Asymptotic Higher Spin symmetry in gravity and Newman's good cuts

Laurent Freidel

Perimeter Institute

From Good cut to Celestial Holography
05-2025

Based on 2410.15219 , 2409.12178   2505.04327
with Nicolas Cresto
Nicolas Cresto   2501.08856

W-symmetry in gravity

[Cachazo, Strominger '14];
[Zlotnikov '14];

The study of subleading soft theorems and its connection with asymptotic symmetries has opened up a new perspective on gravity and YM

[Campiglia, Laddha '16];
Conde, Mao '16]

The study of colinear limit revealed a tower of new symmetries encoded into $LW_{1+\infty}$

Strominger '21
Guevara, Himwich, Pate, Strominger 21

[Adamo, Mason, Sharma '21];
[Ball, Narayanan, Salzer, Strominger '21]

These results have been connected to classical results in twistor theory that relates integrability of Self-dual GR to the existence of $LW_{1+\infty}$ symmetries

These has led to a new perspective on holomorphic quantization of self-dual theories

[Costello, Paquette '22]
Bittleston,Skinner,Sharma 22

In canonical $LW_{1+\infty}$ was found in terms of an algebra of Noether charges build up from the knowledge of the asymptotic dynamics and expressed in terms of the gravitational phase space

Pranzetti, Raclariu, LF 21-23
Geiller 24
Donnay, Herfray, LF 24

W-symmetry in gravity

So far the connection between canonical symmetries and soft theorems was mostly perturbative and lacking a first principle derivation from Noether. The purpose of our work was to remedy this

Cresto, LF 24,25

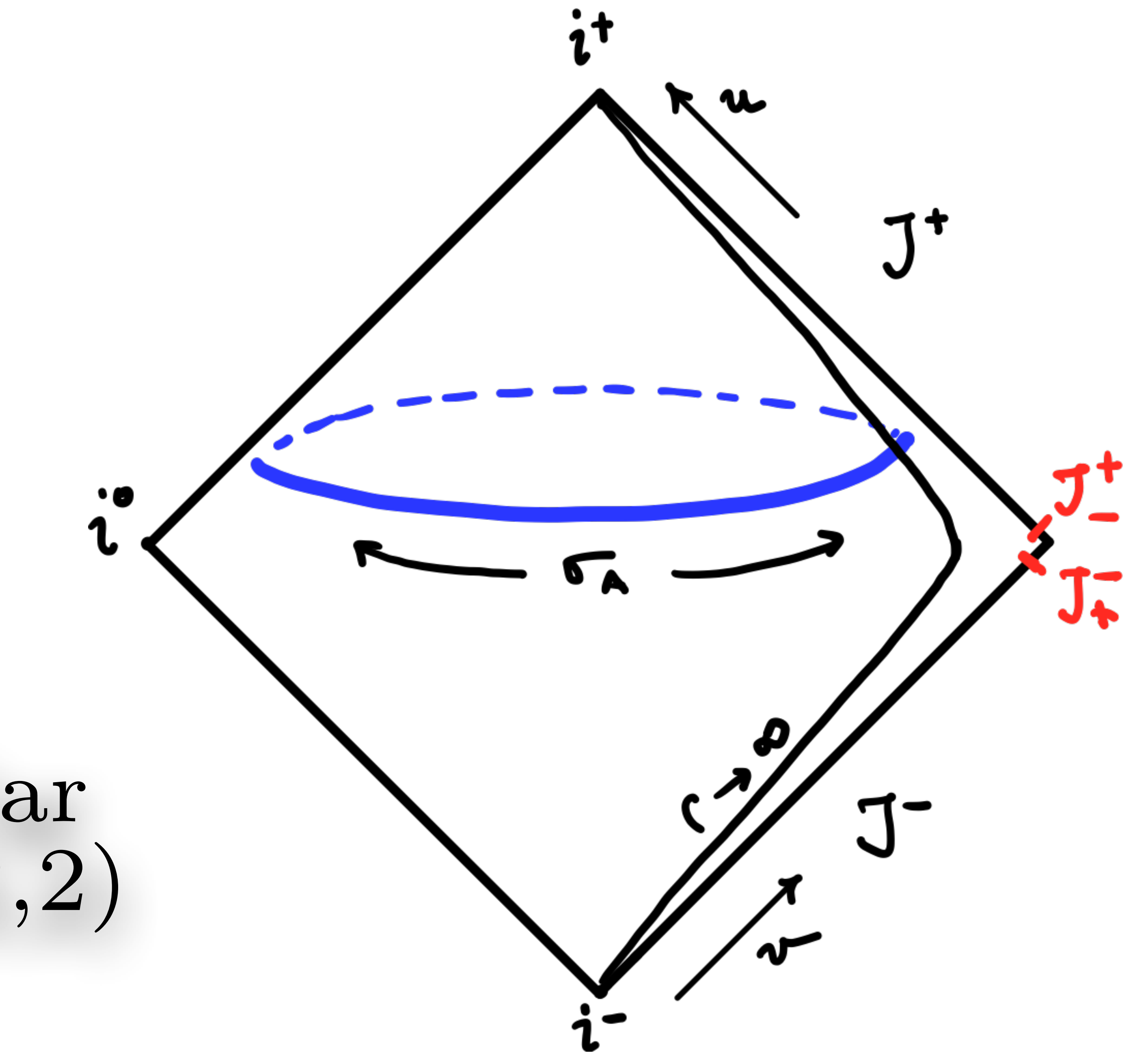
Our results are deeply connected to the beautiful recent work which provided a non-perturbative twistor persective on the construction of charges

Kmec, Mason, Ruzziconi, Srikant 24
Kmec, Mason, Ruzziconi, Sharma 25

Loop/log corrections modifies the charge expressions and challenges our understanding of symmetries.

Sahoo, Sen et al. 18-24
Donnay, Nguyen, Ruzziconi ’22-23
Choi Ladha, Puhm ’24-25

Sub-leading theorems and Charge conservation



The gravitational phase space can be defined at $\mathcal{J}^{\pm}$
It posseses two types of data:

Radiative: complex shear $(C, \bar{C})(u, z, \bar{z})$ and news $N = \partial_u C$

Coulombic: Charge aspects $\tilde{Q}_s(u, z, \bar{z})$

Carrollian weight: $\tilde{Q}_s \in C^{\mathrm{Car}}_{(3,s)}$ $C \in C^{\mathrm{Car}}_{(1,2)}$ $N \in C^{\mathrm{Car}}_{(2,2)}$

where $\Phi \in C^{\mathrm{Car}}_{(\delta,s)}$ when

$$\boxed{\Phi(|a|^2 u, a\lambda_\alpha, \bar{a}\bar{\lambda}_{\dot{\alpha}}) = a^{-(\delta+s)}\bar{a}^{-(\delta-s)}\Phi(u, \lambda_\alpha, \bar{\lambda}_{\dot{\alpha}})}$$

Eastwood-Tod 82
Donnay-Fiorucci-Herfray-Ruzziconi 22

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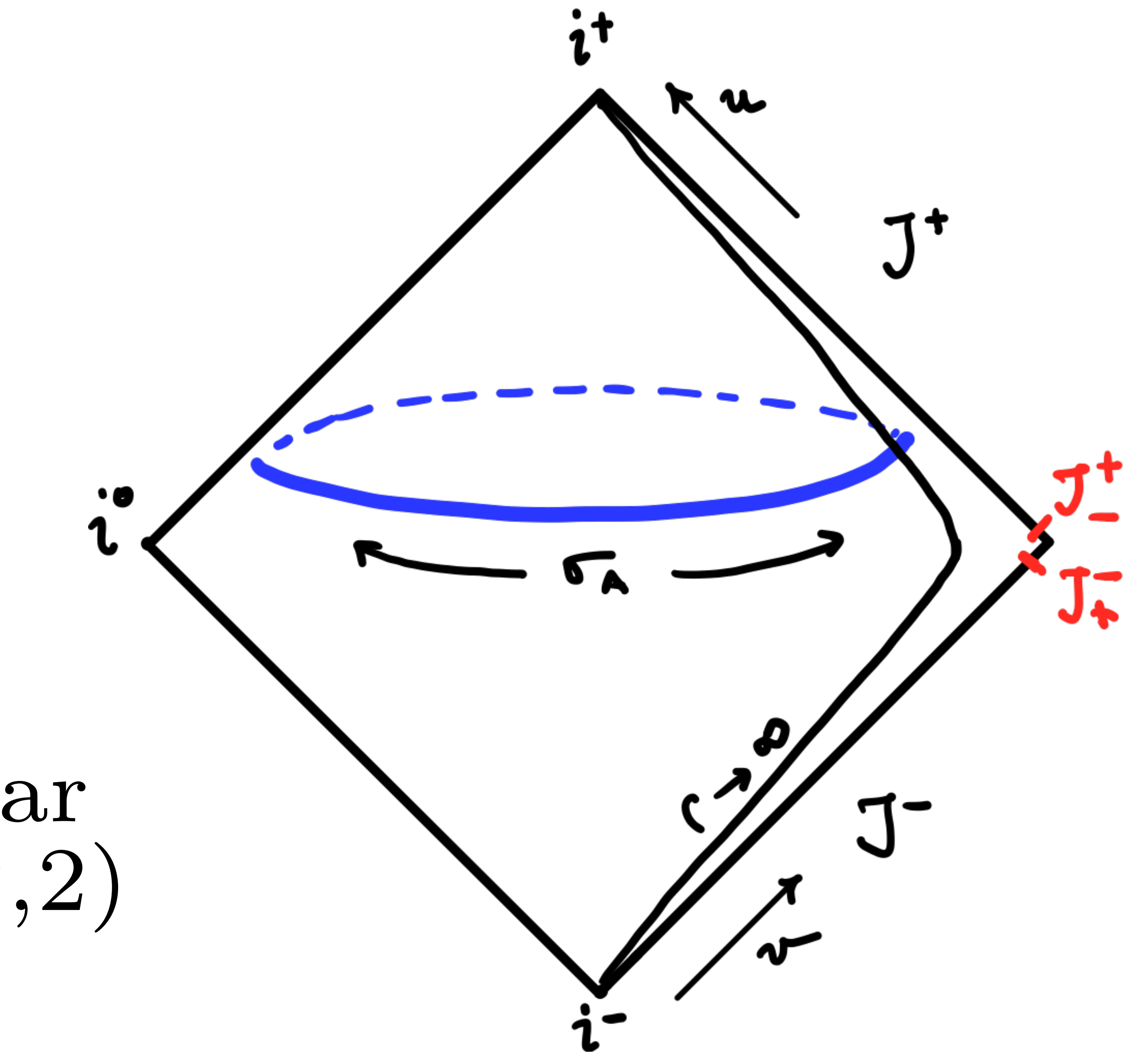

Newman-Penrose 62

The charge aspects encodes the subradiative components of the asymptotic Weyl tensor

LF,Pranzetti, Raclariu 22-23
Geiller. 24

$$\Psi_2 = \sum_{s=0}^{\infty} \frac{(-1)^s \bar{D}^s Q_s}{r^{3+s}} + \cdots$$

Sub-leading theorems and Charge conservation

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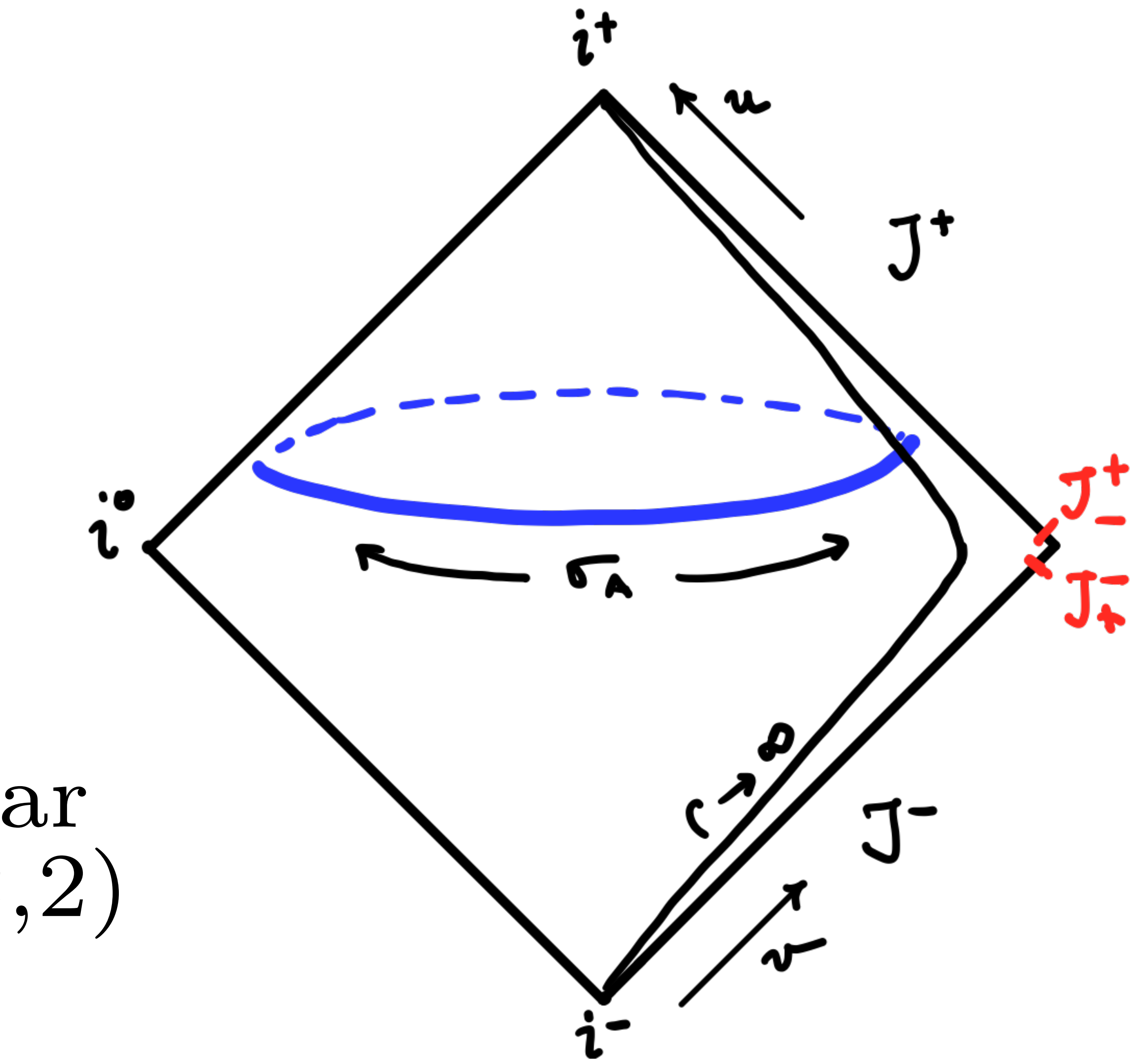

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The charge aspects encodes the components of the asymptotic Weyl tensor

