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Effects of Periodic Disturbances in the Glass Redraw Process

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SCHOTT
glass made of ideas

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1. Introduction

Background

For some specialist glasses, thin sheets are best produced using the redraw process

Modern production methods have made glass into one of the most versatile materials around. Beyond the windows, glass can be found in medical devices, impact sensors, solar panels, and the optical fibres upon which the internet relies. Schott produce glass so thin that it can be wrapped almost entirely around a finger without breaking, leading to the recent experimental developments of bendable screen smartphones.

New touchscreen devices, camera modules, and fingerprint sensors are driving demand for thin glass sheets. The most common method of producing such thin sheets involves drawing molten glass as it cooled, which stretches the glass. However, some specialist glasses have properties that render this process unsuitable for their production. For these cases, the *redraw process* is used.

The Redraw Process

The redraw process involves feeding a thick sheet of glass into a heating zone and drawing out at a faster speed to produce a thin glass sheet

The redraw process is performed on an existing sheet of glass, known as a preform, which has thickness greater than required for the final product. We show a schematic of the redraw process in Figure 1. This sheet is fed into a heating zone, at some feed speed, where the glass softens. It is then drawn from the other side of the heating zone at a faster draw speed. The ratio between the feed and draw speeds is known as the *draw ratio*. It is the draw ratio that determines the thickness of the final sheet of glass.

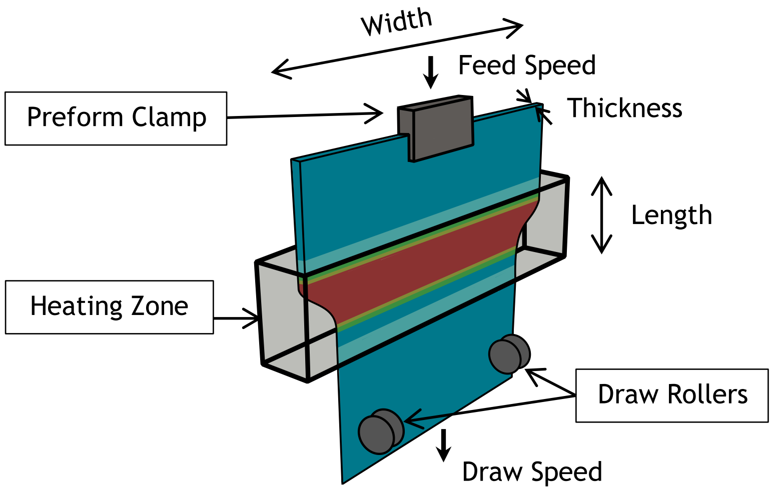

Figure 1: Schematic of the redraw process. A thick sheet of glass is fed into a heating zone and drawn out from the other side at a faster speed.

Thickness variations of just 1% in the final glass sheet can be unacceptable

In a typical redraw process, a glass sheet can be transformed from a thickness of 10mm to 100μm. However, small variations of 1μm in the thickness of the final product can be unacceptable for some applications. Such variations can be caused by an imperfect feed or draw speed, or temperature fluctuations in the surrounding air. To prevent these disturbances causing problems, it is important to understand what types of disturbances have the greatest effect on the final product.

There are three key disturbances: small periodic variations to the feed speed, draw speed, and surrounding air temperature. A small periodic disturbance is one that repeats in time at some frequency and is small relative to the variable that is being disturbed. Certain special frequencies are observed to induce a greater variation in the final glass sheet thickness than others. Our aim is to develop an efficient method to identify these special frequencies by employing a mathematical model of the redraw process.

Glossary of terms

- Draw Ratio: The ratio between the draw speed and feed speed.
- Viscosity: A measure of the resistance of a fluid to movement. For example, honey has a higher viscosity than water.
- Aspect Ratio: The ratio between the length of the heated glass and the thickness of the glass.
- Asymptotic Methods: A set of mathematical methods for describing the behaviour of a system in a particular limit (e.g. as a parameter tends to zero). These methods can be used to gain good approximate solutions when certain parameters are either large or small.
- Linear System: A system of equations in which each variable is linear.
- Nonlinear System: A system that violates the requirements to be a linear system.
- Linearisation: Conversion of a nonlinear system into a corresponding linear system by assuming only small changes from a base state. Solving this linear system will give a good approximation of the solution to the corresponding nonlinear system when the changes from the base state are small.
- Steady State: A solution that does not change in time. In our case, this also corresponds to assuming there are no disturbances to the feed speed, draw speed, and surrounding air temperature.

