Deutsche Bank

Price Signatures

"Electronic FX trading – where Game Theory meets Data Science"

Frontiers in Quantitative Finance Seminar Series

London, May 2019

Roel Oomen

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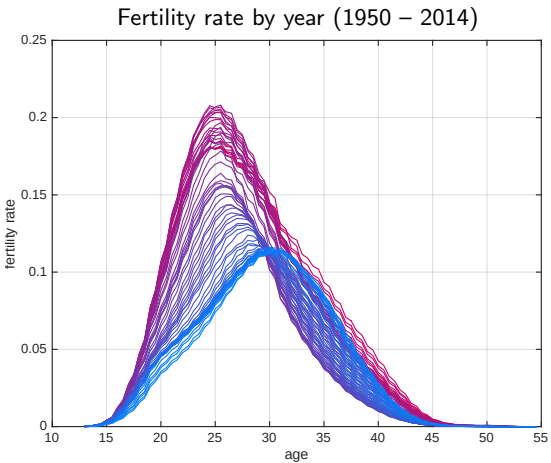
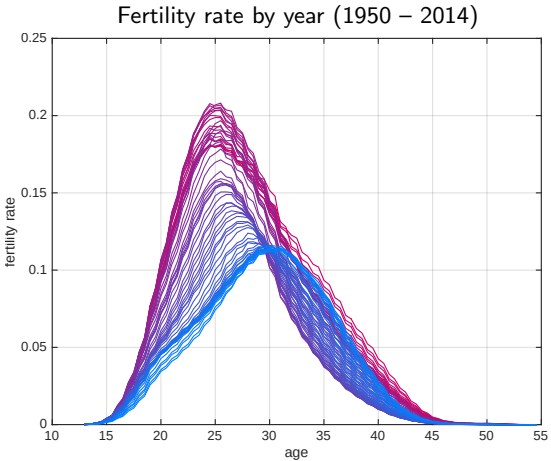
- Reform of young criminals
- Osteoarthritis
- ADHD
- Lip acceleration
- Gender-neutralising exam conditions

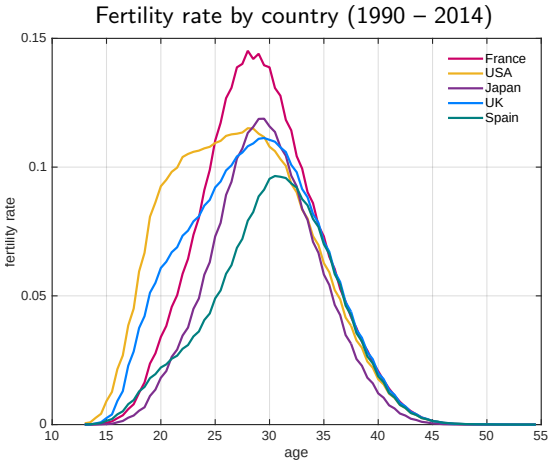
The fertility rate



Source: http://www.humanfertility.org.

The fertility rate





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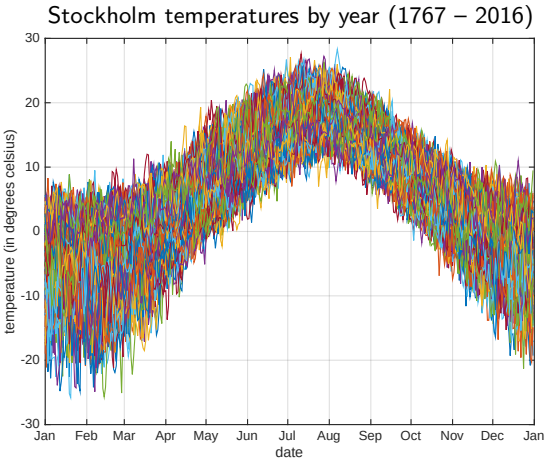

Source: https://bolin.su.se/data/stockholm.

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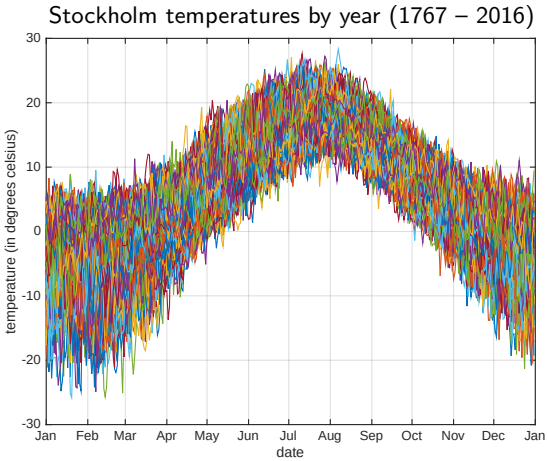

