

Handbook for the Undergraduate Mathematics Courses
 Supplement to the Handbook
 Honour School of Mathematics & Philosophy
 Syllabus and Synopses for Part B 2012–2013
 for examination in 2013

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1 Foreword

This Supplement to the Mathematics Course Handbook specifies the Mathematics courses available for Part B in Mathematics & Philosophy in the 2013 examination. It should be read in conjunction with the Handbook for Mathematics & Philosophy for the academic year 2012–2013, to be issued in Michaelmas Term. The Handbook contains in particular information on the format and rubrics for written examination papers in Mathematics, and the classification rules applicable to Part B.

See the current edition of the *Examination Regulations* for the full regulations governing the examinations.

1.1 Part B of the Honour School of Mathematics & Philosophy

The following is reproduced from the *Examination Regulations* applicable to the 2013 examinations.

The examination for Part B shall consist of units in Mathematics and subjects in Philosophy. The schedule of units in *Mathematics* shall be published in a supplement to the Mathematics Course Handbook by the beginning of the Michaelmas Full Term in the academic year of the examination concerned. The schedule shall be in two parts: Schedule 1 (standard units and half-units) and Schedule 2 (additional units and half-units). In *Philosophy* the subjects shall be subjects 101–118, 120, 122, 124 and 199 from the list given in *Special Regulations for All Honour Schools Including Philosophy*. Each subject in Philosophy other than a Thesis shall be examined in one 3-hour paper. Each candidate shall offer

- (i) Two units of *Mathematics* from Schedule 1, two half units of which shall be B1a *Logic* and B1b *Set Theory*,
- (ii) three subjects in *Philosophy* from 101-118, 120, 122 and 124, of which two must be 122 and **either** 101 **or** 102, and
- (iii) **either** one further unit in *Mathematics* drawn from Schedules 1 and 2 combined **or** one further subject in *Philosophy* from subjects 101-118, 120, 124, and 199: *Thesis*.

Note that the Regulations do not allow candidates who offer from Schedule 1 only the compulsory papers B1a and B1b to offer options from Schedule 2. This means that the units and half-units listed under Schedule 2 are not available to those who wish to offer a total of four Philosophy subjects.

Further information on the units in Mathematics is given below.

1.2 “Units” and “half-units” and methods of examination

Most courses in Mathematics are assessed by examination. Most subjects offered have a ‘weight’ of one unit, and will be examined in a 3-hour examination paper. In many of these subjects it will also be possible to take the first half, or either half, of the subject as a ‘half-unit’. Where this is the case, a half-unit will be examined in an examination paper of $1\frac{1}{2}$ hours duration. Each half-unit paper will contain **3** questions.

1.3 The Schedules of Mathematics units and half-units for Mathematics & Philosophy

All units and half-units in Mathematics are drawn from the list of options for Mathematics Part B.

Schedule 1 comprises those Mathematics Department courses for which the core and options in Mathematics & Philosophy Part A provide the requisite background.

Schedule 2 contains an Extended Essay option and certain further courses from Mathematics Part B appropriate for the Joint School.

In addition you may apply for special approval to be examined in Mathematics Department units and half-units not included under Schedule 1; any such subject approved will be treated as falling under Schedule 2. For the procedure for seeking approval, see Subsection 1.4 below.

For the 2013 examination, the Schedules are as follows. (N.B. All topics listed are half-units unless otherwise stated).

Schedule 1

B1a Logic (Compulsory)

B1b Set Theory (Compulsory)

B2a Introduction to Representation Theory

B2b Group Theory

B3a Geometry of Surfaces

B3b Algebraic Curves

B3.1a Topology and Groups

B4a Banach Spaces

B4b Hilbert spaces (can only be taken in conjunction with B4a)

B9a Galois Theory

B9b Algebraic Number Theory (can only be taken in conjunction with B9a)

B10a Martingales Through Measure Theory

B11a Communication Theory

B11b Graph Theory

Schedule 2 (additional units and half-units)

BE Mathematical Extended Essay	whole unit
O1 History of Mathematics	whole unit
OCS3b Lambda Calculus and Types	
OCS4b Computational Complexity	
OCS5b Knowledge Representation and Reasoning	
OCS6a Computer-aided Formal Verification	
N1 Mathematics Education and Undergraduate Ambassadors' Scheme	whole unit
N1a Mathematics Education	

And also

Any other whole unit or half-unit course from the list of Mathematics Department units in Part B for which special approval has been granted.

1.4 Procedure for seeking approval of additional options where this is required

You may, if you have the support of your Mathematics tutor, apply to the Chairman of the Joint Committee for Mathematics and Philosophy for approval of one or more other options from the list of Mathematics Department units and half-units for Part B. This list can be found in the Supplement to the Mathematics Course Handbook giving syllabuses and synopses for courses in Mathematics Part B and at the end of this Supplement.

Applications for special approval must be made through the candidate's college and sent to the Chairman of the Joint Committee for Mathematics and Philosophy, c/o Academic Administrator, Mathematical Institute, to arrive by **Friday of Week 5 of Michaelmas Term**. Be sure to consult your college tutors if you are considering asking for approval to offer one of these additional options.

Given that each of these additional options, which are all in applied mathematics, presume facility with some or other results and techniques covered in first, second or third year Mathematics courses not taken by Mathematics & Philosophy candidates, such applications will be exceptional. You should also be aware that there may be a clash of lectures for specially approved options and those listed in Schedules 1 and 2 and with lectures in Philosophy; see the section in The Mathematics Part B Synopses on lecture clashes.

1.5 Registration for Part B courses 2012–2013

CLASSES Students will have to register in advance for the classes they wish to take. Students will have to register by Friday of Week 10 of Trinity Term 2012 using the online registration system which can be accessed at <https://www.maths.ox.ac.uk/courses/registration/>.

Students who register for a course or courses for which there is a quota should consider registering for an additional course (by way of a “reserve choice”) in case they do not receive a place on the course with the quota. They may also have to give the reasons why they wish to take a course which has a quota, and provide the name of a tutor who can provide a supporting statement for them should the quota be exceeded. Where this is necessary students will be contacted by email after they have registered. In the event that the quota for a course is exceeded, the Mathematics Teaching Committee will decide who may have a place on the course on the basis of the supporting statements from the student and tutor, and all relevant students will be notified of the decision by email. In the case of the “Undergraduate Ambassadors’ Scheme” students will have to attend a short interview in Week 0, Michaelmas Term.

2 Schedule

2.1 B1a: Logic — Dr Pila — 16 MT

Level: H-level **Method of Assessment:** Written examination.

Weight: Half-unit (OSS paper code 2A40)

Recommended Prerequisites: none.

Overview

To give a rigorous mathematical treatment of the fundamental ideas and results of logic that is suitable for the non-specialist mathematicians and will provide a sound basis for more advanced study. Cohesion is achieved by focussing on the Completeness Theorems and the relationship between provability and truth. Consideration of some implications of the Compactness Theorem gives a flavour of the further development of model theory. To give a concrete deductive system for predicate calculus and prove the Completeness Theorem, including easy applications in basic model theory.