2. Mathematical Model for the Redraw Process

We treat the glass as a slow moving, viscous fluid with temperature-dependent viscosity

In our mathematical model, we treat the glass sheet as a slow-moving *viscous fluid,* where the viscosity of the glass depends on its temperature. When the temperature is low, the viscosity is high and the fluid is more resistant to movement. When the temperature is high, the viscosity is low allowing the fluid to move more freely; this captures the glass behaviour when softened.

The general equations for a slow-moving viscous fluid are called the Stokes equations. These can be solved numerically. However, to do so accurately is computationally expensive. Instead, we take advantage of the small *aspect ratio* of the glass sheet, that is, the fact that the glass sheet is much thinner than it is long. This allows us to apply *asymptotic methods* to remove terms from the equations which we know must be small given a small aspect ratio. The resulting set of equations is known as the *Trouton Model*, which describes conservation of mass along with a force balance.

We use asymptotic methods to simplify the equations, taking advantage of the geometry of the sheet of glass and the dominance of diffusive heat transport. This gives the Extended Trouton Model.

The Trouton Model can be solved to predict the thickness and speed of the glass at every point in the heating zone. However, since the viscosity is strongly dependent on the temperature, we also need to know the temperature of the glass at every point in the heating zone. We therefore extend the model to include an equation describing conservation of energy. For the industrially relevant case, most of the heat energy is transferred by diffusion. We use asymptotic methods to simplify the energy equation by removing terms that are small in this case. With the addition of an empirically obtained viscosity—temperature relation, we now have a full model, which we call the *Extended Trouton Model*, which determines the glass thickness, speed, and temperature at every position in the heating zone.

We linearise the equations about the steady state, which makes them easier to solve. This reduces computation time from days to minutes.

We solve the linearised model for different disturbance frequencies and determine the thickness variation in the final glass sheet thickness for each frequency

Solving the Model Efficiently

The Extended Trouton Model is *nonlinear* and therefore computationally expensive to solve. Since our main aim is to develop an efficient method to study *small* periodic disturbances, we first need to find the thickness, speed, and temperature of the glass in the heating zone in the case where there are no disturbances. This solution does not change in time and is therefore known as the *steady-state* solution; it is found quickly by standard numerical solvers. Given the steady-state solution, we *linearise* the system of equations using the fact our solution is close to the steady state; this simplifies the equations, making them easier to solve.

Comments

- We have_used asymptotic methods to derive the Extended Trouton Model which determines the thickness, speed, and temperature of the glass in the heating zone. output of the model is a prediction of the thickness of the final sheet of glass.
- We have linearised the system of equations about the steady state solution. Solving the linear system rather than the nonlinear system reduces computation time from days to minutes.
- We solve the linearised model for periodic disturbances with different frequencies to determine which frequencies cause the largest variations in the final glass sheet thickness.

3. Results

We consider the redraw process with a feed speed of 20mm/min and draw ratio of 100 performed on a preform sheet of thickness 10mm. This results in a final glass sheet with thickness 100µm. We then impose 5% temporal disturbances to the feed speed, and 1% temporal disturbances to the draw speed, and surrounding air temperature, and calculate the final thickness variation for each disturbance in turn. These disturbances are of the typical magnitude seen in an actual redraw process. We compare our results with those produced by Schott by solving the nonlinear Extended Trouton Model. We note that the results produced by Schott are not necessarily the true solution due to numerical difficulties in solving the nonlinear model. In fact, one motivating factor our investigation is to determine whether Schott's results can be trusted.

