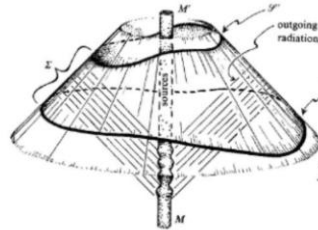
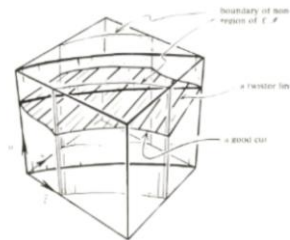
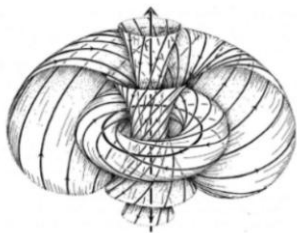


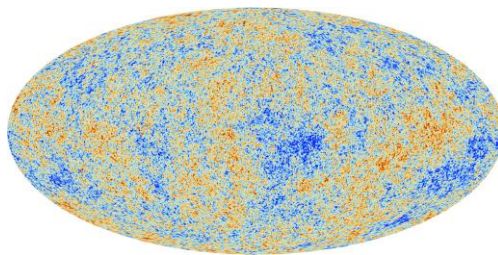
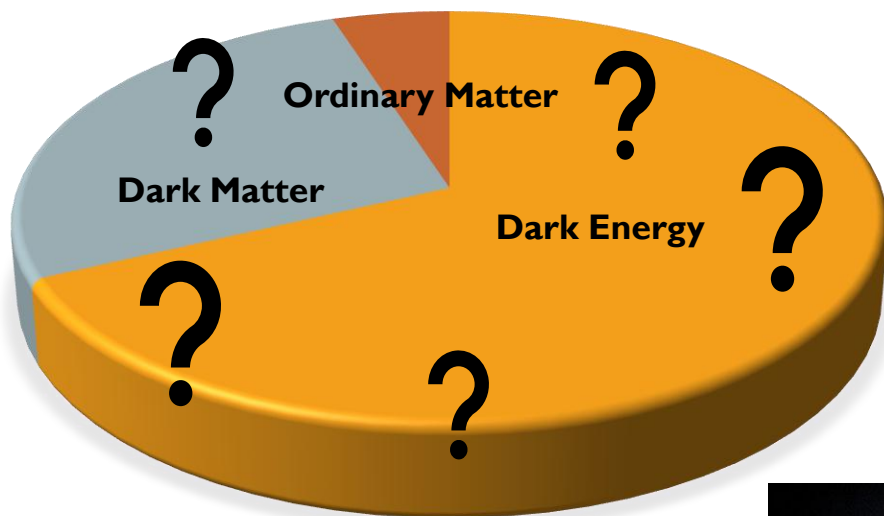
(A)DS DOUBLE COPY FROM TWISTOR SPACE