Learning Outcomes

Students will be able to use the formal language of propositional and predicate calculus and be familiar with their deductive systems and related theorems. For example, they will know and be able to use the soundness, completeness and compactness theorems for deductive systems for predicate calculus.

Synopsis

The notation, meaning and use of propositional and predicate calculus. The formal language of propositional calculus: truth functions; conjunctive and disjunctive normal form; tautologies and logical consequence. The formal language of predicate calculus: satisfaction, truth, validity, logical consequence.

Deductive system for propositional calculus: proofs and theorems, proofs from hypotheses, the Deduction Theorem; Soundness Theorem. Maximal consistent sets of formulae; completeness; constructive proof of completeness.

Statement of Soundness and Completeness Theorems for a deductive system for predicate calculus; derivation of the Compactness Theorem; simple applications of the Compactness Theorem.

A deductive system for predicate calculus; proofs and theorems; prenex form. Proof of Completeness Theorem. Existence of countable models, the downward Löwenheim–Skolem Theorem.

Reading

1. R. Cori and D. Lascar, *Mathematical Logic: A Course with Exercises (Part I)* (Oxford University Press, 2001), sections 1, 3, 4.
2. A. G. Hamilton, *Logic for Mathematicians* (2nd edition, Cambridge University Press, 1988), pp.1–69, pp.73–76 (for statement of Completeness (Adequacy)Theorem), pp.99–103 (for the Compactness Theorem).
3. W. B. Enderton, *A Mathematical Introduction to Logic* (Academic Press, 1972), pp.101–144.
4. D. Goldrei, *Propositional and Predicate Calculus: A model of argument* (Springer, 2005).

Further Reading

1. R. Cori and D. Lascar, *Mathematical Logic: A Course with Exercises (Part II)* (Oxford University Press, 2001), section 8.

2.2 B1b: Set Theory — Dr Pila — 16 HT

Level: H-level

Method of Assessment: Written examination.

Weight: Half-unit (OSS paper code 2B40)

Recommended Prerequisites: none.

Overview

To introduce sets and their properties as a unified way of treating mathematical structures, including encoding of basic mathematical objects using set theoretic language. To emphasize the difference between intuitive collections and formal sets. To introduce and discuss the notion of the infinite, the ordinals and cardinality. The Axiom of Choice and its equivalents are presented as a tool.

Learning Outcomes

Students will have a sound knowledge of set theoretic language and be able to use it to codify mathematical objects. They will have an appreciation of the notion of infinity and arithmetic of the cardinals and ordinals. They will have developed a deep understanding of the Axiom of Choice, Zorn's Lemma and well-ordering principle, and have begun to appreciate the implications.

Synopsis

What is a set? Introduction to the basic axioms of set theory. Ordered pairs, cartesian products, relations and functions. Axiom of Infinity and the construction of the natural numbers; induction and the Recursion Theorem.

Cardinality; the notions of finite and countable and uncountable sets; Cantor's Theorem on power sets. The Tarski Fixed Point Theorem. The Schröder–Bernstein Theorem.

Isomorphism of ordered sets; well-orders. Transfinite induction; transfinite recursion [informal treatment only].

Comparability of well-orders.

The Axiom of Choice, Zorn's Lemma, the Well-ordering Principle; comparability of cardinals. Equivalence of WO, CC, AC and ZL. Ordinals. Arithmetic of cardinals and ordinals; in [ZFC],

Reading

1. D. Goldrei, *Classic Set Theory* (Chapman and Hall, 1996).
2. W. B. Enderton, *Elements of Set Theory* (Academic Press, 1978).
3. R. Cori and D. Lascar, *Mathematical Logic: A Course with Exercises (Part II)*

Further Reading

1. R. Cori and D. Lascar, *Mathematical Logic: A Course with Exercises (Part II)* (Oxford University Press, 2001), section 7.1–7.5.
2. R. Rucker, *Infinity and the Mind: The Science and Philosophy of the Infinite* (Birkhäuser, 1982). An accessible introduction to set theory.
3. J. W. Dauben, *Georg Cantor: His Mathematics and Philosophy of the Infinite* (Princeton University Press, 1990). For some background, you may find JW Dauben's biography of Cantor interesting.
4. M. D. Potter, *Set Theory and its Philosophy: A Critical Introduction* (Oxford University Press, 2004). An interestingly different way of establishing Set Theory, together with some discussion of the history and philosophy of the subject.
5. G. Frege, *The Foundations of Arithmetic : A Logical-Mathematical Investigation into the Concept of Number* (Pearson Longman, 2007).

6. M. Schirn, *The Philosophy of Mathematics Today* (Clarendon, 1998). A recentish survey of the area at research level.
7. W. Sierpinski, *Cardinal and Ordinal Numbers* (Polish Scientific Publishers, 1965). More about the arithmetic of transfinite numbers.

2.3 B2a: Introduction to Representation Theory — Prof Henke — 16 MT

Level: H-level

Method of Assessment: Written examination.

Weight: Half-unit (OSS paper code 2A41)

Recommended Prerequisites: All second year algebra.

Overview

This course gives an introduction to the representation theory of finite groups and finite dimensional algebras. Representation theory is a fundamental tool for studying symmetry by means of linear algebra: it is studied in a way in which a given group or algebra may act on vector spaces, giving rise to the notion of a representation.

We start in a more general setting, studying modules over rings, in particular over euclidean domains, and their applications. We eventually restrict ourselves to modules over algebras (rings that carry a vector space structure). A large part of the course will deal with the structure theory of semisimple algebras and their modules (representations). We will prove the Jordan-Hölder Theorem for modules. Moreover, we will prove that any finite-dimensional semisimple algebra is isomorphic to a product of matrix rings (Wedderburn's Theorem over \mathbb{C}).

In the later part of the course we apply the developed material to group algebras, and classify when group algebras are semisimple (Maschke's Theorem).

Learning Outcomes

Students will have a sound knowledge of the theory of non-commutative rings, ideals, associative algebras, modules over euclidean domains and applications. They will know in particular simple modules and semisimple algebras and they will be familiar with examples. They will appreciate important results in the course such as the Jordan-Hölder Theorem, Schur's Lemma, and the Wedderburn Theorem. They will be familiar with the classification of semisimple algebras over \mathbb{C} and be able to apply this.

Synopsis

Noncommutative rings, one- and two-sided ideals. Associative algebras (over fields). Main examples: matrix algebras, polynomial rings and quotients of polynomial rings. Group

algebras, representations of groups.

Modules over euclidean domains and applications such as finitely generated abelian groups, rational canonical forms. Modules and their relationship with representations. Simple and semisimple modules, composition series of a module, Jordan-Hölder Theorem. Semisimple algebras. Schur's Lemma, the Wedderburn Theorem, Maschke's Theorem.

