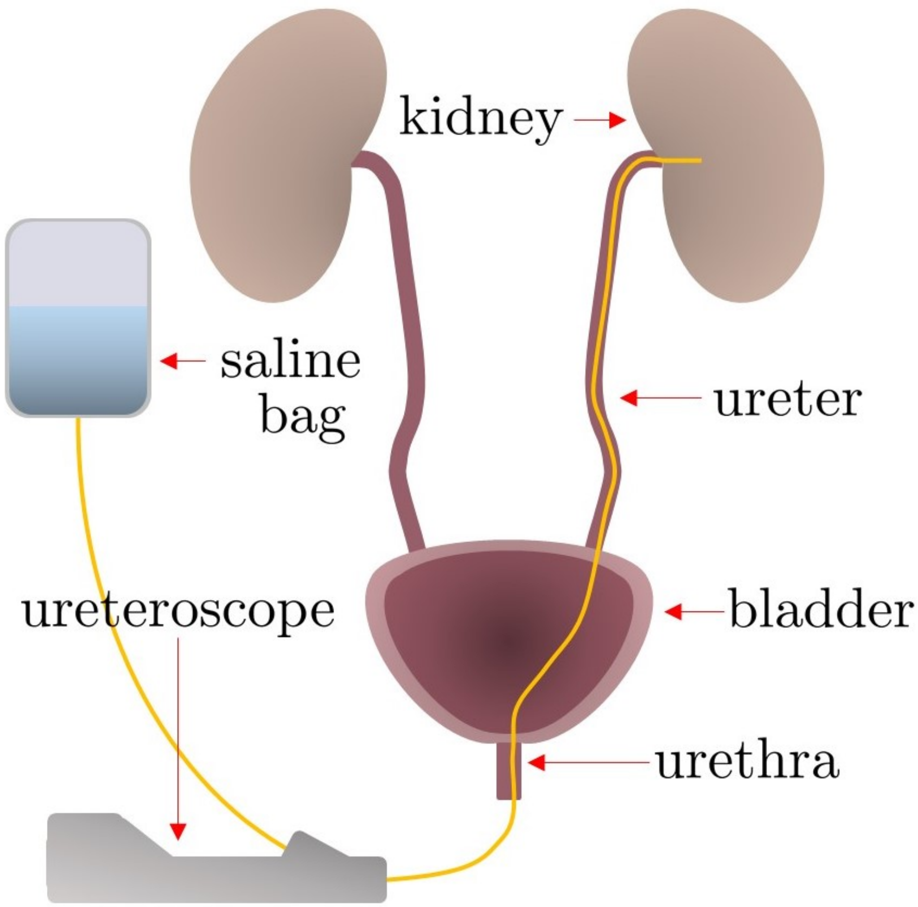
EPSRC

Engineering and Physical Sciences Research Council

InFoMM

Industrially Focused Mathematical Modelling

EPSRC Centre for Doctoral Training in Industrially Focused Mathematical Modelling



Mathematical modelling of unsteady flows through a ureteroscope

Harry Reynolds

UNIVERSITY OF OXFORD

Boston Scientific

Contents

1. Introduction

Background

In the United States, approximately one in eleven people are affected by kidney stones. Stone disease is a great burden on national health resources with the financial cost in the USA estimated to exceed five billion dollars by 2030. Even once the disease is treated, the risk of recurrence is around 40% after 5 years, and 75% after 20 years. These are just some factors that motivate the need for efficient and effective treatment of kidney stones.

Kidney stones are an accumulation of hard material within the kidneys, caused by high levels of certain minerals within the urine. Stones vary in size, and while smaller stones can usually pass naturally, larger ones typically require treatment. One approach which is quickly becoming the preferred method for treating kidney stones is ureteroscopy. This procedure involves passing a flexible medical instrument, known as a ureteroscope, through the urethra, bladder, and ureter, until it reaches the kidney, as shown in Figure 1.

> Kidney stones vary in size, and while smaller stones can usually pass naturally, larger ones typically require treatment. One approach for treating kidney stones is ureteroscopy.

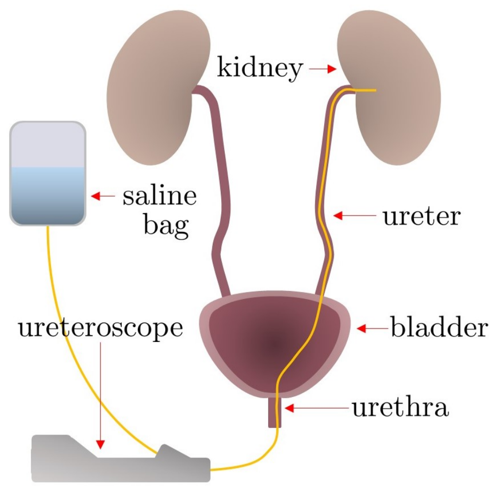


Figure 1 – Diagram of the urinary system, showing a ureteroscope (with hanging saline bag) inserted through the urethra, bladder, and ureter, to reach the kidney (highlighted in yellow).