Mariana Carrillo Gonzalez

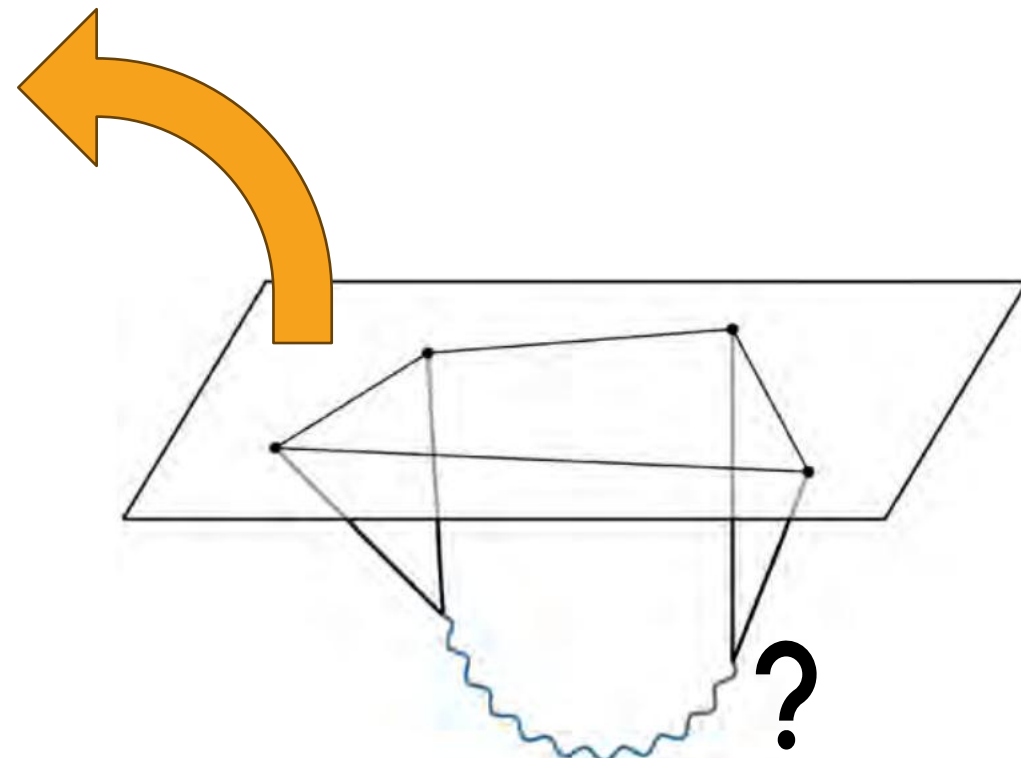
Penrose at 95 and Twistors at 60



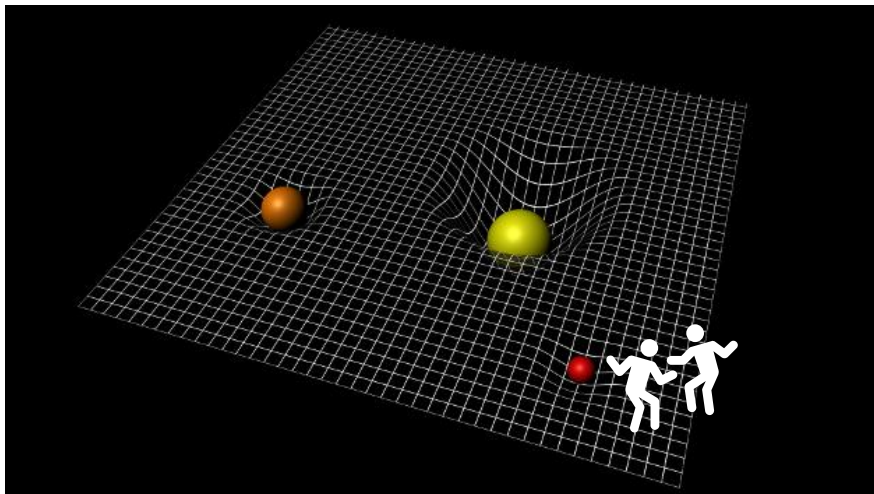
PHYSICS IN CURVED SPACETIMES



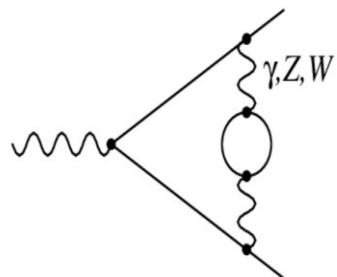
$$\Lambda > 0$$



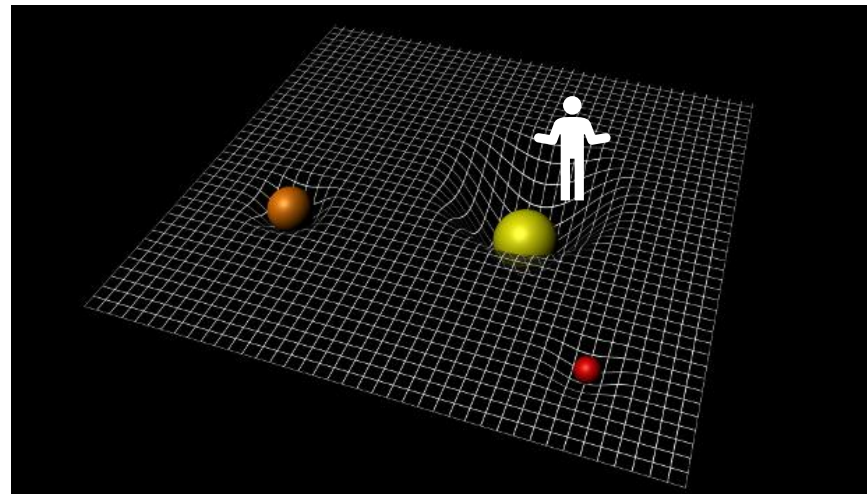
We usually work at small curvature / weak gravity



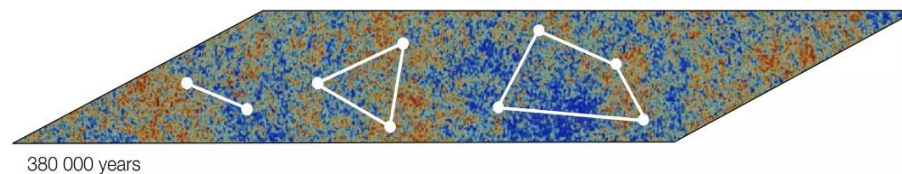
Perturbation theory & EFT
Scattering amplitude methods



Large curvature / strong gravity is much harder!



Perturbation theory & EFT
No S-matrix $\rightarrow$ correlators



LESSONS FROM FLAT SPACE

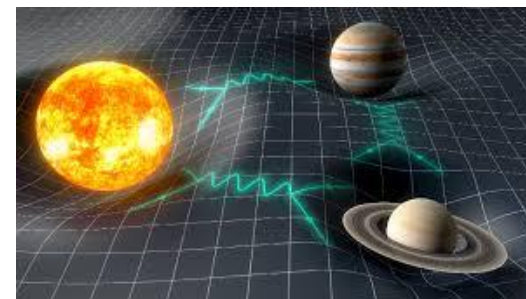
Gauge theories

$$S = \frac{1}{4g^2} \int \text{Tr } F^2$$



Gravity

$$S = \frac{1}{2\kappa} \int R \sqrt{-g} d^4 x,$$



$$\mathcal{L}_{\text{YM}}^{\mathcal{O}(A^3)} = -\frac{1}{4}(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a)(\partial^\mu A^{a\nu} - \partial^\nu A^{a\mu}) - \frac{g}{2} f^{abc}(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a)A^{b\mu}A^{c\nu}$$

(Perturbed[LGR, 3] // ExpandPerturbation // ContractMetric // ToCanonical // ToCanonical) /. e -> 1

