefficients, and therefore the correct footprint, by “measuring” only the average greyscale over two regions. This was far less information than knowing the greyscale value of each pixel, and yet we can “reconstruct” the whole footprint.

Mathematicians continue to research in the area of compressed sensing, developing new and increasingly powerful techniques that help society in a variety of ways.

Footprint number 1



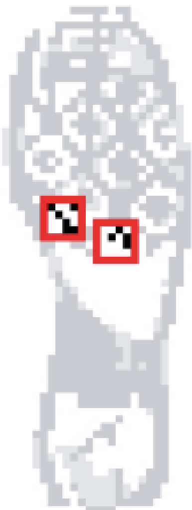
Footprint number 2

Footprint number 3

Footprint number 4

Footprint number 5

Footprint number 6



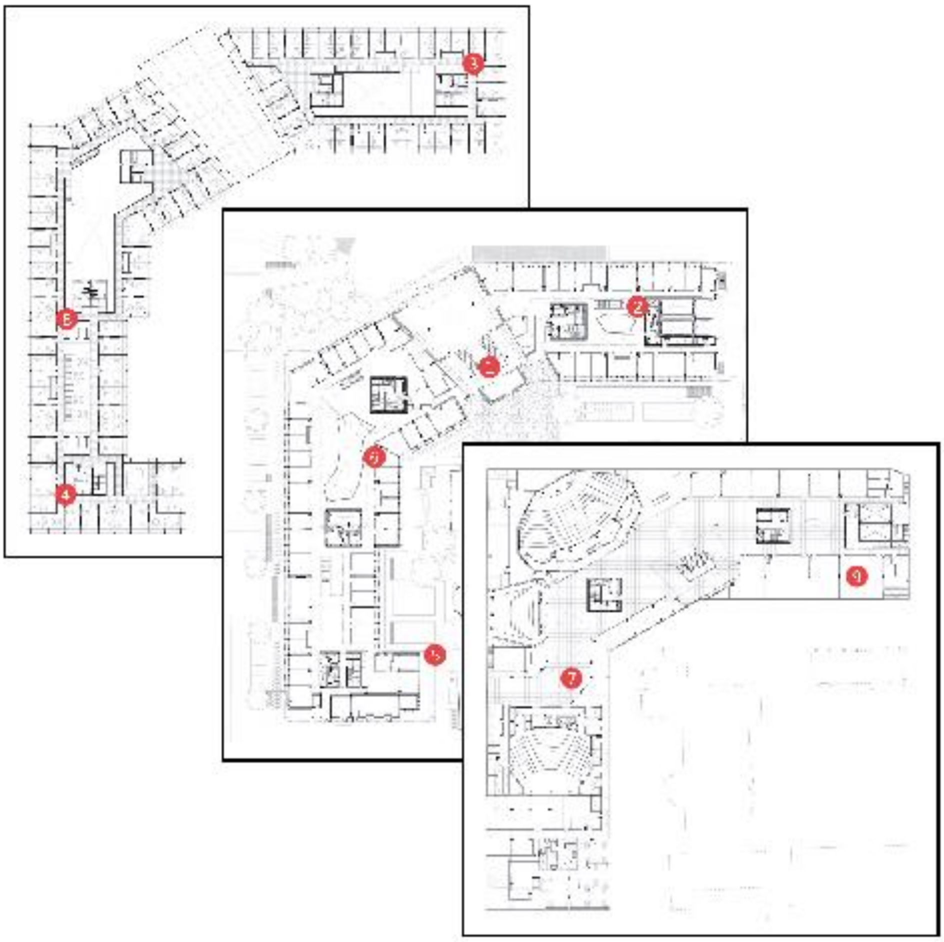
Footprint number 7

Footprint number 8

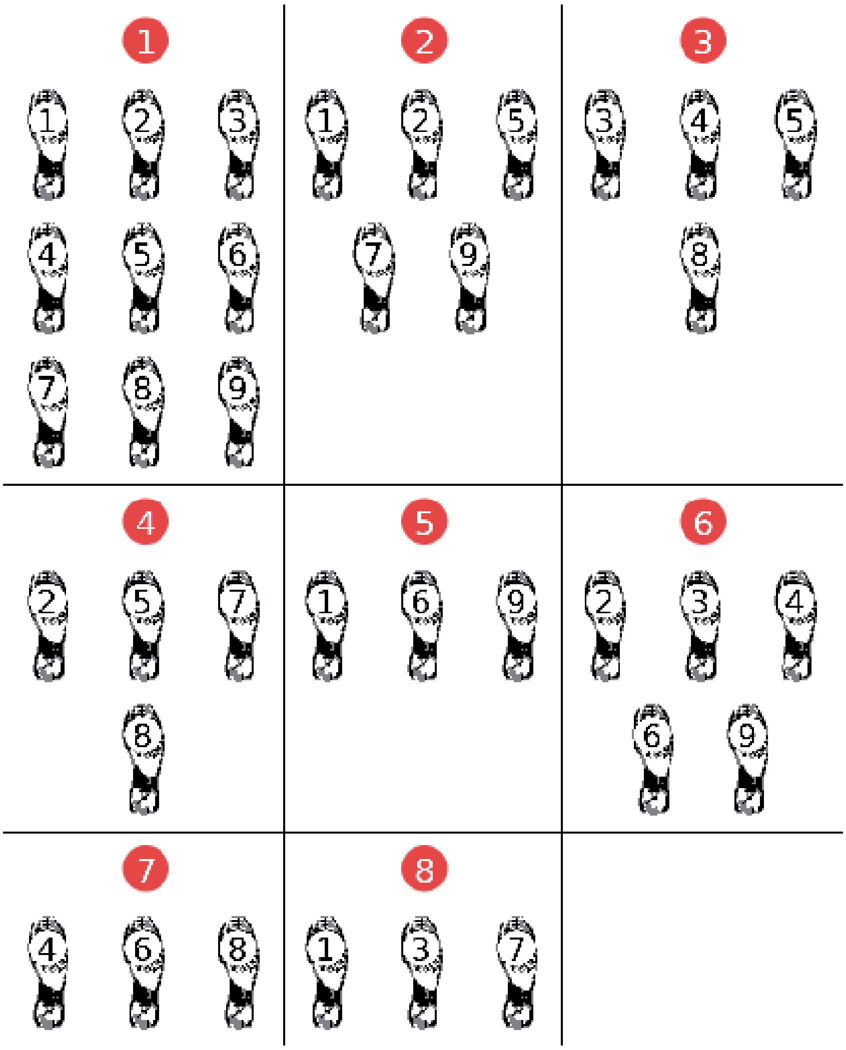
Footprint number 9

The Operations Room

The shortest path



The locations where footprints were discovered



Which footprints were found in which locations

1	0								
2	45	0							
3	110	60	0						
4	100	140	145	0					
5	65	80	125	40	0				
6	45	65	105	60	20	0			
7	60	105	135	130	85	65	0		
8	80	120	120	25	50	70	105	0	
9	30	60	95	130	90	70	55	105	0
	1	2	3	4	5	6	7	8	9

Time in seconds between the locations where footprints were discovered

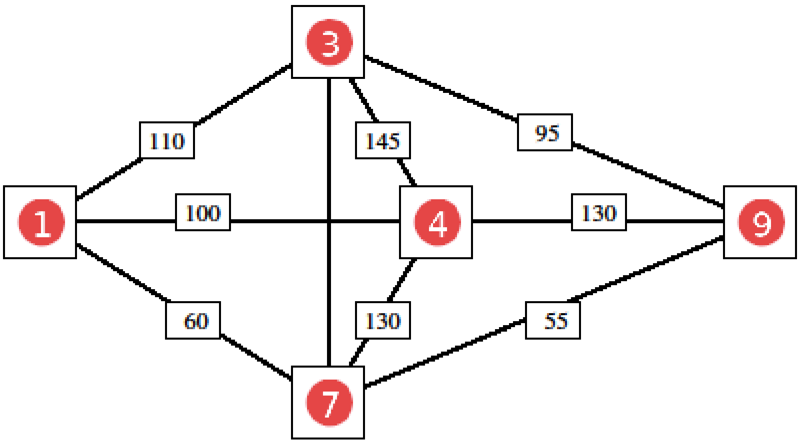
The shortest path: Solution

The thief took 430 seconds (that is, 7 minutes and 10 seconds) to move round the building, so we should look at CCTV for 13:57:50.