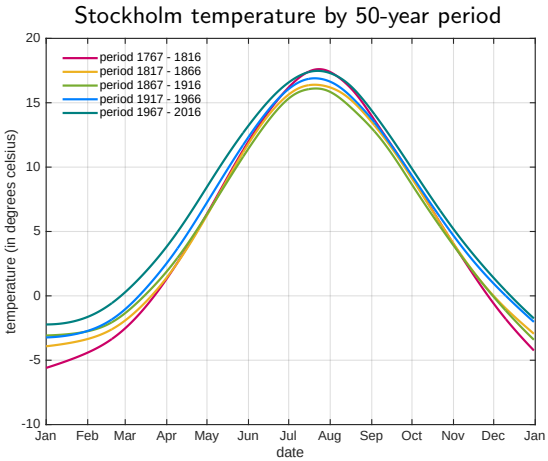

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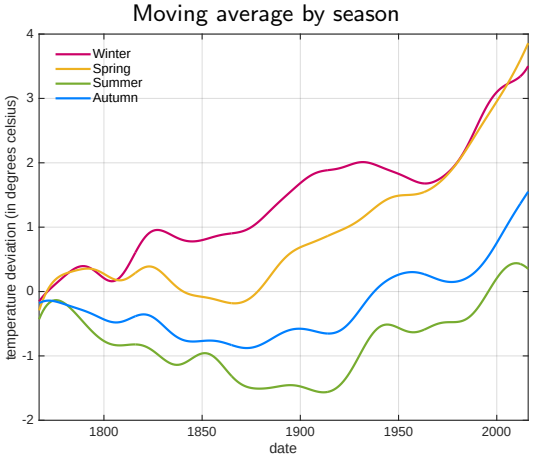
The temperature in Stockholm (cont'd)



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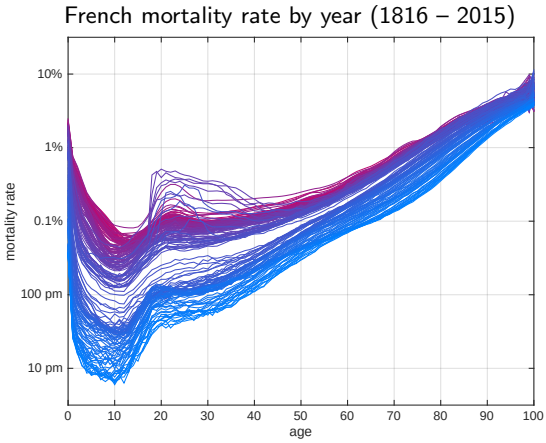
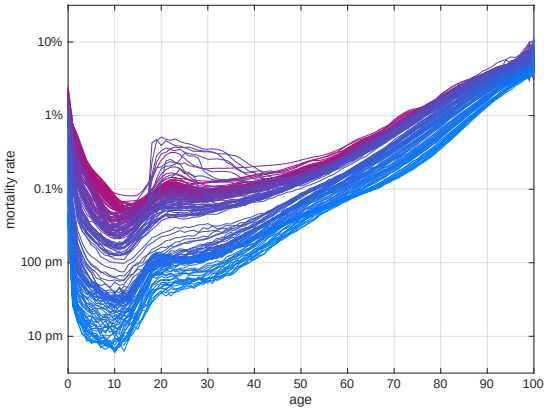
The mortality rate



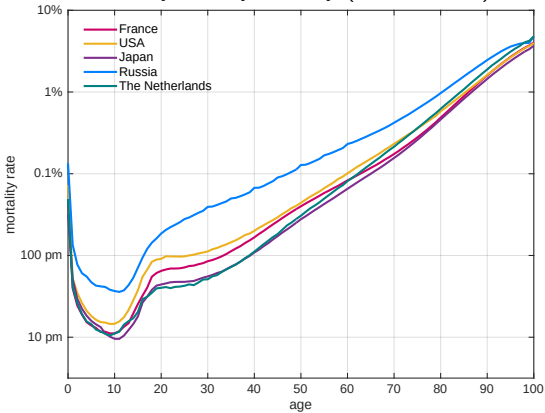
Source: http://www.mortality.org.

