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Swarm Robotics in Bearing-Only Formations

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1. Introduction

In recent decades there has been a marked increase in using UAVs (Unmanned Aerial Vehicles) in areas such as damage assessment after natural disasters, surveillance and detection of coastal hazards, and even large-scale agricultural monitoring. Beyond this, the usefulness of UAVs has also been shown in military and defence capacities, allowing for safer, real-time situational observation and intelligence gathering, which can be used to influence decision-making to reduce operational risk. Leonardo are interested in the general concept of using an autonomous system of UAVs, or *drones*, in such a manner as to complete a certain task under given constraints.

Figure 1: A system of airborne quadcopter drones flies as a group.

Distributed control allows a system to be governed without the need for a global controller, which can often be expensive to implement

In many situations, it can be useful to replace a single complex vehicle by a system of simpler ones that are able to equivalently complete the task at hand by working as a group. This leads to the choice of implementing either a *centralised* approach, or a *distributed* approach to control the system. A centralised approach relies on the availability of a global controller of the system, while distributed control does not require any central governance, but the system can become more complex as a result. However, there is often more flexibility in distributed systems, as well as lower operational costs and greater robustness.

Models focusing on drone formation are often built using *distance-based* information, meaning that the behaviour of the system is dependent on the relative distances between each drone. This is useful because it allows for collisions to be avoided during the evolution of the system, and it also enables the system to be controlled into well-defined formations. However, in the situation of interest to Leonardo, information on the distance between drones is not available. We will instead use *bearing-based* models, meaning that relative distances between each drone are not required, but only the bearing angle from one drone to another. With this restriction, we wish to design the dynamics of the system such that the drones self-align around a target, without explicit instructions to do so.

Consensus refers to a system agreeing upon a common behavior through local interactions between drones

Our aim is to look at the concept of *consensus* in a multi-drone system, hoping for the drones to achieve some kind of equilibrium formation with respect to a chosen target. We will focus on the implementation of a distributed approach to control the behaviour of the drones.

2. Formulating the Problem

We note that if any two drones travel together along similar trajectories, this will cause a redundancy in the system, since the two drones will be measuring very similar information. It is therefore more useful for the drones to encircle the target in some way, as we depict in Figure 2.

Here, we show three drones (D_1, D_2, D_3) along with a target (Y). We present the information that drone D_1 can 'see', namely its own exact position and velocity $([X_{1,1}, X_{1,2}]^T, \boldsymbol{V}_1)$, along with the directions to the target and the other drones $(\boldsymbol{y}_1, \boldsymbol{r}_{1,2}, \boldsymbol{r}_{1,3})$ and the velocity orientations of the other drones $(\boldsymbol{v}_2, \boldsymbol{v}_3)$.

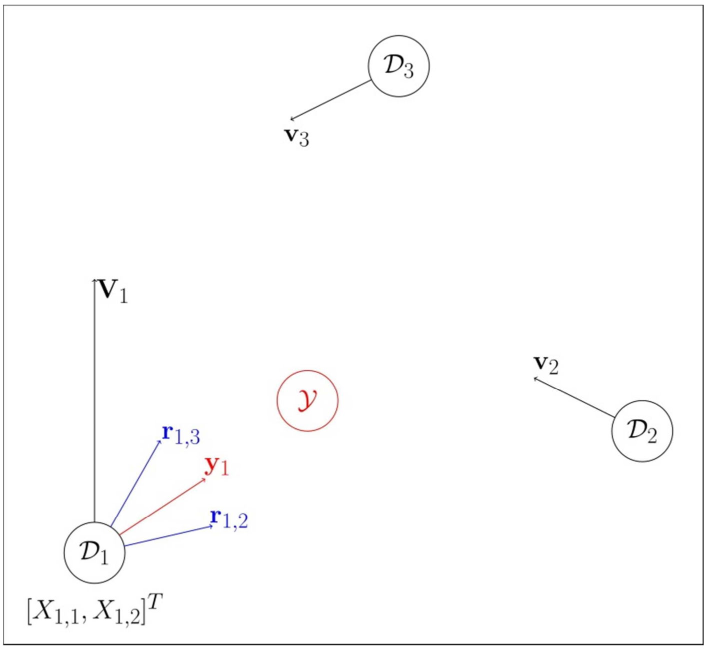

Figure 2: We wish for the drones (black) to move around the target (red) in some way, using available information in the system; velocities (denoted by V and v), positions (denoted by X), and relative directions (denoted by r and y).