From the footprint information, we know that the thief entered the building at position 1, visited locations 3, 4 and 7, and then went to the Savilian Professor's office at 9 to steal the portrait.

The police have collected data on how long it takes to walk between locations. We have to find the shortest path that starts at 1, visits 3, 4 and 7 in some order, and finishes at 9. All journey times are given in seconds.

Here is a network representing the places we have to visit, along with the time taken for each leg in seconds.



There are six possible routes, for the six possible orders of locations 3, 4 and 7. We can calculate the total time each would take:

Route	Time taken in seconds
$1 \to 3 \to 4 \to 7 \to 9$	440
$1 \to 3 \to 7 \to 4 \to 9$	505
$1 \to 4 \to 3 \to 7 \to 9$	435
$1 \to 4 \to 7 \to 3 \to 9$	460
$1 \to 7 \to 3 \to 4 \to 9$	470
$1 \to 7 \to 4 \to 3 \to 9$	430

The shortest path is the final one, which takes 430 seconds, that is, 7 minutes and 10 seconds.

Since the thief arrived at location 9, the Savilian Professor’s office, at exactly 14:05:00, this means that they entered the building 7 minutes and 10 seconds earlier, at 13:57:50, and so this tells us which CCTV shots to look at.

The shortest path: Explanation & discussion

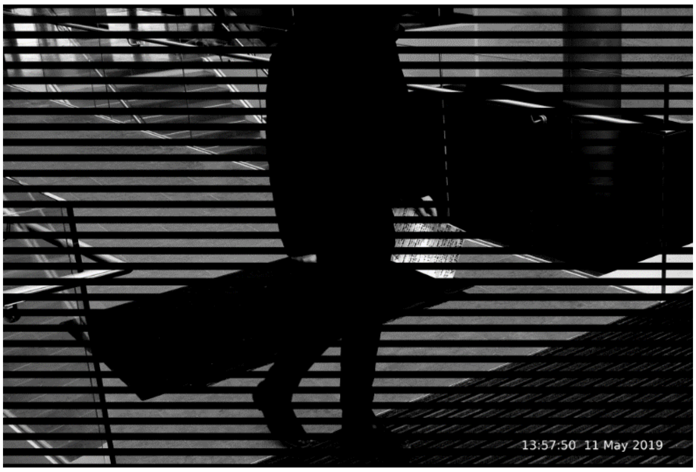
This puzzle is an example of the travelling salesperson problem. In this problem, traditionally we have a list of cities, and we know the distances between all pairs of cities, and we have to find the shortest possible route that visits all the cities. This problem has numerous applications that do not involve physical distance, but instead involve perhaps time or cost, or other notions of "closeness", and there are many variations of the problem.

In our version of the puzzle, there were six possible routes, and it was practical to work out the time for each and so to find the quickest. As the number of calling points increases, the number of possible routes increases extremely fast, and it rapidly becomes impractical to find the lengths of all possible routes. Instead, mathematicians have developed a variety of techniques (algorithms). Some give the best possible solution but work effectively only for small versions of the problem, or versions with special features that make them easier to solve. Others give a route that is guaranteed to be reasonably close to the best, but might not be precisely the best.

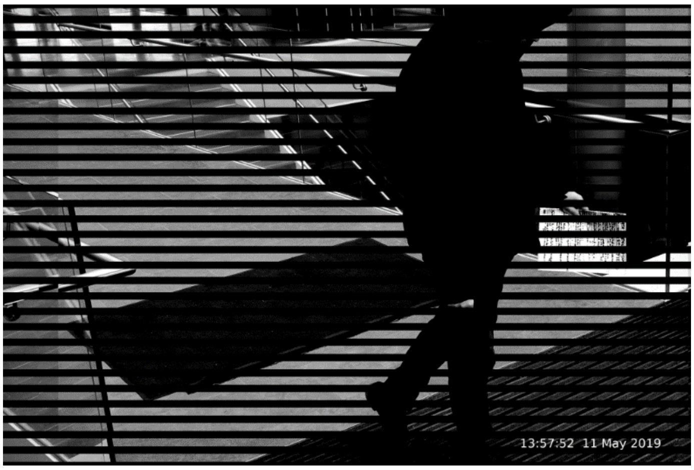
An algorithm is an unambiguous sequence of instructions that will solve a problem. (Knitting patterns are algorithms!) Mathematicians develop algorithms for a wide variety of problems, from blue-sky questions in pure mathematics to real-world applications with everyday practical relevance. Some problems can be solved quickly with efficient algorithms. Others seem intrisically more difficult: either there is no known algorithm that is guaranteed to solve the problem quickly, or in fact it can be proved that no such algorithm exists. The "P versus NP" problem is in the area of computational complexity. In 2000, the Clay Mathematics Institute, whose offices are now here in the Andrew Wiles Building, offered a prize of one million US dollars for the solution of each of seven Clay Millennium Problems, chosen for their mathematical significance. One of these problems is P versus NP, which, like five of the other Clay Millennium Problems, remains unsolved for now. The Poincaré Conjecture is the only one of the problems to have been solved so far.

The Operations Room

CCTV


13:57:50 11 May 2019


13:57:51 11 May 2019


13:57:52 11 May 2019

CCTV: Solution

The CCTV shows the suspect holding their data science homework, with their name Martin Field on it.

The close-ups of the homework show this more clearly.

CCTV: Explanation and discussion

This puzzle relates to the areas of image reconstruction and image alignment. In each image a different part of the paper is shown. By piecing together the information from different images, we can reconstruct the whole text. The human eye is remarkably good at this. We can make sense of the images, even though half of each image has been removed and turned black. Finding ways to get computers to do this automatically is an ongoing subject of research.

Mathematicians have developed algorithms to try to automatically "in-paint" missing areas of photographs. Many of these approaches try to mirror the way the human brain seems to solve the problem. In our puzzle, the eye naturally joins up the broken edges of the piece of paper, imagining one continuous sheet obscured by the black lines. The algorithms similarly look for ways to join up the lines in different parts of the image in the most natural way possible. These algorithms are remarkably successful, and are even being used to help art conservators to restore old masterpieces.

This puzzle involved not just in-painting one image, but also combining the information from different images. To do this, we had to "align" the images, so that the location of a point in the first image can be matched with the corresponding point in the second image. Since the suspect has moved between frames, the images do not align exactly, but the wording on the paper makes it possible for the human brain to identify which parts of one image correspond to which parts of the next.

One application in which such alignment problems arise is medical imaging. For example, tissue samples are often analysed by taking very thin slices, and then staining the slices with chemical markers that identify different components (cell types, for example). The slices can then be stacked together to generate a three-dimensional image. To do this, each sliced needs to be aligned with the slides before and after it in the stack. Mathematicians have developed techniques to automate this alignment process. Usually these algorithms attempt to place each image in such a way as to minimise the different between it and its neighbours in the stack

Note

At this point teams split into two routes. Some went to interview the Savilian Professor and then the Academic Admin Assistant. The rest saw the Academic Admin Assistant and then the Savilian Professor. The puzzles worked the same in either order.