$$\begin{aligned} & -\frac{1}{2} \text{Mp}^2 \sqrt{-g} h^{1ab} R[\nabla]_{ab} - \frac{1}{4} \text{Mp}^2 \sqrt{-g} h^{2ab} R[\nabla]_{ab} - \frac{1}{8} \text{Mp}^2 \sqrt{-g} h^{1ab} h^{2c} R[\nabla]_{ab} - \frac{1}{12} \text{Mp}^2 \sqrt{-g} h^{3ab} R[\nabla]_{ab} + \frac{1}{2} \text{Mp}^2 \sqrt{-g} h^{1a} h^{1ab} R[\nabla]_{bc} - \frac{1}{4} \text{Mp}^2 \sqrt{-g} h^{1a} h^{1bc} R[\nabla]_{bc} + \\ & \frac{1}{2} \text{Mp}^2 \sqrt{-g} h^{1ab} h^{2c} R[\nabla]_{bc} - \frac{1}{8} \text{Mp}^2 \sqrt{-g} h^{1a} h^{2bc} R[\nabla]_{bc} - \frac{1}{2} \text{Mp}^2 \sqrt{-g} h^{1a} h^{1ab} h^{1d} R[\nabla]_{cd} + \frac{1}{4} \text{Mp}^2 \sqrt{-g} h^{1a} h^{1b} h^{1bc} R[\nabla]_{cd} + \frac{1}{8} \text{Mp}^2 \sqrt{-g} h^{1ab} h^{1cd} R[\nabla]_{cd} - \\ & \frac{1}{8} \text{Mp}^2 \sqrt{-g} h^{1a} h^{1b} h^{1cd} R[\nabla]_{cd} + \frac{1}{2} \text{Mp}^2 \sqrt{-g} R[\nabla] + \frac{1}{4} \text{Mp}^2 \sqrt{-g} h^{1a} R[\nabla] - \frac{1}{8} \text{Mp}^2 \sqrt{-g} h^{1ab} h^{1ab} R[\nabla] + \frac{1}{12} \text{Mp}^2 \sqrt{-g} h^{1a} h^{1ab} h^{1bc} R[\nabla] + \frac{1}{8} \text{Mp}^2 \sqrt{-g} h^{1a} h^{1b} R[\nabla] - \\ & \frac{1}{16} \text{Mp}^2 \sqrt{-g} h^{1a} h^{1bc} h^{1bc} R[\nabla] - \frac{1}{48} \text{Mp}^2 \sqrt{-g} h^{1a} h^{1b} h^{1c} R[\nabla] - \frac{1}{8} \text{Mp}^2 \sqrt{-g} h^{1ab} h^{2ab} R[\nabla] + \frac{1}{8} \text{Mp}^2 \sqrt{-g} h^{2a} R[\nabla] + \frac{1}{16} \text{Mp}^2 \sqrt{-g} h^{1a} h^{2b} R[\nabla] + \frac{1}{24} \text{Mp}^2 \sqrt{-g} h^{3a} R[\nabla] + \\ & \frac{1}{16} \text{Mp}^2 \sqrt{-g} h^{1ab} R[\nabla]_{ab} (h^{1a})^2 - \frac{1}{16} \text{Mp}^2 \sqrt{-g} R[\nabla] (h^{1a})^2 + \frac{1}{32} \text{Mp}^2 \sqrt{-g} R[\nabla] (h^{1a})^3 - \frac{3}{8} \text{Mp}^2 \sqrt{-g} h^{1ab} h^{1cd} h^{1cd} + \frac{1}{8} \text{Mp}^2 \sqrt{-g} h^{1ab} h^{1c} h^{1d} + \frac{1}{4} \text{Mp}^2 \sqrt{-g} h^{1ab} h^{2c} + \\ & \frac{1}{2} \text{Mp}^2 \sqrt{-g} h^{1ab} h^{1ab} - \frac{1}{16} \text{Mp}^2 \sqrt{-g} (h^{1a})^2 h^{1ab} h^{1ab} + \frac{1}{2} \text{Mp}^2 \sqrt{-g} h^{1ab} h^{1c} h^{1c} + \frac{1}{4} \text{Mp}^2 \sqrt{-g} h^{2ab} h^{1c} h^{1c} + \frac{1}{4} \text{Mp}^2 \sqrt{-g} h^{1ab} h^{2c} h^{1c} + \frac{1}{12} \text{Mp}^2 \sqrt{-g} h^{3ab} h^{1ab} - \\ & \frac{1}{2} \text{Mp}^2 \sqrt{-g} h^{1a} h^{1b} + \frac{1}{16} \text{Mp}^2 \sqrt{-g} (h^{1a})^2 h^{1a} h^{1b} - \frac{1}{4} \text{Mp}^2 \sqrt{-g} h^{2a} h^{1b} - \frac{1}{12} \text{Mp}^2 \sqrt{-g} h^{3a} h^{1b} - \frac{1}{2} \text{Mp}^2 \sqrt{-g} h^{1ab} h^{1c} h^{1c} - \frac{1}{4} \text{Mp}^2 \sqrt{-g} h^{2ab} h^{1c} h^{1c} - \frac{1}{4} \text{Mp}^2 \sqrt{-g} h^{1ab} h^{2c} h^{1c} - \\ & \frac{1}{8} \text{Mp}^2 \sqrt{-g} h^{1c} h^{1c} h^{1a} h^{1b} - \frac{1}{8} \text{Mp}^2 \sqrt{-g} h^{2c} h^{1a} h^{1b} - \frac{1}{2} \text{Mp}^2 \sqrt{-g} h^{1ab} h^{1d} h^{1c} - \frac{1}{2} \text{Mp}^2 \sqrt{-g} h^{1ab} h^{1c} h^{1c} - \frac{1}{2} \text{Mp}^2 \sqrt{-g} h^{1ab} h^{1c} h^{1d} - \\ & \frac{1}{2} \text{Mp}^2 \sqrt{-g} h^{1ab} h^{2c} h^{1c} + \frac{1}{4} \text{Mp}^2 \sqrt{-g} h^{1a} h^{2c} h^{1c} - \frac{1}{2} \text{Mp}^2 \sqrt{-g} h^{1ab} h^{1c} h^{1c} - \frac{1}{4} \text{Mp}^2 \sqrt{-g} h^{2ab} h^{1c} h^{1c} + \frac{1}{4} \text{Mp}^2 \sqrt{-g} h^{1a} h^{1bc} h^{1bc} + \frac{1}{8} \text{Mp}^2 \sqrt{-g} h^{2a} h^{1bc} h^{1bc} - \\ & \frac{1}{2} \text{Mp}^2 \sqrt{-g} h^{1c} h^{1ab} h^{1d} h^{1bc} + \frac{1}{4} \text{Mp}^2 \sqrt{-g} h^{1a} h^{1bc} h^{1d} h^{1bc} - \frac{1}{4} \text{Mp}^2 \sqrt{-g} h^{1ab} h^{2c} h^{1bc} + \frac{1}{8} \text{Mp}^2 \sqrt{-g} h^{1a} h^{2bc} h^{1bc} + \frac{1}{2} \text{Mp}^2 \sqrt{-g} h^{1ab} h^{1ab} h^{1c} - \frac{1}{4} \text{Mp}^2 \sqrt{-g} h^{2ab} h^{1ab} h^{1c} - \\ & \frac{1}{4} \text{Mp}^2 \sqrt{-g} h^{1a} h^{1b} h^{1c} - \frac{1}{8} \text{Mp}^2 \sqrt{-g} h^{2a} h^{1b} h^{1c} + \frac{1}{4} \text{Mp}^2 \sqrt{-g} h^{1ab} h^{2ab} h^{1c} - \frac{1}{8} \text{Mp}^2 \sqrt{-g} h^{1a} h^{2b} h^{1c} + \frac{1}{2} \text{Mp}^2 \sqrt{-g} h^{1c} h^{1ab} h^{1d} h^{1c} - \frac{1}{4} \text{Mp}^2 \sqrt{-g} h^{1a} h^{1bc} h^{1d} h^{1c} + \\ & \frac{1}{4} \text{Mp}^2 \sqrt{-g} h^{1ab} h^{1d} h^{1ab} h^{1c} - \frac{1}{4} \text{Mp}^2 \sqrt{-g} h^{1ac} h^{1ab} h^{1c} - \frac{1}{4} \text{Mp}^2 \sqrt{-g} h^{2ac} h^{1ab} h^{1c} + \frac{3}{8} \text{Mp}^2 \sqrt{-g} h^{1ab} h^{1ab} h^{1c} + \frac{3}{8} \text{Mp}^2 \sqrt{-g} h^{2ab} h^{1ab} h^{1c} - \frac{1}{16} \text{Mp}^2 \sqrt{-g} h^{1a} h^{1d} h^{1c} h^{1b} + \\ & \frac{1}{2} \text{Mp}^2 \sqrt{-g} h^{1ab} h^{1c} h^{1d} + \text{Mp}^2 \sqrt{-g} h^{1ab} h^{1c} h^{1d} - \frac{1}{4} \text{Mp}^2 \sqrt{-g} h^{1a} h^{1bc} h^{1c} h^{1d} - \frac{1}{2} \text{Mp}^2 \sqrt{-g} h^{1ab} h^{1ab} h^{1c} h^{1d} + \frac{1}{4} \text{Mp}^2 \sqrt{-g} h^{1a} h^{1b} h^{1c} h^{1d} + \\ & \frac{1}{2} \text{Mp}^2 \sqrt{-g} h^{1ab} h^{1cd} h^{1ac} h^{1d} - \frac{1}{2} \text{Mp}^2 \sqrt{-g} h^{1ab} h^{1cd} h^{1ab} h^{1d} + \frac{1}{2} \text{Mp}^2 \sqrt{-g} h^{1a} h^{1bc} h^{1d} h^{1d} - \frac{1}{4} \text{Mp}^2 \sqrt{-g} h^{1a} h^{1bc} h^{1d} h^{1d} - \frac{1}{8} \text{Mp}^2 \sqrt{-g} h^{1ab} h^{1ab} h^{1cd} h^{1d} + \\ & \frac{1}{8} \text{Mp}^2 \sqrt{-g} h^{1a} h^{1b} h^{1cd} h^{1d} - \frac{1}{2} \text{Mp}^2 \sqrt{-g} h^{1c} h^{1ab} h^{1bc} h^{1d} + \frac{1}{4} \text{Mp}^2 \sqrt{-g} h^{1a} h^{1bc} h^{1bc} h^{1d} + \frac{1}{8} \text{Mp}^2 \sqrt{-g} h^{1ab} h^{1ab} h^{1c} h^{1d} - \frac{1}{8} \text{Mp}^2 \sqrt{-g} h^{1a} h^{1b} h^{1c} h^{1d} + \\ & \frac{1}{2} \text{Mp}^2 \sqrt{-g} h^{1ab} h^{1cd} h^{1cd} + \frac{1}{4} \text{Mp}^2 \sqrt{-g} h^{1ab} h^{1bd} h^{1cd} - \frac{3}{4} \text{Mp}^2 \sqrt{-g} h^{1ab} h^{1bc} h^{1cd} - \frac{1}{8} \text{Mp}^2 \sqrt{-g} h^{1a} h^{1bd} h^{1bc} h^{1cd} + \frac{3}{16} \text{Mp}^2 \sqrt{-g} h^{1a} h^{1bc} h^{1bc} h^{1cd} \end{aligned}$$