LF,Pranzetti, Raclariu 22-23
Geiller. 24

The data are not independent: They satisfy constraint equations $D=D_z$

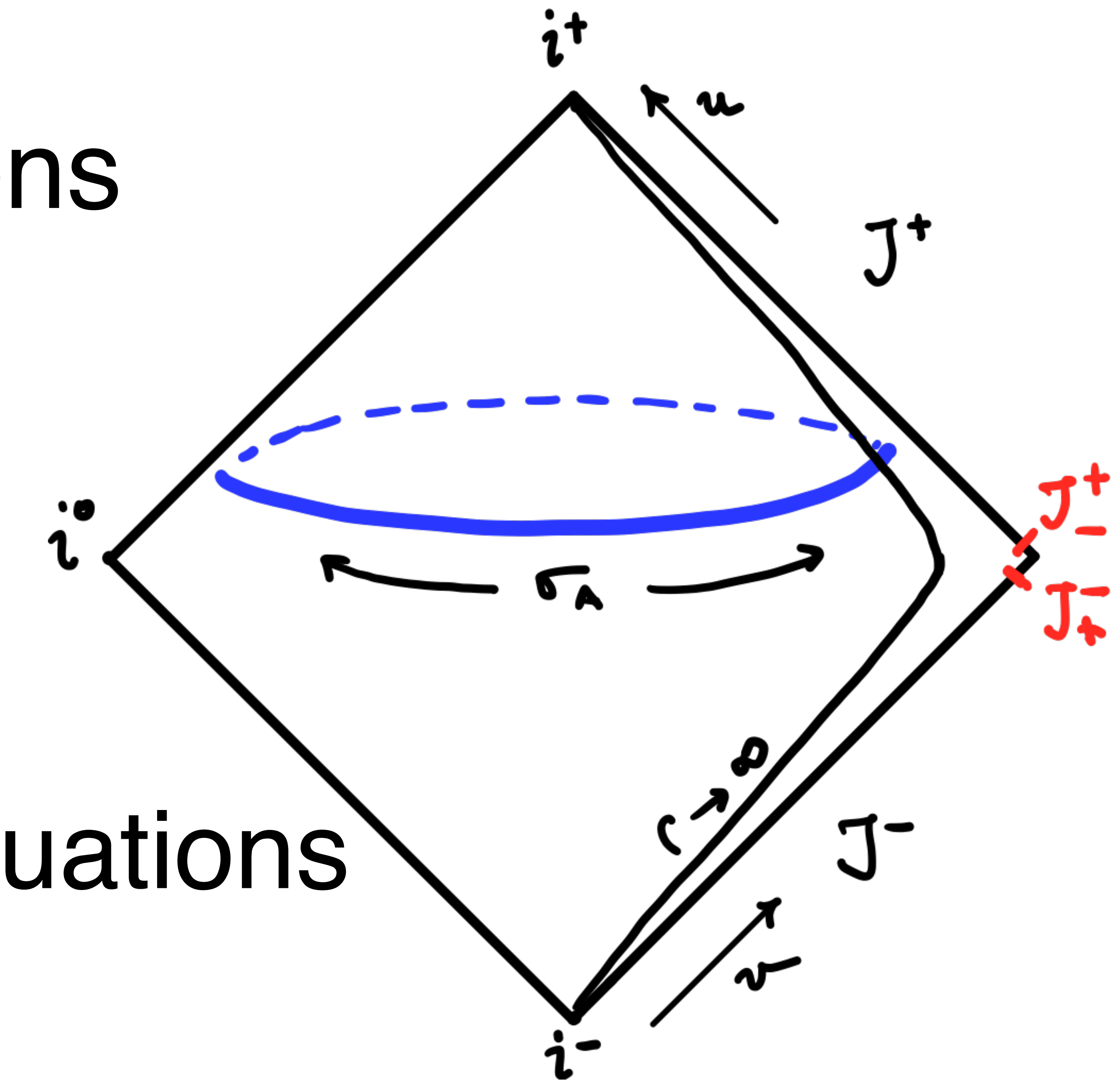
$$\boxed{\partial_u\tilde{Q}_s = D\tilde{Q}_{s-1}+(s+1)C\tilde{Q}_{s-2}} \quad \in C^{\rm Car}_{(4,s)}$$

Sub-leading theorems and Charge conservation

The data are not independent: They satisfy evolution equations

$$\boxed{\partial_u \tilde{Q}_s = D\tilde{Q}_{s-1} + (s+1)C\tilde{Q}_{s-2}}$$

$s=-1,0,1,2,3$ represent the first 5 Asymptotic Einstein's equations



Newman-Adamo 09
Pranzetti, LF 21

Initial value $\tilde{Q}_{-2} = \frac{\dot{\bar{N}}}{4\pi G}$ $\tilde{Q}_{-1} = \frac{D\bar{N}}{4\pi G}$ radiative data

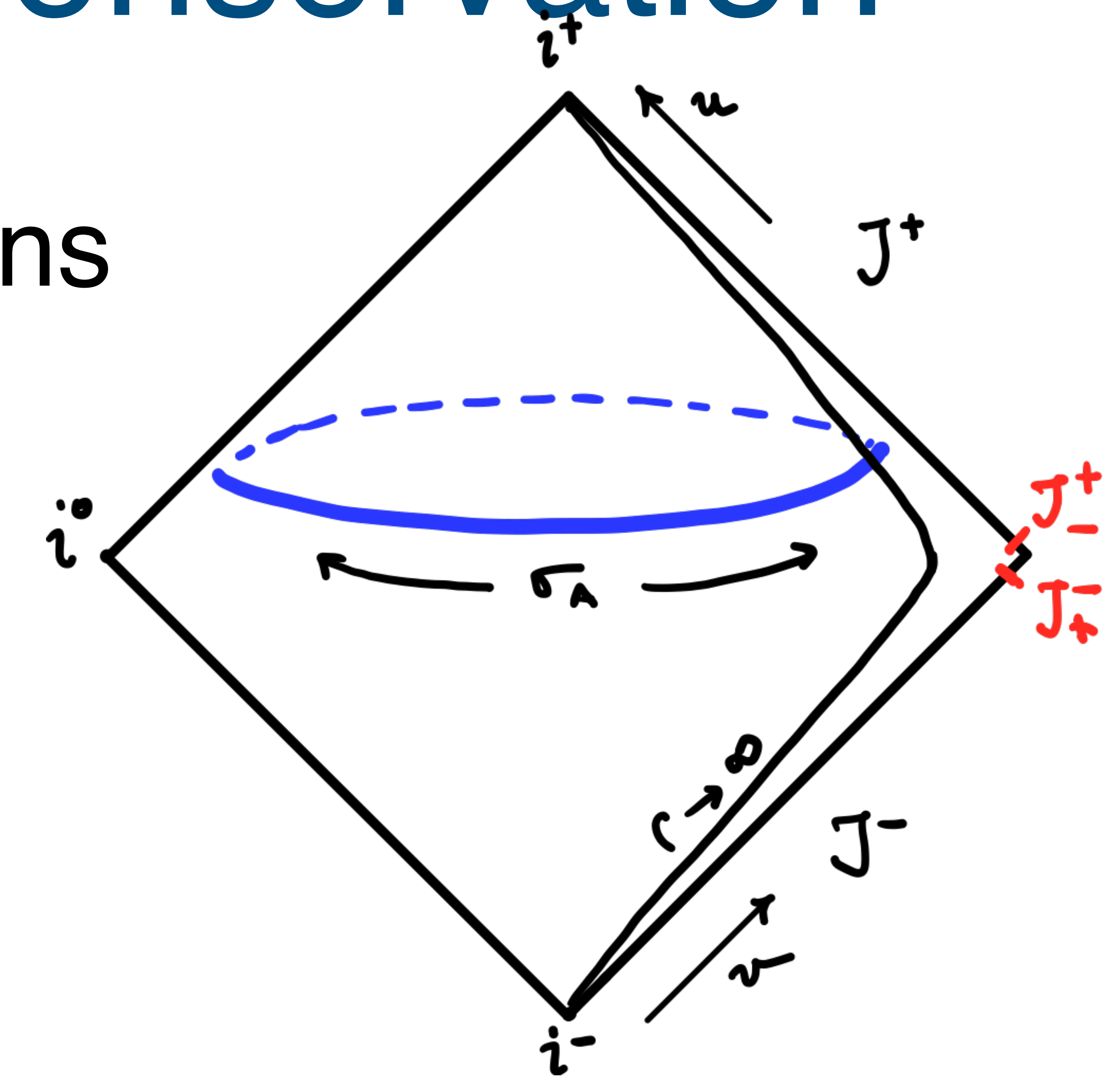
$\tilde{Q}_0$ mass aspect $\tilde{Q}_1$ angular-momentum aspect

$\tilde{Q}_2, \tilde{Q}_3$ spin 2,3 charges

In pure gravity these equation are initialized by the conditions that $\tilde{Q}_s(i^\pm) = 0$

In the presence of massive matter $\tilde{Q}_s(i^\pm) = \tilde{Q}_s(\rho_\pm)$

Sub-leading theorems and Charge conservation



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$$\boxed{\partial_u \tilde{Q}_s = D\tilde{Q}_{s-1} + (s+1)C\tilde{Q}_{s-2}}$$

In pure gravity using $\boxed{\tilde{Q}_s(i^{\pm}) = 0}$

We can integrate these charges as functionals $\tilde{Q}_s(C, \bar{C})$

Convenient to introduce an holomorphic perturbative counting $\boxed{(C, \bar{C}) \to (g_N C, \bar{g}_N \bar{C})}$

$$\tilde{Q}_s = \sum_{k=0}^{1+[s/2]} Q_s^{(k)} \quad \text{with} \quad Q_s^{(k)} \sim \bar{g}_N g_N^k$$

the charge are quadratic $\sim \bar{g}_N g_N$ for spin 0,1 cubic $\sim \bar{g}_N g_N^2$ for spin 2,3

Sub-leading theorems and Charge conservation

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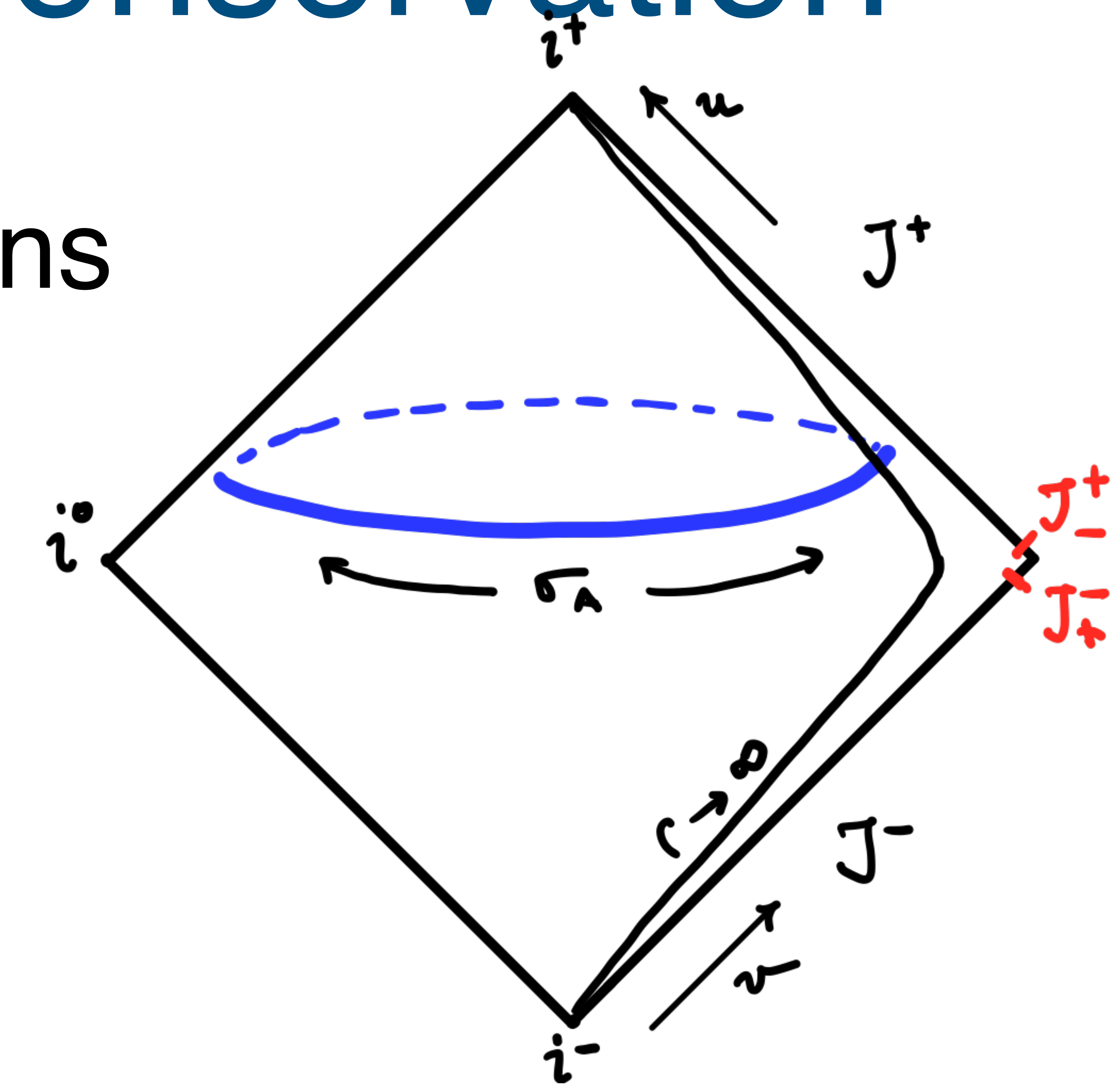

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This leads, beyond spin 2, to an expansion $\tilde{Q}_s = \tilde{Q}_s^{\mathsf{S}} + \tilde{Q}_s^{\mathsf{H}} + \tilde{Q}_s^{\mathsf{SH}}$