Academic Admin Assistant
Lecture timetabling

There are 9 lectures to schedule, with three rooms available (L1, L2 and L3) at times 9am-10am, 10am-11am, and 2pm-3pm. There are three lecturers:

S1-Analytic Topology S2-Infinite Groups S3-Algebraic Geometry	are lectured by	Professor Anand
S4-Complex Analysis S5-Solid Mechanics S6-Numerical Linear Algebra	are lectured by	Professor Brown
S7-General Relativity S8-Combinatorics S9-Elliptic Curves	are lectured by	Professor Chang

L1 has capacity 360, L2 capacity 210, and L3 capacity 110.
The timetable has to satisfy the following constraints:

1. There are 250 students taking S1-Analytic Topology.
2. Prof Chang refuses to lecture in L1.
3. Some students take both S2-Infinite Groups and S7-General Relativity, so these must be scheduled at different times.
4. Prof Brown prefers to lecture S6-Numerical Linear Algebra before S5-Solid Mechanics.
5. There are 150 students taking S9-Elliptic Curves.
6. Some students take both S1-Analytic Topology and S8-Combinatorics, so these must be scheduled at different times.
7. Prof Anand prefers to lecture S3-Algebraic Geometry in the afternoon.
8. Prof Brown does not like to lecture in L1 first thing in the morning.
9. S2-Infinite Groups must be in a smaller room than S3-Algebraic Geometry.
10. S5-Solid Mechanics must take place in the morning.
11. Some students take both S3-Algebraic Geometry and S9-Elliptic Curves, so these must be scheduled at different times.
12. S3-Algebraic Geometry must be in a larger room than S4-Complex Analysis.
13. Prof Chang does not like to lecture in L3 in the afternoon.
14. S7-General Relativity must be in a smaller room than S6-Numerical Linear Algebra.

	9am-10am		10am-11am		2pm-3pm	
	subject	room	subject	room	subject	room
Prof Anand						
Prof Brown						
Prof Chang						

The following grid may be useful. Each row, column and 3x3 box should contain exactly one ticked box (lecture). Use the constraints to eliminate boxes.

		9–10			10–11			2–3		
		L1	L2	L3	L1	L2	L3	L1	L2	L3
	S1									
Anand	S2									
	S3									
	S4									
Brown	S5									
	S6									
	S7									
Chang	S8									
	S9									

Academic Admin Assistant lecture timetabling: Solution

	9am-10am		10am-11am		2pm-3pm	
	subject	room	subject	room	subject	room
Prof Anand	S1	L1	S2	L3	S3	L1
Prof Brown	S6	L2	S5	L1	S4	L3
Prof Chang	S7	L3	S9	L2	S8	L2

Lecture Timetabling: Explanation and discussion

This is a classic type of logic puzzle. As Sherlock Holmes says, the key is to eliminate the impossible. This is made easier by having an effective way to keep track of the information you have and what you have deduced from it. The grid we gave you was one convenient way to do this.

The information given can be used to eliminate boxes. For example, point 1 says that there are 250 students taking S1, so it must be held in L1. The updated grids after points 1 and 2 have been taken into account are shown here, followed by the completed grid.

		9–10			10–11			2–3		
		L1	L2	L3	L1	L2	L3	L1	L2	L3
	S1		X	X		X	X		X	X
Anand	S2									
	S3									
	S4									
Brown	S5									
	S6									
	S7									
Chang	S8									
	S9									

		9–10			10–11			2–3		
		L1	L2	L3	L1	L2	L3	L1	L2	L3
	S1		X	X		X	X		X	X
Anand	S2									
	S3									
	S4									
Brown	S5									
	S6									
	S7	X			X			X		
Chang	S8	X			X			X		
	S9	X			X			X		

		9–10			10–11			2–3		
		L1	L2	L3	L1	L2	L3	L1	L2	L3
	S1	■	X	X	X	X	X	X	X	X
Anand	S2	X	X	X	X	X	■	X	X	X
	S3	X	X	X	X	X	X	■	X	X
	S4	X	X	X	X	X	X	X	X	■
Brown	S5	X	X	X	■	X	X	X	X	X
	S6	X	■	X	X	X	X	X	X	X
	S7	X	X	■	X	X	X	X	X	X
Chang	S8	X	X	X	X	X	X	X	■	X
	S9	X	X	X	X	■	X	X	X	X

We provided more clues than were strictly necessary. Removing clues 1, 3, 6 and 12 would still only leave one solution to the puzzle.

The foundation of all mathematics is the logical deduction of consequences from assumptions.

This particular problem is also related to Operations Research (also called Operational Research), and in particular to scheduling problems and optimisation. In this field it is typical to have some constraints, and an "objective function" (such as cost, or time). The goal is to devise a schedule that satisfies all the constraints, and that minimises the objective function. In this case, we designed the puzzle so that there is only one solution that satisfies all of the constraints.

Savilian professor of Geometry – Marble Run

Reconstruct the run so that it satisfies the following criteria:

entry above base coloured	exit at base coloured
red	orange
orange	purple
yellow	orange
green	orange
blue	yellow
purple	yellow

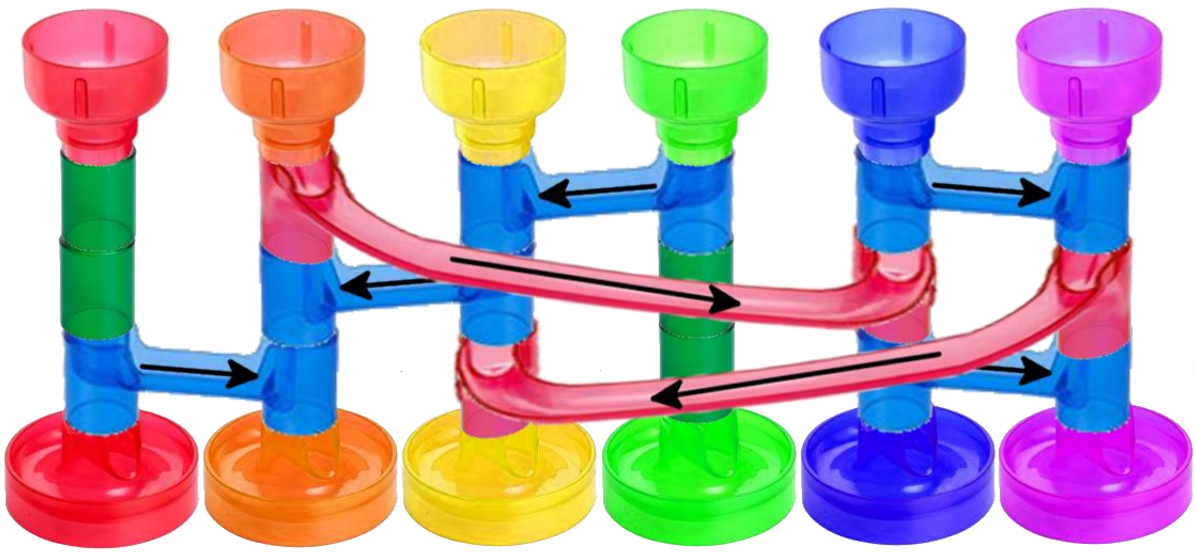
The run is three levels high above every base. The base pieces are in a circle in the order red, orange, yellow, green, blue, purple:

Short chutes couple nearest neighbours:

Long chutes go across the middle and couple opposites:

Savilian Professor marble run: Solution

To make it easier to see, here is the solution stretched into a line rather than a circle.



Marble Run: Explanation and discussion

To create this puzzle, we looked at all possible marble runs that can be constructed using these pieces. For practical reasons, we restricted ourselves to runs that don't have a chute between the red base and the purple base. These runs can be "opened out" and displayed as a line rather than a circle (as in the solution above). This also removes the overcounting where each run effectively occurs 6 times, corresponding to rotating the circle of bases.