ON-SHELL + SPINOR-HELICITY

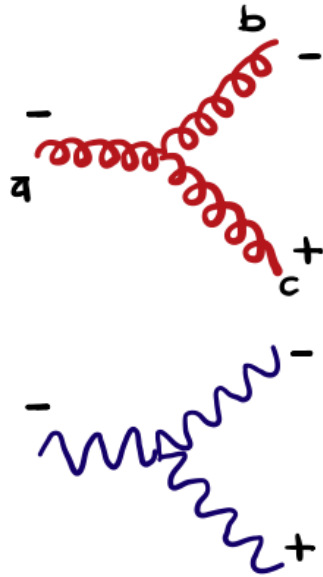
Use different
representations of
the Lorentz group

$$SO(1,3) \cong SL(2)_{\mathbb{C}}/\mathbb{Z}_2$$

$$p^\mu \longrightarrow p_{\alpha\dot{\alpha}} = p_\mu \sigma^\mu_{\alpha\dot{\alpha}} = \lambda_\alpha \tilde{\lambda}_{\dot{\alpha}} = |p\rangle \langle p|$$

Massless $p^2 = 0$

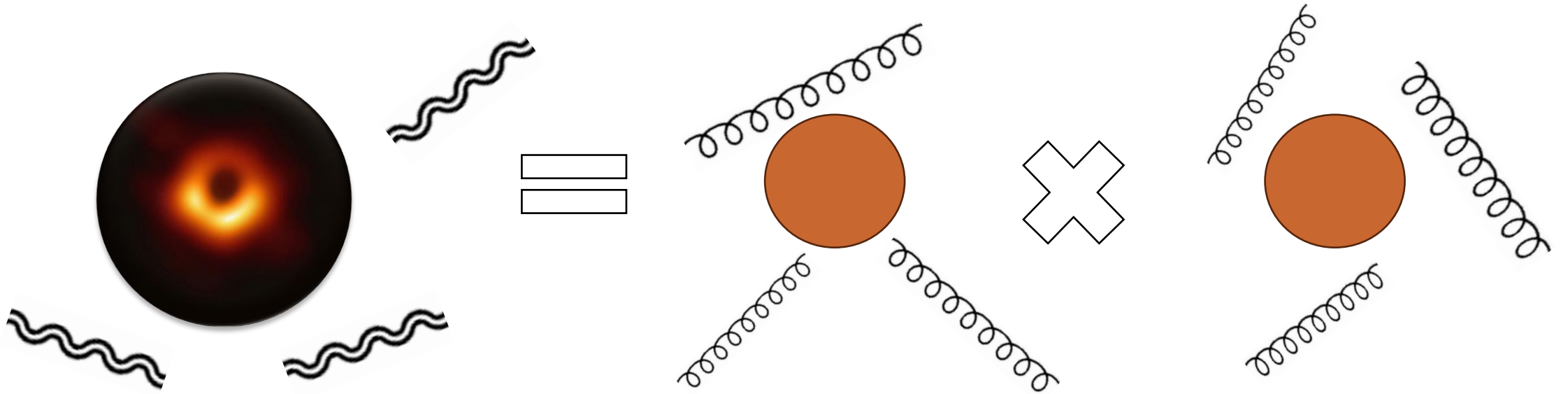
On-shell helicity amplitudes



$$A^{\text{YM}}[1^- 2^- 3^+] = g f^{abc} \frac{\langle 12 \rangle^3}{\langle 23 \rangle \langle 31 \rangle}$$

$$M^{\text{G}}(1^- 2^- 3^+) = \frac{\kappa}{2} \frac{\langle 12 \rangle^6}{\langle 23 \rangle^2 \langle 31 \rangle^2}$$

Double Copy



kinematics $\times$ kinematics

color $\times$ kinematics

color $\times$ kinematics

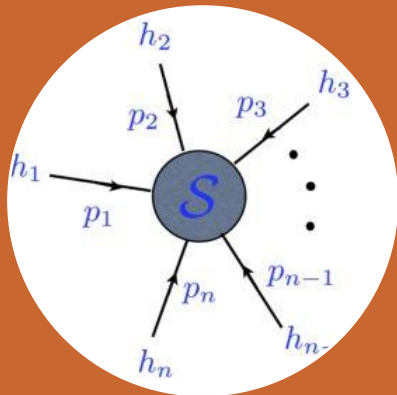
color – **kinematics duality**

color algebra



kinematic algebra

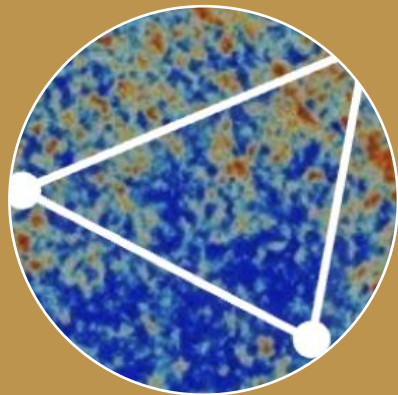
WHAT CAN WE LEARN FROM THE DOUBLE COPY?



Amplitudes

Breakdown of gravity
EFT (divergences in
loops)

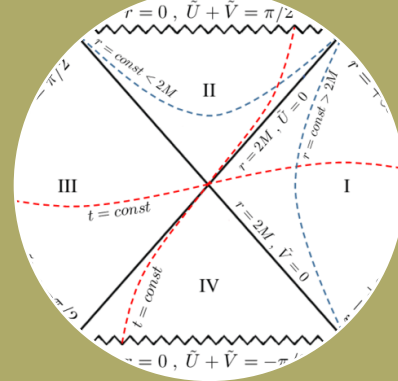
Precision calculations
for gravitational waves



Curved space correlators

Breakdown of gravity
EFT (divergences in
loops)

Precision calculations
of cosmological
correlators



Exact solutions and eom

New gravitational
solutions

On-shell symmetries
of the theory

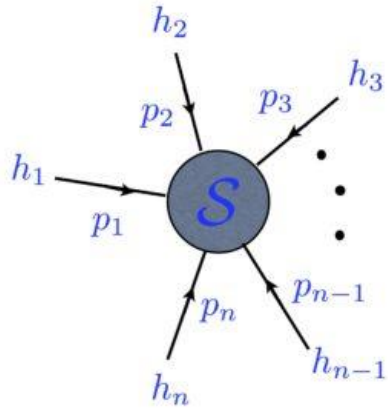
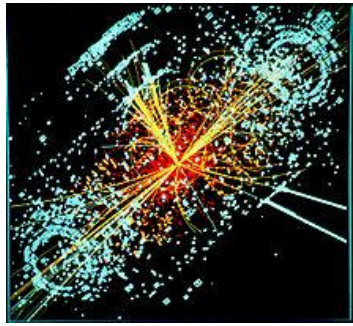
$$\mathcal{L} = \frac{1}{4g^2} \text{Tr } F^2$$

Lagrangian and action

Simplified Feynman
rules

Off-shell symmetries
of the theory

Double copy in different spaces



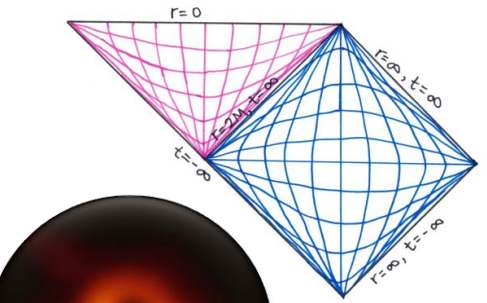
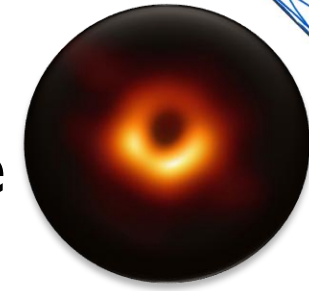
$$\mathcal{M}^{\text{DC}} = \mathcal{A}^{\text{YM}} \otimes S \otimes \mathcal{A}^{\text{YM}}$$

