Oxford Mathematics Alphabet

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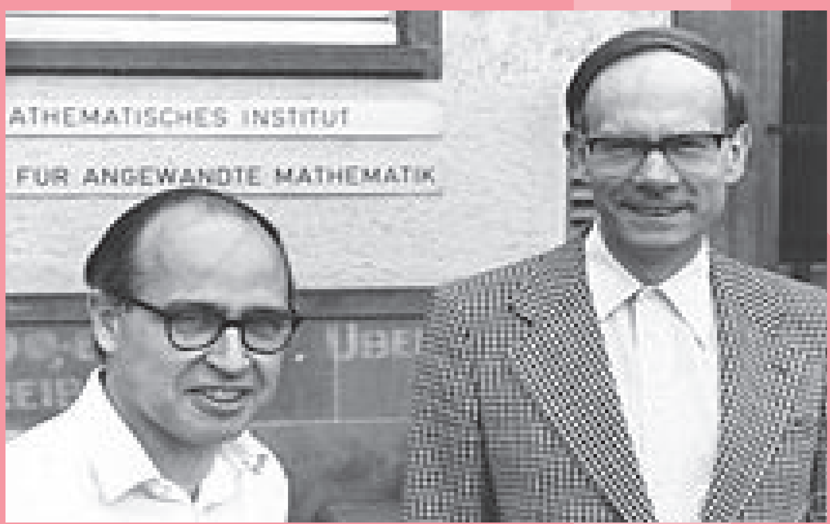
K *is for* K-theory

The origins of K-theory in the late 1950s go back to Alexander Grothendieck, who worked in algebraic geometry, an area in which ideas from algebra and geometry combine to lead to deeper understanding of both. Today there are really several K-theories. The most commonly known is probably the K-theory of topological spaces, which was introduced by Michael Atiyah and Friedrich Hirzebruch. Early successes of topological K-theory include Adams' solution to the vector fields on spheres problems (1962) and Atiyah and Singer's Index Theorem (1963). More recently, a twisted version has played an important role in the study of string theory.

Vector field on a sphere.



Michael Atiyah and Friedrich Hirzebruch

From the point of view of algebraic K-theory, topological K-theory of a space X is the study of its functions C(X). The generalisation from the commutative rings of functions to more general rings is the key idea in non-commutative geometry, a branch of mathematics coined by Alain Connes. For a while, it was a difficult problem to find the right extension to yield higher invariants with good properties. This was finally solved by Dan Quillen (1973). Oxford Mathematicians Atiyah and Quillen both received Fields medals for their work.

There are deep connections between algebraic K-theory and number theory, as expressed in the related and now solved conjectures of Milnor, Quillen-Lichtenbaum, and Bloch-Kato. Waldhausen, motivated by the study of geometric objects called manifolds, extended the algebraic K-theory of rings to the much wider class of ring spectra. This is a very powerful theory and remains an active area of research for topologists and number theorists.

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