Oxbridge PDE Conference 2022
Mathematical Institute, University of Oxford
11th - 13th April 2022

Monday 11th April

LIVE

11:00am Registration and Refreshments — **Mezzanine**

11:25am Welcome Remarks — **L3**
Prof. Gui-Qiang Chen | Website | LinkedIn
Prof. Clément Mouhot | Website | LinkedIn

11:30am – 12:10pm **Prof. José A. Carrillo - University of Oxford** — **L3**
Website | LinkedIn

Nonlocal Aggregation-Diffusion Equations: entropies, gradient flows, phase transitions and applications

This talk will be devoted to an overview of recent results understanding the bifurcation analysis of nonlinear Fokker-Planck equations arising in a myriad of applications such as consensus formation, optimization, granular media, swarming behavior, opinion dynamics and financial mathematics to name a few. We will present several results related to localized Cucker-Smale orientation dynamics, McKean-Vlasov equations, and nonlinear diffusion Keller-Segel type models in several settings. We will show the existence of continuous or discontinuous phase transitions on the torus under suitable assumptions on the Fourier modes of the interaction potential. The analysis is based on linear stability in the right functional space associated to the regularity of the problem at hand. While in the case of linear diffusion, one can work in the L2 framework, nonlinear diffusion needs the stronger Linfty topology to proceed with the analysis based on Crandall-Rabinowitz bifurcation analysis applied to the variation of the entropy functional. Explicit examples show that the global bifurcation branches can be very complicated. Stability of the solutions will be discussed based on numerical simulations with fully explicit energy decaying finite volume schemes specifically tailored to the gradient flow structure of these problems. The theoretical analysis of the asymptotic stability of the different branches of solutions is a challenging open problem. This overview talk is based on several works in collaboration with R. Bailo, A. Barbaro, J. A. Canizo, X. Chen, P. Degond, R. Gvalani, J. Hu, G. Pavliotis, A. Schlichting, Q. Wang, Z. Wang, and L. Zhang. This research has been funded by EPSRC EP/P031587/1 and ERC Advanced Grant Nonlocal-CPD 883363.

12:15pm – 12:45pm **Renato Adolfo Velozo Ruiz - University of Cambridge** — **L3**
Website | LinkedIn

Stability of Schwarzschild for the spherically symmetric Einstein--massless Vlasov system

The Einstein--massless Vlasov system is a relevant model in the study of collisionless many particle systems in general relativity. In this talk, I will present an upcoming stability result for the exterior of Schwarzschild as a solution of this system assuming spherical symmetry. We exploit the normal hyperbolicity of the set of trapped null geodesics to obtain quantitative decay estimates for the stress energy momentum tensor of matter. The main result requires precise estimates of radial derivatives of the energy momentum tensor, which we study by using Jacobi fields on the mass-shell in terms of the Sasaki metric.

12:50pm – 2:10pm Group Photo + Networking Lunch — **Mezzanine**

2:15pm – 2:55pm **Prof. Costante Bellettini - University College London** — **L3**
Website

Existence of hypersurfaces with prescribed-mean-curvature

Let N be a compact Riemannian manifold of dimension 3 or higher, and g a Lipschitz non-negative (or non-positive) function on N. We prove that there exists a closed hypersurface M

whose mean curvature attains the values prescribed by g (joint work with Neshan Wickramasekera, Cambridge). Except possibly for a small singular set (of codimension 7 or higher), the hypersurface M is C^2 immersed and two-sided (it admits a global unit normal); the scalar mean curvature at x is g(x) with respect to a global choice of unit normal. More precisely, the immersion is a quasi-embedding, namely the only non-embedded points are caused by tangential self-intersections: around such a non-embedded point, the local structure is given by two disks, lying on one side of each other, and intersecting tangentially (as in the case of two spherical caps touching at a point). A special case of PMC (prescribed-mean-curvature) hypersurfaces is obtained when g is a constant, in which the above result gives a CMC (constant-mean-curvature) hypersurface for any prescribed value of the mean curvature.