Momentum
space:
Amplitudes

Inverse Fourier transform



Coordinate
space:
classical
solutions +
eom

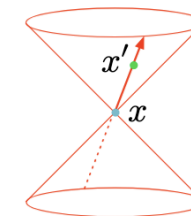


ASD field
strength

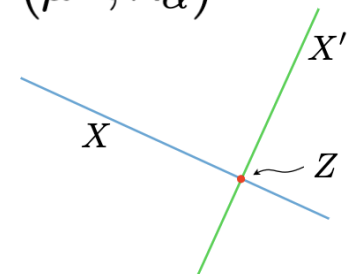
$$W_{\alpha\beta\gamma\delta} = \frac{f_{(\alpha\beta} f_{\gamma\delta)}}{\varphi}$$

ASD Weyl

Penrose transform



$$Z^A = (\mu^{\dot{\alpha}}, \lambda_{\alpha})$$



Twistor
space

Half- Inverse
Fourier
transform

(Witten)



$$p^{\mu} \longrightarrow \lambda_{\alpha} \tilde{\lambda}_{\dot{\alpha}}$$

$$\mathcal{A}(\lambda, \mu) = \int \frac{d^2 \tilde{\lambda}}{(2\pi)^2} e^{i \mu_{\dot{\alpha}} \tilde{\lambda}^{\dot{\alpha}}} \tilde{\mathcal{A}}(\lambda, \tilde{\lambda})$$

DOUBLE COPY IN TWISTOR SPACE

Penrose transform

$$\nabla^{\alpha_1 \dot{\alpha}_1} \phi_{\alpha_1 \dots \alpha_{2s}} = 0$$

Curvature spinors for
linearized YM, GR
(vacuum),..., classical
solutions

(C.D.White; Armstrong-Williams, Chacón, Luna, Nagy; Emond, Moynihan, Rumbutis, MCG,...)

Conformally invariant equations

Scalar is conformally coupled

$$\nabla^2 \varphi - \frac{R}{6} \varphi = 0$$

Hold for (A)dS straightforwardly

Double copy for Čech cohomology class
representatives

$$\check{f}_{-6} = \frac{\check{f}_{-4} \check{f}_{-4}}{\check{f}_{-2}}$$

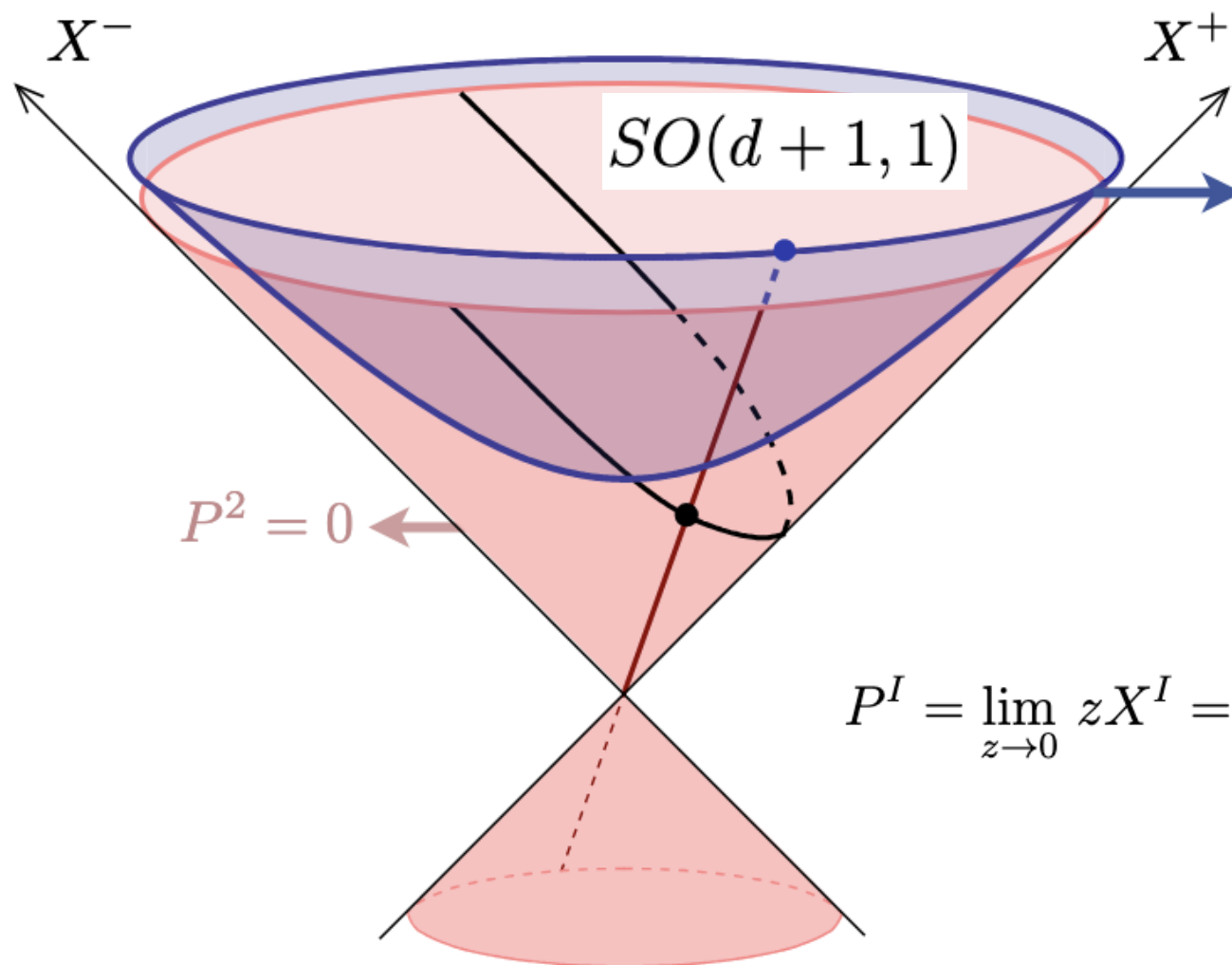
Provides proof of Weyl double copy.
Exact for Kerr-Schild spacetimes

AdS_3 twistor classical
double copy

Embedding Space

D+2 dim.

Linear Lorentz



$$X^2 = -1$$

Poincare coordinates

$$X^I = \frac{1}{z} \left(\vec{x}, \frac{1 - \vec{x}^2 - z^2}{2}, \frac{1 + \vec{x}^2 + z^2}{2} \right)$$

D+1 dim.