The ureteroscope is hollow along its length, creating a working channel through which irrigation fluid can pass into the body, as shown in Figure 2. Once the ureteroscope is inserted into the patient, working tools such as wires, baskets, and laser fibres are used to access and remove the kidney stones. Modern scopes are also fitted with a light source and a camera at the tip, allowing urologists to visualise the renal system and locate the stones.

Ureteroscopy requires constant fluid irrigation to ensure a clear field of view for the camera, which can become obscured by stone dust during laser treatment. The fluid irrigation also keeps the urinary tract dilated, which provides better visualisation and scope maneuverability during the procedure. Traditionally, this constant irrigation is provided by hanging a bag of saline solution above the level of the scope to create a pressure gradient to drive the flow. The irrigation fluid flows from the bag into the scope, through the working channel, and out directly into the urinary system. Recent developments in the field of ureteroscopy involve using an electronic pump to control the flow of fluid, rather than relying on the pressure gradient from an elevated saline bag. This allows the flow to be completely regulated during the entire procedure, with the ability to respond actively to changes in pressure. The use of these pumps means that unsteady flows are possible, which may provide additional benefits during ureteroscopy.

Our interest is in the effect of having an unsteady flow on ureteroscope fluid irrigation. We focus on oscillatory flows, motivated by the type of motion that electronic pumps can produce,

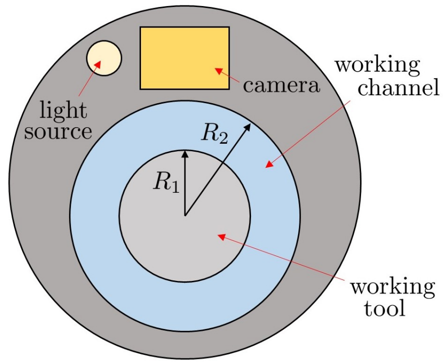


Figure 2 – Cross section of the tip of the ureteroscope showing the position of the light source and camera, as well as the working channel through which the irrigation fluid flows, with a working tool inserted.

and will use mathematical modelling in combination with experimental approaches to explore the effect of oscillating flows.

2. Time dependent annular pipe flow: theory and experiment

The flow is driven by an oscillatory pressure drop, causing it the oscillate in time.

We develop a mathematical model for the flow inside of a ureteroscope containing a working tool. We assume that the flow is driven by a pressure difference, where the upstream pressure oscillates in time, and the downstream pressure is taken to be atmospheric. The fluid flows through an annulus, created by a working tool contained within the working channel of the ureteroscope. We assume the working tool sits in the centre of the working channel. We model the annular region as two concentric cylinders where fluid flows between the inner and outer cylinder. We define R_1 as the radius of the inner cylinder, R_2 as the radius of the outer cylinder, p_1 as the pressure at the inlet of the channel, p_2 as the atmospheric pressure at the outlet of the channel, and L_s as the length of both cylinders, as shown in Figure 3. We assume the pressure p_1 oscillates in time, and can be decomposed into a constant part and an oscillatory part. To ensure there is no backflow, p_1 is always greater than p_2. The fluid is assumed to have the same properties as water, since this closely resembles the weak saline solution used during ureteroscopy. Therefore, we assume a viscous, incompressible, Newtonian fluid of density ρ and dynamic viscosity μ.

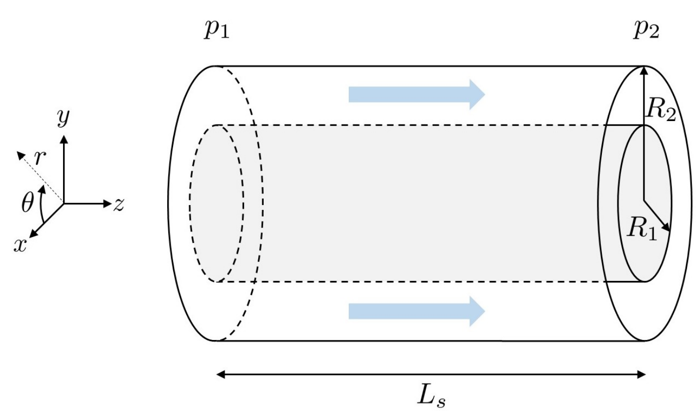
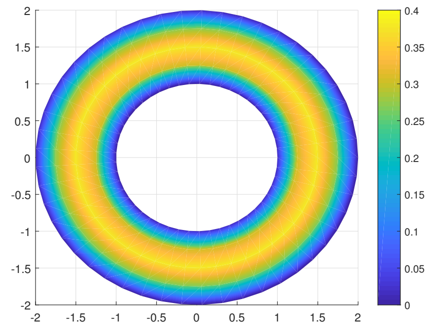