Reading

1. K. Erdmann, *B2 Algebras*, Mathematical Institute Notes (2007).
2. G. D. James and M. Liebeck, *Representations and Characters of Finite Groups* (2nd edition, Cambridge University Press, 2001).

Further Reading

1. J. L. Alperin and R. B. Bell, *Groups and Representations*, Graduate Texts in Mathematics 162 (Springer-Verlag, 1995).
2. P. M. Cohn, *Classic Algebra* (Wiley & Sons, 2000). (Several books by this author available.)
3. C. W. Curtis, and I. Reiner, *Representation Theory of Finite Groups and Associative Algebras* (Wiley & Sons, 1962).
4. L. Dornhoff, *Group Representation Theory* (Marcel Dekker Inc., New York, 1972).
5. I. M. Isaacs, *Character Theory of Finite Groups* (AMS Chelsea Publishing, American Mathematical Society, Providence, Rhode Island, 2006).
6. J.-P. Serre, *Linear Representations of Finite Groups*, Graduate Texts in Mathematics 42 (Springer-Verlag, 1977).

2.4 B2b: Group Theory and an Introduction to Character Theory — Dr Erdmann — 16 HT

Level: H-level

Method of Assessment: Written examination.

Weight: Half-unit (OSS paper code 2B41)

Recommended Prerequisites: Part A Group Theory is essential. Part B Introduction to Representation Theory is useful as “further algebraic thinking”, and some of the results proved in that course will be stated (but not proved). In particular, character theory will be developed using Wedderburn's theorem (with Maschke's theorem assumed in the background). Students should also be well acquainted with linear algebra, especially inner products and conditions for the diagonalisability of matrices.

Overview

A finite group represents one of the simplest algebraic objects, having just one operation on a finite set, and historically groups arose from the study of permutations or, more generally, sets of bijective functions on a set closed under composition. Thus there is the scope for both a rich theory and a wide source of examples.// Some of this has been seen in the Part A course Group Theory, and this course will build on that. In particular, the Jordan–Hölder theorem (covered there but not examined) shows that there are essentially two problems, to find the finite simple groups, and to learn how to put them together. The first of these dominated the second half of the 20th century and has been completed; this proved a massive task, encompassing in excess of 20,000 printed pages, and much remains to be done to distil the underlying ideas.// In this course, our aim will be to introduce some of the very fundamental ideas that made this work possible. Much is classical, but it will be presented in a modern form.

Learning Outcomes

By the end of this course, a student should feel comfortable with a number of techniques for studying finite groups, appreciate certain classes of finite simple groups that represent prototypes for almost all finite simple groups, and have seen the proofs of some of the "great" theorems.

Synopsis

Review of isomorphism theorems (up to Jordan–Hölder), composition series, soluble groups; abelian groups; some examples of groups of (relatively) small order. Constructions of groups; semidirect products, notion of presentations. Cauchy’s theorem, Sylow’s theorems, the Frattini argument. Characters of complex representations. Orthogonality relations, construction of character tables and applications. The character ring as a subring of the algebra of complex-valued class functions. Burnside’s $p^\alpha q^\beta$ -theorem. Some simple groups.

Reading

The bulk of the course is well covered by the web notes written in 2011 by Jan Grabowski, which will be reposted with minor corrections. (A few topics from those notes will not be covered as will be clear from this year’s synopsis.)

Additional Reading

1. Geoff Smith and Olga Tabachnikova, *Topics in Group Theory*, Springer Undergraduate Mathematics Series (Springer-Verlag, 2000). ISBN 1-85233-235-2
2. H. Kurzweil and B. Stellmacher, [The Theory of Finite Groups. An Introduction], Springer Universitext (Springer Verlag, 2004). ISBN 0-387-40510-0. (Up to about page 100).

3. G.D. James and M. Liebeck, *Representations and Characters of Groups* (Second edition, Cambridge University Press, 2001). ISBN 0-521-00392-X

The following more sophisticated books are useful for reference and, in approach, may better represent the spirit of this course:

3. J I Alperin and Rowen B Bell, *Groups and Representations*, Graduate Texts in Mathematics 162 (Springer-Verlag, 1995). ISBN 0-387-94526-1
4. M J Collins, *Representations and Characters of Finite Groups*, Cambridge Studies in Advanced Mathematics 22, Cambridge University Press, 1990; reprinted p/b 2008, ISBN 978-0-521-06764-5, esp pp 48-63.

All these, and many other books on finite group theory and introductory character theory, can be found in most college libraries. You might also try the internet.

2.5 B3a: Geometry of Surfaces — Prof Bridson — 16 MT

Level: H-level

Method of Assessment: Written examination.

Weight: Half-unit (OSS paper code 2A42)

Recommended Prerequisites: 2nd year core algebra and analysis, 2nd year topology. Multivariable calculus and group theory would be useful but not essential. Also, B3a is helpful, but not essential, for B3b.

Overview

Different ways of thinking about surfaces (also called two-dimensional manifolds) are introduced in this course: first topological surfaces and then surfaces with extra structures which allow us to make sense of differentiable functions ('smooth surfaces'), holomorphic functions ('Riemann surfaces') and the measurement of lengths and areas ('Riemannian 2-manifolds').

These geometric structures interact in a fundamental way with the topology of the surfaces. A striking example of this is given by the Euler number, which is a manifestly topological quantity, but can be related to the total curvature, which at first glance depends on the geometry of the surface.

The course ends with an introduction to hyperbolic surfaces modelled on the hyperbolic plane, which gives us an example of a non-Euclidean geometry (that is, a geometry which meets all Euclid's axioms except the axioms of parallels).

Learning Outcomes

Students will be able to implement the classification of surfaces for simple constructions of topological surfaces such as planar models and connected sums; be able to relate the

Euler characteristic to branching data for simple maps of Riemann surfaces; be able to describe the definition and use of Gaussian curvature; know the geodesics and isometries of the hyperbolic plane and their use in geometrical constructions.

Synopsis

The concept of a topological surface (or 2-manifold); examples, including polygons with pairs of sides identified. Orientation and the Euler characteristic. Classification theorem for compact surfaces (the proof will not be examined).

Riemann surfaces; examples, including the Riemann sphere, the quotient of the complex numbers by a lattice, and double coverings of the Riemann sphere. Holomorphic maps of Riemann surfaces and the Riemann–Hurwitz formula. Elliptic functions.

Smooth surfaces in Euclidean three-space and their first fundamental forms. The concept of a Riemannian 2-manifold; isometries; Gaussian curvature.

Geodesics. The Gauss–Bonnet Theorem (statement of local version and deduction of global version). Critical points of real-valued functions on compact surfaces.

The hyperbolic plane, its isometries and geodesics. Compact hyperbolic surfaces as Riemann surfaces and as surfaces of constant negative curvature.