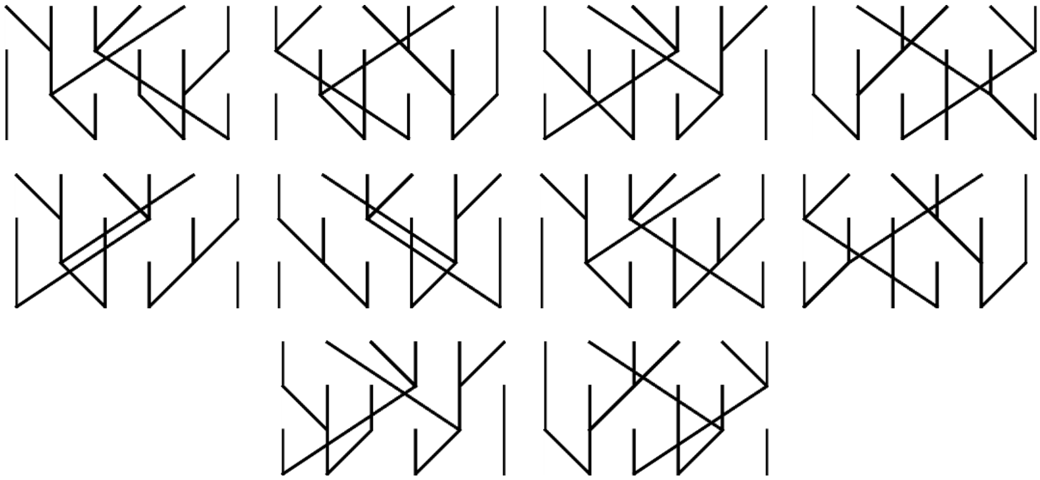
We looked at the input/output pairs for each run. We restricted ourselves to marble runs with a unique solution.

We narrowed the list further by removing marble runs where a marble goes into one colour and comes out of the same colour (red to red, for example).

This left ten possible marble runs, illustrated below.

This is an example of an "inverse problem". Given a set of inputs and the corresponding outputs, and an understanding of the "rules", we have to reconstruct the internal system. This is very similar to problems in medical imaging and seismology. A known input wave is applied, and the resulting reflected/transmitted wave is measured. We have to infer the structure of the unknown material.

Inverse problems are notoriously difficult. Given a marble run, it is easy to discover which inputs lead to which outputs (the “forward” problem): simply put a marble into each input and see what happens. But for the inverse problem you have to keep trying possible structures until you get to the right one. Mathematical techniques for inverse problems usually hinge on a sensible way to update a wrong guess. In this case we might move chutes that carry marbles that end up in the wrong place, but leave those that carry marbles that end up in the right place.



Newsletter


Round up
The Oxford Mathematics Newsletter · Spring 2019
University of Oxford
Mathematical Institute
Spotlight on Sir Henry Savile
Art exhibition
Oxford mathematical alphabet
Research highlights
New starters
2018 highlights
Oxford Mathematics

In This Issue...

Some 2018 highlights and 2019 resolutions from Institute members

My mathematical highlight of 2018 was...

Getting my first paper published

Learning about D-modules and the Bernstein-Sato theorem

Advancing the state of the art in cryptography

My mathematical resolution for 2019 is to...

Solve the dynamics of stratified flows at large scales using statistical mechanical methods

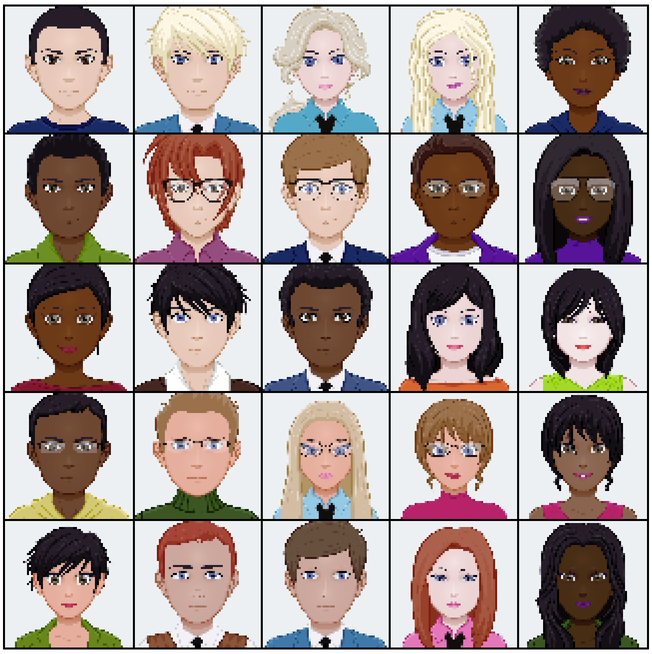
Solve the Riemann hypothesis

Earn myself a million dollars in the process

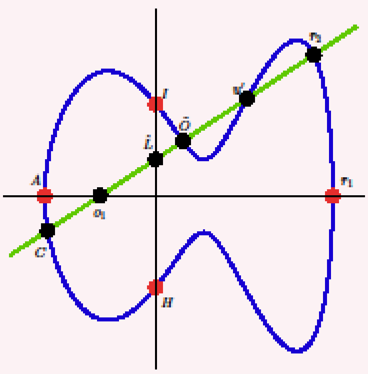
Solve the turbulence problem as an encore

New starters

A big welcome to our new graduate students.



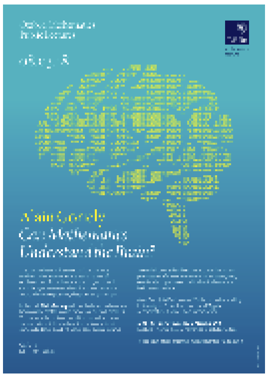
Research highlights



From L-functions to tracking disease via data security and why your cup of coffee sloshes, our researchers continue to expand the boundaries of mathematics. In a funding environment where impact is ever more important, spreading the word about our important and intellectually challenging work is crucial. A full range of our case studies can be found in the Research section of our website at: http://maths.ox.ac.uk/research/case-studies

Public Lectures

The audiences for our Oxford Mathematics Public Lectures are full of aspiring mathematicians still in their teens, and a healthy cohort from the "I was useless at maths at school" crew.

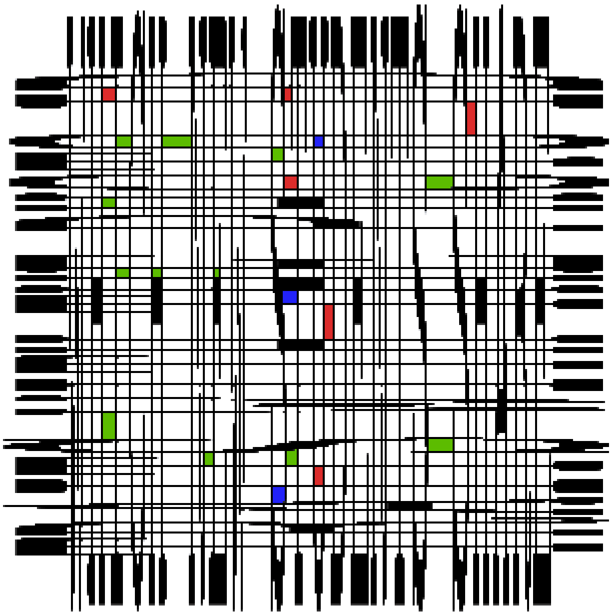


Mathematics is an entry to learning about science and technology today, and our lectures cover as much maths and life as possible—from tackling influenza via the mathematics of architecture to the enduring mystery of prime numbers.

All of our lectures are broadcast live and you can watch them any time on our Oxford Mathematics YouTube Channel.

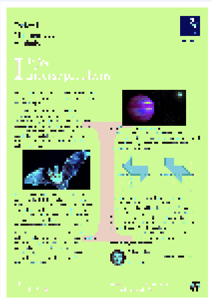
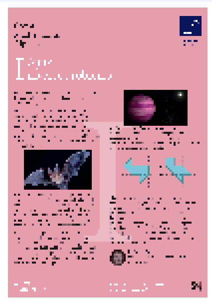
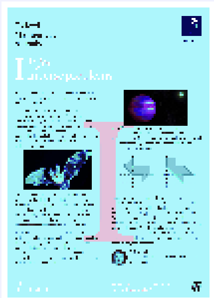
Art exhibition

From 1st May to 9th May the institute will be hosting an exhibition of work by the Dutch artist Piet Mondrian, including *Composition in red, blue and green*, shown below.



Oxford Mathematical Alphabet

The Oxford Mathematics Alphabet posters are a way of promoting our research and introducing potential students to our work. They can all be viewed on, or downloaded from, the Mathematical Institute website.