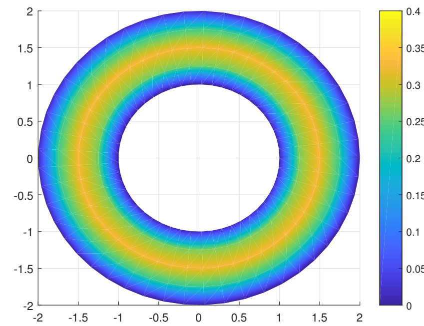
Figure 3 – Flow through two concentric cylinders model sketch.

Using the Navier-Stokes equations, which describe continuity and balance of momentum for a viscous fluid, as our starting point, we neglect inertia and external forces to obtain a reduced model, which has an explicit solution. This allows us to analyse how the velocity profile and flux differ as the frequency of oscillations changes. In Figure 4, we show the flow profile over half a period and we find that the maximum velocity of the flow is in the centre of the channel for all time.

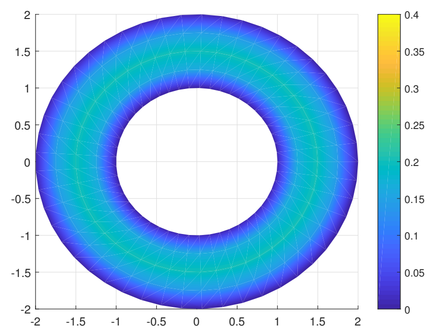
An explicit solution to the mathematical model is found, allowing us to analyse how the flow differs as the frequency of oscillations changes.



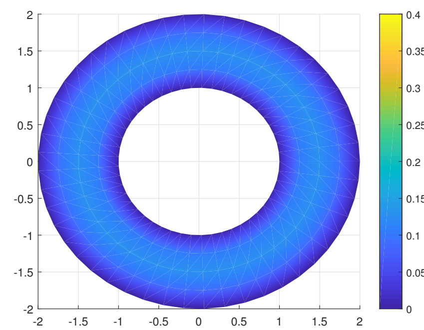
(a) t=0



(b) $t = \frac{1}{3}\pi$



(c) $t = \frac{2}{3}\pi$



(d) $t = \pi$

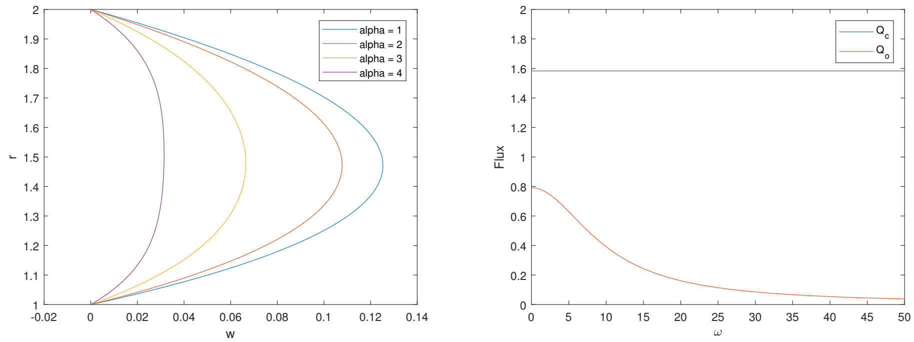
Figure 4 – Axial flow velocity, w, for half a period

We explore how the model depends on the frequency of the oscillations ω. In Figure 5a we plot the flow velocity w for different values of α, where α measures the important of the pulsatile flow frequency compared with viscous effects. We see that, as we increase α, which corresponds to increasing the frequency of the oscillations, the flow profile changes from a parabolic profile to a relatively flat or plug-like one.

The mathematical model is verified by carrying out experiments on the same flows. Comparison between the model and experiments allows us to understand the limitations of the model.

In Figure 5b we show how the total flux of fluid varies with frequency of the oscillations ω. We see that, as the frequency increases, the oscillatory part of the flux decreases and the constant part starts to dominate. This suggest that oscillatory flows with relatively large frequencies will have similar flow rates to constant flows. We want to explore the accuracy of these ideas, and so now we seek to justify our mathematical model experimentally.

