

Introduction to SPDEs from Probability and PDE

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Abstract

The course will aim to provide an introduction to stochastic PDEs from the classical perspective, that being a mixture of stochastic analysis and PDE analysis. We will focus in particular on the variational approach to semi-linear parabolic problems, à la Lions. There will also be comments on other models and approaches.

Structure: 4 x 1.5hr Lectures [starts 20th April 2021 – see <https://www.maths.ox.ac.uk/events/list/3820>]

Suggested Pre-requisites: The course is broadly aimed at graduate students with some knowledge of PDE theory and/or stochastic analysis. Familiarity with measure theory and functional analysis will be useful.

Lecture 1: Introduction and Preliminaries

- Introduction to randomness in PDE
- Stochastic analysis in infinite dimensions

Literature: [DKM⁺09, Hai09, Par07, PR07, DPZ14]

Lecture 2: Variational Approach to Deterministic PDE

- Variational approach to linear parabolic equations
- Variational approaches to non-linear parabolic equations

Literature: [Par07, Eva10]

Lecture 3: Variational Approach to Parabolic SPDE

- Itô's formula in Hilbert spaces
- Variational approach to monotone, coercive SPDE
- Concrete examples

Literature: [PR07, Par07]

Lecture 4: Further Topics and Directions (time permitting)

- Regularity of solutions
- Ergodicity
- Pathwise approach to SPDE

Literature: [Hai09, DKM⁺09, DPZ96, Hai14, GIP15]

References

[DKM⁺09] Robert Dalang, Davar Khoshnevisan, Carl Mueller, David Nualart, and Yimin Xiao. *A minicourse on stochastic partial differential equations*, volume 1962 of *Lecture Notes in Mathematics*. Springer-Verlag, Berlin, 2009.

[DPZ96] G. Da Prato and J. Zabczyk. *Ergodicity for Infinite Dimensional Systems*. *London Mathematical Society Lecture Note Series*. Cambridge University Press, 1996.

[DPZ14] Giuseppe Da Prato and Jerzy Zabczyk. *Stochastic Equations in Infinite Dimensions*. *Encyclopedia of Mathematics and its Applications*. Cambridge University Press, 2 edition, 2014.

[Eva10] Lawrence Craig Evans. *Partial Differential Equations*. American Mathematical Society, 2010.

[GIP15] Massimiliano Gubinelli, Peter Imkeller, and Nicolas Perkowski. *Paracontrolled distributions and singular PDEs*. *Forum Math. Pi*, 3:75, 2015.

[Hai09] Martin Hairer. *An Introduction to Stochastic PDEs*. *Technical report, The University of Warwick / Courant Institute*, 2009. Available at: <http://hairer.org/notes/SPDEs.pdf>

[Hai14] M. Hairer. *A theory of regularity structures*. *Inventiones mathematicae*, 198(2):269–504, 2014.

[Par07] Etienne Pardoux. *Stochastic partial differential equations*. <https://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.405.4805&rep=rep1&type=pdf> 2007.

[PR07] Claudia Prévôt and Michael Röckner. *A concise course on stochastic partial differential equations*. Springer, 2007.