The mortality rate

French mortality rate by year (1816 – 2015)



mortality rate by country (1990 – 2014)



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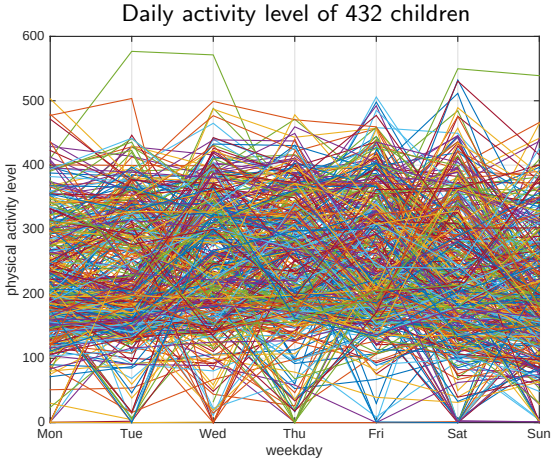
7-year old's physical activity level



I acknowledge: the Centre for Longitudinal Studies, Institute of Education for the use of these data; the UK Data Service for making them available; the MRC Centre of Epidemiology for Child Health (Grant reference G0400546), Institute of Child Health, University College London for creating the accelerometer data resource which was funded by the Wellcome Trust (grant reference 084686/Z/08/A). The institutions and funders acknowledged bear no responsibility for the analysis or interpretation of these data.

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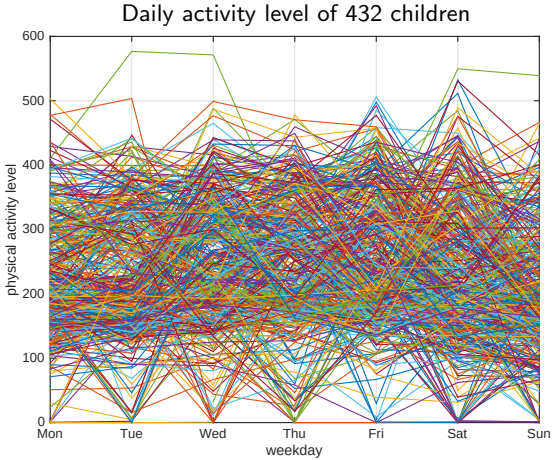

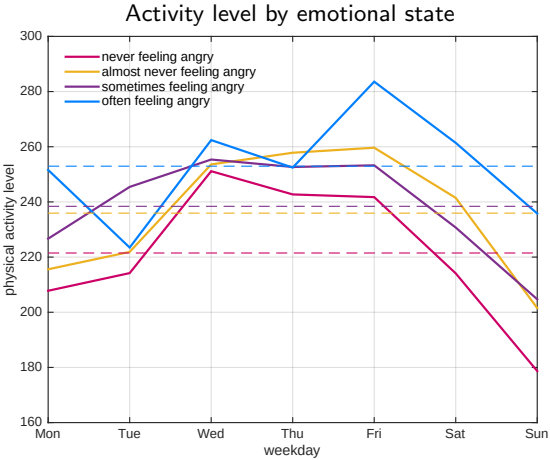

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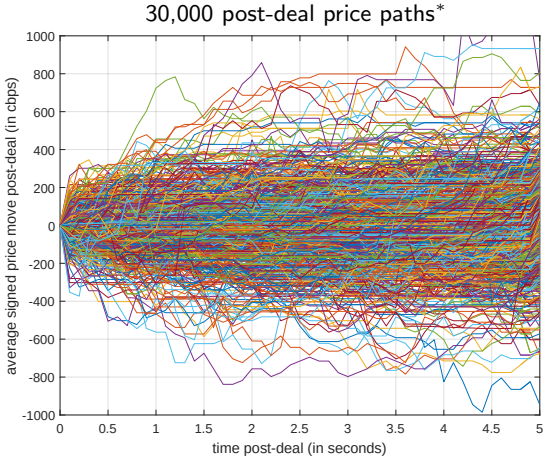
Market impact



Source: Deutsche Bank.

∗ Chart draws only a stratified subset of the full sample. Paths are signed for direction of trade.

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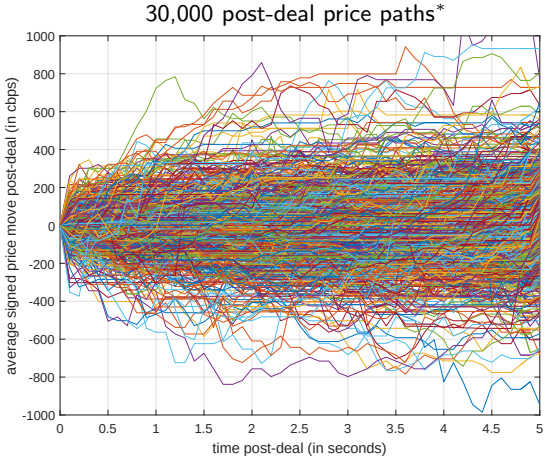

