Quantitative Morse Theory on Loop Spaces

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Theorem of Lusternik and Schnirelmann

Theorem

On any Riemannian 2-sphere there exist at least three simple periodic geodesics.

joint with Y. Liokumovich and A. Nabutovsky

Theorem

Let M be a Riemannian sphere of diameter d and area A. There exist three distinct non-trivial simple periodic geodesics of length at most $20d$. Moreover, there exist three distinct simple periodic geodesics on M of length at most $800d \max\{1, \log \frac{\sqrt{A}}{d}\}$ such that none of these geodesics has index zero.

Summary of the proof of Lusternik and Schnirelmann's theorem

- Consider the space ΠM of non-parametrized simple curves on a 2-dimensional Riemannian sphere. Consider its subspace $\Pi_0 M$ of constant curves of M. Note that $\Pi_0 M$ can be naturally identified with M.

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- Consider the three relative homology classes of the pair $(\Pi M, \Pi_0 M)$ with coefficients in Z_2
- Lusternik and Schnirelmann constructed a curve shortening flow that should not create self-intersections. Thus, they get stuck on simple closed geodesics.

Proof

Based point loops

- The length of a shortest geodesic loop on M^n

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- The length of a shortest geodesic loop at each point $p \in M^n$ of a closed Riemannian manifold.

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- R. R. , "The length of a shortest geodesic loop at a point", JDG, 78 (2008), 497-519

J.-P. Serre's theorem

Theorem

Given a pair of points on a closed Riemannian manifold, there exist infinitely many geodesics connecting them.

J.-P. Serre, "Homologie singuliere des espaces fibres", Annals of Mathematics (1951), 425-505

Question

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Let $p, q \in M^n$, where M^n is a closed Riemannian manifold of dimension n. Is there a function $f(k, d)$, such that for every k there exist at least k geodesics connecting p and q of length at most $f(k, n)d$, where d is the diameter of M^n?

Curvature-free estimates for the lengths of geodesics

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Theorem

Let M^n be a closed Riemannian manifold of diameter d. Then for each pair of points $p, q \in M^n$ there exist at least k geodesics of length at most $4nk^2d$

Linear bounds for the length of geodesics on closed Riemannian surfaces

- Let M be a Riemannian 2-sphere of diameter d. Then for each pair of points p, q there exists at least k geodesics of length at most $22kd$, ($20kd$, when $p = q$).

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- Herng Yi Cheng, "Curvature-free linear length bounds on geodesics in closed Riemannian surfaces", Transactions of the AMS.

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- Cartan-Serre's theorem implies the existence of an even-dimensional real cohomology class u of the loop space $\Omega_p M^n$ such that all of its cup powers are non-trivial. Using rational homotopy theory one can prove that there exists such a class u of dimension at most $2n-2$.

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- Apply Morse theory to produce critical points of the length functional on $\Omega_p M^n$ corresponding to cohomology classes u^i.

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- Therefore, critical point corresponding to u^i also corresponds to c^i. We need to choose a representative of c.
- Let L be such that this representative c is contained in the set of loops of length $\leq L$. Then c^i can be represented by a set of loops of length $\leq iL$.

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- We will use the same pseudo-extension technique.

Proof

General dimension

If there are not many geodesic loops, then any sphere in the loop space can be replaced by a homotopic sphere passing through short loops and our result follows.

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Theorem

Let M^n be a closed Riemannian manifold of dimension nand diameter d. Then either there exist non-trivial geodesic loops with lengths in every interval $(2(i-1)d, 2d]$ for $i \in \{1, ..., k\}$ or all maps $f : S^m \longrightarrow \Omega_p(M^n)$ can be homotoped to a map of loops based at p of length that does not exceed $((4k+2)m+(2k-3))d$, and the length of loops during this homotopy does not increase that much in comparison with the maximal length of loops in the image of f.

Observation

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- the length of curves in the homotopy is at most $L+2d$
- the length of the resulting curve is at most $l+d$
- there exist a family of curves β_t which connect p with $\gamma(t)$ and the maximal length of the curves in this family is at most $l+3d+\delta$

Proof of the observation

Proof

Problem 8

(a) Let M^{2n} be a Riemannian $2n$-sphere for some natural number n with a positive sectional curvature. Let $\gamma : [0,1] \longrightarrow M^{2n}$ be a periodic geodesic on M^{2n}. Show that the index $i(\gamma) \geq 1$.
(b) Let M^n be a closed Riemannian manifold with $Ric \geq (n-1)$ where Ric is the Ricci curvature of M^n. Show that any geodesic loop $\gamma_p \in \Omega_p M^n$ of length $> \pi$ has index $i(\gamma_p) \geq 1$.

Problem 9

Let M^n be a Riemannian manifold, such that $Ric \geq (n-1)H$. Given $r, \epsilon > 0$ and $p \in M^n$ prove that there exists a covering of $B_r(p)$ by balls $B_\epsilon(p_i)$, where $p_i \in B_r(p)$, with the number of balls N bounded in terms of n, H, r, ϵ. Compute some bound on N.