Reading

1. A. Pressley, *Elementary Differential Geometry*, Springer Undergraduate Mathematics Series (Springer-Verlag, 2001). (Chapters 4–8 and 10–11.)
2. G. B. Segal, *Geometry of Surfaces*, Mathematical Institute Notes (1989).
3. R. Earl, *The Local Theory of Curves and Surfaces*, Mathematical Institute Notes (1999).
4. J. McCleary, *Geometry from a Differentiable Viewpoint* (Cambridge, 1997).

Further Reading

1. P. A. Firby and C. E. Gardiner, *Surface Topology* (Ellis Horwood, 1991) (Chapters 1–4 and 7).
2. F. Kirwan, *Complex Algebraic Curves*, Student Texts 23 (London Mathematical Society, Cambridge, 1992) (Chapter 5.2 only).
3. B. O’Neill, *Elementary Differential Geometry* (Academic Press, 1997).

2.6 B3b: Algebraic Curves — Dr Szendroi — 16 HT

Level: H-level

Method of Assessment: Written examination.

Weight: Half-unit (OSS paper code 2B42)

Recommended Prerequisites: 2nd year core algebra and analysis, 2nd year topology. Multivariable calculus and group theory would be useful but not essential. Also, B3a is helpful, but not essential, for B3b.

Overview

A real algebraic curve is a subset of the plane defined by a polynomial equation $p(x, y) = 0$. The intersection properties of a pair of curves are much better behaved if we extend this picture in two ways: the first is to use polynomials with complex coefficients, the second to extend the curve into the projective plane. In this course projective algebraic curves are studied, using ideas from algebra, from the geometry of surfaces and from complex analysis.

Learning Outcomes

Students will know the concepts of projective space and curves in the projective plane. They will appreciate the notion of nonsingularity and know some basic features of intersection theory. They will view nonsingular algebraic curves as examples of Riemann surfaces, and be familiar with divisors, meromorphic functions and differentials.

Synopsis

Projective spaces, homogeneous coordinates, projective transformations.

Algebraic curves in the complex projective plane. Euler's relation. Irreducibility, singular and nonsingular points, tangent lines.

Bezout's Theorem (the proof will not be examined). Points of inflection, and normal form of a nonsingular cubic.

Nonsingular algebraic curves as Riemann surfaces. Meromorphic functions, divisors, linear equivalence. Differentials and canonical divisors. The group law on a nonsingular cubic.

The Riemann–Roch Theorem (the proof will not be examined). The geometric genus. Applications.

Reading

1. F. Kirwan, *Complex Algebraic Curves*, Student Texts 23 (London Mathematical Society, Cambridge, 1992), Chapters 2–6.

2.7 B3.1a: Topology and Groups —Dr Papazoglou — 16 MT

Level: H-level.

Method of Assessment: Written examination.

Weight: Half-unit, OSS paper code 2A63.

Prerequisites

2nd year Groups in Action, 2nd year Topology.

Overview

This course introduces the important link between topology and group theory. On the one hand, associated to each space, there is a group, known as its fundamental group. This can be used to solve topological problems using algebraic methods. On the other hand, many results about groups are best proved and understood using topology. For example, presentations of groups, where the group is defined using generators and relations, have a topological interpretation. The endpoint of the course is the Nielsen–Schreier Theorem, an important, purely algebraic result, which is proved using topological techniques.

Synopsis

Homotopic mappings, homotopy equivalence. Simplicial complexes. Simplicial approximation theorem.

The fundamental group of a space. The fundamental group of a circle. Application: the fundamental theorem of algebra. The fundamental groups of spheres.

Free groups. Existence and uniqueness of reduced representatives of group elements. The fundamental group of a graph.

Groups defined by generators and relations (with examples). Tietze transformations.

The free product of two groups. Amalgamated free products.

The Seifert–van Kampen Theorem.

Cell complexes. The fundamental group of a cell complex (with examples). The realization of any finitely presented group as the fundamental group of a finite cell complex.

Covering spaces. Liftings of paths and homotopies. A covering map induces an injection between fundamental groups. The use of covering spaces to determine fundamental groups: the circle again, and real projective n -space. The correspondence between covering spaces and subgroups of the fundamental group. Regular covering spaces and normal subgroups.

Cayley graphs of a group. The relationship between the universal cover of a cell complex, and the Cayley graph of its fundamental group. The Cayley 2-complex of a group.

The Nielsen–Schreier Theorem (every subgroup of a finitely generated free group is free) proved using covering spaces.

Reading

1. John Stillwell, *Classical Topology and Combinatorial Group Theory* (Springer-Verlag, 1993).

Additional Reading

1. D. Cohen, *Combinatorial Group Theory: A Topological Approach*, Student Texts 14 (London Mathematical Society, 1989), Chapters 1–7.
2. A. Hatcher, *Algebraic Topology* (CUP, 2001), Chapter 1.
3. M. Hall, Jr, *The Theory of Groups* (Macmillan, 1959), Chapters 1–7, 12, 17 .
4. D. L. Johnson, *Presentations of Groups*, Student Texts 15 (Second Edition, London Mathematical Society, Cambridge University Press, 1997). Chapters 1–5, 10,13.
5. W. Magnus, A. Karrass, and D. Solitar, *Combinatorial Group Theory* (Dover Publications, 1976). Chapters 1–4.

2.8 B4a: Banach Spaces — Dr Belyaev — 16 MT

Level: H-level

Method of Assessment: Written examination.

Weight: Half-unit (OSS paper code 2A43)

Recommended Prerequisites: Part A Topology and Integration. [From Topology, only the material on metric spaces, including closures, will be used. From Integration, the only concepts which will be used are the convergence theorems and the theorems of Fubini and Tonelli, and the notions of measurable functions and null sets. No knowledge is needed of outer measure, or of any particular construction of the integral, or of any proofs.]

Learning Outcomes

Students will have a firm knowledge of real and complex normed vector spaces, with their geometric and topological properties. They will be familiar with the notions of completeness, separability and density, will know the properties of a Banach space and important examples, and will be able to prove results relating to the Hahn–Banach Theorem. They will have developed an understanding of the theory of bounded linear operators on a Banach space.

Synopses

Real and complex normed vector spaces, their geometry and topology. Completeness. Banach spaces, examples (ℓ^p , ℓ^∞ , L^p , $C(K)$, spaces of differentiable functions).

Finite-dimensional normed spaces; equivalence of norms and completeness. Separable spaces; separability of subspaces.

Continuous linear functionals. Dual spaces. Hahn–Banach Theorem (proof for real separable spaces only) and applications, including density of subspaces and separation of convex sets. Stone-Weierstrauss Theorem.

Bounded linear operators, examples (including integral operators). Adjoint operators. Spectrum and resolvent. Spectral mapping theorem for polynomials.