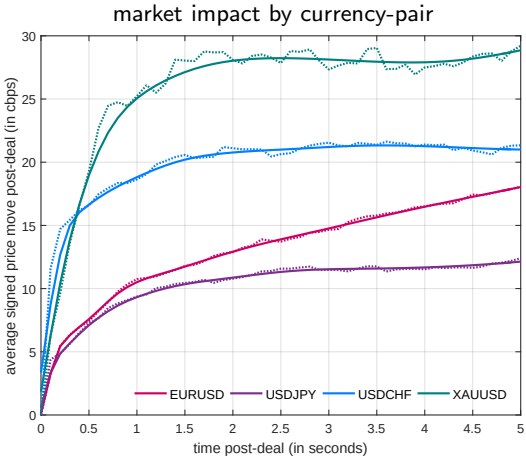

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All these examples can be studied using Functional Data Analysis or FDA

The price signature

I define a price "signature" as:

$$S(\delta) = \frac{1}{q'\iota} \sum_n q_n d_n (P_{t_n+\delta} - P_{t_n}), \qquad \text{for } \delta \in [-\underline{\delta}, \overline{\delta}].$$

It is the *volume weighted (q), trade direction adjusted (d), average price movement, over an interval (δ) centred around the point of trading (t).*

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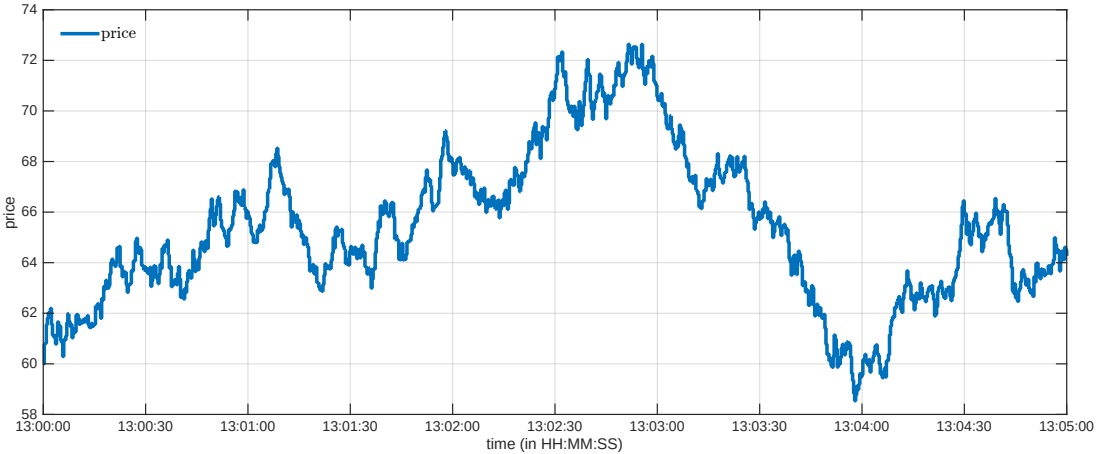
  - ... by time of the day, by trader / user, etc
- it can be applied more generally
  - ... to quotes, to rejects, to hypothetical backtest trading signals, etc
  - ... to construct volume signatures, spread signatures, liquidity or activity signatures, etc

Signature construction



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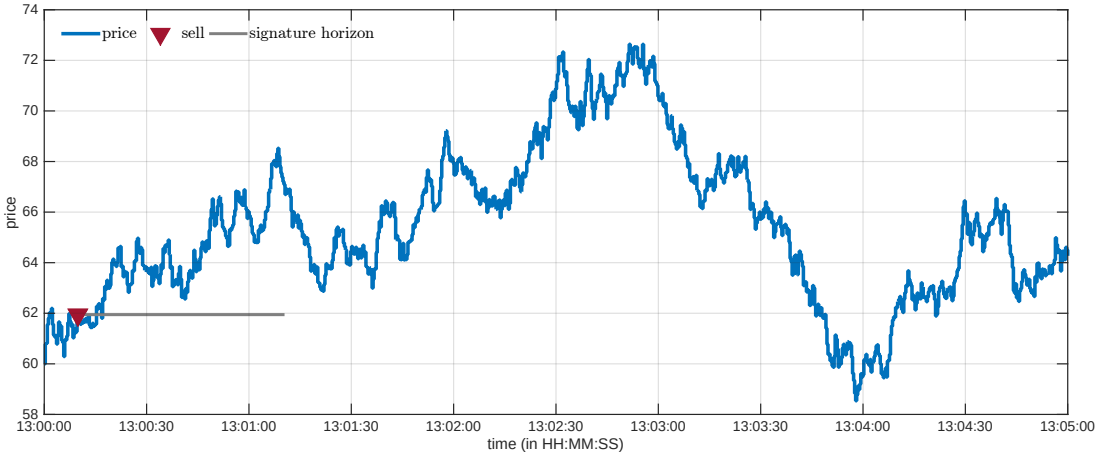