⟶ a non-linear generalization of the soft theorems

Sub-leading theorems and Charge conservation

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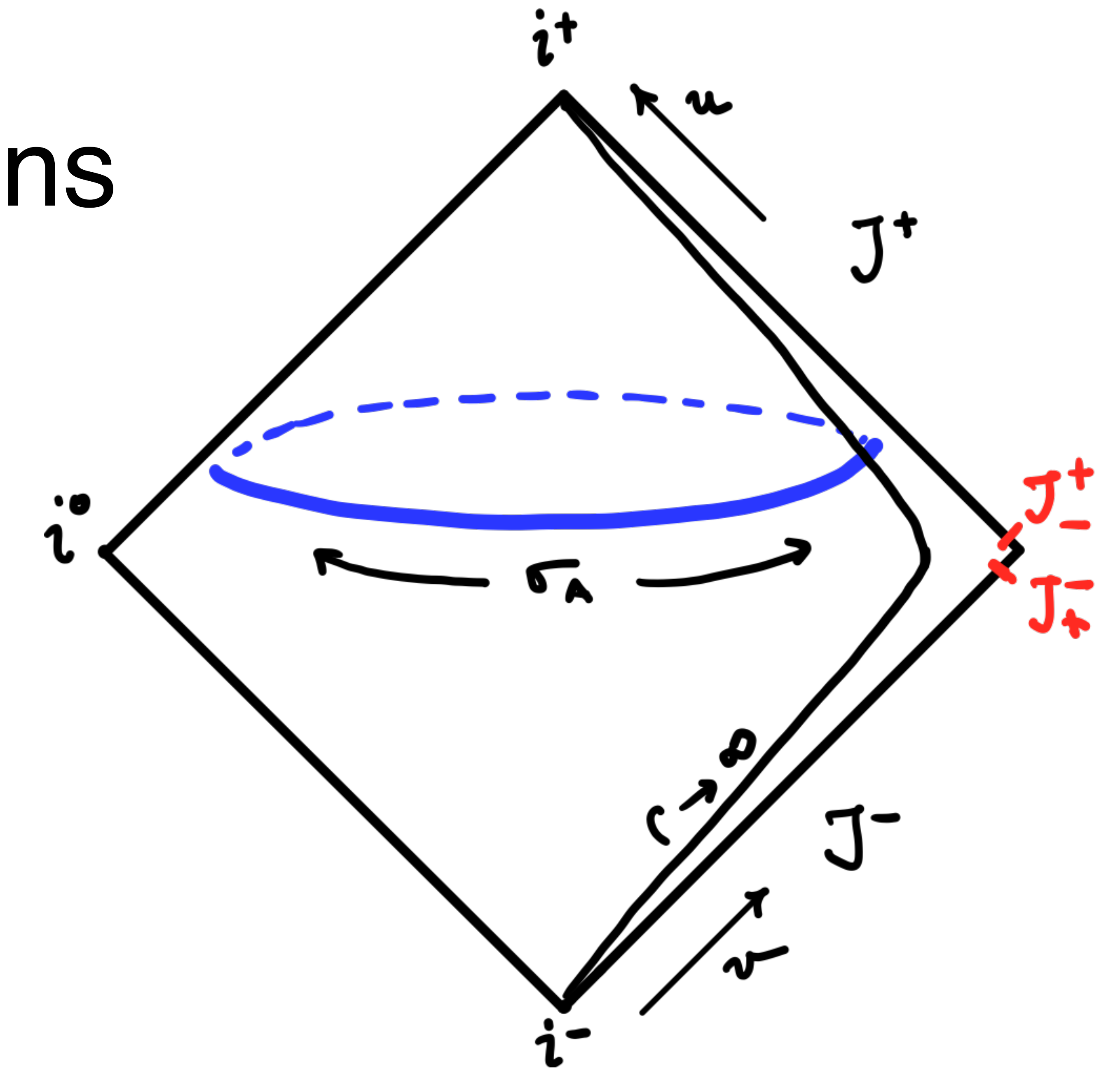

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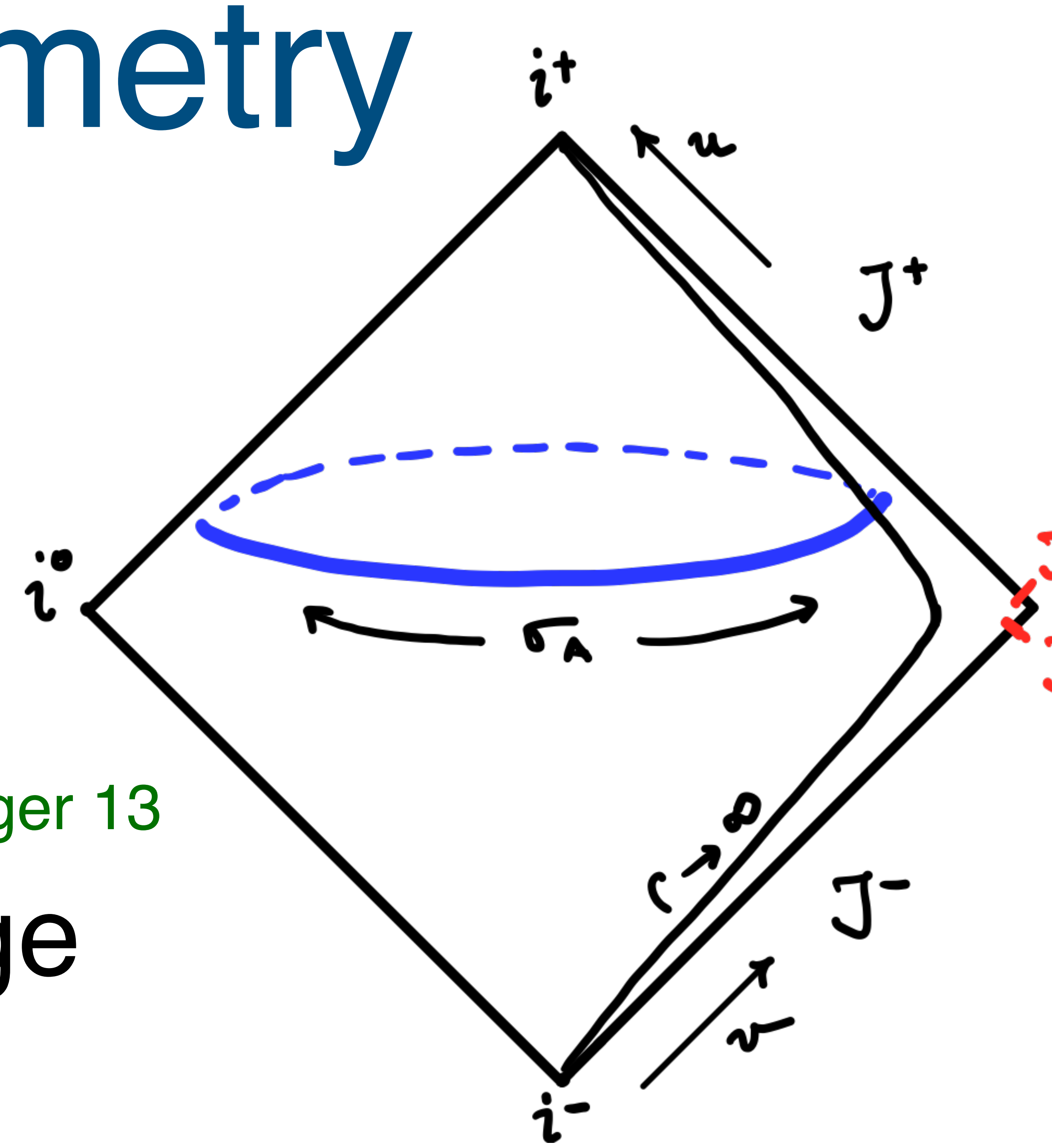
Beyond spin4 they represent a truncation of EE linear in $\bar{g}_N$ non-linear in g_N

They represent the Gauss conservation law associated with self-dual gravity

Kmec, Mason, Ruzziconi, Srikant 24

Subleading Theorem from Symmetry



The goal of this talk is to show that the system of equations

$$\boxed{\partial_u \tilde{Q}_s = D\tilde{Q}_{s-1} + (s+1)C\tilde{Q}_{s-2}}$$

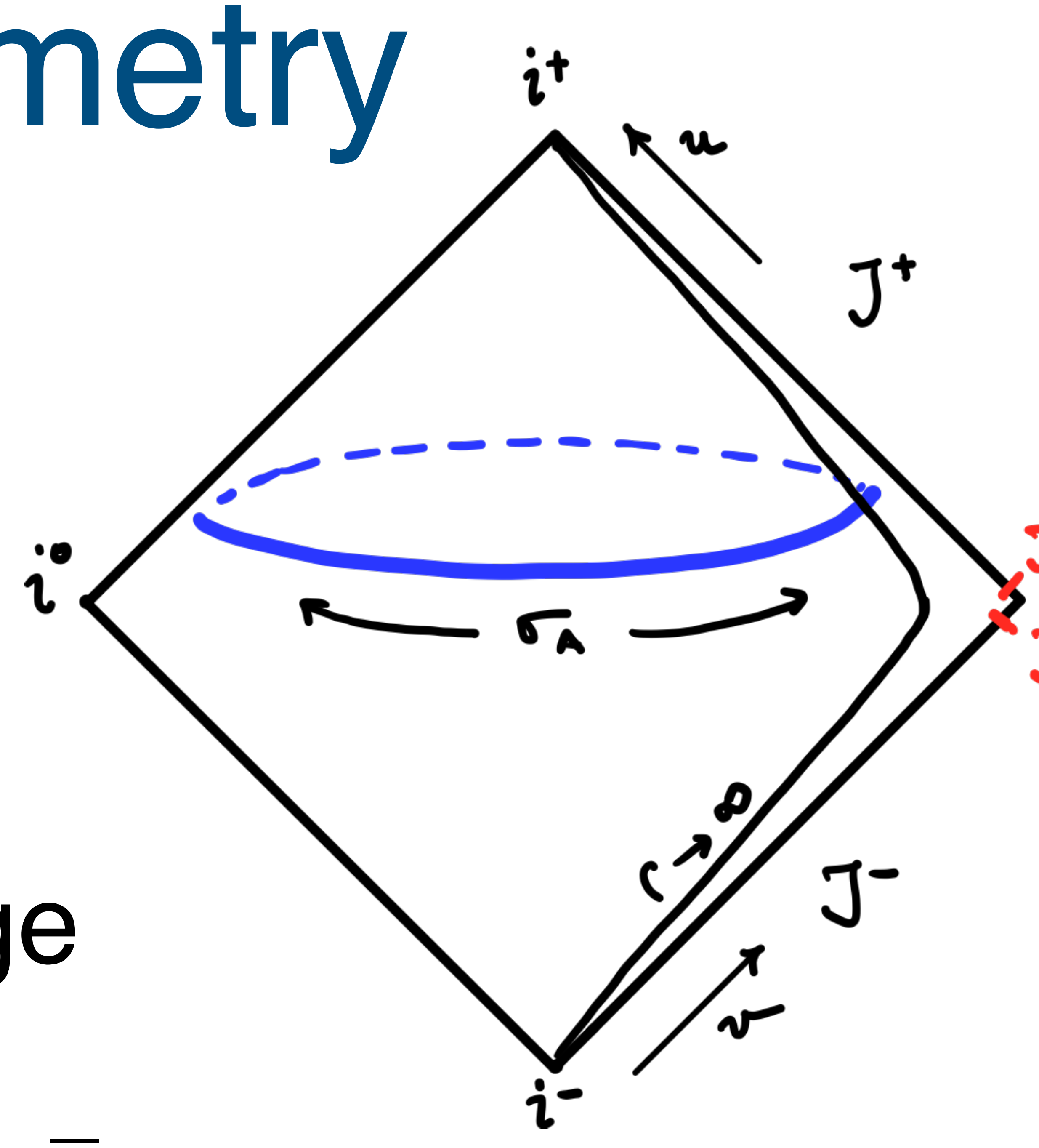
supplemented by the conservation law $\tilde{Q}_s(\mathcal{J}^+_-) = \tilde{Q}_s(\mathcal{J}^-_+)$ Strominger 13

represent the charge conservation equation of a symmetry charge

So far the proof of the subleading theorem is valid only at order $\bar{g}_N g_N$

Can we extend this to all semi non-linear orders $\bar{g}_N g_N^n$? Pranzetti,Raclariu, LF 23

Subleading Theorem from Symmetry



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Strominger

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Can we extend this to all semi non-linear orders $\bar{g}_N g_N^n$?

Pranzetti,Raclariu, LF 23

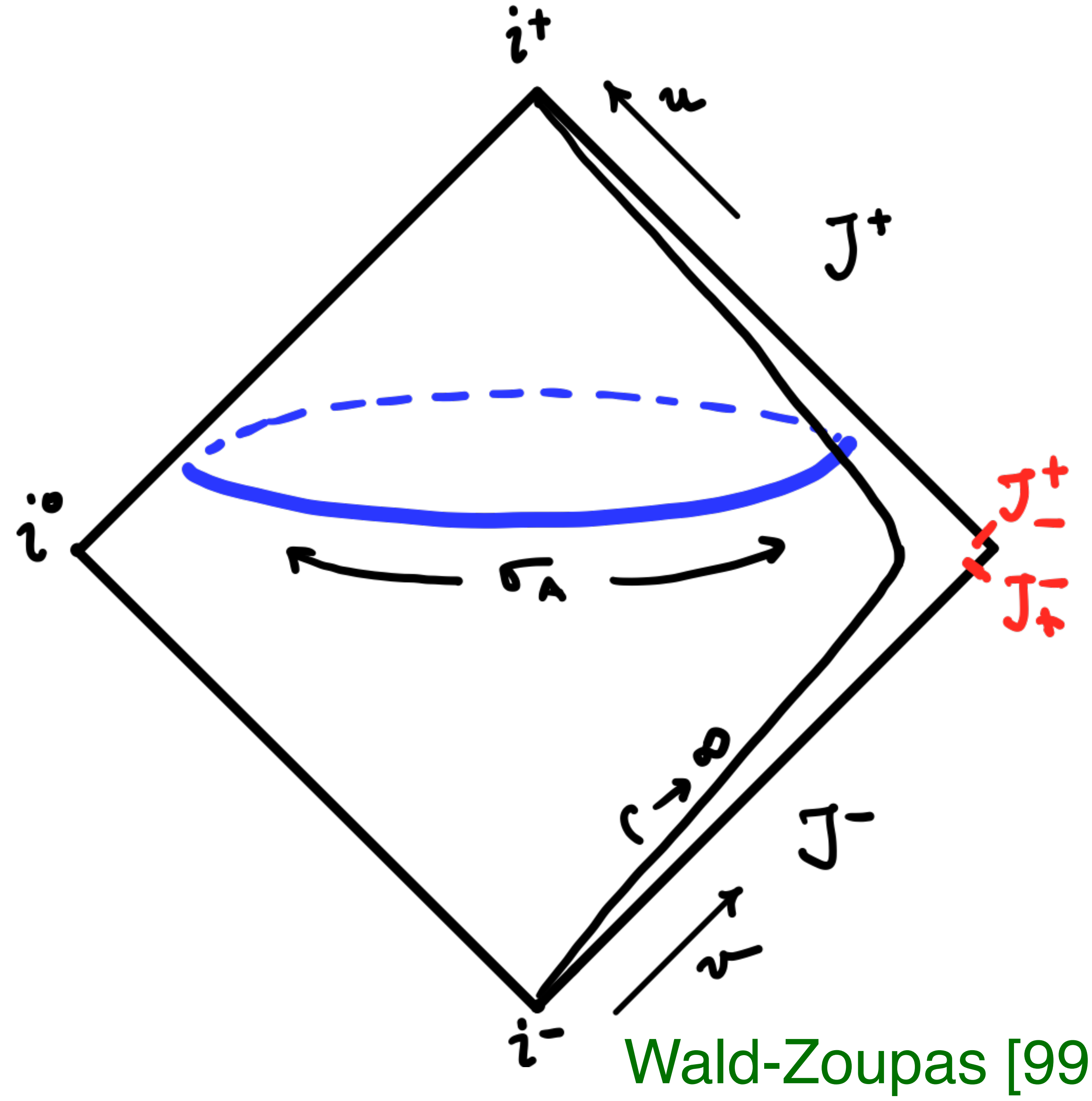
To do this we need to

identify the symmetry algebra ?

understand its (non-linear) action on the gravitational Phase space

Construct the corresponding Noether charge and prove the charge conservation

Noether Charges



Wald-Zoupas [99]
Harlow-Wu[19]
LF,Pranzetti, Oliveri, Speziale [21]
Odak-Rignon-Bret-Speziale [22]

Symmetries are linked to conservation laws

$$\boxed{\partial_u \tilde{Q}_s = D\tilde{Q}_{s-1} + (s+1)C\tilde{Q}_{s-2}}$$

We have to show that these leads to charge conservation under non-radiative boundary conditions.