Outreach

Our Mathematical Institute Outreach team visits and hosts schools from across the country, especially those from under- represented groups and areas.

In 2016/17 we spent more than 500 hours on outreach activities, interacting with over 15,000 students. Over 300 of these individual students would be the first in their family to go to higher education, over 600 came from neighbourhoods where the fewest number of people have historically entered higher education, and over 700 came from high deprivation neighbourhoods. Over 1500 different schools attended our events, or were visited across the UK, and we interacted with students from nearly every local authority.

The 2017 UNIQ, PROMYS Europe and Sutton Trust summer schools were all very successful, with 64 students applying and 20 students being made admissions offers.

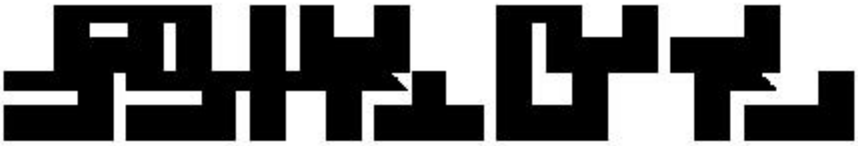
Sir Henry Savile

Sir Henry Savile (1549-1622) was a well-known English scholar and mathematician. He served as the Warden of MerTon College, Oxford, and the Provost of Eton.

He endowEd the Savilian chairs of Astronomy and of Geometry at Oxford University in 1619. With a bit of detective work, Savile was one of the scholaRs who translated the New Testament from Greek into English. Savile was keen to impart his uNderstanding of mathematics to his students at Oxford, and in founding the Geometry chair he gave thirteen prepAratory lectures on the original books of Euclid's Elements in 1620.

There have been 20 Savilian Professors of Geometry, including John Wallis, who introduced the use of ∞ for infinity, and Edmond Halley, who successfully pRedicted the return of the comet named in his honour. The painting of Savile shown here still hangs in the office of the current Savilian Professor of GeometrY, Dame Frances Kirwan.

8 26 7 1 71 4 7 67 26 68



On the representation of numbers in different bases

S.J. Chapman[1]

[1]OCIAM, Mathematical Institute, Andrew Wiles' Building,
Radcliffe Infirmary Quarter, Woodstock Road, Oxford OX2 6GG (UK)

(Dated: March 13, 2019)

Numbers are usually written in decimal format, otherwise known as base 10. However, it is possible to use other bases. Here it is shown how to translate a number from one base into another.

I. INTRODUCTION

In place-value notation the position of a numeral within a number indicates its value. For example, in the number 11 the first digit "1" represents the number 10, since it lies in the second ("tens") column, while the second digit "1" represents just 1, since it lies in the first ("units") column. Not all number systems follow this rule: in Roman numerals, for example, 1, 10, 100 and 1000 are each given separate, distinct symbols (I, X, C and M, respectively [1]). However, calculations with numbers are much easier using positional notation, and the system has now been universally adopted.

On the other hand, the convention that the second column represents *tens*, and the third *hundreds*, etc. is somewhat arbitrary (and in fact probably arose because we have ten fingers and thumbs). It is quite possible to use a number other than ten as the *base* of the system. Here we investigate the representation of numbers in some other bases.

binary representation					decimal representation	
2^4	2^3	2^2	2^1	2^0	10^1	10^0
16	8	4	2	1	10	1
				1		1
			1	0		2
			1	1		3
		1	0	0		4
		1	0	1		5
		1	1	0		6
		1	1	1		7
	1	0	0	0		8
	1	0	0	1		9
	1	0	1	0	1	0
	1	0	1	1	1	1
	1	1	0	0	1	2
	1	1	0	1	1	3
	1	1	1	0	1	4
	1	1	1	1	1	5
1	0	0	0	0	1	6
1	0	0	0	1	1	7
1	0	0	1	0	1	8
1	0	0	1	1	1	9

TABLE I: Binary representation of the first 19 numbers.

II. BINARY NOTATION

Perhaps the simplest base to use is the number two. Base two is also known as binary notation (in the same way that base 10 is known as decimal notation) [2]. In this case the second column represents the "twos", the third column the "fours" ($4 = 2 \times 2 = 2^2$), the fourth column the "eights" ($8 = 2 \times 2 \times 2 = 2^3$), etc. Now each column can contain only the digits 0 or 1, since the number 2 is represented by a "1" in the twos column. The binary representation of the first 19 numbers is shown in Table I.

We see from this table, that the decimal number 13 for example (which represents $1 \times 10 + 3 \times 1$) is represented in binary by the number 1101 (which represents $1 \times 8 + 1 \times 4 + 1 \times 1$).

III. TERNARY NOTATION

The next simplest base to use is the number three. Base three is also known as ternary notation. In this case the second column represents the "threes", the third column the "nines" ($9 = 3 \times 3 = 3^2$), the fourth column the "twenty-sevens" ($27 = 3 \times 3 \times 3 = 3^3$), etc. Now each column can contain only the digits 0, 1, or 2, since the number 3 is represented by a "1" in the threes column. The ternary representation of selected numbers is shown in Table II [3].

We see from this table, that the decimal number 14 for example (which represents $1 \times 10 + 4 \times 1$) is represented in ternary by the number 112 (which represents $1 \times 9 + 1 \times 3 + 2 \times 1$).

IV. HEXADECIMAL NOTATION

It is also possible to use bases which are greater than ten. One base which is in common use in computing is base 16, which is also known as hexadecimal notation [4].

When we use a base greater than 10 we need some more symbols, since the numbers 10 to 15 must now be

ternary representation				decimal representation	
3^3	3^2	3^1	3^0	10^1	10^0
27	9	3	1	10	1
			1		1
			2		2
		1	0		3
		1	1		4
		1	2		5
		2	0		6
		2	1		7
		2	2		8
	1	0	0		9
	1	0	1	1	0
	1	0	2	1	1
	1	1	0	1	2
	1	1	1	1	3
	1	1	2	1	4
	1	2	0	1	5
	1	2	1	1	6
	1	2	2	1	7
	2	0	0	1	8
	2	0	1	1	9
	2	0	2	2	0
	2	1	0	2	1
	2	1	1	2	2
	2	1	2	2	3
	2	2	0	2	4
	2	2	1	2	5
	2	2	2	2	6
		⋮		⋮	
2	1	1	1	6	7
2	1	1	2	6	8
2	1	2	0	6	9
2	1	2	1	7	0
2	1	2	2	7	1

TABLE II: Ternary representation of the selected numbers.

represented by a single digit in the "units" column. It is common to adopt the symbols A to F for these numbers.

In base 16 the second column represents the "six-teens", the third column the "two-hundred-and-fifty-sixes" ($256 = 16 \times 16 = 16^2$), etc. The hexadecimal representation of the first 19 numbers is shown in Table III.

We see from this table, that the decimal number 18 for example (which represents $1 \times 10 + 8 \times 1$) is represented in hexadecimal by the number 12 (which represents $1 \times 16 + 2 \times 1$).

hexadecimal representation			decimal representation	
16^2	16^1	16^0	10^1	10^0
256	16	1	10	1
		1		1
		2		2
		3		3
		4		4
		5		5
		6		6
		7		7
		8		8
		9		9
		A	1	0
		B	1	1
		C	1	2
		D	1	3
		E	1	4
		F	1	5
	1	0	1	6
	1	1	1	7
	1	2	1	8
	1	3	1	9

TABLE III: Hexadecimal representation of the first 19 numbers.

V. CONCLUSION

Positional notation in which the value of a digit depends on its position within a number is an incredibly useful concept. However, the use of ten as the base of such a numbering system, though common, is somewhat arbitrary. We have demonstrated in this paper how to translate between numbers written in different bases.

Acknowledgments

The author acknowledges useful discussions with Drs Vicky Neale and Mareli Grady.