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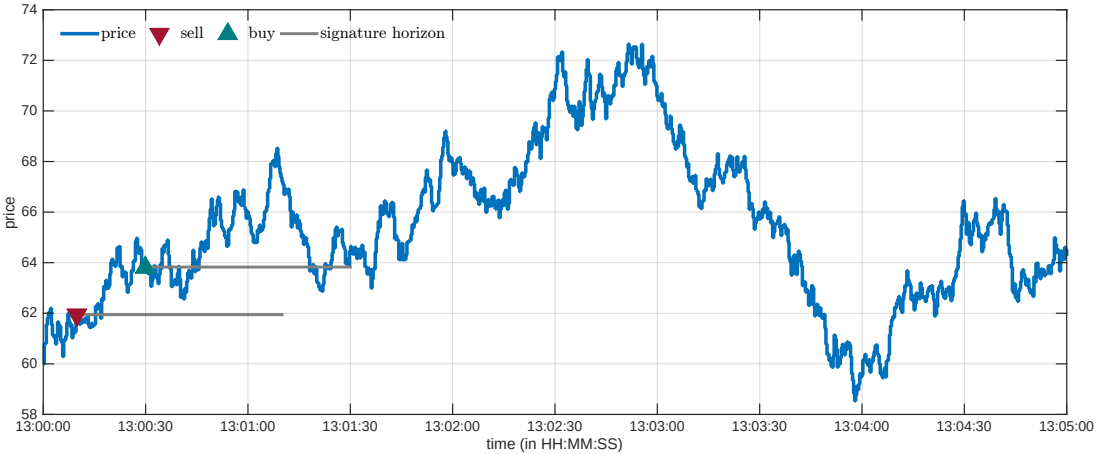

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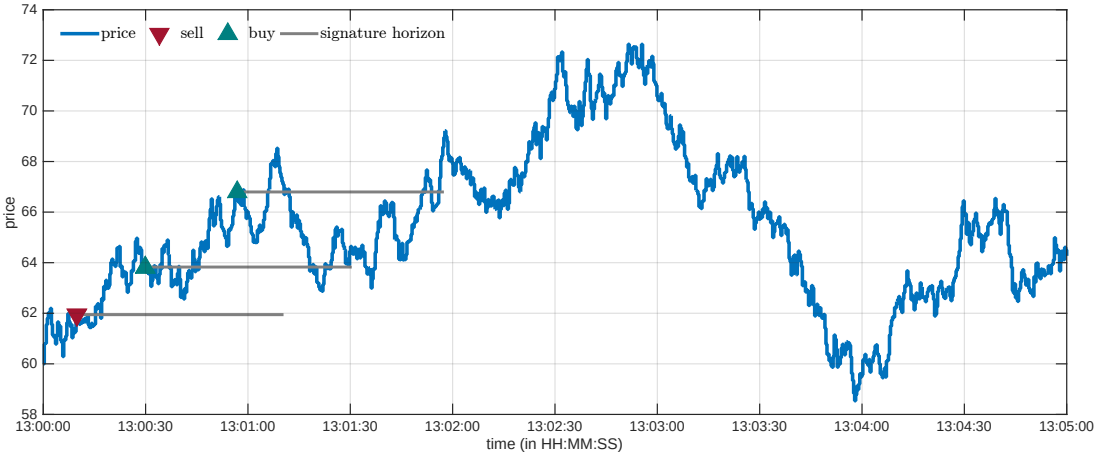

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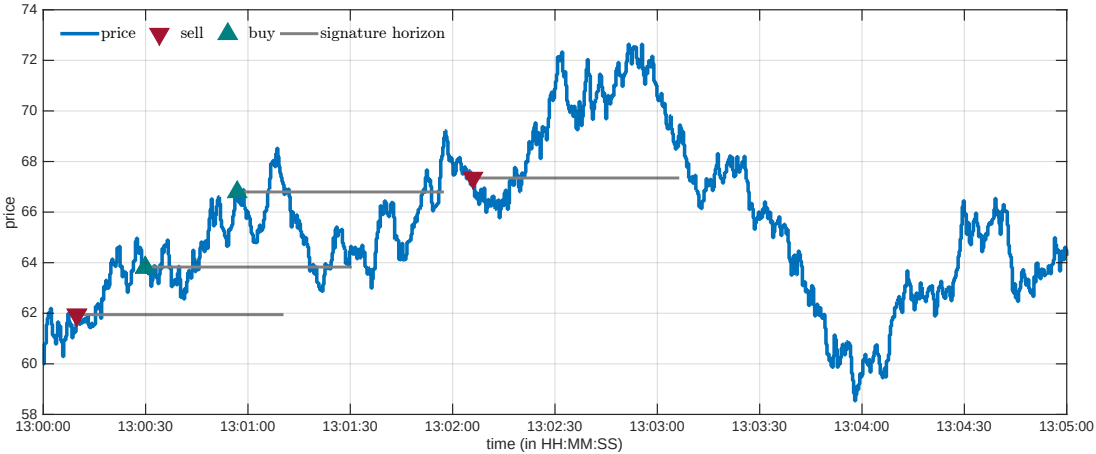