Admissible bdy conditions are Boundary eom defined as $\Theta = 0$

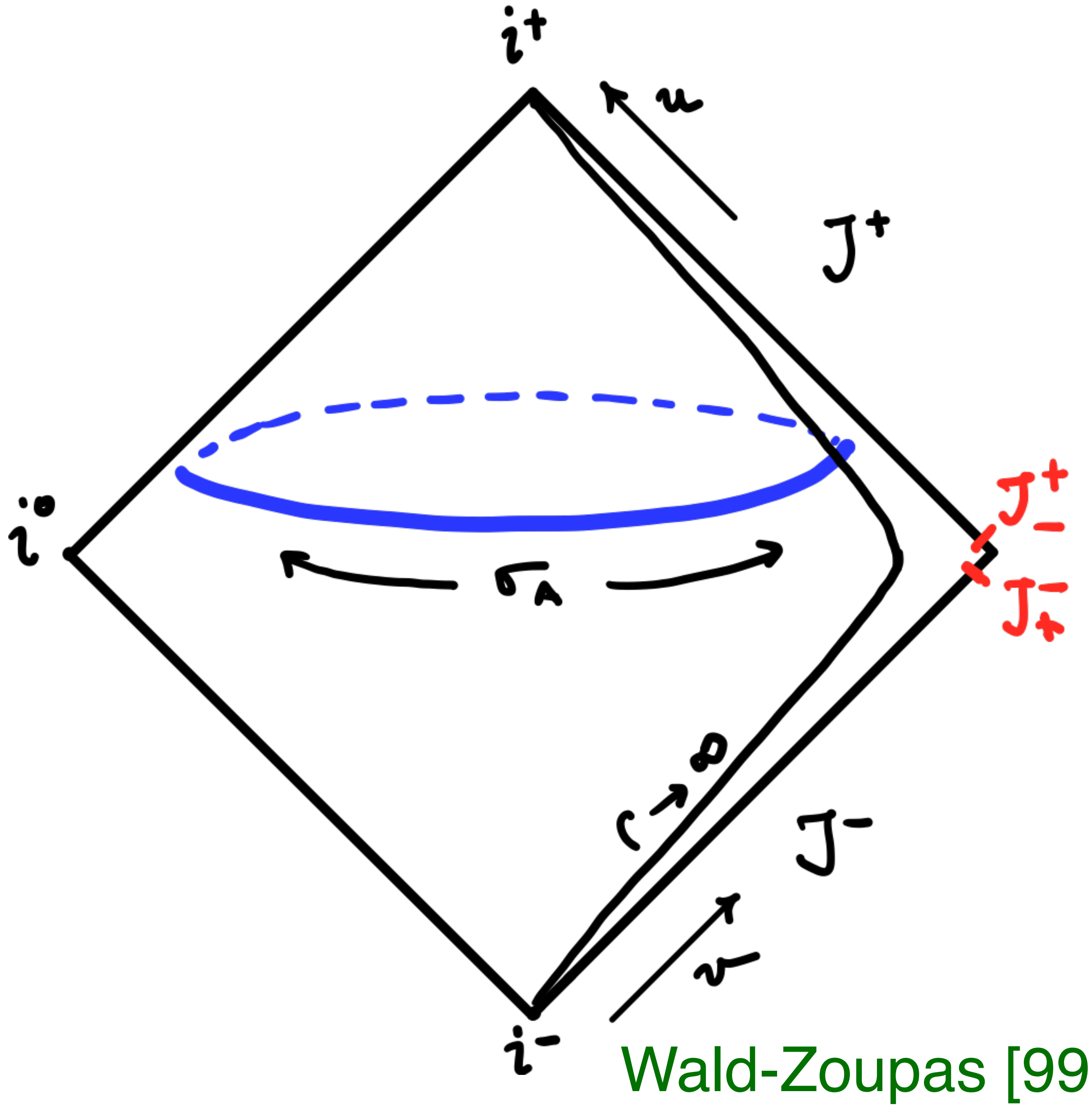
What symplectic potential? $\Theta^{\text{AS}} = \frac{1}{8\pi G}\int_{\mathcal{J}} (\bar{N}\delta C + N\delta\bar{C}) \longrightarrow \quad N = \bar{N} = 0$

usual notion of no-radiation but too strong

Change of boundary conditions are achieved through $\Theta \to \Theta + \delta\ell - \mathrm{d}\vartheta$

Key idea: allow complex canonical transformations.

Noether Charges



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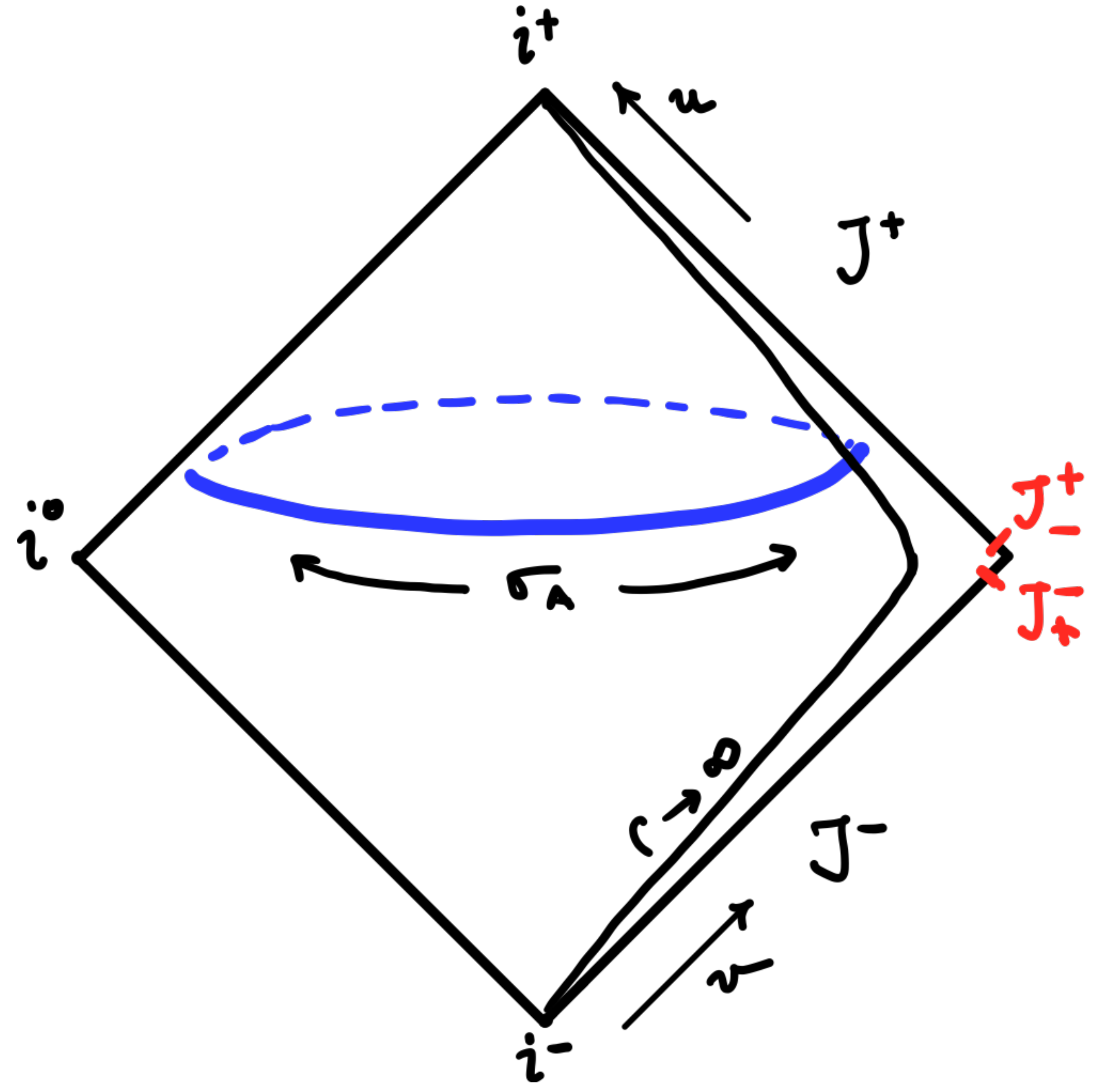
Speziale [24]

Admissible bdy conditions are Boundary eom defined as $\Theta = 0$

We chose instead $\Theta^{\text{HAS}} = \frac{1}{4\pi G}\int_{\mathcal{J}} \bar{N}\delta C$ $\longrightarrow$ $\bar{N} = 0, N$ arbitrary

We are therefore looking for a symmetry charge conserved under this holomorphic no-radiation condition

Symmetry Charges



The symmetry parameters are dual to the charges

$$\boxed{\tau = (\tau_0, \tau_1, \tau_2, \cdots)}$$

τ_0: super-translation

τ_1: super-rotation

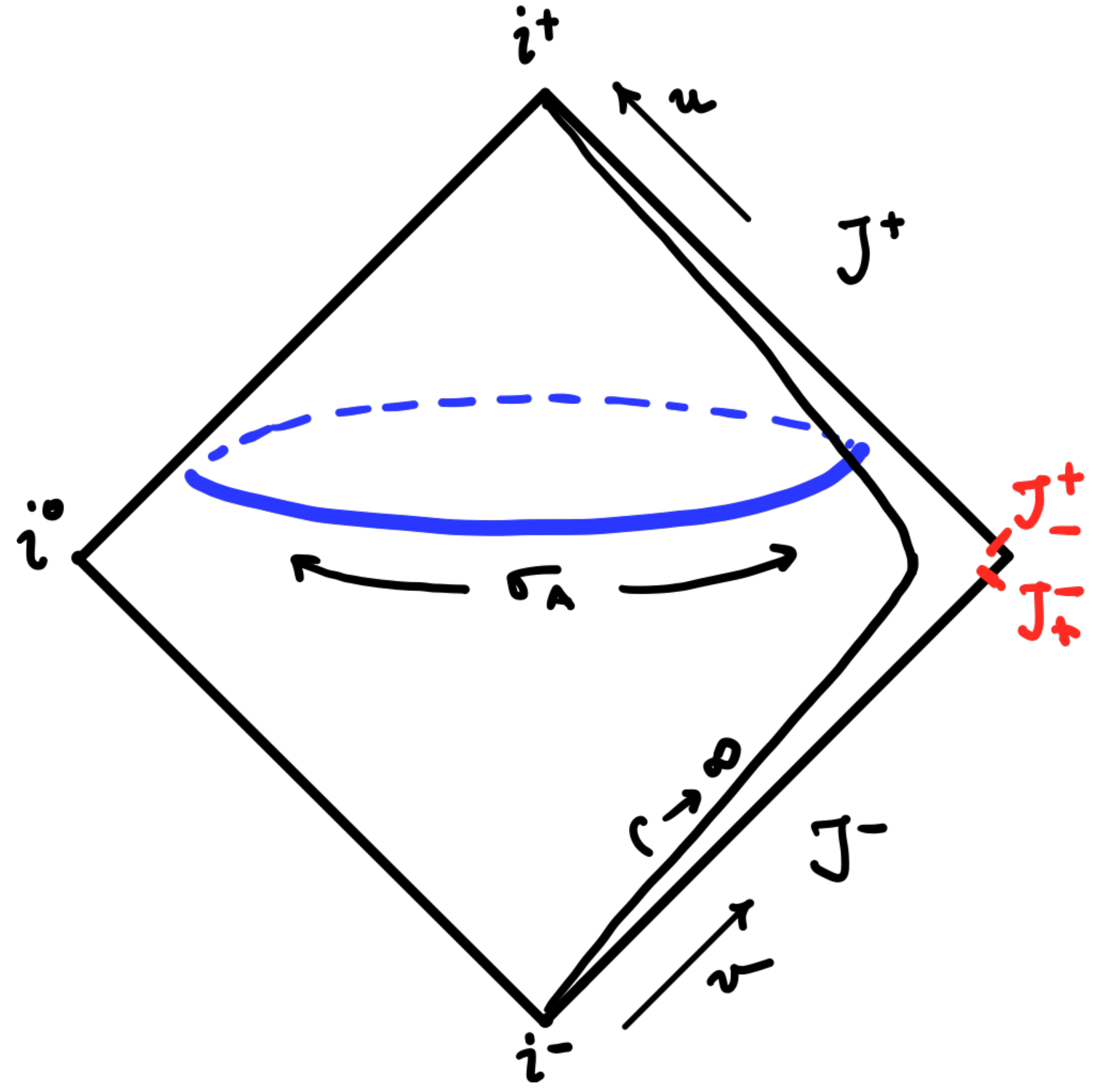
Since $\tilde{Q}_s \in C^{\mathrm{Car}}_{(3,s)} \quad \longrightarrow \quad \tau_s \in C^{\mathrm{Car}}_{(-1,-s)}$

This allows us to define a master charge aspect

$$\boxed{\widehat{Q}_\tau = \sum_{s=0}^{\infty} \tau_s (Q_s \mathrm{d}^2 z + Q_{s-1} \mathrm{d}u \mathrm{d}\bar{z})}$$