Glossary of terms

- Stability: For a solution to be stable, the trajectory should not change too much under small disturbances. If a solution is stable, then a different solution that is very close to it will be close for all time.
- Asymptotic Stability: This requires that the perturbed solution is stable, and will eventually tend back to the original solution.
- Robustness: For our model to be robust, we require that it is not sensitive to internal disturbances, e.g., the failure of a single drone should not cause the collapse of the entire system.

Mathematical model

We assume that we can capture the behaviour of each drone by tracking the motion of its centre of mass, with the dynamics of each drone being governed by a set of two differential equations; one for its position, and one for its velocity. Our aim is to determine whether the behaviour of a swarm of drones can be robustly controlled without using distance information.

We make the following assumptions:

1. *Drones move in only two dimensions.* We simplify the true, three-dimensional situation by imagining that the system of drones exists on a plane which is perpendicular to a very distant target.

2. *The target is stationary.* This makes it easier to analyse the behaviour of the drones reacting to a simple target. We locate the target at the origin of our coordinate system.

3. *Each drone knows its exact position and velocity.* This is akin to each drone having a perfect inertial measurement device, which is unlikely in reality, but allows us to proceed without the influence of external noise or measurement error.

4. *Drones are unable to communicate.* Leonardo wish for the system to evolve without the need for communication (communication could be intercepted and would also make the drones more technologically complex).

Each drone can detect the *direction* of its neighbours, i.e., the relative angular bearing of each drone in the system. They can also measure the *orientation* of each neighbour, meaning that they can see the direction that each drone is travelling

5. *Drones are unable to compute relative distances.* Leonardo mainly works with systems that have bearing-only capabilities, meaning relative distances are not able to be measured accurately enough for use in distance-based models. This provides the key restriction on our control system.

6. *Drones can detect the orientation and direction of their neighbours.* This is a great simplification and is physically unrealistic. However, it allows for some information to be shared in the system without being explicitly communicated.

With these assumptions we develop an initial model, which we call 'Model I'. The motion of each drone is governed by $\boldsymbol{F} = m\boldsymbol{a}$, where $\boldsymbol{a}$ is the drone's acceleration, m is the drone's mass, and $\boldsymbol{F}$ is a combination of the external forces acting on the drone and the forces due to the control system. We assume that $\boldsymbol{F}$ depends on an attraction to the target (dependent on a strength factor α), repulsion from neighbouring drones (dependent on a strength factor β), and a damping term (dependent on a strength factor λ) to ensure that the drone velocities do not grow to enormous magnitudes. We scale the model to remove the dimensions, and in particular we choose a timescale such that the rescaled λ becomes equal to 1.

Initial conditions for each drone are drawn randomly, to represent the inability to deploy the system of drones in a controlled manner

We assume that the initial drone positions are randomly distributed around the target, using the normal distribution. This mimics the real situation, in which the deployment of drones would likely not be well-controlled (for example, they could be deployed from a moving aircraft). Similarly, the initial velocities of the drones are assumed to be random, taking values from a uniform distribution with a minimum of 1 and a maximum of 10.

We solve Model I numerically to determine the possible dynamics for systems of different sizes. We also analyse the model analytically to find attributes of the system for certain solutions.

3. Results

We run simulations for systems of different sizes, and find a variety of different behaviours. Our initial focus is on a system of four drones, since this is the smallest system size which exhibits interesting behaviour, as we show in Figure 3.

There are three main behaviours of the system of four drones; 'Consensus' (left), 'Opposite' (middle), and 'Triangular' (right). 'Consensus' has same-direction, circular trajectories, 'Opposite' has pairwise-opposite, circular trajectories, while 'Triangular' has three elliptic trajectories, with a counter-rotating ellipse.