The construction of M is carried out largely by means of PDE principles: (i) a mountain pass construction for an Allen--Cahn energy, involving a parameter that, when sent to 0, leads to an interface from which the desired PMC hypersurface is extracted; (ii) quasi-linear elliptic PDE and geometric-measure-theory arguments, to obtain regularity conclusions for said interface; (iii) parabolic semi-linear PDE (together with specific features of the Allen-Cahn framework), to tackle cancellation phenomena that can happen when sending to 0 the Allen-Cahn parameter.

3:00pm – 3:40pm **Prof. Manuel Del Pino - University of Bath** **L3**

Website | LinkedIn

Interacting concentrated vorticities in incompressible Euler flows

A classical problem that traces back to Helmholtz and Kirchhoff is the understanding of the dynamics of solutions to the Euler equations of an inviscid incompressible fluid when the vorticity of the solution is initially concentrated near isolated points in $2d$ or vortex lines in $3d$. We discuss some recent results on these solutions' existence and asymptotic behaviour. We describe, with precise asymptotics, interacting vortices, and travelling helices. We rigorously establish the law of motion of "leapfrogging vortex rings", originally conjectured by Helmholtz in 1858

3:45pm – 4:10pm Refreshments **Mezzanine**

4:15pm – 4:55pm **Prof. Felix Schulze – University of Warwick** **L3**

Website

Initial stability estimates for Ricci flow and three dimensional Ricci-pinched manifolds

We investigate the question of stability for a class of Ricci flows which start at possibly non-smooth metric spaces. We show that if the initial metric space is Reifenberg and locally bi-Lipschitz to Euclidean space, then two solutions to the Ricci flow whose Ricci curvature is uniformly bounded from below and whose curvature is bounded by c t^{-1} converge to one another at an exponential rate once they have been appropriately gauged. As an application, we show that smooth three dimensional, complete, uniformly Ricci-pinched Riemannian manifolds with bounded curvature are either compact or flat, thus confirming a conjecture of Hamilton and Lott. This is joint work with A. Deruelle and M. Simon

5.00pm – 5:30pm **Catalina Pesce Reyes - University of Oxford** **L3**

Website | LinkedIn

How do degenerate mobilities determine singularity formation in Cahn-Hilliard equations?

Cahn-Hilliard models are central for describing the evoluNon of interfaces inphase separaNon processes and free boundary problems. In general, they have non-constant and o`en degenerate mobiliNes. However, in the la]er case, the spontaneous appearance of points of vanishing mobility and their impact on the soluNon are not well understood.

In this talk I will present the work done by myself and Andreas Münch on the axially symmetric two dimensional seang. We develop a singular perturbaNon theory to idenNfy a range of degeneracies for which the soluNon of the Cahn-Hilliard equaNon forms a singularity |u|-> 1 in infinite Nme. Moreover, we obtain two different approximate self-similar profiles which divide this range of degeneracies into 1/2 < n <2 and 2<n. We also draw a parallel with a related result for a one dimensional thin-film equaNon.

5:35pm – 6:05pm **Paul Minter - University of Cambridge** **L3**

Website | LinkedIn

Branch points in stable varifolds

Almgren introduced the notion of varifold in the 1960's to find critical points of the area functional in general closed Riemannian manifolds. However, aside from Allard's regularity theory --- which gives that the smoothly embedded part of a stationary integral varifold is open and dense in the support of the varifold --- very little is known regarding the regularity of these stationary integral varifolds. In particular, nothing is known regarding the dimension of the singular set. A key difficulty lies within understanding a certain type of degenerate singularity known as a branch point. Very few techniques are known for understanding branch points; Almgren was able to prove dimension bounds on the branch set for area minimisers in part by utilising his frequency function; almost all other results in the regularity theory of the area functional need to a priori rule out branch points. In this talk, I will discuss a novel new approach for understanding branch points using the frequency function instead as a means for proving an epsilon-regularity theorem for a certain class of stable varifolds. This result has applications to understanding branch points in area minimising hypercurrents mod p. This talk is based on work joint with N. Wickramasekera.