Which can be integrated along arbitrary cuts $u = T(z, \bar{z})$

Symmetry Charges



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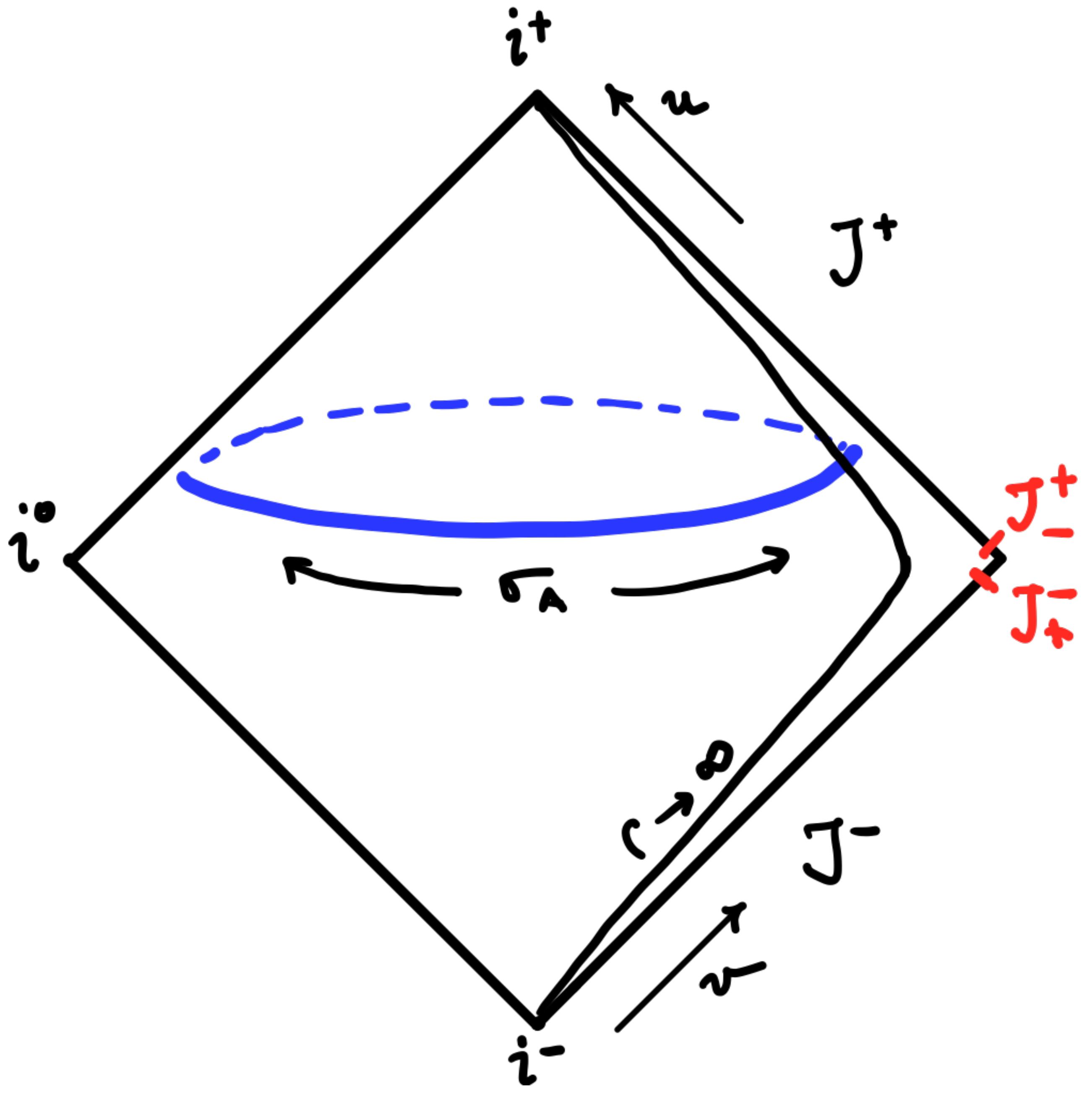
$$\boxed{Q^u_\tau = \sum_{s=0}^{\infty} \int_{S_u} \tilde{Q}_s \tau_s - \frac{1}{4\pi G}\int_{S_u} \tau_0 \bar{N} C}$$

This charge is conserved under the Holomorphic radiation condition $\bar{N} = 0$ provided τ_s satisfy the dual eom $\mathsf{E}_s(\tau) = 0$

$$\boxed{\mathsf{E}_s(\tau) = \partial_u \tau_s - D\tau_{s+1} + (s+3)C\tau_{s+2}}$$

Symmetry Charges



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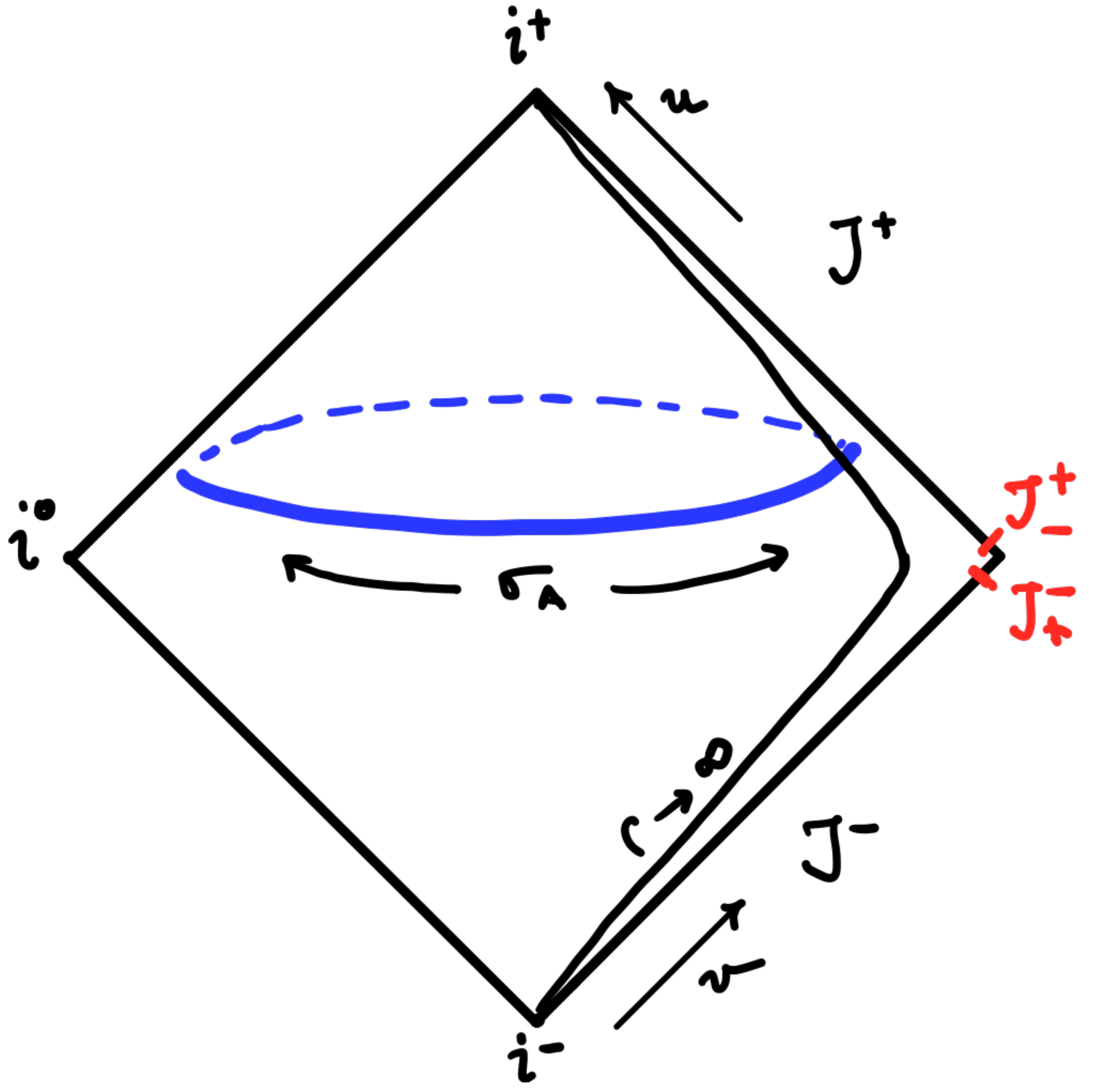


The symmetry parameters are dual to the charges

$$\boxed{\tau = (\tau_0, \tau_1, \tau_2, \cdots)}$$

satisfy the dual eom $\quad \partial_u \tau_s = D\tau_{s+1} - (s+3)C\tau_{s+2}$

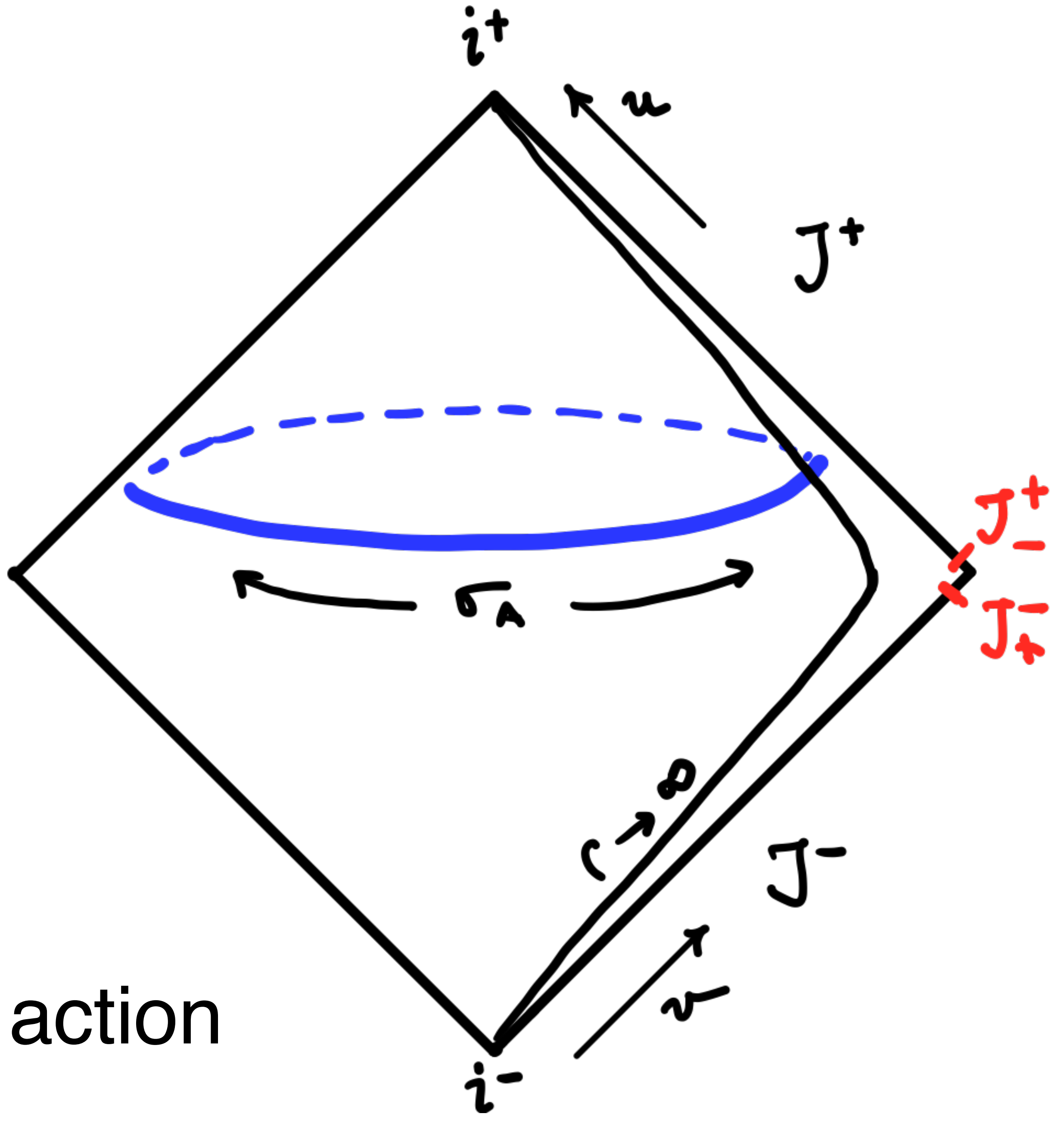
These equation determines τ_s in terms of the celestial parameters T_s which appears as initial conditions $\quad T = \tau|_{u=0} \quad \longrightarrow \quad \tau(T)$

For instance for BMS $\quad \tau_0 = T + \frac{u}{2}DY \quad \tau_1 = \frac{1}{2}Y \quad \tau_2 = 0$

$$\boxed{Q_{(T,Y)} = \int_{\mathcal{J}} \bar{N}(-D^2T - \tfrac{u}{2}D^3Y + TN + YDC + \tfrac{3}{2}DYN)}$$

Symmetry action

The W symmetry parameters τ acts on the sphere as



$$\boxed{\delta_\tau C = \tau_0 N - D^2\tau_0 + 2\tau_1 DC + 3CD\tau_1 - 3C^2\tau_2}$$

super-translation

super-rotation on $C^{\text{Car}}_{(1,2)}$

Spin2 non-linear action

infinite dimensional and non-linear generalization of the BMS action.

When $s \geq 2$ τ_s depends on the shear: non-linearity of the action

It controls the time evolution of the generating charge

$$\partial_u Q^u_\tau = -\frac{1}{4\pi G}\int_S \bar{N}\delta_\tau C$$

Symmetry action

The W symmetry acts on the sphere as

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Three remarkable and non-trivial facts:

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Three remarkable and non-trivial facts: The action closes

$$[\delta_\tau, \delta_{\tau'}]C = -\delta_{[\![\tau,\tau']\!]}C$$

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The symmetry preserves the asymptotic eom $\tilde{\mathsf{E}}_s = \partial_u \tilde{Q}_s - D\tilde{Q}_{s-1} - (s+1)C\tilde{Q}_{s-2}$

$$\delta_\tau \tilde{\mathsf{E}}_s = \sum_{n\geq s} L_s{}^n(\tau)\tilde{\mathsf{E}}_n$$

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The symmetry is canonical $\delta_\tau C = \{Q_\tau, C\}$ $Q_\tau = I_{\delta_\tau}\Theta^{\mathrm{HAS}}$

Symmetry Charges

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We have a compact expression for the charges assuming that $Q_\tau^{i+} = 0$

$$\widehat{Q}_\tau = \int_{\mathcal{J}} \bar{N}\delta_\tau C$$

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$$\widehat{Q}_\tau = \int_{\mathcal{J}} \left(\tau_0(-D^2\bar{N} + \bar{N}N) + \tau_1(\bar{N}DC - 3D(\bar{N}C)) + \tau_2\bar{N}C^2\right)$$

Solving the dual eom provides the explicit charge expressions

Symmetry Bracket

The symmetry bracket is $$[\![\tau, \tau']\!]_s = [\tau, \tau']_s + \delta'_\tau \tau_s - \delta_\tau \tau'_s$$

where $$\boxed{[\tau, \tau']_s = \sum_n (n+1)(\tau_n D\tau'_{s+1-n} - \tau'_n D\tau_{s+1-n}) - (s+3)C(\tau_0 \tau'_{s+2} - \tau'_0 \tau_{s+2})}$$

flat space contribution ↑

deformation to accomodate any SD background

Three non trivial properties:

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The Charges provides a canonical representation of the algebra

$$\{Q_\tau, Q_{\tau'}\} = Q_{[\![\tau,\tau']\!]}$$

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deformation to accomodate any SD background

Three non trivial properties: it preserve the dual eom $\mathsf{E}_s = \partial_u \tau_s - D\tau_{s+1} + (s+3)C\tau_{s+2}$

$$\mathsf{E}_s([\![\tau,\tau']\!]) = 0 \quad \text{when} \quad \mathsf{E}_s(\tau) = \mathsf{E}_s(\tau') = 0$$