Essential Reading

1. B.P. Rynne and M.A. Youngson, *Linear Functional Analysis* (Springer SUMS, 2nd edition, 2008), Chapters 2, 4, 5.
2. E. Kreyszig, *Introductory Functional Analysis with Applications* (Wiley, revised edition, 1989), Chapters 2, 4.2–4.3, 4.5, 7.1–7.4.

2.9 B4b: Hilbert Spaces — Prof Haydon — 16 HT

Level: H-level

Method of Assessment: Written examination.

Weight: Half-unit (cannot be taken unless B4a is taken)

Prerequisites: B4a Banach Spaces

Recommended Prerequisites: Part A Topology and Integration. [From Topology, only the material on metric spaces, including closures, will be used. From Integration, the only concepts which will be used are the convergence theorems and the theorems of Fubini and Tonelli, and the notions of measurable functions and null sets. No knowledge is needed of outer measure, or of any particular construction of the integral, or of any proofs.]

Learning Outcomes

Students will appreciate the role of completeness through the Baire category theorem and its consequences for operators on Banach spaces. They will have a demonstrable knowledge of the properties of a Hilbert space, including orthogonal complements, orthonormal sets, complete orthonormal sets together with related identities and inequalities. They will be familiar with the theory of linear operators on a Hilbert space, including adjoint operators, self-adjoint and unitary operators with their spectra. They will know the L^2 -theory of Fourier series and be aware of the classical theory of Fourier series and other orthogonal expansions.

Synopses

Baire Category Theorem and its consequences for operators on Banach spaces (Uniform Boundedness, Open Mapping, Inverse Mapping and Closed Graph Theorems). Strong convergence of sequences of operators.

Hilbert spaces; examples including L^2 -spaces. Orthogonality, orthogonal complement, closed subspaces, projection theorem. Riesz Representation Theorem.

Linear operators on Hilbert space, adjoint operators. Self-adjoint operators, orthogonal projections, unitary operators, and their spectra.

Orthonormal sets, Pythagoras, Bessels inequality. Complete orthonormal sets, Parseval.

L^2 -theory of Fourier series, including completeness of the trigonometric system. Discussion of classical theory of Fourier series (including statement of pointwise convergence for piecewise differentiable functions, and exposition of failure for some continuous functions). Examples of other orthogonal expansions (Legendre, Laguerre, Hermite etc.).

Reading

Essential Reading

1. B.P. Rynne and M.A. Youngson, *Linear Functional Analysis* (Springer SUMS, 2nd edition, 2008), Chapters 3, 4.4, 6.
2. E. Kreyszig, *Introductory Functional Analysis with Applications* (Wiley, revised edition, 1989), Chapters 3, 4.7–4.9, 4.12–4.13, 9.1–9.2.
3. N. Young, *An Introduction to Hilbert Space* (Cambridge University Press, 1988), Chs 17.

Further Reading

1. E.M. Stein and R. Shakarchi, *Real Analysis: Measure Theory, Integration & Hilbert Spaces* (Princeton Lectures in Analysis III, 2005), Chapter 4.

2.10 B9a: Galois Theory — Dr de la Ossa — 16 MT

Level: H-level

Method of Assessment: Written examination.

Weight: Half-unit (OSS paper code 2A48)

Recommended Prerequisites: All second-year algebra and arithmetic. Students who have not taken Part A Number Theory should read about quadratic residues in, for example, the appendix to Stewart and Tall. This will help with the examples.

Overview

The course starts with a review of second-year ring theory with a particular emphasis on polynomial rings, and a discussion of general integral domains and fields of fractions. This is followed by the classical theory of Galois field extensions, culminating in some of the classical theorems in the subject: the insolubility of the general quintic and impossibility of certain ruler and compass constructions considered by the Ancient Greeks.

Learning Outcomes

Understanding of the relation between symmetries of roots of a polynomial and its solubility in terms of simple algebraic formulae; working knowledge of interesting group actions in a nontrivial context; working knowledge, with applications, of a nontrivial notion of finite group theory (soluble groups); understanding of the relation between algebraic properties of field extensions and geometric problems such as doubling the cube and squaring the circle.

Synopsis

Review of polynomial rings, factorisation, integral domains. Reminder that any nonzero homomorphism of fields is injective. Fields of fractions.

Review of group actions on sets, Gauss' Lemma and Eisenstein's criterion for irreducibility of polynomials, field extensions, degrees, the tower law. Symmetric polynomials.

Separable extensions. Splitting fields and normal extensions. The theorem of the primitive element. The existence and uniqueness of algebraic closure (proofs not examinable).

Groups of automorphisms, fixed fields. The fundamental theorem of Galois theory.

Examples: Kummer extensions, cyclotomic extensions, finite fields and the Frobenius automorphism. Techniques for calculating Galois groups.

Soluble groups. Solubility by radicals, solubility of polynomials of degree at most 4, insolubility of the general quintic, impossibility of some ruler and compass constructions.

Reading

1. J. Rotman, *Galois Theory* (Springer-Verlag, NY Inc, 2001/1990).
2. I. Stewart, *Galois Theory* (Chapman and Hall, 2003/1989)
3. D.J.H. Garling, *A Course in Galois Theory* (Cambridge University Press I.N., 1987).
4. Herstein, *Topics in Algebra* (Wiley, 1975)

2.11 B9b: Algebraic Number Theory — Prof. Flynn — 16 HT

Level: H-level

Method of Assessment: Written examination.

Weight: Half-unit (cannot be taken unless B9a is taken) (OSS paper code to be confirmed)

Prerequisites: B9a Galois Theory

Recommended Prerequisites: All second-year algebra and arithmetic. Students who have not taken Part A Number Theory should read about quadratic residues in, for example, the appendix to Stewart and Tall. This will help with the examples.

Overview

An introduction to algebraic number theory. The aim is to describe the properties of number fields, but particular emphasis in examples will be placed on quadratic fields, where it is easy to calculate explicitly the properties of some of the objects being considered. In such fields the familiar unique factorisation enjoyed by the integers may fail, and a key objective of the course is to introduce the class group which measures the failure of this property.

Learning Outcomes

Students will learn about the arithmetic of algebraic number fields. They will learn to prove theorems about integral bases, and about unique factorisation into ideals. They will learn to calculate class numbers, and to use the theory to solve simple Diophantine equations.

Synopsis

1. field extensions, minimum polynomial, algebraic numbers, conjugates, discriminants, Gaussian integers, algebraic integers, integral basis
2. examples: quadratic fields
3. norm of an algebraic number
4. existence of factorisation
5. factorisation in $\mathbb{Q}(\sqrt{d})$
6. ideals, \mathbb{Z} -basis, maximal ideals, prime ideals
7. unique factorisation theorem of ideals
8. relationship between factorisation of number and of ideals
9. norm of an ideal
10. ideal classes
11. statement of Minkowski convex body theorem
12. finiteness of class number
13. statement of Dirichlet's theorem on finite generation of units
14. computations of class number to go on example sheets

Reading

1. I. Stewart and D. Tall, *Algebraic Number Theory* (Chapman and Hall Mathematics Series, May 1987).

Further Reading

1. D. Marcus, *Number Fields* (Springer-Verlag, New York–Heidelberg, 1977). ISBN 0-387-90279-1.

2.12 B10a: Martingales Through Measure Theory — Prof Riordan — 16 MT

Level: H-level

Method of Assessment: Written examination.