6.30pm Drinks Reception + Dinner ******

Somerville College (Directions)

Tuesday 12th April

9:00am – 9:25am Networking Coffee **Mezzanine**

9:30am – 10:10am **Prof. Clément Mouhot - University of Cambridge** **L3**

Website | LinkedIn

Quantitative Hydrodynamic Limits of Stochastic Lattice Systems

In this talk, I will present a simple abstract quantitative method for proving the hydrodynamic limit of interacting particle systems on a lattice, both in the hyperbolic and parabolic scaling. In the latter case, the convergence rate is uniform in time. This "consistency-stability" approach combines a modulated Wasserstein-distance estimate comparing the law of the stochastic process to the local Gibbs measure, together with stability estimates a la Kruzhkov in weak distance, and consistency estimates exploiting the regularity of the limit solution. It avoids the use of "block estimates" and is self-contained. We apply it to the simple exclusion process, the zero range process, and the Ginzburg-Landau process with Kawasaki dynamics. This is a joint work with Daniel Marahrens and Angeliki Menegaki (IHES).

10:15am – 10:55am **Dr Michele Coti Zelati – Imperial College London** **L3**

Website | LinkedIn

Quantitative stability results in fluid mechanics and kinetic theory

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We discuss recent results on the nonlinear asymptotic stability of certain stationary solutions to fluids and kinetic equations. We show how various modifications of classical techniques involving vector fields and hypocoercivity can be used to obtain sharp quantitative stability results. Examples include the Navier-Stokes equations, the Boltzmann equations and models describing active particles and suspensions.

11:00am – 11:30am **Christopher Irving - University of Oxford** **L3**

Website

Quasiconvexity in the general growth setting

The quasiconvexity condition of Morrey plays a fundamental role in the calculus of variations, both in establishing the existence and partial regularity of minima in the vector-valued setting. I will discuss some recent and ongoing work on variational problems satisfying a quasiconvexity and general growth condition, which will include the degenerate case when the integrand grows 'almost linearly.'

11:35am – 12:00pm Refreshments **Mezzanine**

12:05pm – 12:35pm **Assoc. Prof. Jingwei Hu - INI / University of Washington** **L3**

Website | LinkedIn

Fast Fourier Spectral Methods for Nonlinear Boltzmann Kinetic Equations — Algorithm and Analysis

Boltzmann equation is one of the central equations in kinetic theory and finds applications in many science and engineering disciplines. Numerical approximation of the Boltzmann equation is a challenging problem due to its high-dimensional, nonlocal, and nonlinear collision integral. I will discuss recent progress on development of fast Fourier-Galerkin spectral methods for Boltzmann type collisional kinetic equations. I will also present a new stability and convergence result of the method.

12:40pm – 1:20pm **Assoc. Prof. Cyril Imbert - INI / École Normale Supérieure Paris** **L3**

Website

Decay estimates for large velocities in the Boltzmann equation without cut-off

We consider solutions to the full (spatially inhomogeneous) Boltzmann equation with periodic spatial conditions for hard and moderately soft potentials without the angular cutoff assumption, and under the a priori assumption that the main hydrodynamic fields, namely the local mass and local energy and local entropy, are controlled along time. We establish quantitative estimates of propagation in time of "pointwise polynomial moments", i.e. $\sup_{x,v} f(t,x,v)(1+|v|)^q$, $q \geq 0$. In the case of hard potentials, we also prove appearance of these moments for all $q \geq 0$. In the case of moderately soft potentials we prove the appearance of low-order pointwise moments. Joint work with Clément Mouhot and Luis Silvestre.

1:25pm – 2:05pm **Assoc. Prof. Maria Pia Gualdani - INI / The University of Texas at Austin** **L3**

Website | LinkedIn

Global regularity estimates for the homogeneous Landau equation.

Kinetic equations describe evolution of interacting particles. The most famous kinetic equation is the Boltzmann equation: formulated by Ludwig Boltzmann in 1872, this equation describes motion of a large class of gases. Later, in 1936, Lev Landau derived a new mathematical model for motion of plasma. One of the main features of the Landau equation is nonlocality, meaning that particles interact at large, non-infinitesimal length scales. In this talk I will focus on the homogeneous Landau equation, and on the question of global regularity vs finite-time blow-up for smooth initial data. This is a joint work with Nestor Guillen.

