Oxford Mathematics

UNIVERSITY OF OXFORD
Mathematical Institute

Guide to Part A Short Options

Number Theory

Take two odd primes p and q. Suppose that there is a square number that is p more than a multiple of q. Is there also a square number that is q more than a multiple of p? How does this answer depend on the primes p and q? This question is answered by the beautiful theorem of *quadratic reciprocity*. The importance of this theorem is demonstrated by the amount of time that Gauss spent thinking about it: he published at least six proofs of the theorem himself. In this course, you'll meet the theorem and a proof, having developed the relevant and very elegant theory first.

In addition to the mathematical attractions of this subject material, there are also practical applications to cryptography, and the course will spend some time looking at these applications as well as developing the underlying theory.

This course develops in detail the ideas of modular arithmetic and Fermat's Little Theorem which were briefly discussed during the Prelims Groups and Group Actions course, and in turn leads on to Part B Algebraic Number Theory and other related courses.

Group Theory

Groups are best thought of as symmetries of geometric objects. As such they play an important role in large parts of mathematics and its applications. In this course we will study their internal structures and start to classify them: How many groups of a given size are there? Given two groups how can we combine them to form a new group? What are the primary building blocks of groups?

This course will build on your knowledge from the Prelims course Groups and Group Actions. We will break down groups via their normal groups and quotient groups, and study their subgroups of prime powers. It will prepare you for several third year courses. Both B3.1 Galois Theory and B3.5 Topology and Groups will lead directly on from this.

Projective Geometry

Ever wondered whether the parallel lines in the plane meet at infinity or not? Come to this course to find out – in Projective Geometry, they most certainly do! Projective geometry is a version of geometry which includes, but is subtly different from, our ordinary (Euclidean) notion of geometry; it can be thought of as the geometry of perspective drawing. It possesses a pleasant non-Euclidean symmetry called *duality*, which turns points into lines and vice versa, and leads to most theorems appearing in attractive pairs.

The course builds on linear algebra, giving an illustration of how ideas of that algebraic subject can be applied in a different context. It develops the notion that there is not just one notion of "geometry". Other geometries will be introduced in later courses in relativity theory and B3.2 Geometry of Surfaces, while non-linear projective geometry will be developed in the courses B3.3 Algebraic Curves and more generally in C3.4 Algebraic Geometry.

Introduction to Manifolds

The notion of a curved geometry is fundamental to our modern understanding of the mathematics and physics of space (and space-time) around us. This course gives the first glimpse of the technical underpinnings of this subject, by developing a geometrically meaningful notion of differentiation of functions of several variables. The theory leads to a general notion of invertible functions, as well as a formalisation of the tangent space to a curved geometry.

Integral Transforms

How do we describe a point mass, force, charge, or heat source mathematically? We can define the cumulative distribution function of a discrete random variable, but what is the density function? Can the derivative of a step function make sense? All these questions are handled in the beautiful and elegant framework of *distributions* or *generalised functions*. This is the first topic of the course.

The second topic is a generalisation of some of the Fourier series ideas from Prelims. We can represent a function defined on a finite interval as a weighted sum of basis functions – a Fourier series – but what if the interval is infinite? The sum becomes an integral and we study two versions, called the *Laplace* and *Fourier transforms*. We see how to use them to solve problems in differential equations, and we see how they interact with the distributions introduced in the first part of the course.

This short option is directly relevant to a wide range of later courses across the whole spectrum of mathematics, from analysis and partial differential equations to probability and a host of physical application areas.

Calculus of Variations

We all know that the shortest distance between two points is a straight line – but how do you prove this, and what happens if the points are on a curved surface? What is the shape of a curve of a fixed prescribed length that maximises the area under the curve? Problems like these are known as *variational problems*, and occur in many areas of mathematics. For example, the motion of the simple pendulum can be framed as a variational problem.

The aim of this course is to formulate questions of the above form in terms of variational problems and then derive a system of differential equations, known as the *Euler-Lagrange equations*, to determine the solution.

Graph Theory

Networks are everywhere in our world: transportation networks (e.g. roads), communication networks (e.g. phones), social networks (e.g. Facebook). What kind of mathematical questions arise when we think about networks? One such question that gripped the popular imagination in the mid 20th century, and was the subject of a famous experiment by Stanley Milgram, was the "small world problem", also known as "six degrees of separation": can any two people be linked by a chain of people, of length at most six, such that any link in the chain is a pair of people who know each other?

This course develops the theory of *graphs* (the mathematical abstraction of networks) through the lens of various practical problems: minimum cost spanning trees (making cheap connections), shortest paths (getting from A to B), bipartite matching (assigning jobs to contractors) and the Chinese Postman Problem (delivering items). The solutions of these problems, and the proofs that these are correct, require various elegant mathematical ideas - indeed, the great variety of proof techniques in Graph Theory is at once a challenge in the beginning and a reward at the end. These ideas in turn are fundamental to more advanced courses, such as B8.5 Graph Theory, C8.3 Combinatorics and C8.4 Probabilistic Combinatorics within the Oxford Mathematics curriculum, and more generally to any further study of networks from a theoretical or practical viewpoint.

Special Relativity

In 1905, Albert Einstein revolutionized humanity's understanding of space and time when he published his seminal paper, "On the Electrodynamics of Moving Bodies". In a stroke, Einstein showed that the apparent invariance of the speed of light under changes in reference frame could be reconciled with the philosophical (if not the technical) principle of relativity by radically altering one's view on the relationship between space and time. The consequences of this discovery have been far-reaching, including the development of atomic energy and a complete reformulation of the theory of gravity in the form of Einstein's later theory of general relativity.

Starting from basic postulates, we derive the structure of a unified, relativistic space-time known as *Minkowski space*. We address various apparent paradoxes (the twin paradox and barn door paradox) and their dissolution in an appropriate relativistic treatment. Elementary considerations of how to generalize the rules of dynamics to a relativistic setting leads to the famous relation between the mass and energy of an object, $E=mc^2$.

This course builds on first-year algebra, geometry, and dynamics, and in turn leads on to Part C General Relativity and many MMathPhys courses such as Quantum Field Theory and String Theory.

Mathematical Modelling in Biology

Consider two populations, say rabbits and foxes. How do these populations evolve over time? We will explore different modelling frameworks including discrete and continuous-time models. Specifically we will focus on the predator-prey models describing the dynamics of rabbits and foxes. In this course we will introduce different mathematical models and mathematical techniques to analyse these systems.

This course builds on ideas of differential equations and in turn leads on to Part B Further Mathematical Biology as other modelling and differential equation courses.