Weight: Half-unit (OSS paper code to be confirmed)

Recommended Prerequisites: Part A Integration is a prerequisite, so that the corresponding material will be assumed to be known. Part A Probability is a prerequisite.

Overview

Probability theory arises in the modelling of a variety of systems where the understanding of the “unknown” plays a key role, such as population genetics in biology, market evolution in financial mathematics, and learning features in game theory. It is also very useful in various areas of mathematics, including number theory and partial differential equations. The course introduces the basic mathematical framework underlying its rigorous analysis, and is therefore meant to provide some of the tools which will be used in more advanced courses in probability.

The first part of the course provides a review of measure theory from Integration Part A, and develops a deeper framework for its study. Then we proceed to develop notions of conditional expectation, martingales, and to show limit results for the behaviour of these martingales which apply in a variety of contexts.

Learning Outcomes

The students will learn about measure theory, random variables, independence, expectation and conditional expectation, product measures and discrete-parameter martingales.

Synopsis

A branching-process example. Review of σ -algebras, measure spaces. Uniqueness of extension of π -systems and Carathéodory’s Extension Theorem [both without proof], monotone-convergence properties of measures, \limsup and \liminf of a sequence of events, Fatou’s Lemma, reverse Fatou Lemma, first Borel–Cantelli Lemma.

Random variables and their distribution functions, removed from synopses as on problem sheet and not lectured representation of a random variable with prescribed distribution

function [proved in problem sheet], σ -algebras generated by a collection of random variables. Independence of events, random variables and σ -algebras, π -systems criterion for independence, second Borel–Cantelli Lemma. The tail σ -algebra, Kolmogorov’s 0–1 Law. Convergence in measure and convergence almost everywhere.

Integration and expectation, review of elementary properties of the integral and L^p spaces [from Part A Integration for the Lebesgue measure on \mathbb{R}]. Scheffé’s Lemma, Jensen’s inequality, orthogonal projection in L^2 . The Kolmogorov Theorem and definition of conditional expectation, proof as least-squares-best predictor, elementary properties. The Radon–Nikodym Theorem [without proof, not examinable].

Filtrations, martingales, stopping times, discrete stochastic integrals, Doob’s Optional-Stopping Theorem, Doob’s Upcrossing Lemma and “Forward” Convergence Theorem, martingales bounded in L^2 , Doob decomposition.

Uniform integrability and L^1 convergence, Levy’s “Upward” and “Downward” Theorem, corollary to the Kolmogorov’s Strong Law of Large Numbers, Doob’s submartingale inequalities.

Examples and applications, including branching processes, and harmonic functions with boundary conditions on connected finite subsets of \mathbb{Z}^d .

L^p submartingale inequalities [covered in problem sheet].

Reading

1. D. Williams, *Probability with Martingales*, Cambridge University Press, 1995.
2. P. M. Tarres Lecture notes, *Appendix : Notes on Fubini’s theorem on \mathbb{R} , Product measures, infinite products of probability triples*, Mathematical Institute, 2009.

Further Reading

1. Z. Brzeźniak and T. Zastawniak, *Basic stochastic processes. A course through exercises*. Springer Undergraduate Mathematics Series. (Springer-Verlag London, Ltd., 1999) [more elementary than D. Williams’ book, but can provide with a complementary first reading].
2. M. Capinski and E. Kopp, *Measure, integral and probability*, Springer Undergraduate Mathematics Series. (Springer-Verlag London, Ltd., second edition, 2004).
3. R. Durrett, *Probability: Theory and Examples*. (Second Edition Duxbury Press, Wadsworth Publishing Company, 1996).
4. A. Etheridge, *A Course in Financial Calculus*, (Cambridge University Press, 2002).
5. J. Neveu, *Discrete-parameter Martingales*. (North-Holland, Amsterdam, 1975).
6. S. I. Resnick, *A Probability Path*, (Birkhäuser, 1999).

2.13 B11a: Communication Theory — Dr Stirzaker — 16 MT

NB: B22a: Integer Programming is a very suitable complement to this course.

Level: H-level

Method of Assessment: Written examination.

Weight: Half-unit (OSS paper code 2650).

Recommended Prerequisites: Part A Probability would be helpful, but not essential.

Overview

The aim of the course is to investigate methods for the communication of information from a sender, along a channel of some kind, to a receiver. If errors are not a concern we are interested in codes that yield fast communication; if the channel is noisy we are interested in achieving both speed and reliability. A key concept is that of information as reduction in uncertainty. The highlight of the course is Shannon's Noisy Coding Theorem.

Learning Outcomes

- (i) Know what the various forms of entropy are, and be able to manipulate them.
- (ii) Know what data compression and source coding are, and be able to do it.
- (iii) Know what channel coding and channel capacity are, and be able to use that.

Synopsis

Uncertainty (entropy); conditional uncertainty; information. Chain rules; relative entropy; Gibbs' inequality; asymptotic equipartition and typical sequences. Instantaneous and uniquely decipherable codes; the noiseless coding theorem for discrete memoryless sources; constructing compact codes.

The discrete memoryless channel; decoding rules; the capacity of a channel. The noisy coding theorem for discrete memoryless sources and binary symmetric channels.

Extensions to more general sources and channels.

Reading

1. D. J. A. Welsh, *Codes and Cryptography* (Oxford University Press, 1988), Chapters 1–3, 5.
2. T. Cover and J. Thomas, *Elements of Information Theory* (Wiley, 1991), Chapters 1–5, 8.

Further Reading

1. R. B. Ash, *Information Theory* (Dover, 1990).
2. D. MacKay, *Information Theory, Inference, and Learning Algorithms* (Cambridge, 2003). [Can be seen at: <http://www.inference.phy.cam.ac.uk/mackay/itila>. Do not infringe the copyright!]
3. G. Jones and J. M. Jones, *Information and Coding Theory* (Springer, 2000), Chapters 1–5.

2.14 B11b: Graph Theory — Prof. Riordan — 16HT

Level: H-level

Method of Assessment: Written examination.

Weight: Half-unit (OSS paper code tbc).

Recommended Prerequisites: None

Overview

Graphs (abstract networks) are among the simplest mathematical structures, but nevertheless have a very rich and well-developed structural theory. Since graphs arise naturally in many contexts within and outside mathematics, Graph Theory is an important area of mathematics, and also has many applications in other fields such as computer science.

The main aim of the course is to introduce the fundamental ideas of Graph Theory, and some of the basic techniques of combinatorics.

Learning Outcomes

The student will have developed a basic understanding of the properties of graphs, and an appreciation of the combinatorial methods used to analyze discrete structures.