Feed Speed Disturbances

In Figure 3, we plot the thickness variation (the difference between the maximum and minimum thickness of the final glass sheet) for a 5% disturbance to the feed speed found by solving our linearised model and compare this with the results from Schott's nonlinear model. We observe that our results capture the same behaviour as that predicted by Schott although there are small differences. An important result for Schott is that our results also do not detect any thickness variations significantly larger than Schott's current results for disturbances at frequencies greater than 0.03Hz. Therefore, Schott can be more confident that disturbances at higher frequencies will not cause problems and instead focus on preventing feed speed disturbances at frequencies between 0.01Hz and 0.03Hz, which we can see produce the greatest thickness variations apart from low frequencies disturbances (below 0.01Hz) which can be dealt with by control systems and so are not an issue for Schott.

For feed speed disturbances we capture the same behaviour as the solution provided by Schott in particular for higher frequencies where Schott do not trust their solution

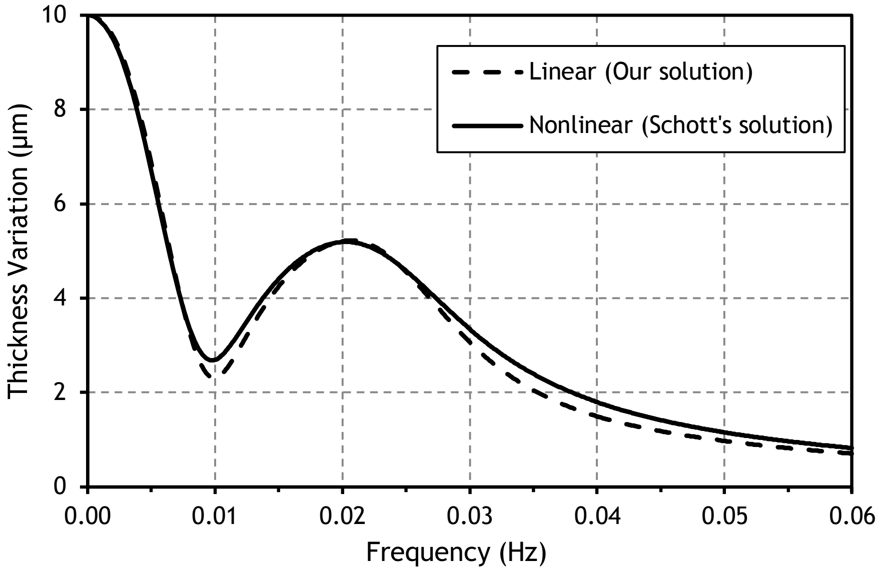

Figure 3: Thickness variation in the final sheet of glass measured in microns (µm) for a 5% disturbance to the feed speed where the draw ratio is 100. The thickness of the sheet in the absence of the disturbance is 100µm.

Draw Speed Disturbances

In Figure 4, we consider a 1% disturbance to the draw speed. Our results capture the same key behaviour as Schott's results. As we noted in the case of feed speed disturbances, the two results differ slightly. Again, disturbances at frequencies below 0.01Hz can be dealt with by control systems. However if thickness variations must be kept below 1µm, draw speed disturbances are a problem at all frequencies. If the tolerance is increased to 2µm then draw speed will not cause problems.

For draw speed disturbances we capture the same behaviour as the solution provided by Schott in particular for higher frequencies where Schott do not trust their solution