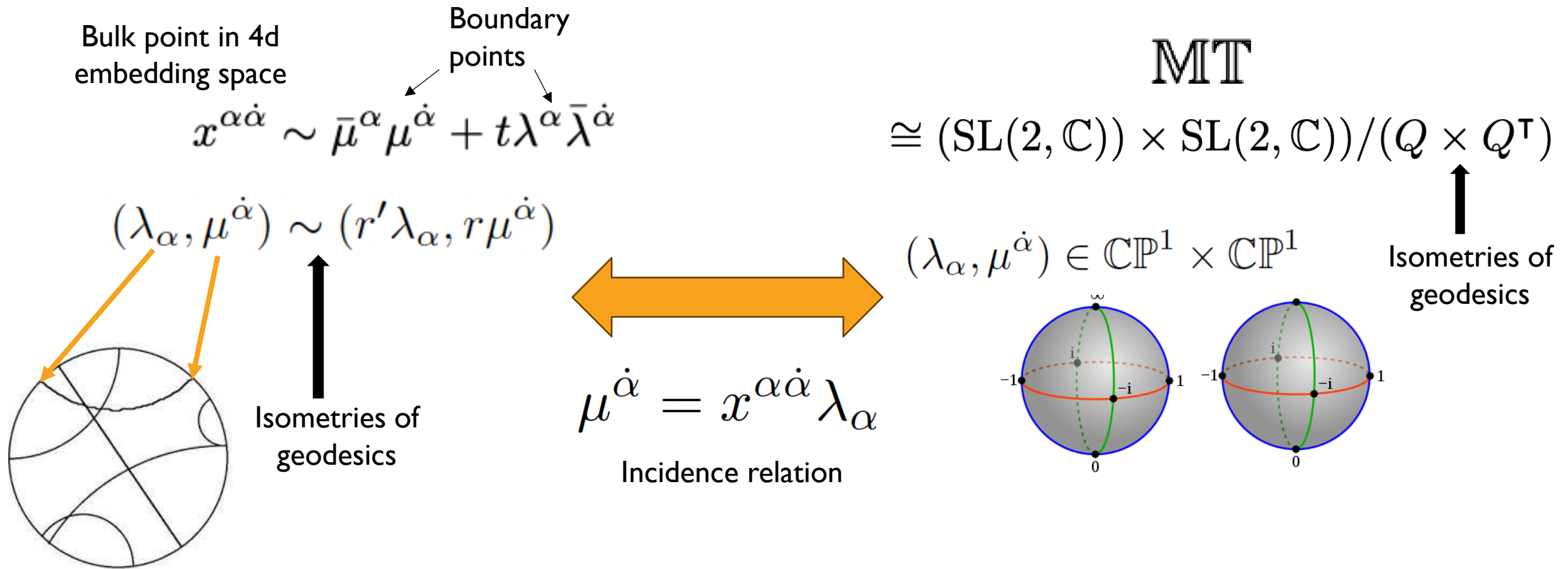
$$P^I = \lim_{z \rightarrow 0} z X^I = \left(\vec{x}, \frac{1 - \vec{x}^2}{2}, \frac{1 + \vec{x}^2}{2} \right)$$

D dim.

Non-linear Conformal

3d-mini-twistor space

Geodesics in 3d hyperbolic space



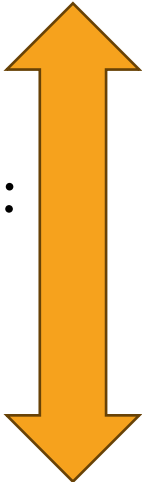
$$H^1(U, \mathcal{O}(-s-1-m|-s-1+m))$$

Cech cohomology class rep.

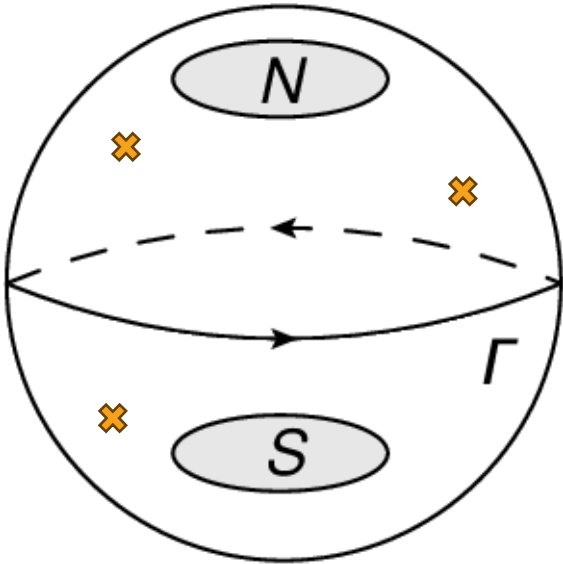
Isometries of geodesics

$$\check{f}(\alpha\lambda, \alpha'\mu) = \alpha^{m-s-1} \alpha'^{-m-s-1} \check{f}(\lambda, \mu)$$

Isomorphism:
Penrose
transform



$$\phi_{\alpha_1 \dots \alpha_{2s}} = \oint_{\Gamma} \langle \lambda d\lambda \rangle \lambda_{\alpha_1} \dots \lambda_{\alpha_{2s}} \check{f}(\lambda_{\alpha}, \mu^{\dot{\alpha}}) \Big|_X$$



$$\nabla_{\alpha}^{\beta} \Psi_{\beta \alpha_2 \dots \alpha_{2s}} = m \Psi_{\alpha_1 \dots \alpha_{2s}}$$

$$\nabla^2 \psi = (m^2 - 1) \psi$$

Linearized eom of massive spin s fields in H_3

ADS3 MINI-TWISTOR SPACE DOUBLE COPY

(Beetar, MCG, Jaitly, Keseman)

In flat space

(Emond,
Moynihan,
Rumbutis,
MCG, White)

$$\check{f}_{s=2,m} = \frac{\check{f}_{s=1,m} \check{f}_{s=1,m}}{\check{f}_{s=0,m}}$$

In AdS_3

$$so(2,2) \simeq sl(2, \mathbb{R}) \oplus \overline{sl(2, \mathbb{R})}$$

$$\Delta_{\pm} = h + \bar{h} = 1 \pm m$$

$$\check{f}_{s=2,m_{\phi}+2} = \frac{\check{f}_{s=1,m_{\phi}+1} \check{f}_{s=1,m_{\phi}+1}}{\check{f}_{s=0,m_{\phi}}}$$

DOUBLE COPY FOR:

- Waves / Bulk-to-boundary correlators*
- Warped black holes
- CFT dual currents

$$\Delta_s = \Delta_{\phi} + s$$

* Can arise as dim. reduction: (Bu, Seet)
See also (Roehrig, Skinner)

“TOPOLOGICALLY” MASSIVE THEORIES

$$\text{TMYM}^2 = \text{TMG}$$

Parity breaking
one d.o.f.

$$S_{\text{TMYM}} = -\frac{1}{4g^2} \int \text{Tr} \left(F \wedge *F + m \left(A \wedge dA + \frac{2}{3} A \wedge A \wedge A \right) \right)$$

$$\varepsilon_\mu^a + \frac{i}{m} \epsilon_{\mu\nu\rho} p^\nu \varepsilon_\rho^a = 0$$

$$S_{\text{TMG}} = -\frac{1}{2\kappa} \int \left(*R + m \left(\Gamma \wedge d\Gamma + \frac{2}{3} \Gamma \wedge \Gamma \wedge \Gamma \right) \right)$$

$$\varepsilon_{\mu\nu} = \varepsilon_\mu \varepsilon_\nu$$

WARPED BLACK HOLES

(Beetar, MCG, Jaitly, Keseman)

Warped AdS_3

$$SL(2, \mathbb{R}) \times U(1)$$

$$ds^2 = (L_{\text{AdS}_2})^2 \left[\frac{-dt^2 + dx^2}{x^2} + \lambda^2 \left(dz + \frac{dt}{x} \right)^2 \right]$$

Squashing
parameter

Quotient by
isometry Γ



Warped Black Holes



(Aninos, Li, Padi, Song,
Strominger)

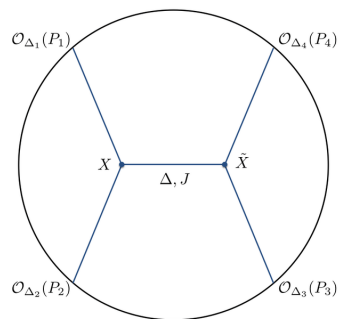
Mini-twistor space:

Showed that it can be written as Kerr-Schild

Hence, it is an exact double copy

Double Copy

$$\check{f}_{s=2, \Delta=4} = \frac{\check{f}_{s=1, \Delta=3} \check{f}_{s=1, \Delta=3}}{\check{f}_{s=0, \Delta=2}}$$



Perturbative

Momentum
space ?