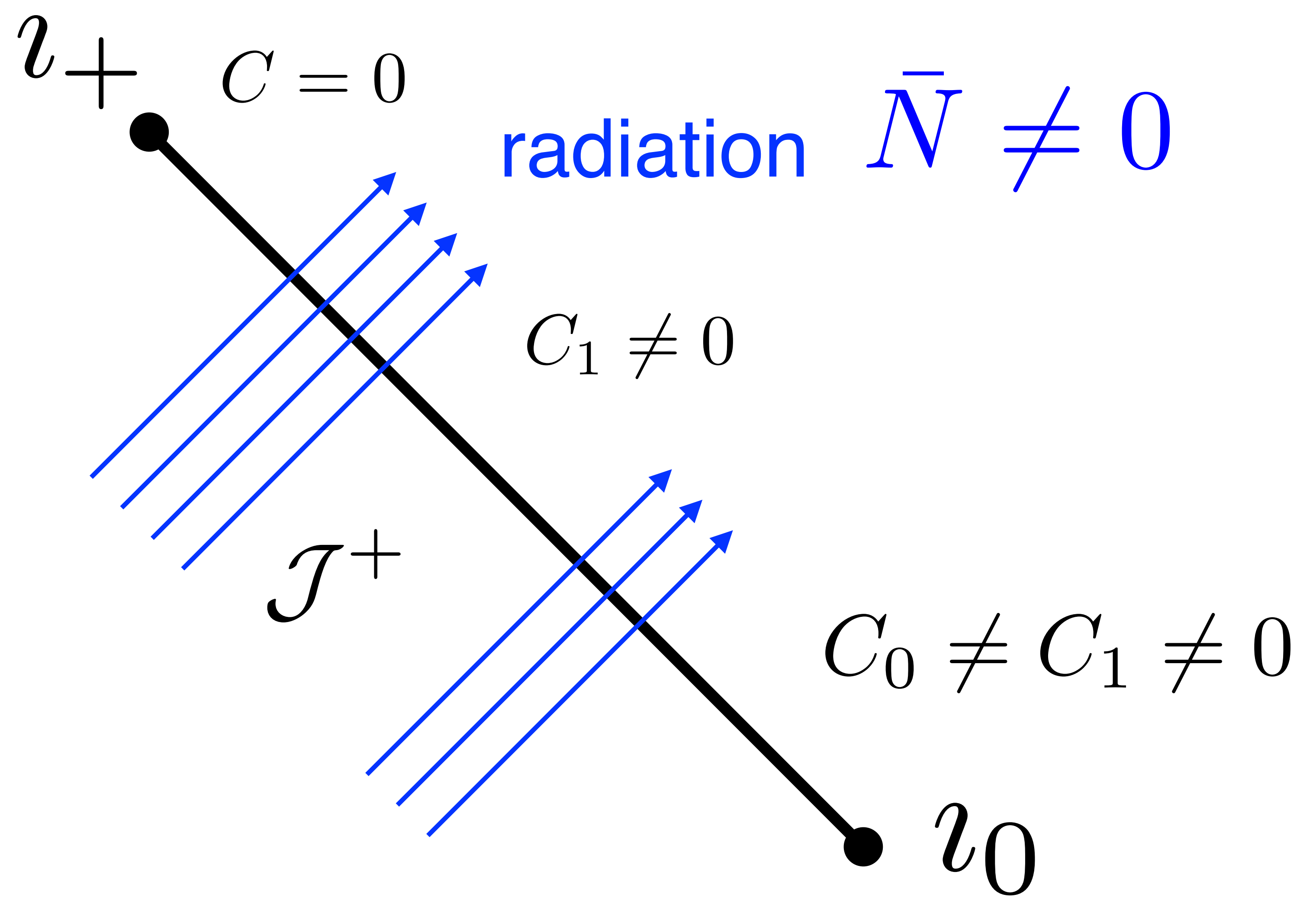
It satisfies Jacobi identity

$[\![\tau,\tau']\!]$ defines an algebroid bracket

Puzzle: Symmetries are about algebra not algebroids !

From algebroid to algebra

Algebroid are the algebraic expression of dynamical symmetries when radiation is present.
But true symmetries expressed through algebras only exist at non-radiative cuts



Due to the memory effect different non-radiative cuts carry different values of the shear

One needs to identify a symmetry algebra $W_C(S)$ the wedge algebra that depends on the value of C at non-radiative cuts

Wedge algebra

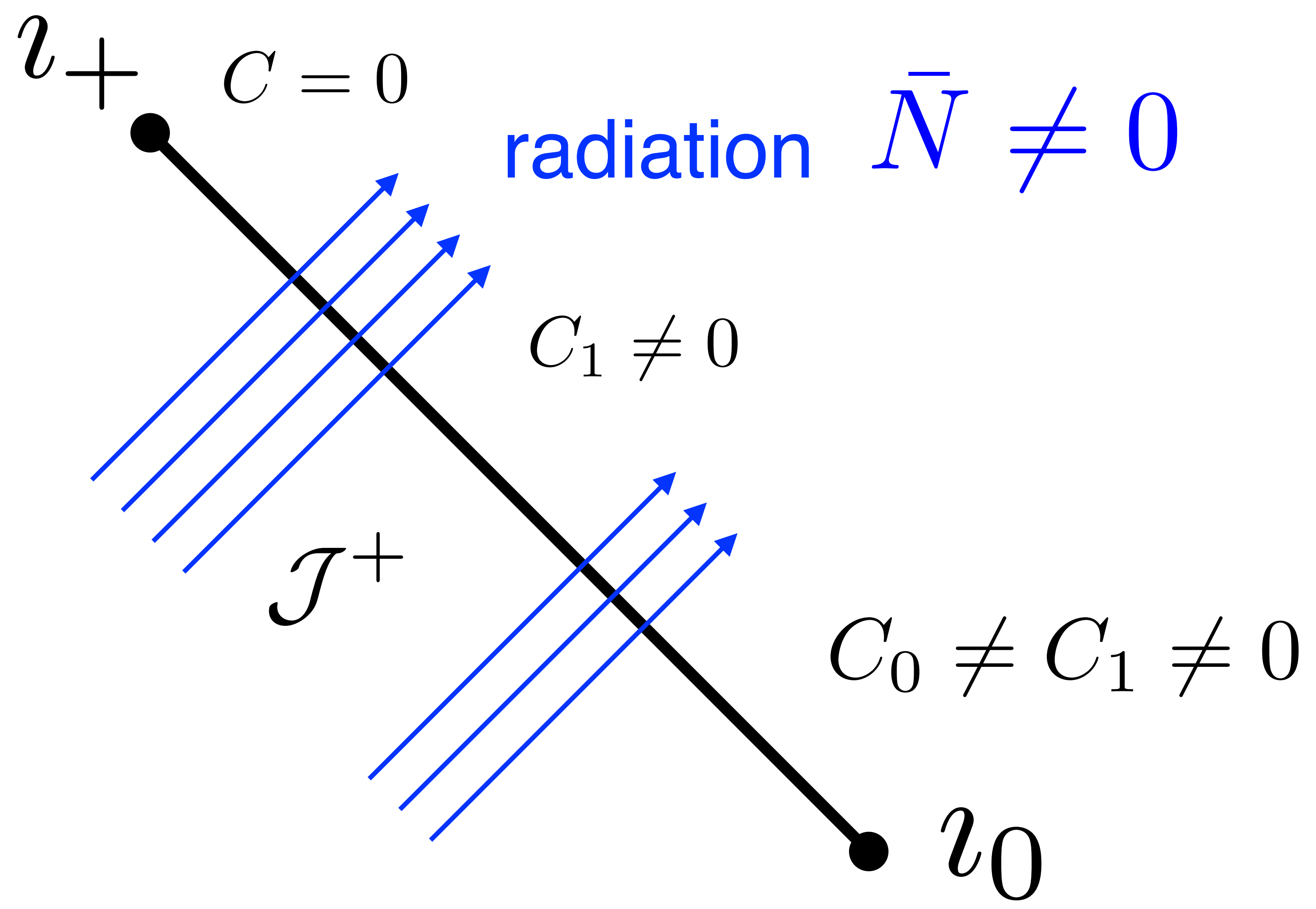
The wedge algebra $W_C(S)$

is defined as the symmetry algebra that preserves C

$$\delta_\tau C = 0$$

for $\tau \in W_C(S)$ C is a field independent parameter

$$\boxed{W_0(S) = \{T \in C_{(-1,-s)}(S) | D^{s+2}T_s = 0\}}$$



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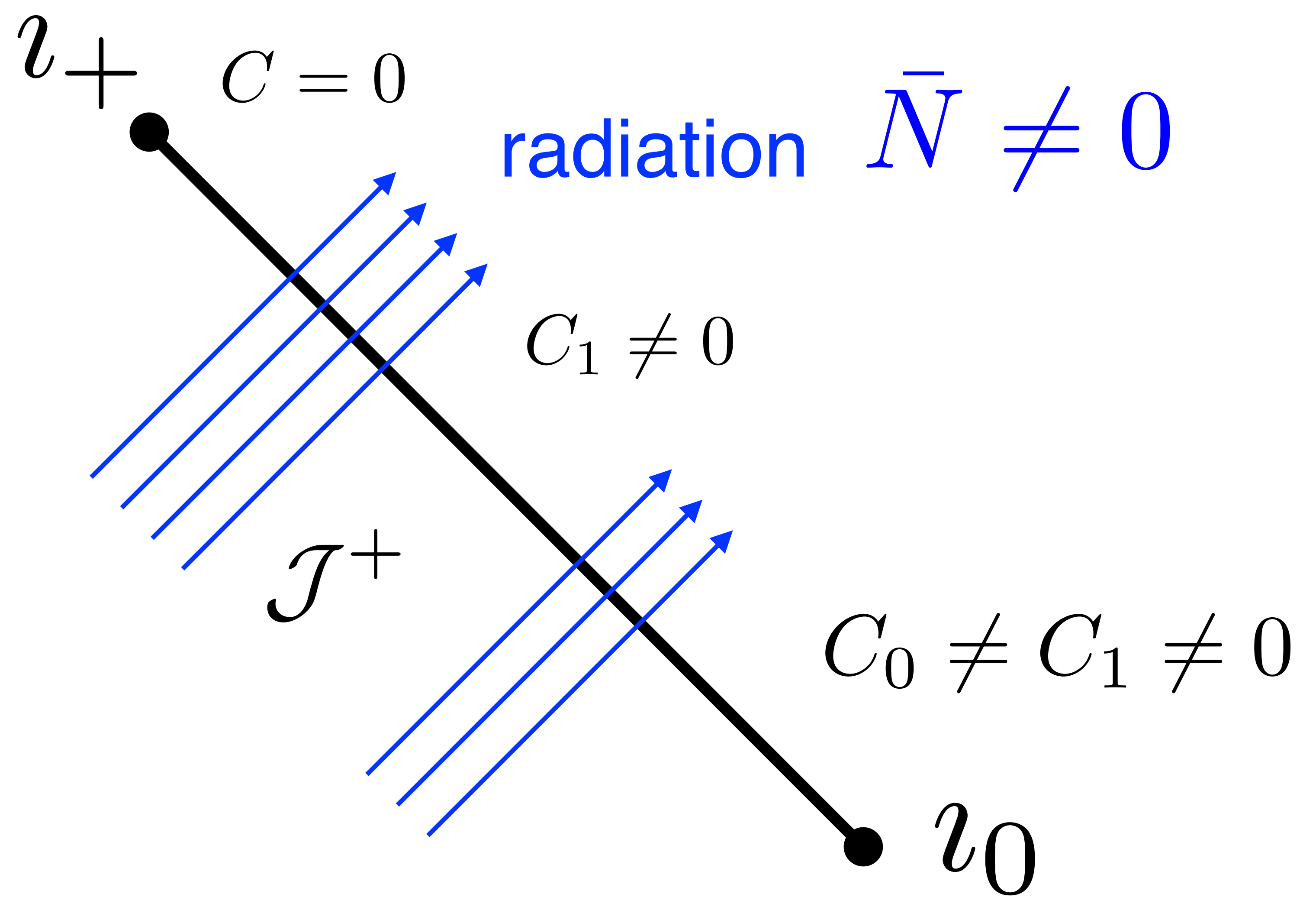

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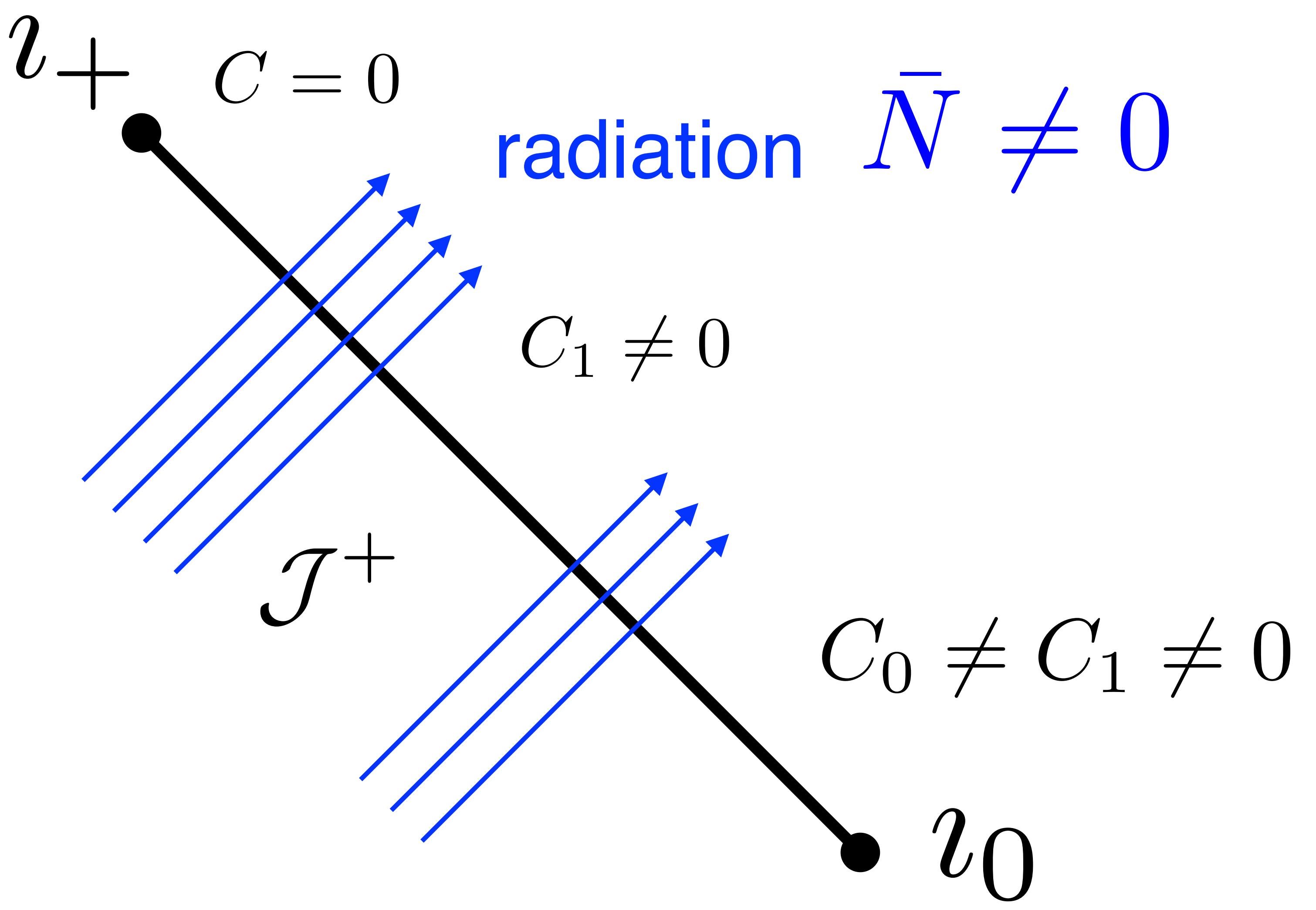
$$W_0(S_0) = SL^+(2) \ltimes \mathbb{C}^4 = \mathrm{Poinc}^+$$