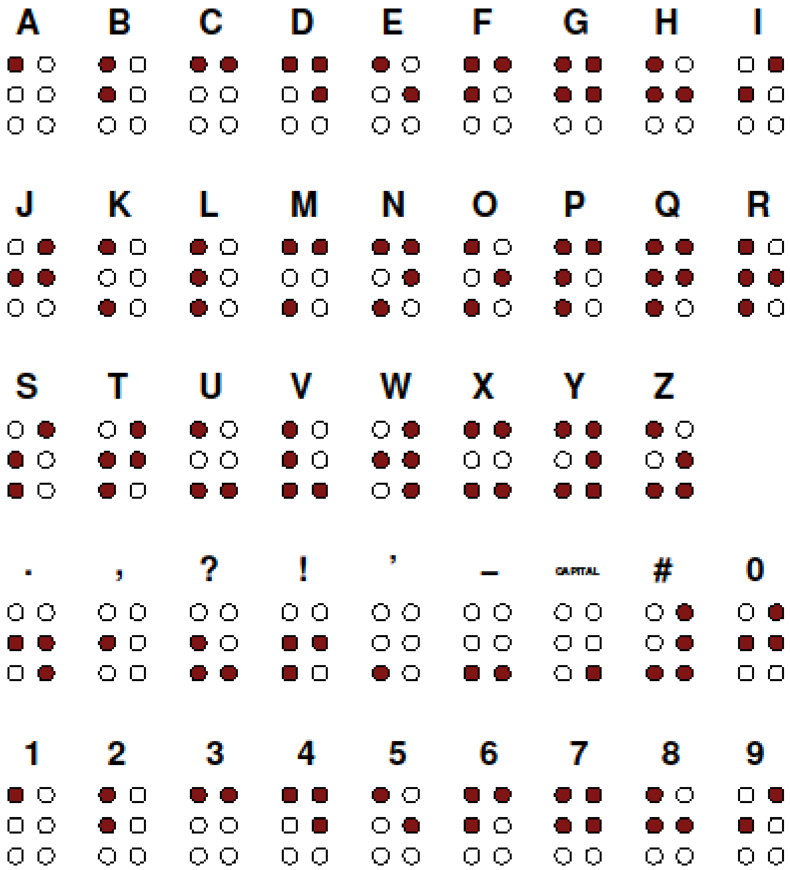
[1] *Encyclopaedia Britannica* (Encyc. Brit. Inc., 2010).
[2] A. Other, Journal of Hard Sums **4**, 20 (2001).
[3] J. Bloggs, Journal Eng. Math. Soc. **100**, 2290 (2012).
[4] A. Goriely, Oxford J. Comp. Sci. **245**, 112 (2015).

HANDBOOK OF SIGNALLING SYSTEMS

BECKERLEG
ROBERTS
AND
YING

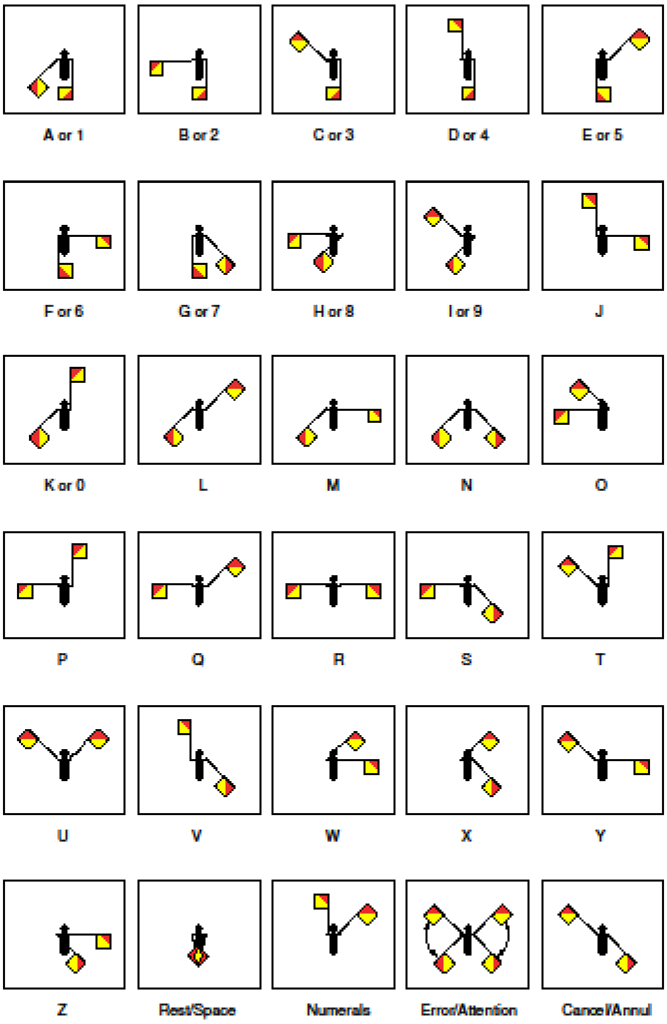
This version was created on February 26, 2019.

1 | Braille



2

2 | Semaphore

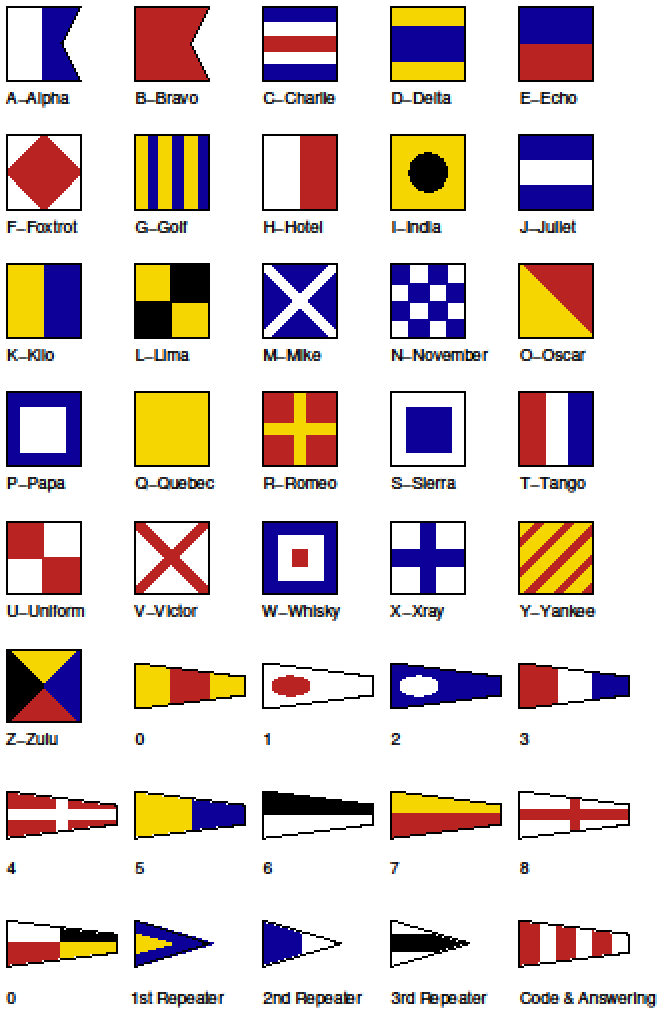


3

3 | International Morse Code

A	● ▬	U	● ● ▬
B	▬ ● ● ●	V	● ● ● ▬
C	▬ ● ▬ ●	W	● ▬ ▬
D	▬ ● ●	X	▬ ● ● ▬
E	●	Y	▬ ● ▬ ▬
F	● ● ▬ ●	Z	▬ ▬ ● ●
G	▬ ▬ ●		
H	● ● ● ●		
I	● ●		
J	● ▬ ▬ ▬		
K	▬ ● ▬	1	● ▬ ▬ ▬ ▬
L	● ▬ ● ●	2	● ● ▬ ▬ ▬
M	▬ ▬	3	● ● ● ▬ ▬
N	▬ ●	4	● ● ● ● ▬
O	▬ ▬ ▬	5	● ● ● ● ●
P	● ▬ ▬ ●	6	▬ ● ● ● ●
Q	▬ ▬ ● ▬	7	▬ ▬ ● ● ●
R	● ▬ ●	8	▬ ▬ ▬ ● ●
S	● ● ●	9	▬ ▬ ▬ ▬ ●
T	▬	0	▬ ▬ ▬ ▬ ▬

4 Nautical signal code flags



Newsletter – Sir Henry Savile coded message: Solution

The message read “money in box”.