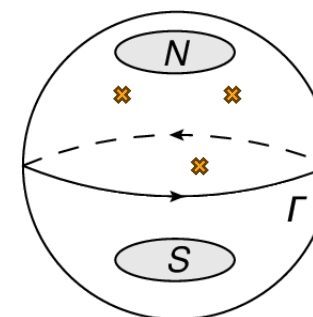
AdS_3
double copy

Coordinate
space

Mini-
twistor
space

On-shell solutions

Exact
in some cases



Special solutions



Twistor double copy for
boundary correlators in

$(A)dS_4/\text{CFT}_3$ &

$(A)dS_5/\text{CFT}_4$

Embedding Space Correlators

Costa, Penedones, Poland, Rychkov

Building blocks

$$\begin{aligned}P_{ij} &= -2P_i \cdot P_j , \\ H_{ij} &= -2((W_i \cdot W_j)(P_i \cdot P_j) - (W_i \cdot P_j)(W_j \cdot P_i)) , \\ V_i &= V_{i,jk} = \frac{(W_i \cdot P_j)(P_k \cdot P_i) - (W_i \cdot P_k)(P_j \cdot P_i)}{P_j \cdot P_k}\end{aligned}$$

Scalings

$$O_i(r_i P_i^I, q_i W_i^I) = r_i^{-\Delta} q_i^s O_i(P_i^I, W_i^I)$$

Conservation

$$\frac{\partial}{\partial P^M} \left(\left(h - 1 + W \cdot \frac{\partial}{\partial W} \right) \frac{\partial}{\partial W^M} - \frac{1}{2} W_M \frac{\partial^2}{\partial W \cdot \partial W} \right) \langle \cdots \rangle = 0$$

Three-Point Embedding Space Correlators for Conserved Currents

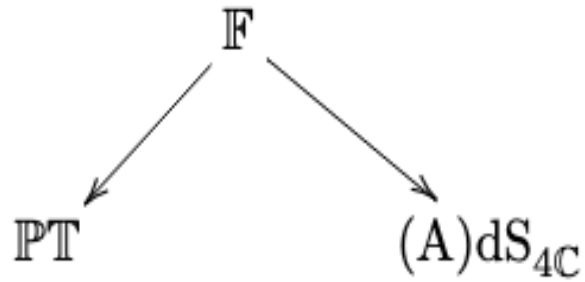
$$\langle \tilde{J}_1^{a_1} \tilde{J}_2^{a_2} \tilde{J}_3^{a_3} \rangle \propto f^{a_1 a_2 a_3} \frac{V_1 H_{23} + V_2 H_{31} + V_3 H_{12} + V_1 V_2 V_3}{(P_{12} P_{23} P_{31})^{3/2}},$$

$$\langle \tilde{T}_1 \tilde{T}_2 \tilde{T}_3 \rangle \propto \frac{(6V_1^2 H_{23}^2 + 16V_2 V_3 H_{31} H_{12} + 4H_{23} V_1^2 V_2 V_3 - 3V_1^2 V_2^2 V_3^2) + \text{cyclic}}{(P_{12} P_{23} P_{31})^{5/2}}$$

$$\begin{aligned} \langle \mathcal{O}_1^{s=3} \mathcal{O}_1^{s=3} \mathcal{O}_3^{s=3} \rangle &= \frac{c_B}{(P_{12} P_{23} P_{31})^{7/2}} \sum_{\text{cyc}} \left[\frac{1}{6} H_{12}^2 H_{13} H_{23} V_3 - \frac{1}{540} H_{12}^3 V_3^3 - \frac{1}{12} H_{12}^2 H_{13} V_2 V_3^2 \right. \\ &\quad \left. - \frac{1}{12} H_{12}^2 H_{23} V_1 V_3^2 + \frac{1}{36} H_{12}^2 V_1 V_2 V_3^3 + \frac{1}{4} H_{12} H_{13} V_1 V_2^2 V_3^2 - \frac{1}{24} H_{12} V_1^2 V_2^2 V_3^3 \right] \\ &\quad - H_{12} H_{13} H_{23} V_1 V_2 V_3 + \frac{1}{216} V_1^3 V_2^3 V_3^3. \end{aligned}$$

(A)DS₄ TWISTORS

Binder, Freedman, Pufu



Choice of infinity twistor:

$$I_{MN} = \Omega_{MN}$$

Breaks conformal sym.
down to AdS isometries

5d embedding space
massive spinor-helicity
SO(5) ~ SP(4)

$$Z^A = \sqrt{z} \delta_M^A T^{M\alpha} \lambda_\alpha = (\lambda_\alpha, \mu^{\dot{\alpha}}) \quad \mu^{\dot{\alpha}} = (x^{4d})^{\dot{\alpha}\alpha} \lambda_\alpha$$

$$W_A = \sqrt{z} \delta_A^M \bar{T}_M^{\dot{\alpha}} \tilde{\lambda}_{\dot{\alpha}} = (\tilde{\mu}^\alpha, -\tilde{\lambda}_{\dot{\alpha}}) \quad \tilde{\mu}^\alpha = (x^{4d})^{\dot{\alpha}\alpha} \tilde{\lambda}_{\dot{\alpha}}$$

5d massless spinor-helicity

$$P^{MN} = \Lambda^{Na} \Lambda_a^M$$

Boundary limit

$$\Lambda^M \equiv \lim_{z \rightarrow 0} Z^A \delta_A^M = \pi^a \Lambda_a^M$$

Complexification
of variables used in
(Baumann, Mathys,
Pimentel, Rost)

(A) DS_4 PENROSE TRANSFORM

Self-dual and anti-self-dual fields

$$H^1(\mathbb{PT}, \mathcal{O}(-2s-2))$$

$$\nabla^{\alpha_1 \dot{\alpha}_1} \phi_{\alpha_1 \dots \alpha_{2s}} = 0 \quad \phi_{\alpha_1 \dots \alpha_{2s}}^- = \oint D\lambda \lambda_{\alpha_1} \dots \lambda_{\alpha_{2s}} f^-(\lambda, \mu)|_X$$

$$\nabla^{\alpha_1 \dot{\alpha}_1} \phi_{\dot{\alpha}_1 \dots \dot{\alpha}_{2s}} = 0 \quad \phi_{\dot{\alpha}_1 \dots \dot{\alpha}_{2s}}^+ = \oint D\tilde{\lambda} \tilde{\lambda}_{\dot{\alpha}_1} \dots \tilde{\lambda}_{\dot{\alpha}_{2s}} \tilde{f}^+(\tilde{\lambda}, \tilde{\mu})|_X$$

Boundary limit gives conserved currents

$$\nabla^{a_1 a_2} J_{a_1 a_2 \dots a_{2s}} = 0 \quad J_{a_1 \dots a_{2s}} = \oint D\pi \pi_{a_1} \dots \pi_{a_{2s}} f^J(\Lambda)$$

$$O_i(r_i P_i^I, q_i W_i^I) = r_i^{-\Delta} q_i^s O_i(P_i^I, W_i^I) \quad P^{MN} = \Lambda^{Na} \Lambda_a^M \quad \Delta_c = s + 1$$

3-POINT BOUNDARY CORRELATORS

(MCG, Keseman)

