

Kovalevskaya Ladd-Franklin Mirzakhani Noether



Mathematical Institute



## Sofya Kovalevskaya (Russian Empire, 1850–1891)

Although prevented from entering university in Imperial Russia because of her gender, Sofya Kovalevskaya had a talent for mathematics and was determined to pursue higher education. She arranged a sham marriage in 1868 to circumvent laws requiring fathers or husbands to permit foreign travel, then convinced Karl Weierstrass (German mathematician, 1815–1897) to begin privately teaching her in 1870. Kovalevskaya later presented three papers in support of her doctoral thesis at the University of Göttingen. The first contained a proof of the Cauchy-Kovalevskaya Theorem, which established the existence of solutions for a general class of PDEs; the other two papers analysed abelian integrals and the dynamics of Saturn's rings. The success of her work enabled her to become the first woman to obtain a modern doctorate in mathematics, and in 1889 the first woman since Laura Bassi (Italian physicist, 1711–1778) and Maria Gaetana Agnesi (Italian mathematician, 1718–1799) to be appointed to a full professorship in Europe. Her life as a feminist, political activist, and female mathematician has made her the subject of several books, short stories, and films.

## **Christine Ladd-Franklin** (USA, 1847–1930)

Christine Ladd-Franklin was an American scholar who contributed to mathematical logic and psychology. She attended Vassar College for her undergraduate studies in the 1860s and displayed talents for physics and mathematics. Since US physics laboratories did not permit women in the 1800s, however, Ladd-Franklin chose to study mathematics. Although she was admitted to Johns Hopkins University in 1878, and even received a fellowship to fund her graduate studies, she was refused a PhD due to the university's policy against granting degrees to women. Nevertheless, she did achieve the distinction of becoming the first American woman to complete a university's requirements for a PhD in mathematics, and was eventually awarded her PhD in 1926, 44 years after submission. Ladd-Franklin was also the first woman to publish in *The Analyst* (since renamed the Annals of *Mathematics*, this journal has come to be considered one of the most prestigious in the field). Her other achievements were wide ranging and included pedagogical material, psychology research, and an evolution-based theory of colour vision.

## Maryam Mirzakhani (Iran, 1977–2017)

Maryam Mirzakhani was born in Tehran and experienced a childhood framed by the Iran-Iraq War (1980–1988). In 1994, she and Roya Beheshti (Iranian mathematician, b. 1977) became the first women to compete on the Iranian Mathematical Olympiad team. At the Olympiads of 1994 and 1995, Mirzakhani succeeded in attaining two gold medals, and even won her second with a perfect score. Following her undergraduate studies in Tehran, in 2004 she completed her PhD at Harvard with a thesis on geodesics that solved a major open problem in geometry and combinatorics. Mirzakhani is most recognised for being the first woman and first Iranian to be awarded a Fields Medal – an award widely regarded as the most prestigious in mathematics. In the press release for the award, the International Mathematical Union said of Mirzakhani: "... she embodies a rare combination of superb technical ability, bold ambition, far-reaching vision, and deep curiosity." Her life inspired the creation of the Mirzakhani Society, a group for female and non-binary students studying mathematics at Oxford.

## **Emmy Noether** (Germany, 1882–1935)

By the start of the 20th century, Emmy Noether's contributions to mathematics were a tour de force. As well as contributing to variational calculus and mathematical physics, her work drove the emergence of abstract algebra as a field in itself. She distinguished herself from her male peers by her creativity: by making











Physical model of a Kovalevskaya top, discovered by Kovalevskaya while studying the mathematical dynamics of a spinning top in a gravitational field.

Oxford Mathematics

Trigonometry .- HAMILTON does not give the quaternion equivalents for all the transformations of Trigonometry, and those which he does give are widely scattered through his works. I have found a resume of the subject fruitful in suggesting new relations between the symbols of quatern The following formulæ from the *Elements* will be made use of :--210. XI.  $S.q^2 = (Sq)^2 + (Vq)^2.$  $V.q^2 = 2Sq. Vq.$ XII.  $Tq^2 = (Sq)^2 - (Vq)^2$ XV.  $SU(q)^2 = 2(SUq)^2 - 1.$ XVII. XVIII. Sq'q = Sq'.Sq + S(Vq'.Vq).XIX. Vq'q = Vq'.Sq + Vq.Sq' + V(Vq'.Vq).274. XII.  $(TV:S)\sqrt{q} = \sqrt{[(Tq-Sq)+(Tq+Sq)]}$ .  $TV_{V}q = \sqrt{\frac{1}{2}}(Tq-Sq).$ 199. XII.  $S_V q = \sqrt{\frac{1}{2}}(Tq + Sq)$  $V\hat{\Sigma} = \Sigma V, \ S\Sigma = \Sigma S.$  $V.\gamma V\beta a = aS\beta\gamma - \beta S\gamma a.$ 210. XXIX. Tq' + Tq = T(q'+q) if q' = xq.  $S.\gamma V\beta a = \gamma\beta a - \gamma S\beta a + \beta S\gamma a - aS\beta\gamma.$ We have, for the trigonometrical functions,  $\sin \ \angle q = \ TVUq, \quad \cos \ \angle q = \ SUq$  $\sin n \angle q = I V U(q^n), \qquad \cos n \angle q = S U(q^n),$  $\tan \ \angle q = (TV:S)q.$ Putting Uq for q in (1) and taking the tensor  $SU(q^2) = (SUq)^2 + (TVUq)^2$  $\cos 2x = \cos^2 x - \sin^2 x$ ; or from (4)  $\cos 2x = 2\cos^2 x - 1.$ By treating (2) in a similar way we have  $\sin 2x = 2\sin x \cos x$  $1 = (SUq)^2 - (VUq)^2$ From (3) $1 = \sin^2 x + \cos^2 x.$ Introducing in (6) the condition that q' and q be complanar, and substitu ting versors, we have VUq'q = VUq'SUq + VUqSUq'. Taking the tensor of this equation and observing that  $VUq' \parallel VUq$ , we have, by (12), TVUq'q = TVUq'SUq + SUq'TVUq $\sin(x+y) = \sin x \cos y + \cos x \sin y.$ We have from (5), since

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The first page of Ladd-Franklin's Analyst article, published 1877.



A geodesic is a generalisation of "shortest distance" from Euclidean space to curved space. If you imagine a ladybird on a surface continually walking "forward", the path it traces will be a geodesic. Scan the above QR code to see an example.

An example of *Noether's Theorem*: the laws of motion for balls in a Newton cradle do not depend on the time from which we begin measuring them, thus there is a quantity (which we call energy) that is conserved.

Poster sources Kovalevskava: Wikipedia, mathshistory.st-andrews.ac.uk, mathwomen.agnesscott.org. Ladd-Franklin: Wikipedia, Pioneering Women in American Mathematics: The Pre-1940 PhD's by Green & LaDuke. Mirzakhani: Wikipedia, International Mathematical Union, Stanford, icm2014.org, New York Times: Maryam Mirzakhani, Only Woman to Win a Fields Medal, Dies at 40, mirzakhanisociety.org.uk

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