We complement the mathematical study by also carrying out some experiments. Our set up is shown in Figure 6. The fluid is initially contained within a reservoir, where it is drawn out using an electric peristaltic pump. The fluid passes through a flow meter which records pressure readings upstream of the scope. The fluid enters the ureteroscope and travels down the length of the working channel, before exiting the scope tip into a container placed upon a mass balance, which records the mass of the fluid downstream of the scope. A working tool is placed within the working channel of the ureteroscope during the experiment, creating



(a) **(b)**

Figure 5 – (a) Flow profile w for increasing α, (b) Comparison of the constant (Q_c) and oscillatory (Q_o) parts of the flux as the oscillations increase.

an annulus for the fluid to pass through. The fluid used during the experiments is water as this closely resembles the weak saline solution used in practice. The ureteroscope is held horizontally, allowing us to neglect the effects of gravity on the fluid.

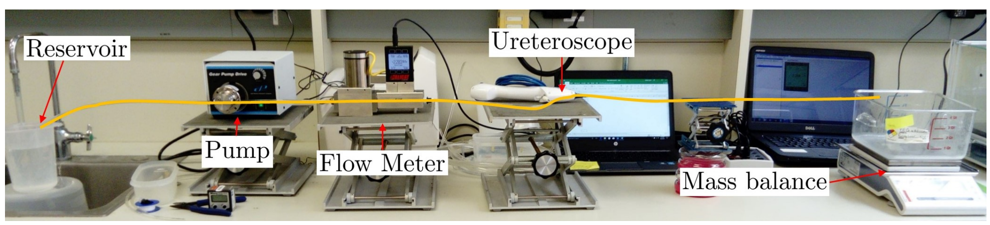


Figure 6 – Experimental set-up, with the path the flow takes from left to right indicated in yellow.

Analysis of data and comparison with mathematical model

Data was collected for six separate runs of the experiment where the working channel contained a working tool. Data collecting began when the pump was turned on. The pressures within the set-up were given time to increase until the initial transience had decayed away.

The model consistently under predicts the data.

Using the experimental data we determine how the upstream scope pressure changes in time. For each sample, we first interpolate the collected data, to give a curve that represents the change in pressure over time. In order to allow comparison with the mathematical model, we seek to divide this curve into its frequency components. This is achieved by performing a fast Fourier transform (FFT) on the interpolated data. Once this is done, it is used in our mathematical model to predict what the total mass is over the same time period as the data. The prediction from the mathematical model is compared with the experimental data for each sample. The results of this analysis are shown in Figure 7 for all six samples, with each pair of data (dotted line) and corresponding mathematical model prediction (solid line) being the same colour. The lines are almost completely linear, highlighting the minimal impact that the oscillations have on the total mass over time. The primary result shown by the analysis is that the mathematical model under-predicts the data for all six samples by an average of 30%.

3. Improving the mathematical model

To find an explanation for the discrepancy between the mathematical model and experiments, we revisit the assumptions made in our theory which may be physically inaccurate.

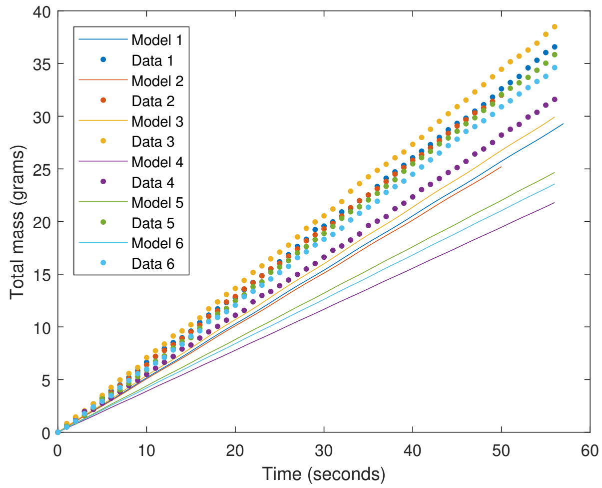


Figure 7 – Total mass of six data sample compared against the corresponding model prediction.

Additional tubing

One explanation for the error may be that the mathematical model assumes the pressure values are at the beginning of the scope. However, the flow meter which records the pressure has a length of additional tubing before it reaches the beginning of the scope. This suggests that the assumption made in the model is not representative of our experiments. After extending the mathematical model to account for the additional tubing, the model predicts a small decrease in pressure from the flow meter to the scope, with the constant part decreasing by 0.007% and the oscillatory part decreasing by 0.171%. This suggests that the additional tubing did not have a significant influence on our experimental results.