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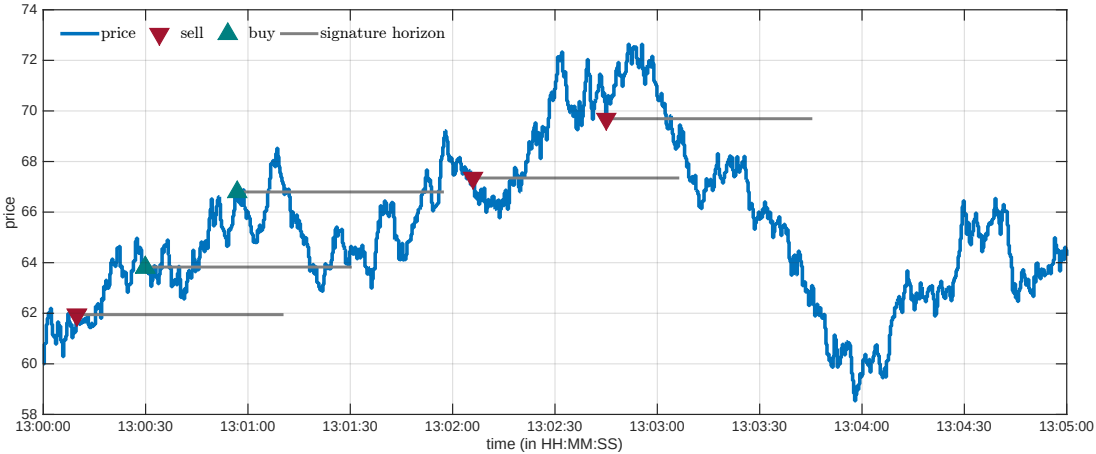

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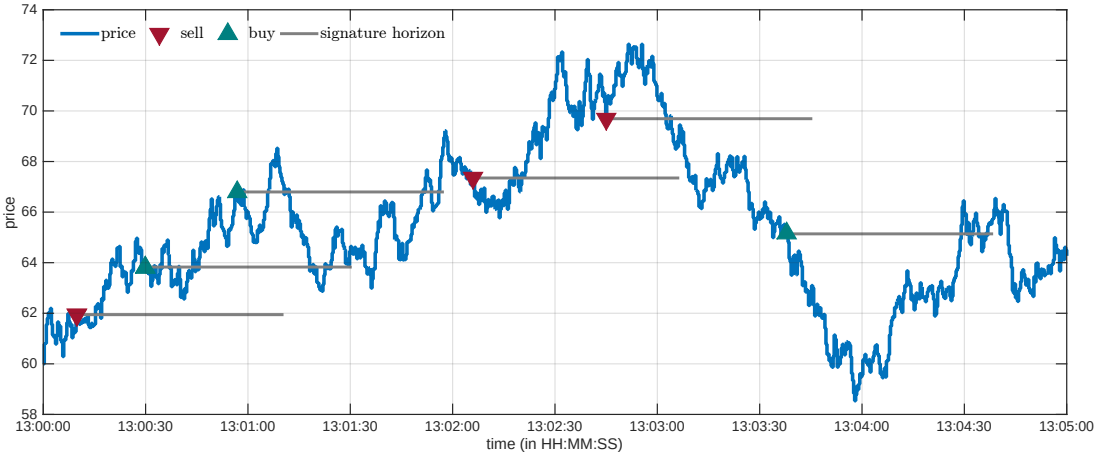