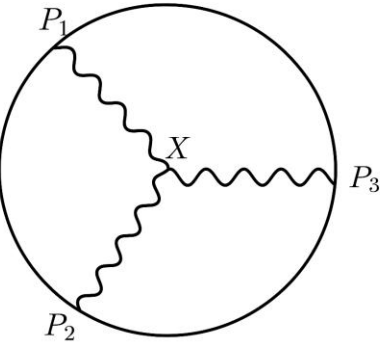
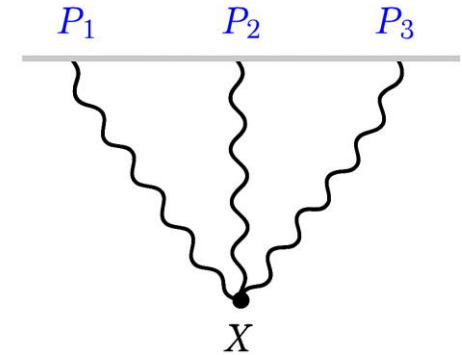
$$\Lambda^M \equiv \lim_{z \rightarrow 0} Z^A \delta_A^M = \pi^a \Lambda_a^M$$

$$F^3, W^3 \quad +++$$

$$\langle O_1^s O_2^s O_3^s \rangle = \frac{1}{(\Lambda_1 \cdot \Lambda_2)^{1+s} (\Lambda_2 \cdot \Lambda_3)^{1+s} (\Lambda_3 \cdot \Lambda_1)^{1+s}}$$

$$\text{YM, GR} \quad +- -$$

$$\langle \tilde{O}_1^{s_1} \tilde{O}_2^s \tilde{O}_3^s \rangle = \frac{(\Lambda_2 \cdot \Lambda_3)^{s-1} (\Lambda_3 \cdot \Lambda_1)^{s-1}}{(\Lambda_1 \cdot \Lambda_2)^{3s+1}} \log(\Lambda_2 \cdot \Lambda_3) \log(\Lambda_3 \cdot \Lambda_1)$$



Double copy $\check{f}_{s=2} = \frac{\check{f}_{s=1} \check{f}_{s=1}}{\check{f}_{s=0}}$

Twistor building blocks: cleanly track symmetries and their breaking

Complexification of results in (Baumann, Mathys, Pimentel, Rost)

3-POINT CORRELATORS: WARD IDENTITIES

(MCG, Keseman)

The YM, GR correlators satisfy the
inhomogeneous Ward Identity

$$\nabla_{\mu}^{x_3} \langle \tilde{J}_1 \tilde{J}_2 \tilde{J}_3^{\mu} \rangle \propto \delta^3(x_3 - x_1) \langle (\delta \tilde{J}_1) \tilde{J}_2 \rangle + \delta^3(x_3 - x_2) \langle \tilde{J}_1 (\delta \tilde{J}_2) \rangle$$

In twistor space

$$\int \langle \lambda_3 d\lambda_3 \rangle \int dS_{x_3}^{ab} \frac{\partial}{\partial \mu_3^a} \frac{\partial}{\partial \mu_3^b} f_s^{\langle \tilde{O}_1^{s1} \tilde{O}_2^s \tilde{O}_3^s \rangle}(\Lambda_1, \Lambda_2, \Lambda_3) = f_s^{\langle \tilde{O}_1^{s1} \tilde{O}_2^s \rangle}(\Lambda_1, \Lambda_2)$$

Only satisfied for complexified reps.

CFT4 AMBITWISTOR CORRELATORS

(MCG, Keseman)

$$\mathbb{A}_{d=4} = \{(Z^A, W_,) \in \mathbb{CP}^3 \times \mathbb{CP}^3 \mid Z \cdot W = 0\}$$

2pt see also
(Adamo, Skinner,
Williams)

$$H^2(\mathbb{A}, \mathcal{O}(-s-2, -\bar{s}-2)).$$

$$f(rZ, \tilde{r}W) = r^{-s-2} \tilde{r}^{-\bar{s}-2} f(Z, W)$$

3pt boundary correlator of
conserved currents

$$\Delta = 2 + \frac{s + \bar{s}}{2}$$

$$f_s = \frac{\log(t_{12}) \log(t_{23}) \log(t_{31})}{(t_{13} t_{32} t_{21})^{s+2}}$$

$$t_{ij} \equiv Z_i \cdot W_j$$

$$\psi^{s, \bar{s}}(x) = \oint_{\mathbb{CP}^1 \times \mathbb{CP}^1} D\lambda D\pi \langle \xi \lambda \rangle^s [\sigma \pi]^{\bar{s}} f(Z, W)|_X$$

$$\partial^{\beta \dot{\beta}} O_{\beta \alpha_2 \dots \alpha_s, \dot{\beta} \dot{\alpha}_2 \dots \dot{\alpha}_{\bar{s}}} = 0$$

Double
copy $\check{f}_{s=2} = \frac{\check{f}_{s=1} \check{f}_{=1}}{\check{f}_{s=0}}$

TWO-TWISTOR REPRESENTATION

(MCG, Keseman)

$$\mathbb{A}_{d=4} = \{ (Z^A, W,) \in \mathbb{CP}^3 \times \mathbb{CP}^3 \mid Z \cdot W = 0 \}$$

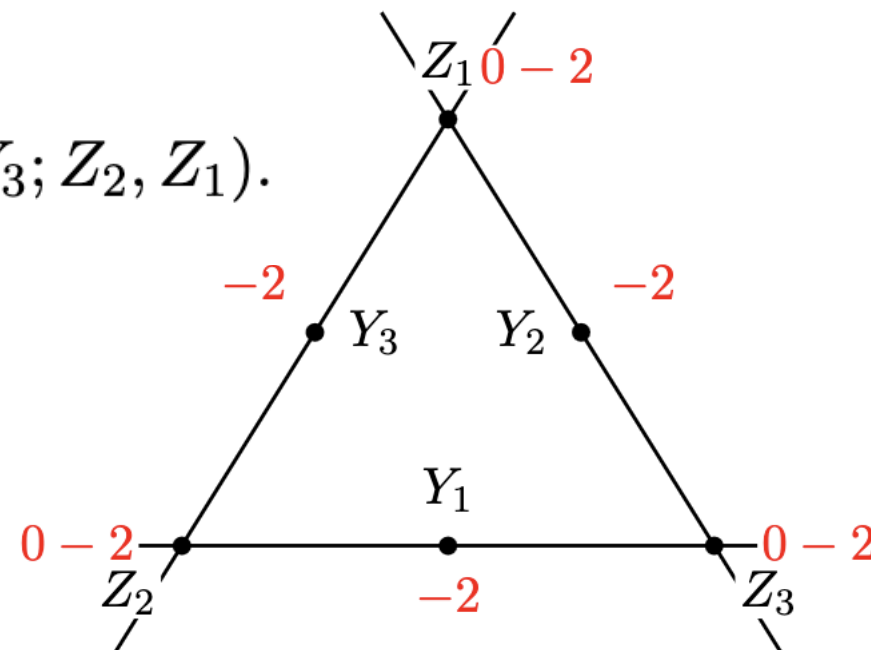
Work in split signature and
Fourier transform the dual twistor

$$\tilde{f}(Y) = \int d^4 W e^{iY \cdot W} f(W)$$

3pt scalar correlator

$$\tilde{f}_3(Z_i, Y_i) = \tilde{\delta}_{-2,0}^{(2)}(Y_1; Z_3, Z_2) \tilde{\delta}_{-2,0}^{(2)}(Y_2; Z_1, Z_3) \tilde{\delta}_{-2,0}^{(2)}(Y_3; Z_2, Z_1).$$

$$\tilde{\delta}_{p,q}^{(2)}(Z_1; Z_2, Z_3) = \int \frac{ds}{s^{1+p}} \frac{dt}{t^{1+q}} \delta^4(Z_1 - sZ_2 - tZ_3)$$



Happy 95th birthday!