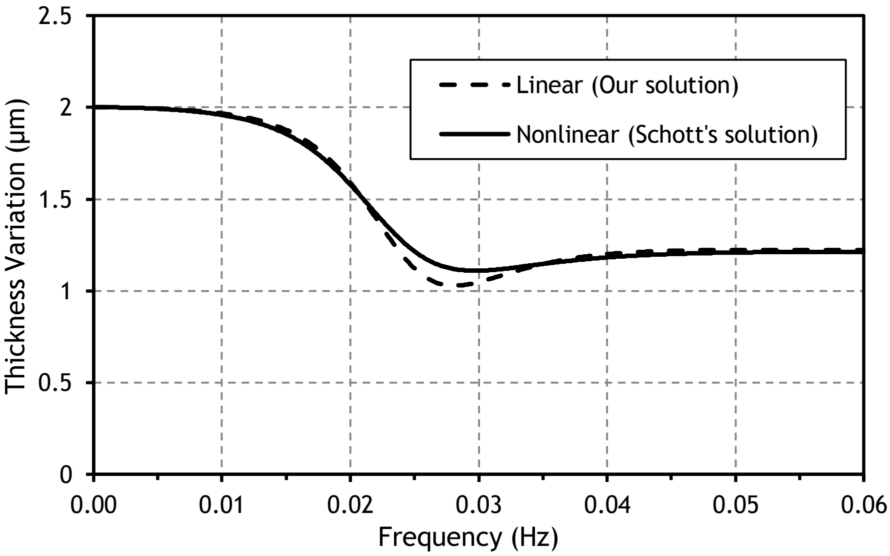

Figure 4: Thickness variation in the final sheet of glass measured in microns (µm) for a 1% disturbance to the draw speed where the draw ratio is 100. The thickness of the sheet in the absence of the disturbance is 100µm.

Surrounding Air Temperature Disturbances

We now consider 1% (of temperature in Kelvin) disturbances to the surrounding air temperature localised to a small region in the heating zone. We first vary the location of this small region and show in Figure 5 the resulting thickness variation for a disturbance at a frequency of 0.025Hz. The greatest thickness variation occurs when the small region within which the temperature disturbance occurs is co-located with the maximum temperature of the steady-state glass and therefore where the glass deforms from a thick sheet to a thin sheet. In the example in Figure 5, this is located at three-tenths of the distance along the full heating zone.

A disturbance to the temperature causes the greatest thickness variation when it occurs at the location where the glass is at its highest temperature

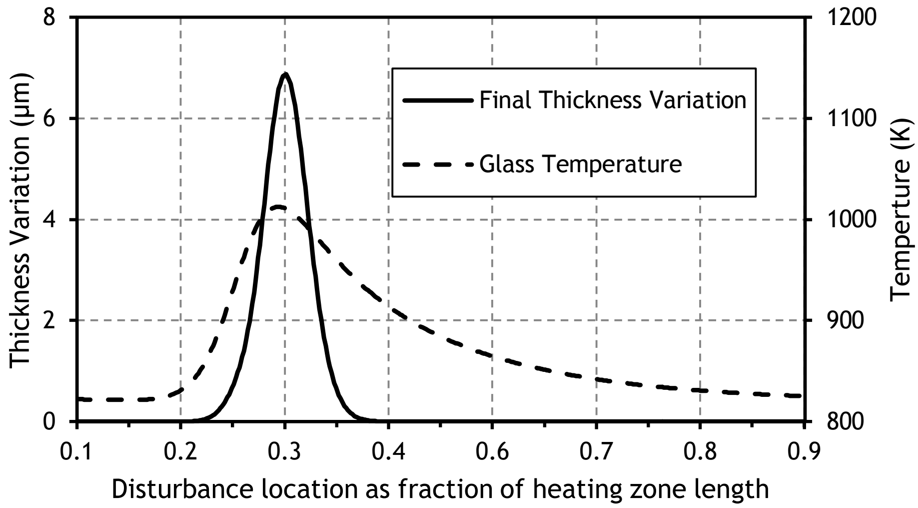

Figure 5: Thickness variation in the final glass sheet due to a temperature disturbance concentrated in a small region which is located at a range of locations in the heating zone. The draw ratio is 100 and the disturbance at a frequency of 0.025Hz.

In Figure 6, we vary the frequency of a disturbance which is co-located with the maximum temperature of the steady-state glass. When the temperature disturbance is at this location, the thickness variation is greater than 2μm for a large range of frequencies, and takes a maximum of just under 7μm for disturbances at a frequency of 0.024Hz. This suggests preventative measures should be taken to protect this region from convection in the surrounding air. In fact, Schott has already taken such measures on the basis of non-periodic disturbance results.