Working tool position

We now explore what happens when we relax the assumption that the working tool is concentric with the working channel. We assume the working tool and working channel are still parallel to one another, but the working tool has been displaced laterally in the channel. We aim to find how the total flux changes as the position of the working tool varies.

We consider a toy model in which the flow is steady (since in the experiments we see the oscillations have minimal impact on the total mass). In our toy model, we prescribe channel flow between two parallel plates, in which a working tool is present, which decouples the flow into two distinct regions. We define the region above the working tool as the upper region and the region below the working tool as the lower region. The centre of working tool is allowed to be at any position within the channel. We assume that the liquid is a viscous, incompressible, Newtonian fluid of density ρ and dynamic viscosity μ.

We find that the flux is maximum when the working tool is pressed up against either channel wall. This suggests the assumption of a concentric working tool may be what is causing the model to under predict the data.

Applying classical results of Poiseuille flow yields an analytical solution in both regions. We are interested in how the total flux varies as the working tool changes position. The total flux for different values of κ, the position of the working tool for the length of the channel, is shown in Figure 8. We see that the flux is minimum when the working tool is sitting in the centre of the channel, and that it is maximum when the working tool is pressed against either channel wall. The maximum flux is roughly four times larger than the minimum, suggesting the position of the tool has a large influence on the total flux.

We conclude that our earlier discrepancy between theory and experiment may be due to the assumption that the working tool is concentric in the working channel. The result in Figure 8 suggests that the working tool may be sitting only slightly offset from the centre of the channel.

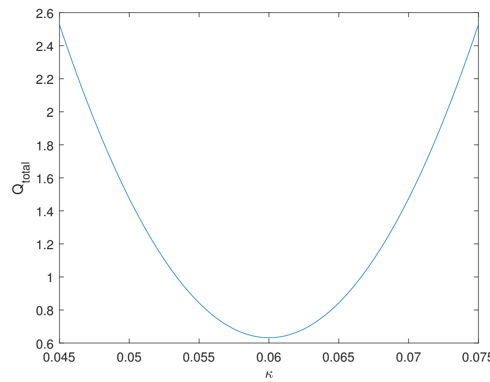


Figure 8 – Total flux of fluid for varying working tool centre position κ.

4. Discussion, conclusions & recommendations

We have established a mathematical model for unsteady flows in a ureteroscope, and solved this model analytically. To validate the mathematical model, we have carried out experiments, analysed the data collected, and made comparisons with the predictions from the mathematical model. We found a discrepancy between the model and experimental data, which motivated us to explore possible explanations for this error. We reviewed the experimental set-up, and adjusted our model to account for additional tubing, but concluded that this was not the source of the error. We then postulated that our assumption of the working tool being concentric with the working channel may in practice be inaccurate, and presented a model to explore what happens when the tool changes position within the channel. We found that this assumption is likely to be the cause of the discrepancies between experiments and model.

A next logical step for this work would be to return to the first model presented and remove the assumption that the working tool is concentric in the working channel, creating a three dimensional model for unsteady flows with an offset tool.

Thinking of the wider applications of unsteady flows during ureteroscopy, it would be interesting to explore what effect these flows have outside of the working channel of the scope, for example, when the irrigation fluid is exiting the tip of the scope into the kidney. Within the kidney, the stone dust created from laser treatment can often become stagnant and obscure the view of the camera, and so it is possible that unsteady flows may counteract this.

5. Potential impact

Our work serves as the initial modelling steps towards developing a generic framework for describing unsteady flow in ureteroscopes. Such a model provides insights into the effects unsteady flows have during ureteroscopy, allowing for optimisation of the procedure.

Niraj Rauniyar, Principal development Engineer - R&D at Boston Scientific said: "*Harry's work on understanding system level modelling for fluid flow with laminar flow and well as impact/effect of pulsation is likely be very impactful to the company, and we are excited about the results. His research will aim at modelling unsteady flow through a ureteroscope in order to determine the role of time dependent flows. His system level research will not only compare unsteady flow to steady flow results but also try to understand how we can harness the oscillations. His follow-on research project, in essence, will be a system level view and will aggregate several other projects (Intracavity pressure impact, tissue compliance model with flow, flow patterns with various flow and annulus shape) that are happening across Oxford University sponsored by us. Based on all the work that is happening across multiple projects within oxford, he will also understand effects on flow characteristics (such as with inter-renal pressure, flux, deformability etc.)*"