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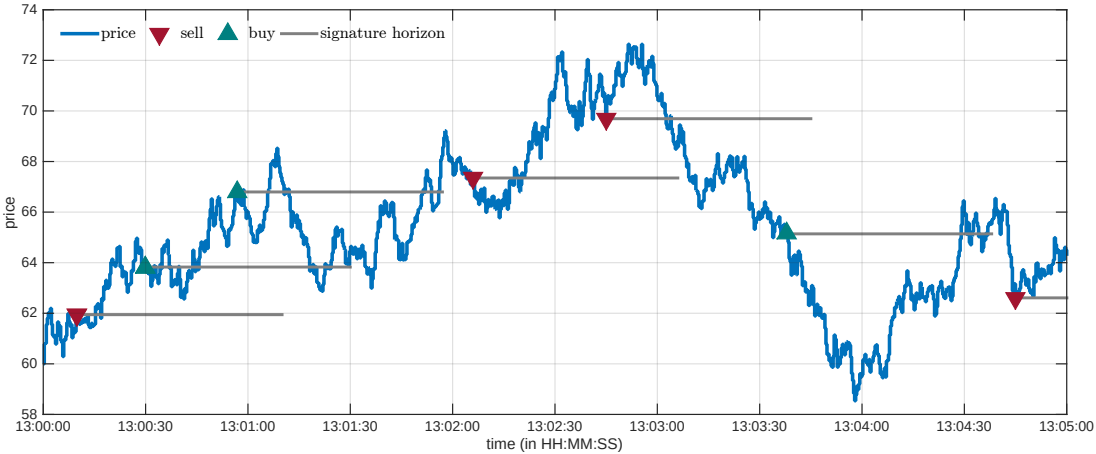
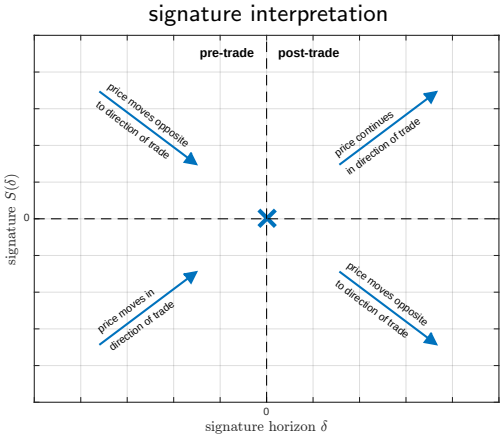


Signature construction

Signature interpretation



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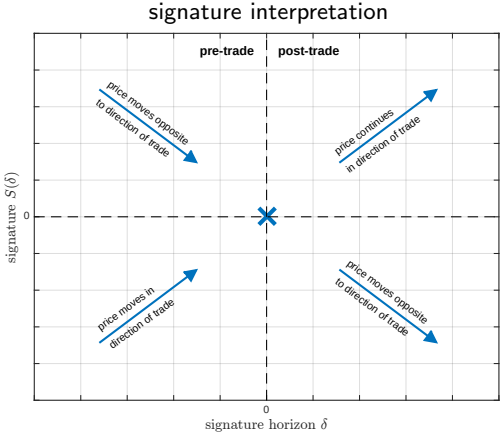

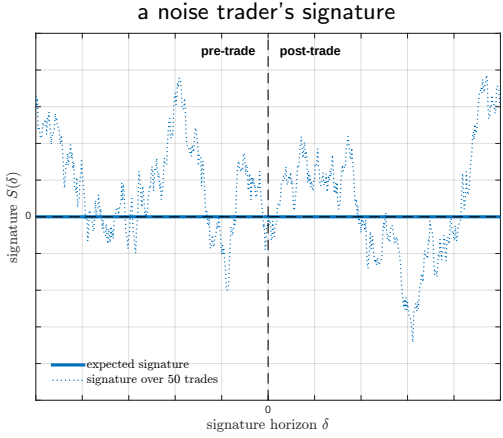

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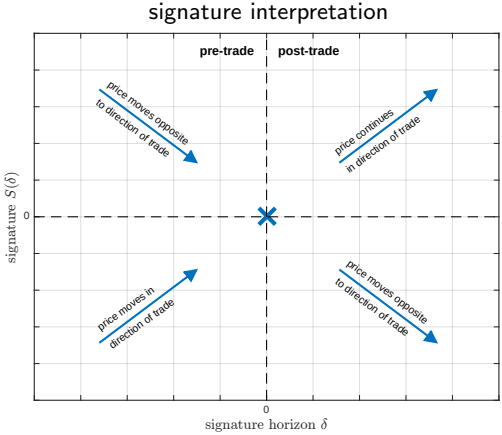

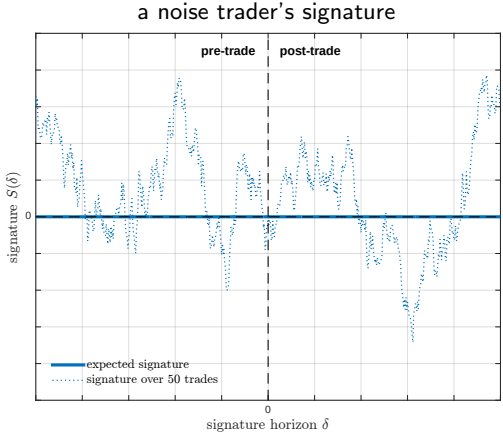

- post-deal ($\delta > 0$), the signature measures the marked-to-market revenues or margin