Synopsis

Introduction: basic definitions and examples. Trees and their characterization. Euler circuits; long paths and cycles. Vertex colourings: Brooks theorem, chromatic polynomial. Edge colourings: Vizing's theorem. Planar graphs, including Euler's formula, dual graphs. Maximum flow - minimum cut theorem: applications including Menger's theorem and Hall's theorem. Tutte's theorem on matchings. Extremal Problems: Turan's theorem, Zarankiewicz problem, Erdos-Stone theorem.

Reading

B. Bollobas, *Modern Graph Theory*, Graduate Texts in Mathematics 184 (Springer-Verlag, 1998)

Further Reading

J. A. Bondy and U. S. R. Murty, *Graph Theory: An Advanced Course*, Graduate Texts in Mathematics 244 (Springer-Verlag, 2007).

R. Diestel, *Graph Theory*, Graduate Texts in Mathematics 173 (third edition, Springer-Verlag, 2005).

D. West, *Introduction to Graph Theory* (second edition, PrenticeHall, 2001).

3 Schedule 2 (additional units and half-units)

3.1 BE “Mathematical” Extended Essay

Level: H-level

Method of Assessment: Written extended essay.

Weight: Whole unit (7,500 words). OSS code 9921.

An essay on a mathematical topic may be offered as a whole unit. It is equivalent to a 32-hour lecture course. Generally, students will have approximately 8 hours of supervision distributed over Michaelmas and Hilary terms. In addition there are lectures on writing mathematics and using LaTeX in Michaelmas and Hilary terms. See the lecture list for details.

Students considering offering an essay should read the *Guidance Notes on Extended Essays and Dissertations in Mathematics* by the Projects Committee available at:

<http://www.maths.ox.ac.uk/current-students/undergraduates/projects/>.

Application Students must apply to the Mathematics Project Committee for approval of their proposed topic in advance of beginning work on their essay. Proposals should be addressed to The Chairman of the Projects Committee, c/o Mrs Helen Lowe, Room DH61, Dartington House, and are accepted from the end of Trinity Term. All proposals must be received before 12 noon on Friday of Week 0 of Michaelmas Full Term. Note that a BE essay must have a substantial mathematical content. The application form is available at .

Once your title has been approved, it may only be changed by approval of the Chairman of the Projects Committee.

Assessment Each project is blind double marked. The marks are reconciled through discussion between the two assessors which is overseen by the examiners. Please see the *Guidance Notes on Extended Essays and Dissertations in Mathematics* for detailed marking criteria and class descriptors.

Submission THREE copies of your essay, identified by your candidate number only, should be sent to the Chairman of Examiners, FHS of Mathematics Part B, Examination Schools,

Oxford, to arrive no later than **12noon on Friday of week 9, Hilary Term 2013**. An electronic copy of your dissertation should also be submitted via the Mathematical Institute website. Further details may be found in the *Guidance Notes on Extended Essays and Dissertations in Mathematics*.

3.2 O1: History of Mathematics — Dr Stedall — 16 lectures in MT and reading course of 8 seminars in HT

Level: H-level

Assessment: 2-hour written examination paper for the MT lectures and 3000-word essay for the reading course.

Weight: Whole unit.

Recommended prerequisites: None.

Quota: The maximum number of students that can be accepted for 2012–13 will be 20.

Learning outcomes

This course is designed to provide the historical background to some of the mathematics familiar to students from A-level and the first four terms of undergraduate study, and looks at a period from approximately the mid-sixteenth century to the end of the nineteenth century. The course will be delivered through 16 lectures in Michaelmas Term, and a reading course consisting of 8 seminars (equivalent to a further 16 lectures) in Hilary Term. Guidance will be given throughout on reading, note-taking, and essay-writing.

Students will gain:

- an understanding of university mathematics in its historical context;
- an enriched understanding of the mathematical content of the topics covered by the course

together with skills in:

- reading and analysing historical mathematical sources;
- reading and analysing secondary sources;
- efficient note-taking;
- essay-writing (from 1000 to 3000 words);

- construction of references and bibliographies;
- oral discussion and presentation.

Lectures

The Michaelmas Term lectures will cover the following material:

- Introduction.
- Seventeenth century: analytic geometry; the development of calculus; Newton's *Principia*.
- Eighteenth century: from calculus to analysis; functions, limits, continuity; equations and solvability.
- Nineteenth century: group theory and abstract algebra; the beginnings of modern analysis; sequences and series; integration; complex analysis; linear algebra.

Classes to accompany the lectures will be held in Weeks 3, 5, 6, and 7. For each class students will be expected to prepare one piece of written work (1000 words) and one discussion topic.

Reading course

The Hilary Term part of the course is run as a reading course during which we will study two or three primary texts in some detail, using original sources and secondary literature. Details of the books to be read in HT 2013 will be decided and discussed towards the end of MT 2012. Students will be expected to write two essays (2000 words each) during the first six weeks of term. The course will then be examined by an essay of 3000 words to be completed during Weeks 7 to 9.

Recommended reading

Jacqueline Stedall, *Mathematics emerging: a sourcebook 1540–1900*, (Oxford University Press, 2008).

Victor Katz, *A history of mathematics* (brief edition), (Pearson Addison Wesley, 2004), or:

Victor Katz, *A history of mathematics: an introduction* (third edition), (Pearson Addison Wesley, 2009).

Benjamin Wardhaugh, *How to read historical mathematics*, (Princeton, 2010).

Supplementary reading

John Fauvel and Jeremy Gray (eds), *The history of mathematics: a reader*, (Macmillan, 1987).

Assessment

The Michaelmas Term material will be examined in a two-hour written paper at the end of Trinity Term. Candidates will be expected to answer two half-hour questions (commenting on extracts) and one one-hour question (essay). The paper will account for 50% of the marks for the course. The Reading Course will be examined by a 3000-word essay at the end of Hilary Term. The title will be set at the beginning of Week 7 and two copies of the project must be submitted to the Examination Schools by midday on Friday of Week 9. This essay will account for 50% of the marks for the course.

3.3 Computer Science: Half units

There are other half units that students in Part B Mathematics and Philosophy may take which are drawn from Part B of the Honour School of Mathematics and Computing. For full details of these half units see the syllabus and synopses for Part B of the Honour School Mathematics and Computing, which are available on the Web at

<http://www.cs.ox.ac.uk/teaching/mcs/PartB//>.

The Computer Science half units available are as follows:

- OCS3b Lambda Calculus and Types
- OCS4b Computational Complexity
- OCS5b Knowledge Representation and Reasoning
- OCS6a Computer-aided Formal Verification

3.4 N1: Mathematics Education and Undergraduate Ambassadors' Scheme

Level: H-level **Method of Assessment:** See individual synopses for each half unit

Weight: Whole-unit (OSS paper code: tbc), or N1a may be taken as a half-unit.