In the article about Sir Henry Savile, there were several mistakenly capitalised letters. Taken in order, these spell the word TERNARY.

This indicates that the numbers under the article must be translated into ternary. The “research paper” *On the representation of numbers in different bases* helps with this. The result is

22 222 21 1 2122 11 21 2111 222 2112

Also, there are three words in the article that appear in a different font (san serif). Taken in order, these read well-known Oxford detective. This was a clue to Morse.

The numbers written in ternary represent letters written in Morse code. Here a 2 represents a dash – and 1 represents a dot · so the message becomes

– – – – – – · · – · – – · · – · – · · · – – – – · · –

The booklet of signalling systems helps to decode the letters, which translate as

M O N E Y I N B O X

Newsletter Coded Message: Explanation and discussion

Mathematicians have contributed to the study of cryptography for a very long time, both by designing tools that can be used to encrypt messages and by developing techniques for decrypting messages. Famously mathematicians such as Alan Turing and Bill Tutte, amongst many others, contributed to the work at Bletchley Park, breaking the Nazis’ Enigma and Lorenz ciphers. Encryption is now fundamental to everyday life: not only do governments and large companies need secure communication methods, but also we rely on encryption for safe online banking and shopping, for example.

Modern “public key, private key” cryptography draws on a variety of mathematical ideas. Some relate to prime numbers, especially the idea that it is (with the help of a computer) relatively quick to multiply two very large prime numbers, but (even with the help of a computer) it is very time-consuming to break up such a product into the original prime numbers. For example, it is quick to multiply 29 and 37 to get the answer 1073, but given the answer 1073 it takes much longer to factorise to find the primes 29 and 37 – and this effect increases when the primes have many digits. Another approach to modern cryptography uses elliptic curves, also famous for being one of the objects of study in Andrew Wiles’s proof of Fermat’s Last Theorem.

When sufficiently powerful quantum computers become a practical reality, these cryptosystems are not going to be strong enough. While a normal computer takes a long time to factorise a number, there is an algorithm that will allow a quantum computer to do it sufficiently quickly to threaten the integrity of the code. For this reason, and because cryptography is so important to us all, researchers in the Oxford Mathematical Institute are actively working on “post-quantum cryptography”.

Newsletter faces board: Solution

The faces board reveals the name SMITH, which belongs on the ID card found by the Academic Admin Assistant.

Newsletter faces board: Explanation and discussion

The “Mondrian painting” was a clue to understanding the faces board. Viewed from a shallow angle, it shows the words

WHICH NEW STUDENTS SHARE   TRAITS WITH THE PHOTO ID

There were five clues, one for each blank letter space on the ID card, hidden in different places in the Newsletter. Each clue is to a trait, and each trait gives a letter. The five traits are glasses, eye colour, tie, clothes colour, and hair colour. We need to identify which of the new students share each trait with the photo ID (that is, no glasses, brown eyes, wearing a tie, blue clothes, dark brown hair). For each trait, the position of the students in the grid spells out a letter. The letters are S, M, I, T, H, giving the name Smith.

The articles that give clues to the five traits are the articles mentioned on the front cover of the Newsletter.

In the box “In This Issue…”, the first (coloured) letters of the 2018 highlights and 2019 resolutions of Mathematical Institute members spell out GLASSES.

In the box “Oxford Mathematical Alphabet”, there are three posters. Each one is an I (eye), but they are in different colours. This is a clue for EYE COLOUR.

In the box “Art exhibition”, the Mondrian painting is called *Composition in red, blue and green*. Looking at the colour squares in the painting, the red squares form the letter T, the blue squares form the letter I, and the green squares form the letter E. Taken in order, these spell TIE.

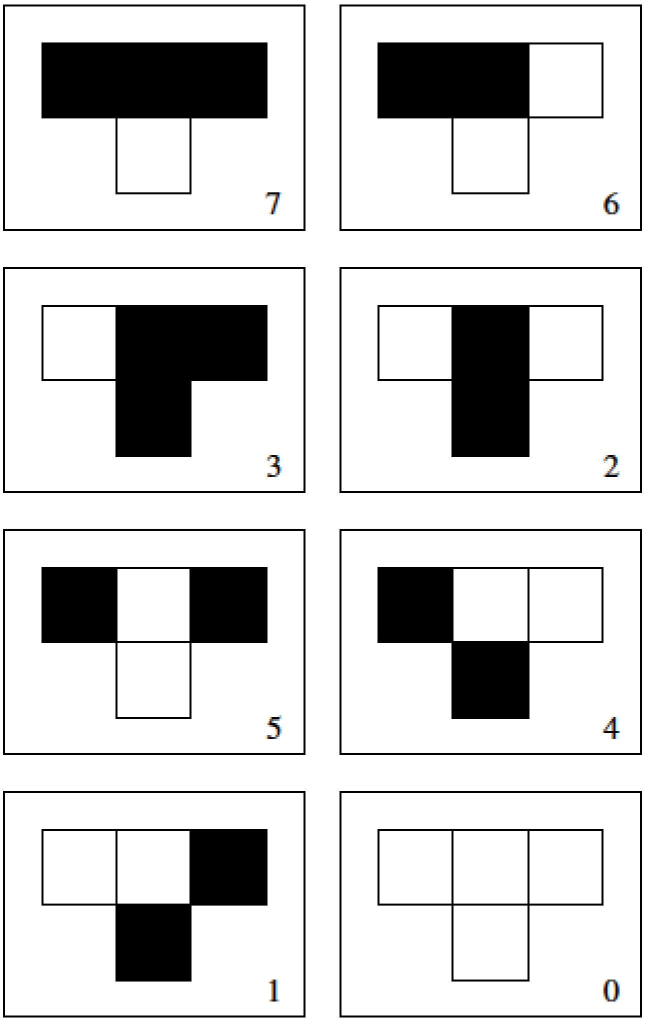
On the back page, the geometric shapes at the top and bottom of the page correspond to two words that have been sliced in half (and turned upside down). The words can be identified by folding the newsletter to match the edges. This gives CLOTHES COLOUR.

In the box "Research highlights", the labelled points on the figure spell out HAIR (red points taken clockwise) COLOUR (black points taken from left to right).

This puzzle is inspired by ideas of data visualisation. Each image has a lot of associated data: position in the grid, glasses, eye colour, hair colour, hair style, etc. We could represent each image by a point in a high-dimensional space, based on its attributes. Each of the letters corresponds to picking out just one of the variables, ignoring the others. This is a form of data visualisation. Repeatedly doing this, for different variables, helps us to understand what the data looks like in the high-dimensional space. However, clusterings that form when we consider just one or two variables can be misleading: images may seem close together based on one or two attributes, but are in fact far apart in the high-dimensional space where we consider all the attributes. Such data clustering is an active area of research, particularly in the field of bioinformatics, where the "images" may be cells and the attributes may be active genes, for example.

Professor Smith's office: The locked padlock

									[illegible]
									[illegible]
	?	?	?	?	?	?	?		[illegible]
	?	?	?	?	?	?	?		[illegible]
	?	?	?	?	?	?	?		144
	?	?	?	?	?	?	?		[illegible]
	?	?	?	?	?	?	?		[illegible]
	?	?	?	?	?	?	?		[illegible]


7
6
3
2
5
4
1
0

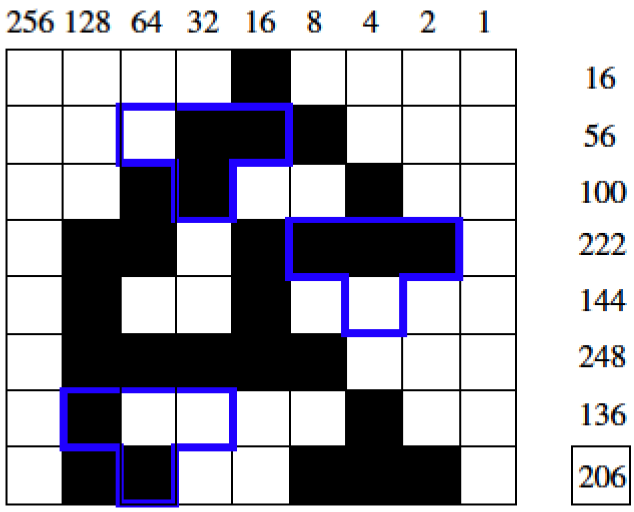
The locked padlock: Solution

To unlock the padlock we need the three-digit code 206.