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2:05pm – 3:20pm	Networking Lunch	**Mezzanine**
3:25pm – 3:55pm	**Prof. François Golse - INI / École Polytechnique** Website **From the N-body Schrödinger equation to the Euler-Poisson system** *This talk discusses the validity of the joint mean-field and classical limit of the bosonic quantum N-body dynamics leading to the pressureless Euler-Poisson system for any factorized initial data whose first marginal has a monokinetic Wigner measure. The interaction is given by the repulsive Coulomb potential. The validity of this derivation is limited to finite time intervals on which the Euler-Poisson system has a rapidly decaying at infinity, smooth solution. One key ingredient in the proof is a remarkable inequality proved by S. Serfaty (Duke Math. J. 169 (2020), 2887–2935).* *Joint work with Thierry Paul (CNRS & Sorbonne université)*	**L3**
4:00pm – 4:30pm	**Prof. Emer. Claude Bardos – INI / Universidad de París VII Denis Diderot** Website | LinkedIn **Zero viscosity limit for solutions of the Navier Stokes Equations in a domain with curved boundary and no slip boundary condition.** *This is a report on a work done with Toan Nguyen, Trinh Nguyen and Edriss Titi. Starting from the Kato theorem and using recent contributions of Mayekawa, Nguyen and Nguyen and Kukavica, Vicol and Wang we prove the convergence to the solution of the Euler equation for short time and analyticity hypothesis concerning the boundary and the initial data. Proofs can be adapted both to interior , exterior or non simply connected domain.*	**L3**
4:35pm	Visit Oxford (in-person and online)	**

Wednesday 13th April

9:00am – 9:25am	Networking Coffee	**Mezzanine**
9:30am – 10:10am	**Dr Matthew Schrecker - University College London** Website **Self-similar gravitational collapse for Newtonian star** *The Euler-Poisson equations give the classical model of a self-gravitating star under Newtonian gravity. It is widely expected that, in certain regimes, initially smooth initial data may give rise to blow-up solutions, corresponding to the collapse of a star under its own gravity. In this talk, I will present recent work with Yan Guo, Mahir Hadzic and Juhi Jang that demonstrates the existence of smooth, radially symmetric, self-similar blow-up solutions for this problem. At the heart of the analysis is the presence of a sonic point, a singularity in the self-similar model that poses serious analytical challenges in the search for a smooth solution.*	**L3**
10:15am – 10:55am	**Dr Zoe Wyatt - University of Cambridge** Website **Coupled wave and Klein-Gordon equations in two and three spatial dimensions** *Semilinear wave equations in three spatial dimensions with wave--wave nonlinearities exhibit interesting and well-studied phenomena: from John's famous blow-up examples, to*	**L3**

the null condition of Christodoulou and Klainerman, and more recently to the weak null condition of Lindblad and Rodnianski. The study of coupled semilinear wave and Klein-Gordon equations is less well-developed, and interesting problems occur across the possible spectrum of wave--wave, wave--KG and KG--KG interactions. In this talk I will discuss some recent results, in collaboration with Shijie Dong (SUSTech), on such mixed systems. This includes a recent proof of small data global-existence and sharp asymptotics for a Dirac--Klein-Gordon system in two spatial dimensions.

11:00am – 11:40am **Alexis Michelat - University of Oxford** **L3**

Website

Quantization of the Willmore Energy in Riemannian Manifolds

The integral of mean curvature squared is a conformal invariant of surfaces reintroduced by Willmore in 1965 whose study exercised a tremendous influence on geometric analysis and most notably on minimal surfaces.

Amongst the challenging problems raised by the associated fourth order non-linear elliptic partial differential equation, the question of compactness of sequences of critical immersions (called Willmore immersions) with uniformly bounded energy is one of the most natural ones. We show that Bernard-Rivière's energy quantization theorem generalises under similar assumptions to closed Riemannian manifolds, and we will explain during this presentation how (approximate) conservation laws and Lorentz (or Orlicz) spaces play a central role in the analysis of similar problems, ranging from harmonic maps to the Ginzburg-Landau functional.

Joint work with Andrea Mondino (University of Oxford).

11:45am Concluding Remarks **L3**

12:10pm Close ******

Organisers

Prof. Gui-Qiang G. Chen (University of Oxford)

Prof. Endre Suli (University of Oxford)

Prof. Clément Mouhot (University of Cambridge)

Prof. Neshan Wickramasekera (University of Cambridge)

All lectures will take place live in-person and online.

Access to the programme with broadcast link will be available to participants who have registered in advance.

Please direct any enquiries to

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Conference website https://www.maths.ox.ac.uk/node/40583