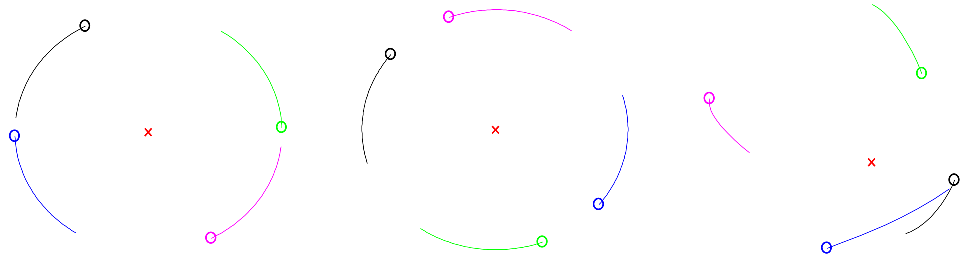

Figure 3: Numerical plots of the trajectories of four drones: 'Consensus' (left), 'Opposite' (middle), and 'Triangular' (right). The circles represent current positions, while the lines indicate the trajectories.

For larger systems, it is often found that the drones collide with each other, because they end up travelling in opposite directions along the same circular trajectories. The system is also able to settle into formations in which drones can be extremely close to one another, as we show in Figure 4. It is clear that the control system we incorporate into our model does not always produce desirable behaviour.

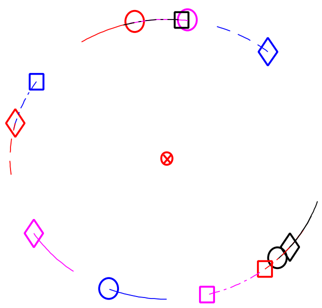

Figure 4: Drones may settle into a state very close to a neighbour, which is not ideal behaviour for a physical system.

Influence of initial conditions and parameters

Due to the randomness of the initial conditions, no prediction can be made about what behaviour a system will tend to given a certain choice of α and β, i.e., the final behaviour of the system is heavily dependent on the initial positions and velocities. However, if the system tends to either 'Consensus' or 'Opposite' behaviour, we find that the choice of α and β explicitly determine the final solution, such that all drones tend to the same speed (prescribed by β) at a shared orbit radius (prescribed by both α and β). However, for both of these behaviour types, the geometric distribution of drones around the circular orbit is non-unique, as we show in Figure 5. For 'Consensus' behaviour especially, it would instead be more useful to force the drones into positions such that they are symmetrically distributed around the origin.

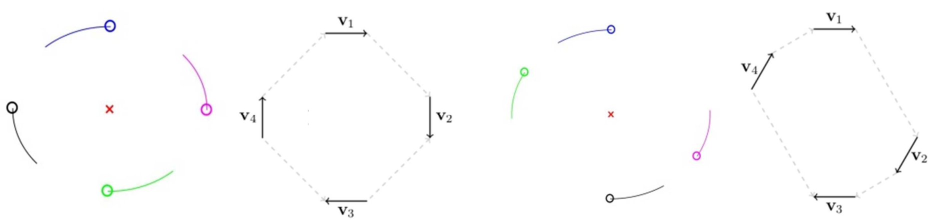

Figure 5: For Model I, the final state is set by the vector sum of velocities being zero and so the solution for four drones is non-unique.

Improvement of Model I

For the system to evolve into a more ordered formation, it is necessary to include additional controls in the model which enable the drones to space themselves equally around the circular solution.

We improve the simple model by allowing each drone to respond to how quickly a neighbour's angular bearing changes

The control that we implement uses the temporal rate-of-change of the bearing between drones; for each drone we sum up these rates-of-change over all neighbouring drones in the system. We then use this summation to act as a repulsion term such that if any drone measures a neighbour's bearing changing quickly, it will infer that the neighbour is close. Although this summation creates a repulsion from all neighbouring drones, the largest repulsion is from the (inferred) closest neighbour, such that it attempts to avoid a direct collision.

We find that adding this control to the system leads to swarms that would otherwise have found a non-unique 'Consensus' solution being forced into a symmetrically distributed orbit, as we show in Figure 6. We refer to this improved model as 'Model II'. We note that this additional control does not guarantee that the system tends to 'Consensus' behaviour for all parameter values and initial conditions.

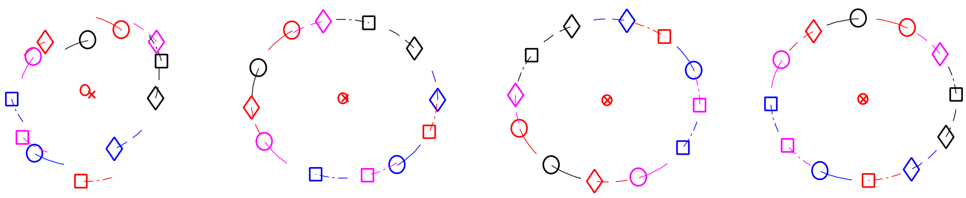

Figure 6: Using the additional control of Model II, the system is able to self-align into a perfectly symmetric distribution around the target. These four sets of trajectories are taken from a single simulation at different times, with time increasing from left to right.