$\mathrm{HConf}(S_0) = \mathrm{SL}(2, \mathbb{C})$

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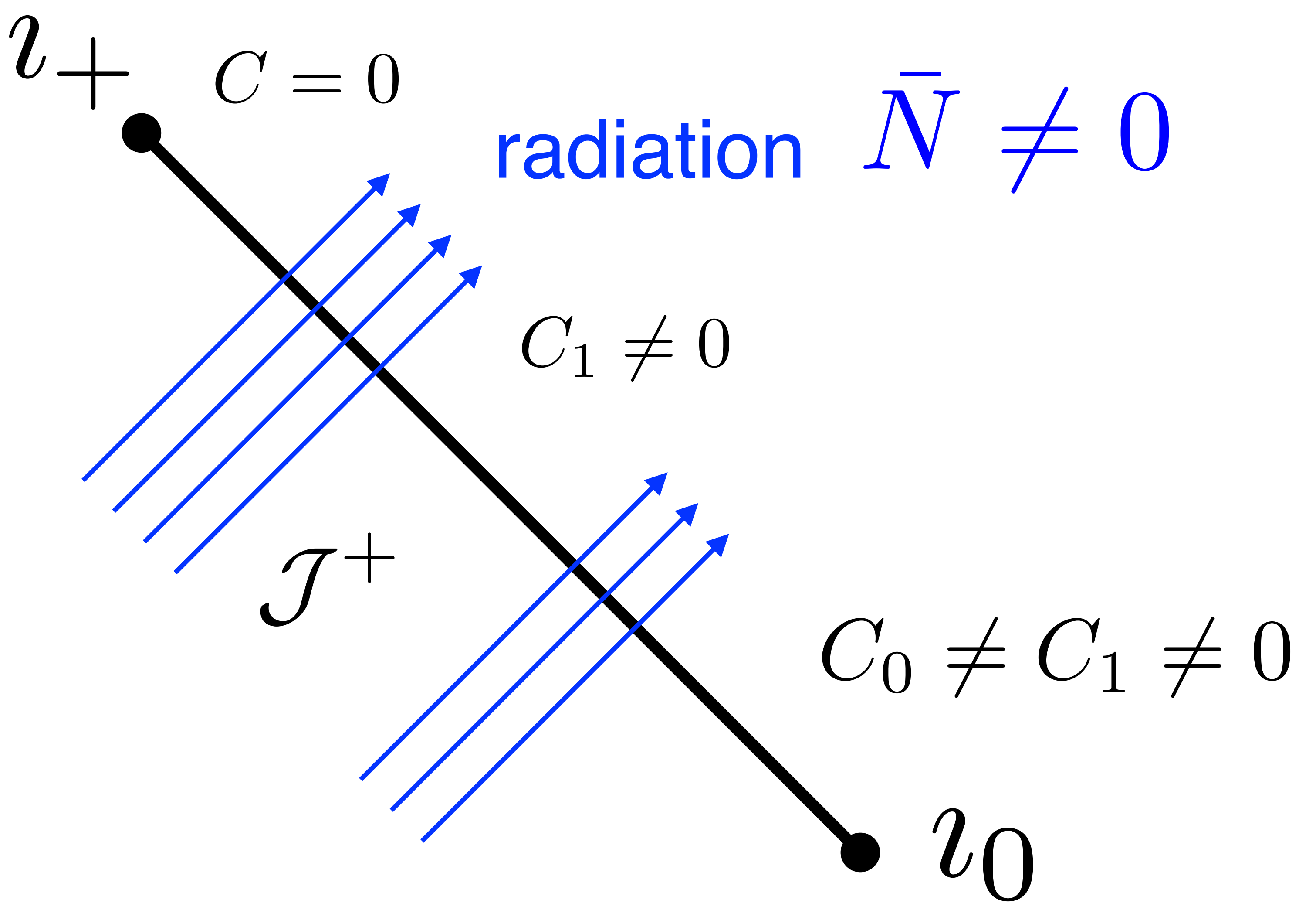
$$W_0(S_2) = LW_\infty$$

localisation of Poinc like Vir is a localisation of SL(2,C) $\mathrm{HConf}(S_2) = \mathrm{Vir}$



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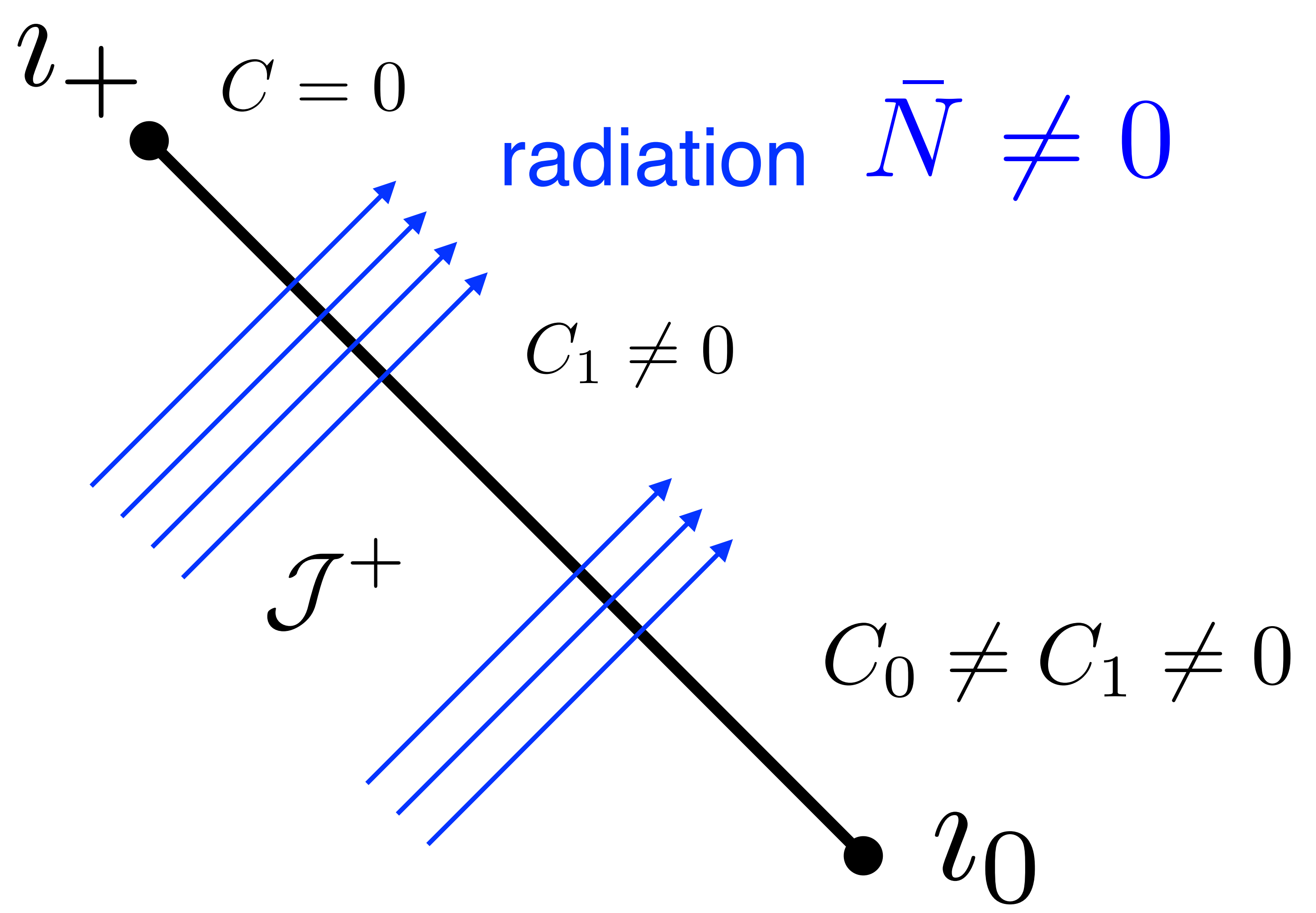

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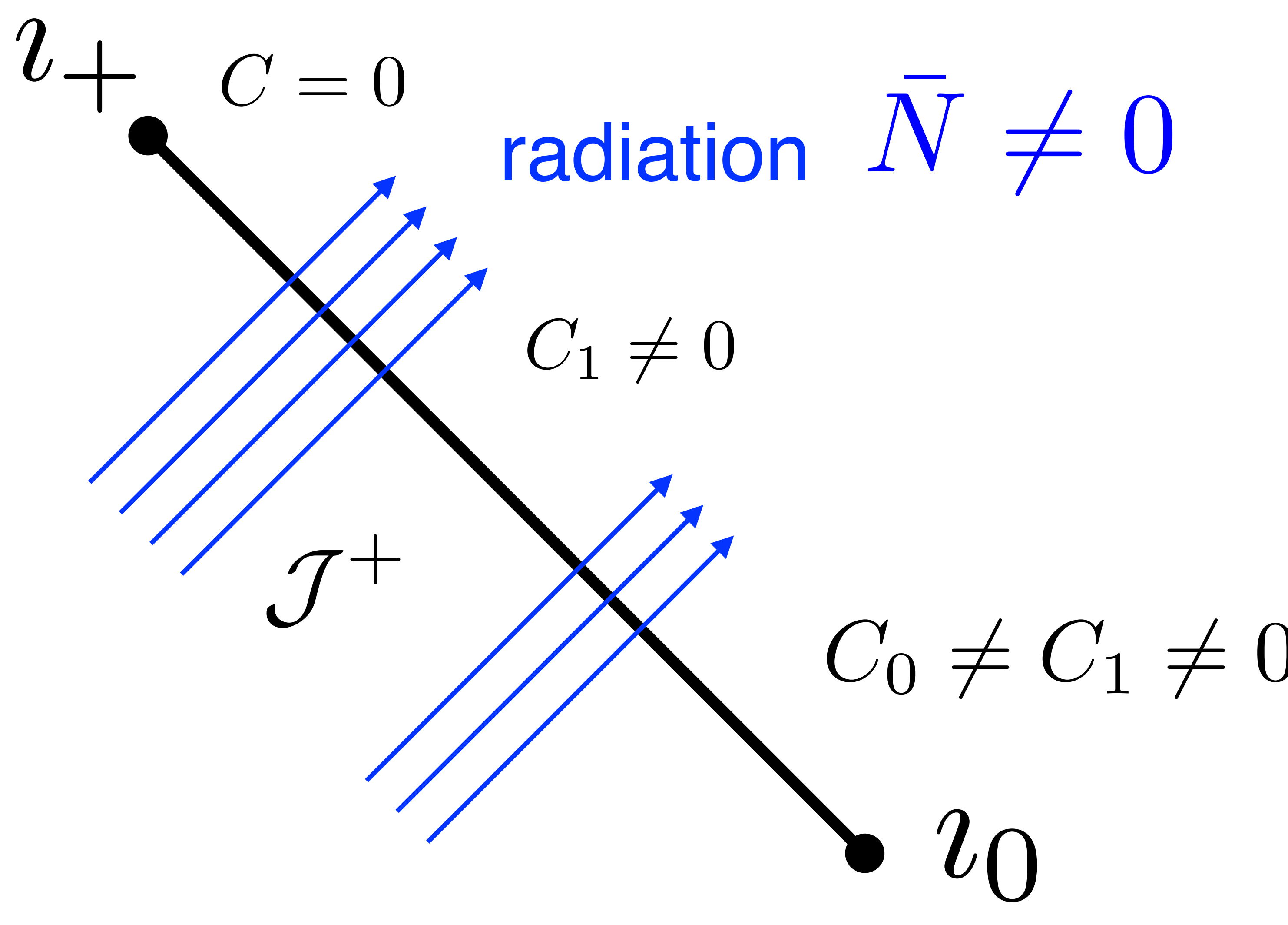
$W_0(S_n)$ Krichever-Novikov like generalisation

Good cut Wedge algebra

the wedge algebra $W_C(S)$ for C is isomorphic to $W_0(S)$



$$D^2 T_0 = 2D(CT_1) + CDT_1 - 3C^2 T_2$$

$$D^3 T_1 = 3D^2(CT_2) + 2D(CDT_2) + CD^2 T_2 + \cdots$$

$$\vdots$$

More generally the twisted wedge algebra is defined as $(\mathcal{D}^n T)_{-2} = 0$ where $\mathcal{D}$ is a covariant derivative

$$(\mathcal{D}T)_s := DT_{s+1} - (s+3)CT_{s+2}$$

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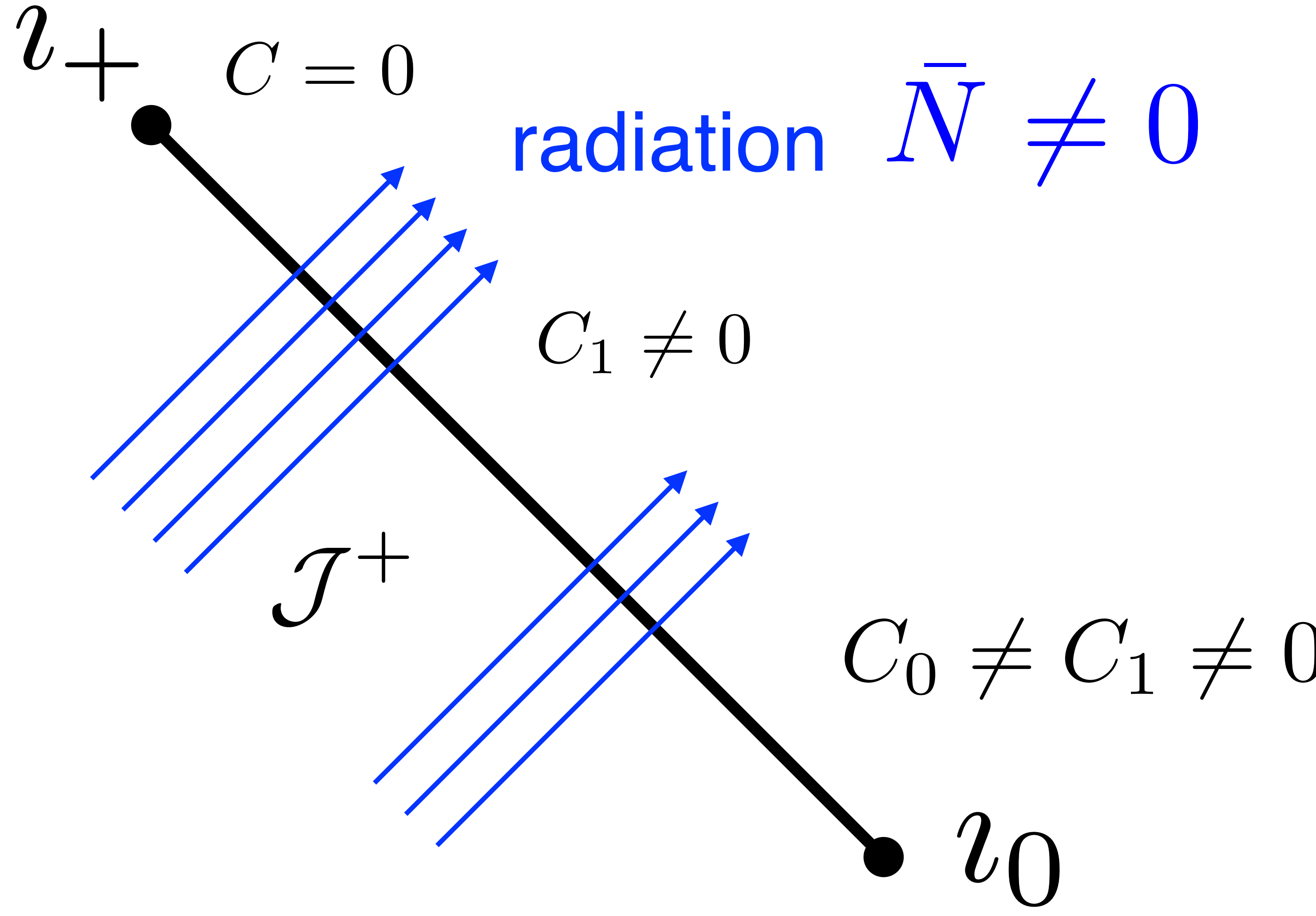

$$D^2 T_0 = 2D(CT_1) + CDT_1 - 3C^2 T_2$$

$$D^3 T_1 = 3D^2(CT_2) + 2D(CDT_2) + CD^2 T_2 + \cdots$$

$$\vdots$$

The isomorphism $T^G : W_C(S) \to W_0(S)$ involves the Goldstone G: $D^2 G = C$

$$T^G(\tau) = P\overrightarrow{\exp} \int_0^1 (\mathrm{ad}_{tG} G)(\tau)$$

It intertwines the naive and covariant derivative $DT^G(T) = T^G(\mathcal{D}T)$

From Carrollian to Twistor

Whats the connection with twistor theory?