A disturbance to the temperature at the location where the glass is at its highest temperature produces large thickness variations for many frequencies. This region should therefore be protected

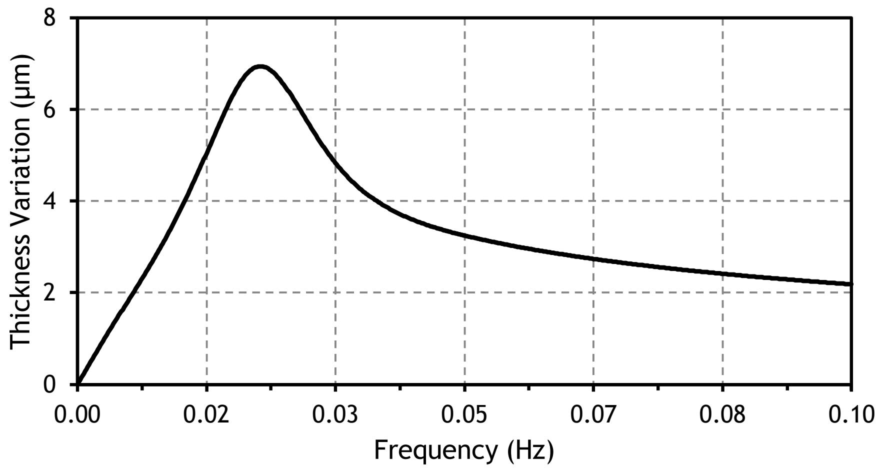

Figure 6: Thickness variation in the final sheet of glass due to a temperature disturbance in a small region centred at three tenths of the distance along the heating zone. The draw ratio is 100.

4. Discussion, Conclusions and Recommendations

We have investigated the effect of small periodic disturbances to the feed speed, draw speed, and surrounding air temperature on thickness variations in the final glass sheet in the glass redraw process. We have achieved this by applying an extended version of the Trouton Model which includes temperature-dependent viscosity. We then solved the resulting system of equations by linearising about the steady state. This approach allowed computation time to be reduced from days to minutes.

Linearising the Extended Trouton Model is an efficient method to obtain a useful approximation

Comparing our results with those provided by Schott, we found that the simplifications we made in solving the model did not affect our ability to replicate the key behaviour. Furthermore, we were able to use our results to give Schott confidence in their current set of results, particularly for high frequency disturbances.

We have provided Schott with a program which implements our approach to solving the Extended Trouton Model. We recommend that this tool be used to quickly analyse different situations and then be complemented by results from other approaches such as a direct computation of the Stokes equations when a full analysis is conducted.

A two-dimensional version of the Trouton model can be derived following a similar approach to that taken here. This allows for the effects on the edges of the glass sheet to be investigated. Furthermore, effects such as surface tension should be incorporated into the model to more closely match reality. The model and linearisation approach could be adapted to investigate instabilities in the drawing of glass tubes.

5. Potential Impact

The plots can be read by engineers to influence design decisions.

The program we provide will allow Schott to quickly analyse different glass types.

The program can be adapted and applied to other glass shaping processes (e.g. drawing glass tubes).

The plots shown in Figures 3-6 are an important output of our investigation. These can be read by an engineer and used to influence design decisions to avoid creating disturbances with frequencies that generate large variations in the final thickness of the glass sheet. For the temperature case, it can be seen in Figure 5 that a temperature disturbance at the location where the bulk of the glass deformation occurs, will result in the largest thickness variations in the final glass sheet. This has been observed by Schott in other models for non-periodic disturbances and preventative measures have already been taken and have proven successful.

Another important output is a computer program that implements our approach to solving the Extended Trouton Model for small periodic disturbances about the steady state. This program will produce a similar set of results to those included here, acting as a tool which can be used to quickly analyse new types of glass which are candidates for the redraw process. Furthermore, the source code can be adapted to other processes such as the drawing of glass tubes.

Dr Ulrich Lange, Senior Scientist at Schott A.G, commented: *"Understanding the impact of time-periodic disturbances of a production process, such as vibrations of mechanical components or ambient air convection patterns, is crucial in order to achieve high quality products, such as drawn glass sheets with very uniform thickness. The assessment of these disturbances by experimental methods (or numerical case studies) can be a tedious task as there may be an abundance of potential sources with different frequencies, locations, etc. The results of the mini-project on the redraw process help establish linearised methods as a much more efficient alternative, thus enabling a more comprehensive analysis of potential process disturbances. We expect that these methods will also prove to be very valuable in the improvement of other drawing processes at Schott in the future."*