The locked padlock: Explanation and discussion

This is an example of a cellular automaton. Each row represents a state of the system. To move from one row to the next, we must follow rules given by the T-shapes. The colour of each box at the next step depends on its colour and also the colours of its two neighbours either side. Only the highlighted squares have to be filled in, because the boxes at the edge of the grid are known to be white.

Here is the full grid, with some of the T-shapes highlighted.



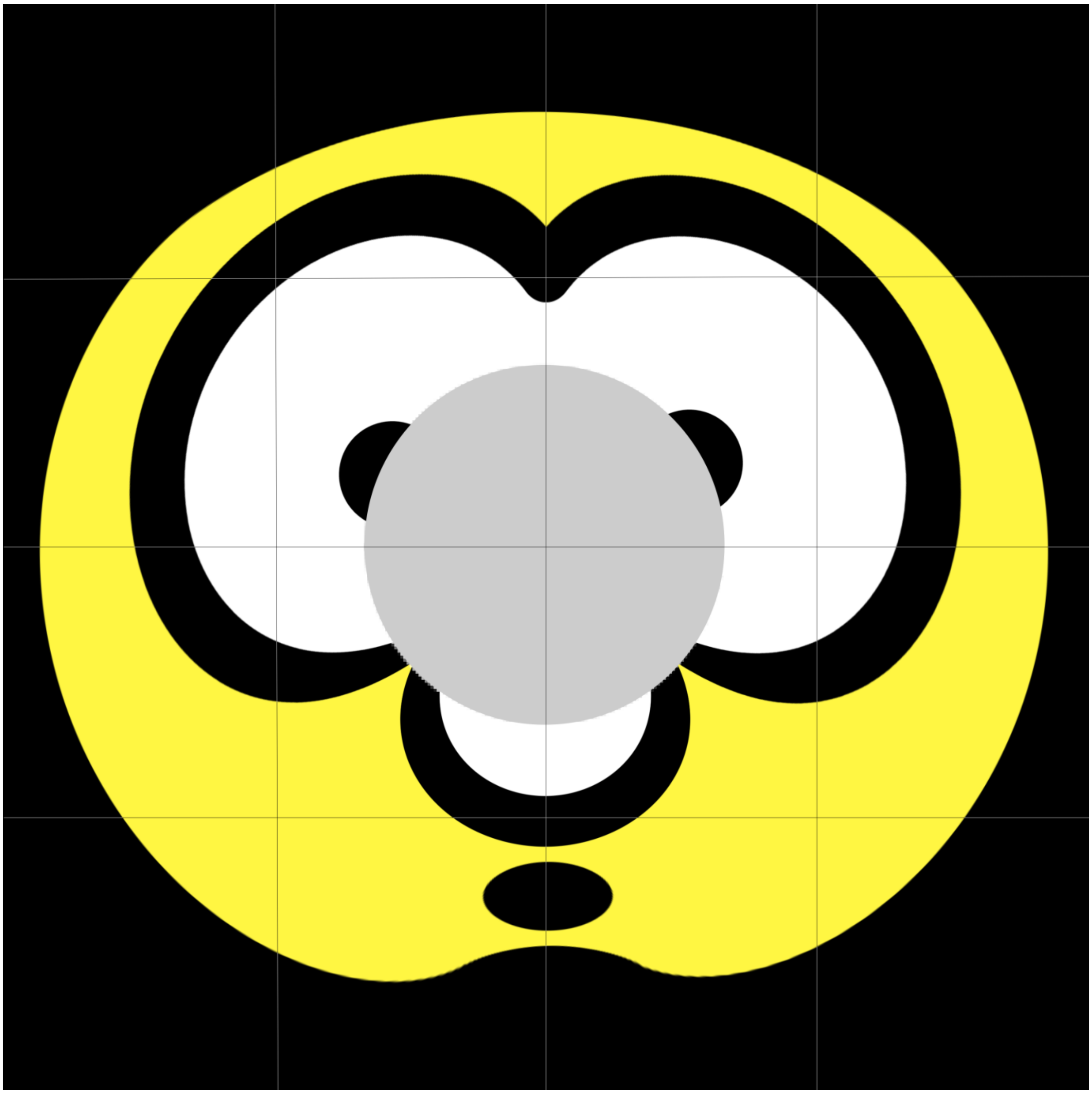
To find the numbers in the column on the right, we think of each row as giving the binary representation of a number. Here black represents 1 and white represents 0. There was a clue in the numbering of the T-shapes, because each is labelled with the number corresponding to the top row of the T with the same binary representation.

As well as being a mathematical curiosity, other types of cellular automata are used to model a wide variety of systems in biology and physics. For example, they can describe the propagation of disease or of rumours, the evolution of a population of organisms, and the flow of granular materials. The states of a box might be “healthy”, “diseased” or “recovered”, for example, with the rules from one row to the next (corresponding to a period of time) being chosen based on the transmission characteristics of the disease.

The rules for the cellular automaton we used in our puzzle are specified by the T-shapes. There are eight possibilities for the colours in the top row of the T-shapes. To define a simple black-white nearest-neighbour cellular automaton, we need to say what the new colour of the centre box will be for each of these possible combinations (that is, which colour goes in the square at the base of the T). So each such cellular automaton corresponds to a sequence of 8 coloured boxes, each black or white. There are $2^8 = 256$ such sequences, and so 256 such cellular automata.

It turns out that many of these give regular patterns, but a few look more random. The rule we used in our puzzle is known to be chaotic – the pattern it gives never repeats.

Professor Smith's office: The jigsaw



The jigsaw: Solution

The jigsaw pieces must be assembled to create the image on the previous page. We then place the conical mirror (that we were given earlier) onto the grey circle. The reflected image fills in the centre:

This shows that the portrait has been hidden in the “Plant Room”, which is labelled on the map with the same warning triangle.

The jigsaw: Explanation and discussion

This image is an example of anamorphosis. The image has been distorted, so that it can only be viewed from a certain angle and in this case also with the help of a specially shaped mirror. The “Mondrian painting” in the Newsletter was also an example of anamorphosis. By viewing it from particular angles, the elongated text became visible.

Artists have used anamorphosis in various ways for centuries. Advertising on sports pitches is sometimes created using anamorphosis, so that from the angle of the television camera the text will appear clearly.

Mathematicians have contributed to the study of anamorphosis, and more broadly matters of perspective. In fact there is a branch of mathematics called “projective geometry”, which explores this. There are different perspectives (pun intended) on geometry. Sometimes, what matters is the precise distance between two points, or how curved a particular shape is (as in differential geometry). On other occasions, what matters is whether the objects have “holes”: the old joke is that for a topologist a coffee mug is the same as a doughnut, because both have one hole, and if the mug was made of a sufficiently stretchy material then it could be reformed into a doughnut. Sometimes, what matters is not the lengths of lines or the angles between them, but just whether they are lines and whether they meet, and this is what projective geometry studies.

There is a theorem from projective geometry, often called Pascal’s Mystic Hexagon, built into the Andrew Wiles Building. The north crystal is a glass area over the seating in the north wing of the Mezzanine (outside the lecture theatre that acted as the Savilian Professor’s office for the Escape Room). The design of the glass and frame includes a diagram representing Pascal’s Mystic Hexagon, as well as a theorem from graph theory.