Twistor space provide a fibration of $\mathcal{J}$ complexified

$$\mathbb{PT} \to \mathcal{J}_{\mathbb{C}}$$

$$(\mu^{\alpha}, \bar{\lambda}_{\dot{\alpha}}) \to (u = [\mu\lambda], z, \bar{z})$$

Remember that $u \in C^{\text{Car}}_{(-1,0)}$

$$\lambda \propto (1, z)$$

What does the fiber coordinate represents ?

$$q = [\mu\check{\lambda}] \in C^{\text{Car}}_{(0,1)} \qquad [\lambda\check{\lambda}] = 1.$$

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Adamo-Newman 10

represents the angle at which a congruence of null geodesic intersect $\mathcal{J}$

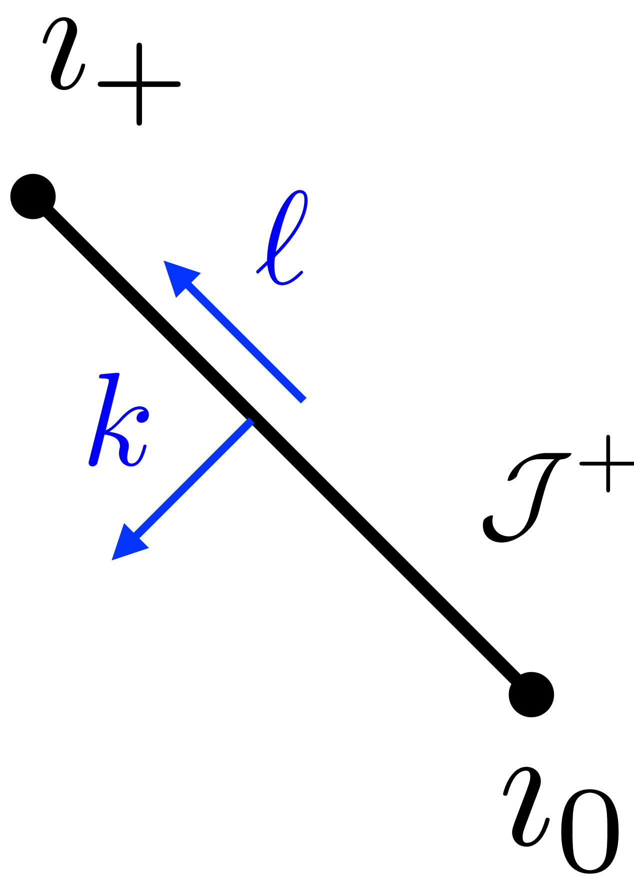
Carrollian complex structure on $\mathcal{J}$: $\ell^a q_{ab} = 0$ $q_{ab} = m_{(a}\bar{m}_{b)}$

needs to be completed with a Ehresman connection k

A null geodesic congruence is characterised by a null vector k_q

Such that $k_q \cdot \ell = 1$ $k_q \cdot m = q$

$m_q = m - q\ell$ holomorphic frame transverse to k and l



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Adamo-Newman 10

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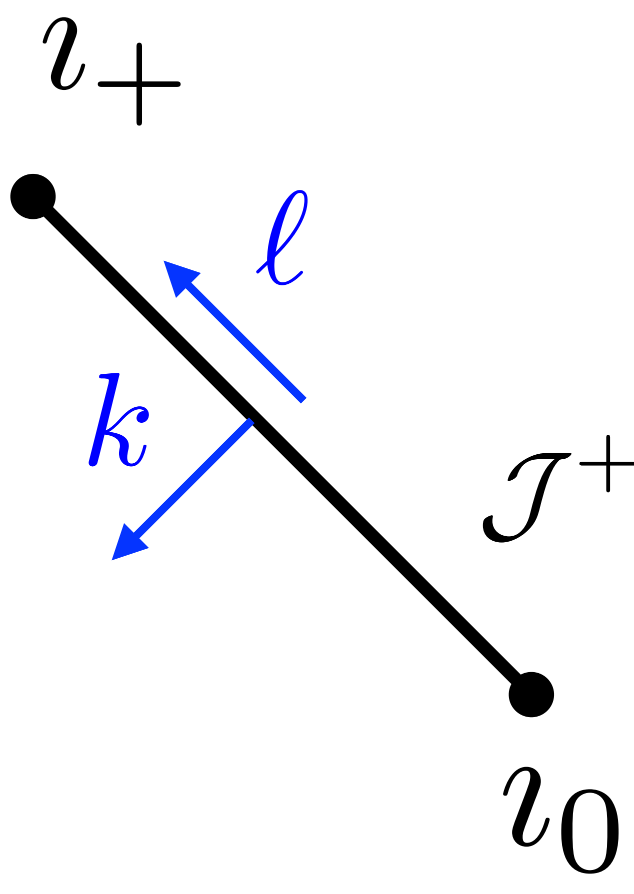
The transformation $(k, m, \ell) \to (k_q, m_q, \ell)$ corresponds to a null boost with angle q



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$$(\mu^{\alpha}, \bar{\lambda}_{\dot{\alpha}}) \to (u = [\mu\lambda], z, \bar{z})$$

The total space isomorphic to twistor space is the space of null rays which reaches scri at a cut u and at an angle q

$\lambda \propto (1, z)$

Adamo-Newman 10

It is the total space of the bundle $\mathcal{N} = O(1,-1) \oplus \mathcal{J} \to \mathcal{J}$

with coordinates (q, u, λ_{α})

Like scri is the total space of $\mathcal{J} = O(1,1) \oplus \mathbb{CP}_1 \to \mathbb{CP}_1$

Newman 10

From Carrollian to Twistor

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Adamo-Newman 10

Note that the ``Newman” bundle $\mathcal{N} = O(1, -1) \oplus O(1, 1) \to \mathbb{CP}^1$

with coordinates (q, u, λ_{α})

is isomorphic to but has a different complex structure than twistor space

$$\mathbb{PT} = O(1) \oplus O(1) \to \mathbb{CP}^1$$

From Carrollian to twistor

The symmetry parameters can be converted into a function of $q \in C^{\mathrm{Car}}_{(0,1)}$

$$\tau = (\tau_0, \tau_1, \tau_2, \cdots) \to \hat{\tau}(q) = \sum_{s=0}^{\infty} \tau_s q^{s+1} \qquad \in C^{\mathrm{Car}}_{(0,1)}$$

The W bracket on can be recasted as a Poisson bracket on-shell of the dual eom

$$[\tau, \tau']_s = \sum_{n=0}^{s+1} (n+1)(\tau_n D\tau'_{s+1-n} - \tau'_n D\tau_{s+1-n}) - (s+3)C(\tau_0\tau'_{s+2} - \tau'_0\tau_{s+2})$$

From Carrollian to twistor

The symmetry parameters can be converted into a function

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The W bracket on can be recasted as a Poisson bracket on-shell of the dual eom

$$\sum_s [\tau, \tau']_s q^{s+1} = \{\hat{\tau}, \hat{\tau}'\} + \text{dual EOM}$$

where $\boxed{\{\hat{\tau}, \hat{\tau}'\} = \partial_q \tau \partial_u \tau' - \partial_q \tau' \partial_u \tau}$

is the twistor Poisson bracket.

From Carrollian to twistor

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The dual eom can be simply written as

$$\boxed{q\partial_u \hat{\tau} - D\hat{\tau} + C\hat{\tau} = 0}$$

They can be equivalentely be written in terms of the twistor potential $h \in C^{\mathrm{Car}}_{(0,2)}$

as

$$\boxed{D\hat{\tau} + \{h, \hat{\tau}\} = 0}$$

where

$$h = -\frac{q^2}{2} + \partial_u^{-1} C$$

change in complex structure $\mathcal{N} \to \mathbb{PT}$ from gravity

$\hat{h} = hD\lambda \in \Omega^{(1,0)}(\mathbb{PT}, O(2)).$

Supertranslation and Good cut

The supertranslations $T(z,\bar{z})$ acts non trivially on the pair (u,q)

$$\delta_T u = T \qquad \delta_T q = -DT$$

The transformation for q follows from the fact that while m is a vector tangent to the cut $u=\mathrm{cst}$, the vector $m_{(q=-DT)} = m + DT\ell$ is the vector tangent to the cut $u - T = \mathrm{cst}$

Given a shear C we can construct a Goldstone $G(u,z,\bar{z})$ solution of the good cut equation

$$D^2 G = C \circ \widehat{G}$$

Where $\widehat{G}:\mathcal{N}\to\mathcal{N}$ is $\boxed{\widehat{G}(q,u,z,\bar{z}) = (q - DG, G, z, \bar{z})}$

Supertranslation and Good cut

Under this map we have that $\widehat{G}_*(\partial_q, \ell, m) = (\partial_{q_G}, \ell^G, m^G)$

And the radiative evolution equations are mapped onto the non radiative one !

$$(q_G \ell^G - m^G)\widehat{G}^*[\tau] = \widehat{G}^*[(q\ell - m + C\partial_q)\tau] = 0$$

Non-linearity

We have seen that $\tilde{Q}_s = \tilde{Q}_s^{\mathsf{S}} + \tilde{Q}_s^{\mathsf{H}} + \tilde{Q}_s^{\mathsf{SH}}$ for $s \geq 2$

The super-Hard contributions requires an extension of the amplitude that includes collinear external states and extension of the subsubleading theorems

Can we obtain these SH contribution from amplitudes?

Narayanan, Jorstad, To appear

We have seen that $\mathsf{E}_3 = 0$ is valid in Einstein gravity

$\longrightarrow$ One expect sub^3 leading theorem to be valid. Amplitude proof?

Quantum

What is the quantization of the W-algebra ?

Preliminary investigation of the quantum commutator algebra shows that there exist potential Lie algebra anomalies

$$[\hat{Q}_\tau, \hat{Q}_{\tau'}] = Q_{[\tau,\tau']} + A_{(\tau,\tau')}$$

where $A_{(\tau,\tau')}$ depends only on C not on $\bar{N}$

First investigation suggests that A=0 for the wedge.

Lie algebra anomaly versus VOA anomalies?

Costello-Paquette 23
Blitteston 24

Anomalies in SD gravity due to all + one loop amplitudes.

Conclusion

We have shown that there exists a semi-perturbative sector of GR $\bar{g}_N g_N^n$ which is integrable and carry the representation of an infinite dimensional symmetry algebra W_C(S) with bracket

$$[\tau, \tau']_s = \sum_{n=0}^{s+1} (n+1)(\tau_n D\tau'_{s+1-n} - \tau'_n D\tau_{s+1-n}) - (s+3)C(\tau_0\tau'_{s+2} - \tau'_0\tau_{s+2})$$

which can be viewed as a non-linear deformation of $LW_{1+\infty}$

Costello-Paquette 23
Blitteston 24

And corresponds to the quantization of SD gravity

Can we use this semi-perturbative sector as a basis for a new form of perturbative quantization in the same way we use free theory as an asymptotic basis?

s=2 and s=3 are exact symmetry of Einstein gravity. If no quantum anomaly are present

$\longrightarrow$ Non-trivial Ward identities beyond the linear orders ?

Generalization to Einstein-Yang-Mills

Agrawal, Chralambous, Donnay 25

Self-dual Polarization

If instead of $\Theta^{\mathsf{HAS}} = \frac{1}{4\pi G}\int_{\mathcal{J}} \bar{N}\delta C \quad \longrightarrow \quad \bar{N} = 0, N$

Self-dual Polarization

we introduce the potential $h \in C^{\mathrm{Car}}_{(0,2)}$ such that $\partial_u h = C$

And work with $$\Theta^{\mathsf{SD}} = \frac{1}{4\pi G}\int_{\mathcal{J}} \dot{\bar{N}}\delta h$$

The no-radiation condition is $\dot{\bar{N}} = 0,\ N$ arbitrary

The symmetry parameters are $\tau = (\tau_{-1}, \tau_0, \tau_1, \tau_2, \cdots)$

subject to $$\mathsf{E}_s = 0,\ s \geq -1$$

The master charge is $$Q^u_\tau = \sum_{s=-1}^{\infty}\int_{S_u} \tilde{Q}_s \tau_s$$

The mass is the covariant mass not the bounday mass

Self-dual Polarization

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The no-radiation condition is $\dot{N} = 0,\ N$ arbitrary

The symmetry action on the potential is $\delta_\tau h = \tau_0 C - D\tau_{-1}$

The symmetry Charge is $$Q_\tau = \int_{\mathcal{J}} \dot{N}\delta_\tau h$$

The global symmetry algebra: $W_0(S_0) =$ central extension of Poinc^+

$$W_0(S_2) = LW_{1+\infty}$$

From Carrollian to twistor

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They can be equivalentely be written in terms of the twistor potential

$$\partial h + \{h, \tau\} = 0$$

For the twistor potential $h = (\partial_u^{-1} C) D\lambda \in \Omega^{(1,0)}(\mathbb{PT}, O(2))$.