Problem 10

We will say that a point q on a manifold M^n is critical with respect to p, if for all vectors v in the tangent space T_qM, there exists a minimal geodesic γ from q to p with the absolute value of the angle between $\gamma'(0)$ and v at most $\frac{\pi}{2}$
Let q_1 be critical point with respect to p and let q_2 satisfy $d(p,q_2) \geq \alpha d(p,q_1)$ for some $\alpha > 1$. Let γ_1, γ_2 be minimal geodesics from p to q_1, q_2 respectively, and let θ be the angle between $\gamma_1'(0)$ and $\gamma_2'(0)$. If sectional curvature K_M of a closed Riemannian manifold M is bounded from below by -1 show that

$$\cos\theta \leq \frac{\tanh\frac{d}{\alpha}}{\tanh d}$$

. Here d denotes the diameter of M.

Hint: Use Toponogov comparison theorem twice.

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- A. Nabutovsky, R. Rotman, "Volume, diameter and the minimal mass of a stationary 1-cycle", GAFA, 14 (2004), 748-790

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- K. Irie, "Dense existence of periodic Reeb orbits and ECH spectral invariants", J. Mod. Dyn. 9 (2015)

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- Y. Liokumovich, F. C. Marques, A. Neves, "Weyl law for the volume spectrum, Ann. Math., vol. 187 (3) (2018), 933-961.
- Similar result for minimal hypersurfaces, $3 \leq n \leq 7$.

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Let S be a 2-sphere with a smooth Riemannian metric with positive curvature. There exists a geodesic net G partitioning S into three components shaped either as a θ-graph, "figure 8", or "glasses".

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- Given a graph, a positively curved 2-sphere and target total curvatures, there is a length-minimizing graph (θ, figure 8, glasses), which divides the two-sphere into regions with those total curvatures.

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- F. Morgan, "Soap bubbles in R^2 and in surfaces", Pc. J. Math 96 (1989), 333-348

Questions

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- Does the conclusion of Theorem 1 hold for arbitrary metrics on the 2-sphere?

Problem 11

Let M be a Riemannian 2-sphere. Let $\mathcal{N}$ be a geodesic net modelled either on θ-graph, figure 8, or glasses, that subdivides M into three regions $R_i, i = \{1, 2, 3\}$.
(a) Evaluate the total curvature K_i of each of R_i, when $\mathcal{N}$ is either a θ-graph or glasses;
(b) Find the bounds for K_i, when $\mathcal{N}$ is a figure 8 with vertex angles $\frac{\pi}{3} \leq t \leq \frac{2\pi}{3}$.

Poincare problem and related questions

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- Find length bounds on closed curves that subdivide surfaces into pieces of comparable areas.

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- P. Papazoglou, "Cheeger constants of surfaces and isoperimetric inequalities", Trans. Amer. Math. Soc. 361 (2009), no. 10, 5139-5162.

Problem 12

Let M be a Riemannian 2-sphere of area A and diameter d.
(a) Show that for any $\delta > 0$ there exist a closed curve of length at most $2d$ that subdivides M into two pieces of area at least $\frac{A}{3} - \delta$.
(b) Besicovitch Lemma. Let D be a Riemannian 2-disk. Consider a subdivision of ∂D into four consecutive sub-arcs with disjoint interiors, i.e. $\partial D = a \cup b \cup c \cup d$. Let l_1 denote the length of a minimizing geodesic between a and c, l_2 denote the length of a minimizing geodesic between b and d. Then the area of D, $A \geq l_1 l_2$.
Use Besicovitch Lemma to show that there exists a closed curve of length at most $4\sqrt{A}$ that subdivides M into two pieces of area at least $\frac{A}{4}$.

Subdividing n-dimensional manifolds into pieces of comparable volume

- Question of P. Papazoglou: Let M be a Riemannian 3-disk with diameter d, boundary area A, volume V. Is there a function $f(d, A, V)$ such that there exists a homotopy S_t contracting the boundary to a point, so that the area of S_t is bounded by $f(d, A, V)$? Is it possible to subdivide M by a disk D into two regions of volume at least $\frac{M}{4}$ so that the area of D is bounded by $h(d, A, V)$?

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- The answer is NO.

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- based on D. Burago, S. Ivanov, "On asymptotic constant of tori", GAFA 8 (1998), no. 5, 783-787

- P. Papazoglou, E. Swenson, ”A surface with discontinuous isoperimetric profile and expander manifolds”, Geometriae Dedicata, 206 (2020), 43-54

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- For any $\epsilon, M > 0$ there exists a Riemannian 3-sphere S of volume 1 such that any, not necessarily connected surface separating S into two regions of volume $> \epsilon$ has area greater than M

Geodesic flowers and wide loops

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- There exists a geodesic net with one vertex and at most $3^{(n+1)^2}$ geodesic loops of total length at most $2(n+1)!^{\frac{5}{2}}3^{(n+1)^3}(n+1)n^n vol(M^n)^{\frac{1}{n}}$, where $vol(M^n)$ is the volume of M^n

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- Also there exists a wide geodesic loop of length at most $n(n+1)!a^{(n+1)^3} vol(M^n)^{\frac{1}{n}}$