Stability analysis

Balanced circular formation refers to the solution whereby drones travel along the same circular trajectory, in the same direction, and are symmetrically distributed around the target

Knowing that a system of drones can evolve into a balanced circular formation, we now analyse whether these formations are stable; indeed, for Leonardo, it is imperative for such a system to have stable behaviour. The symmetric formation is a solution for both Model I and II, and so we analyse the stability of both systems to determine why Model I does not immediately find the balanced solution.

To simplify the analysis, we initially look at a system of only two drones operating under Model I to understand the behaviour in a symmetric formation. Numerical simulations suggest that the symmetric two-drone solution is *asymptotically stable*, and the same result is found for systems of size three.

However, for a system of four or more drones, a balanced circular formation using Model I is verified (numerically) to be stable, and is proven (analytically) to not be asymptotically stable. This is a useful result for Leonardo, as it means that for most systems the symmetric solution will not be found using the simplest model, and so additional control terms must be utilised.

We also test the stability of Model II and we find that it is (numerically) asymptotically stable for systems of size four, and likely the same holds for larger systems too. We have only checked the stability numerically at this stage.

The simplest model is proven to not be asymptotically stable if the system has more than four drones, while the improved model appears to be asymptotically stable for systems of any size

Future research should focus not only on the mathematical analysis for the stability of these two models, but also look at making the dynamics and the measurement models more realistic

4. Discussion, Conclusions, & Recommendations

We have developed two models for controlling the motion of drones in a two-dimensional space, finding that emergent behaviour occurs in the form of circular motion, and that final positions can be made to be symmetric around the origin. We have numerically verified the stability properties of solutions to Model II, with our results suggesting that balanced circular formations can be found and maintained. It is not guaranteed that these formations will always be attained though, due to the reliance on initial conditions. However, if the drones are initialised with an agreed rotation, such that all initial velocities are (counter)clockwise with respect to the target, then it is highly likely that a balanced circular formation will be achieved.

We were able to prove the non-asymptotic stability of solutions to Model I, to show that it is not a viable way of controlling a physically realistic system of drones. We note that the stability analysis of Model II was only performed numerically. However, it will be useful in the future to *prove* that solutions to Model II are asymptotically stable, rather than relying on numerical analysis.

To extend beyond these two simple models, it would be ideal to relax one of the key simplifications and not allow drones to know their neighbour's orientations; this will make the model slightly more realistic. To also improve the realism of the models, it would be useful to include effects of measurement noise, to determine what impact they can have on the long-term dynamics of the system. We should also incorporate moving targets.

We recommend that these initial models be implemented in a ground-based robotic system, using imperfect measurement and control, to hopefully validate our findings with a physical proof-of-concept, before progressing into an airborne system.

5. Potential Impact

We have found useful results that suggest bearing-only formations can be achieved in a way which will guarantee collision avoidance in final system behaviour. Using only simple models, we have been able to create balanced circular formations at specified radii and final speeds, without needing to constrain the drone behaviour in the initial period of system evolution. This research should be a strong foundation upon which further work on bearing-only formation control can be built.

Neil Cade, Lead Systems Specialist at Leonardo, commented *"Remote surveillance systems will almost always require multiple sensors that are distributed in space. This is particularly true of passive sensors that invariably are not able to directly measure range. For long range surveillance such sensing has to be airborne, simply to avoid obscuration by closer objects. Moreover, to gain any overall sense of range it is necessary to have sensor separations to be not too much less than the ranges of interest. It is therefore impractical to use a single aircraft and the sensors must be distributed across multiple platforms. Thus, if we are going to perform such sensing autonomously then we are necessarily led to the need for coordination of multiple UAVs (drones) carrying our angle-only sensors.*

This short research project is an important step in defining the best approach to this problem. Here we have investigated a non-linear control approach to the problem where there is no attempt to explicitly estimate drone separations. These arise as an emergent behaviour from the dynamics itself. Indeed this work has established that such an approach does work and that asymptotically stable drone constellations can be obtained. This is an important step and it opens the way for further exploration under the constraints of more realistic sensing and implementable control."