3.4.1 N1a Mathematics Education —Prof Watson & Dr Stylianides —

Level: H-level **Method of Assessment:** Two examined written assignments and a short presentation.

[1500 word written assignment(not examined) due: end of week 2] [2500 word written assignment (examined) due: end of week 5] [Presentation (examined) week 8] [3000 word written assignment (examined) due: week 1 Hilary Term].

Weight: Half Unit. OSS paper code tbc.

Quota: There will be a quota of approximately 20 students for this course.

Recommended Prerequisites: None

Overview

The Mathematics Education option will be a half-unit, run in Michaelmas Term. The course is appropriate for all students of the appropriate degree courses, whether or not they are interested in teaching subsequently. Final credit will be based on two examined written assignments (35 % each), one of which will be submitted at the start of Hilary Term, and a presentation (30%). Teaching will be 22 hours of contact time which will include lecture, seminar, class and tutorial formats as follows:

- A two-hour lecture/class per week. These will be interactive and involve discussion and other tasks as well as input from the lecturer.
- Four hours of tutorial workshops in small groups to review ideas from the course and preparation for written assignments.

Learning Outcomes

1. Understanding:
 - the psychology of learning mathematics;
 - the nature of mathematics and the curriculum;
 - relations between teaching and learning at primary, secondary and tertiary level;
 - the role of mathematics education in society;
 - issues associated with communicating mathematics.
2. Understanding connections between mathematics, education issues and the mathematical experience of learners.
3. The ability to express ideas about the study and learning of mathematics in writing, verbally, and in other forms of communication.

Synopsis

1. Introduction to mathematics education as a field of study.
 - Issues and problems; relation to learning mathematics. Introduction to course, the education library, and the expected forms of study including pair work. Setting assignment.
2. Relations between teaching and learning
 - Theories about teaching and learning, e.g. variation theory; deep/surface approaches; observable characteristics in comparative studies; task design.
3. Mathematics education in society.
 - Achievement according to class, gender, ethnicity in UK and elsewhere
4. Communicating mathematics.
 - Verbal; symbolic; diagram; gestural; dynamic representation.

Reading

Main Texts:

1. Tall, D. (1991) *Advanced Mathematical Thinking*. (Mathematics Education Library, 11). Dordrecht: Kluwer
2. Gates, P. (ed.) (2001) *Issues in Mathematics Teaching*. London: RoutledgeFalmer
3. Mason, J., Burton, L. & Stacey, K. (2010) *Thinking Mathematically*. Any edition by any publisher will do.
4. Polya, G. (1957) *How to Solve It*. Any edition by any publisher will do.
5. Davis, P. And Hersh, R. (1981) *The Mathematical Experience*. Any edition by any publisher will do.
6. Mason, J. & Johnston-Wilder, S. (2004) *Fundamental Constructs in Mathematics Education*, London: RoutledgeFalmer.
7. Carpenter, T., Dossey, J. & Koehler, J. (2004) *Classics in Mathematics Education Research*. Reston, VA: National Council of Teachers of Mathematics.

Important websites:

1. ncetm.org.uk

3.4.2 N1b Undergraduate Ambassadors' Scheme — Dr Griffiths — mainly HT

hp1406 I don't see any point in saying this If it is included to keep in step with Maths Symopses it needs some punctuation ; added [**This course is not longer available as a half unit; it must be taken alongside N1a Mathematics Education**]

Method of Assessment: Journal of activities, Oral presentation, Course report and project, Teacher report.

Quota: There will be a quota of approximately 10 students for this course.

Co-ordinator: Dr Griffiths

hp1403 Note wanted or needed here Option available to Mathematics, Mathematics & Statistics, Mathematics & Philosophy students.

Learning Outcomes

The Undergraduate Ambassadors' Scheme (UAS) was begun by Simon Singh in 2002 to give university undergraduates a chance to experience assisting and, to some extent, teaching in schools, and to be credited for this. The option focuses on improving students' communication, presentation, cooperation and organizational skills and sensitivity to others' learning difficulties.

Course Description and Timing:

The Oxford UAS option, N1, is a half-unit, mainly run in Hilary Term. A quota will be in place, of approximately 10 students, and so applicants for the UAS option will be asked to

name a second alternative half-unit. The course is appropriate for all students, whether or not they are interested in teaching subsequently.

A student on the course will be assigned to a mathematics teacher in a local secondary school (in the Oxford, Kidlington, Wheatley area) for half a day per week during Hilary Term. Students will be expected to keep a journal of their activities, which will begin by assisting in the class, but may widen to include teaching the whole class for a part of a period, or working separately with a smaller group of the class. Students will be required at one point to give a presentation to one of their school classes relating to a topic from university mathematics, and will also run a small project based on some aspect of mathematics education with advice from the course co-ordinator and teacher/s. Final credit will be based on the journal (20%), the presentation (30%), an end of course report (approximately 3000 words) including details of the project (35%), together with a report from the teacher (15%).

Short interviews will take place on Thursday or Friday of 0th week in Michaelmas term to select students for this course. The interview (of roughly 15 minutes) will include a presentation by the student on an aspect of mathematics of their choosing. Students will be chosen on the basis of their ability to communicate mathematics, and two references will be sought from college tutors on these qualities. Applicants will be quickly notified of the decision.

Those on the course will also need to fill in a CRB form, or to have done so already. By the end of Michaelmas term students will have been assigned to a teacher and have made a first, introductory, visit to their school. The course will begin properly in Hilary term with students helping in schools for half a day each week. Funds are available to cover travel expenses. Support classes will be provided throughout Hilary for feedback and to discuss issues such as the planning of the project. The deadline for the journal and report will be noon on Friday of 0th week of Trinity term.

Any further questions on the UAS option should be passed on to the option's co-ordinator, Martin Griffiths (martin.griffiths@maths.ox.ac.uk).

Reading List

Clare Tickly, Anne Watson, Candia Morgan, *Teaching School Subjects: Mathematics* (Routledge Falmer, 2004).

3.5 List of Mathematics Department half-units available only if special approval is granted

For details of these courses, and prerequisites for them, please consult the Supplement to the Mathematics Course Handbook, Syllabus and Synopses for Mathematics Part B 2012–2013, for examination in 2013,

<http://www.maths.ox.ac.uk/current-students/undergraduates/handbooks-synopses>

B5a: Techniques of Applied Mathematics

B5b: Applied Partial Differential Equations

B5.1a: Dynamics and Energy Minimization

B6a: Viscous Flow

B6b: Waves and Compressible Flow

B7.1a: Quantum Mechanics

C7.1b: Quantum Theory and Quantum Computers (M-level)

B7.2b: Special Relativity and Electromagnetism

B8a: Mathematical Ecology and Biology

B8b: Nonlinear Systems

B10b: Mathematical Models of Financial Derivatives

B12a: Applied Probability

B21a Numerical Solution of Differential Equations I

B21b Numerical Solution of Differential Equations II

B22a Integer Programming