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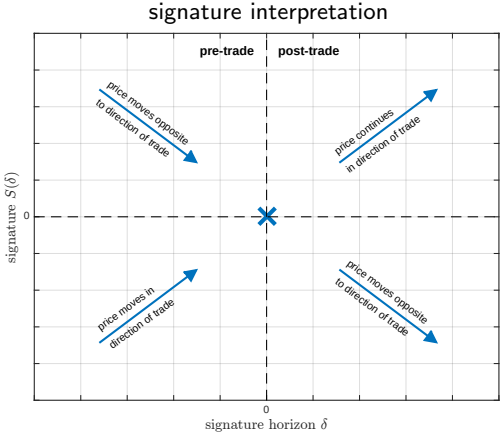
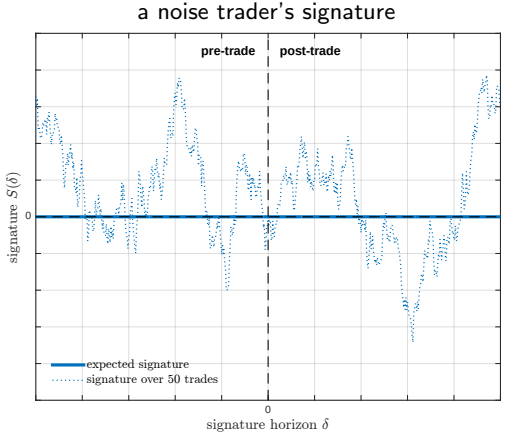
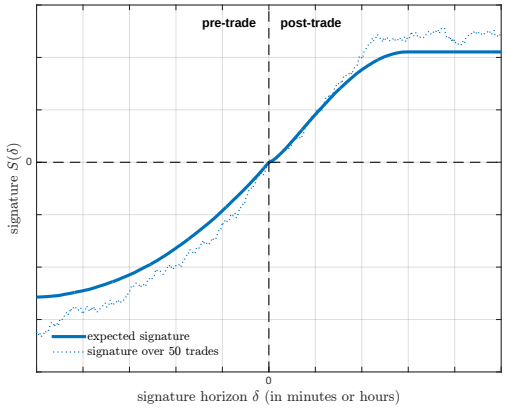


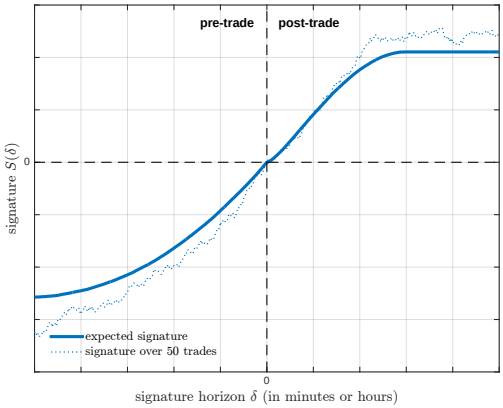


- post-deal ($\delta > 0$), the signature measures the marked-to-market revenues or margin
- pre-deal ($\delta < 0$), the signature measures the opportunity cost of not having traded earlier

Signature examples at macroscopic level

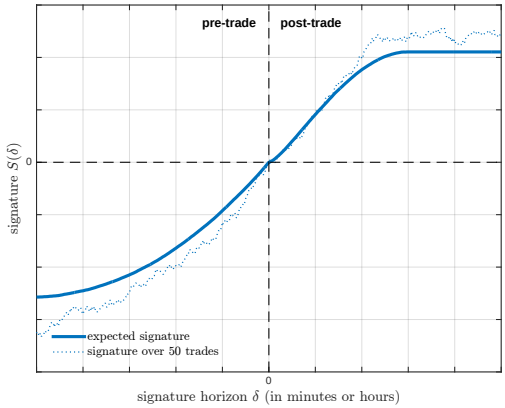


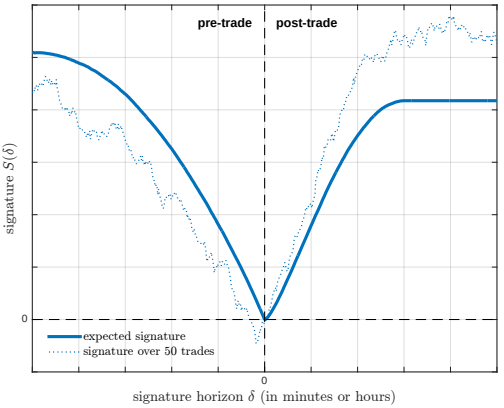
Signature examples at macroscopic level



Momentum strategy

Signature examples at macroscopic level





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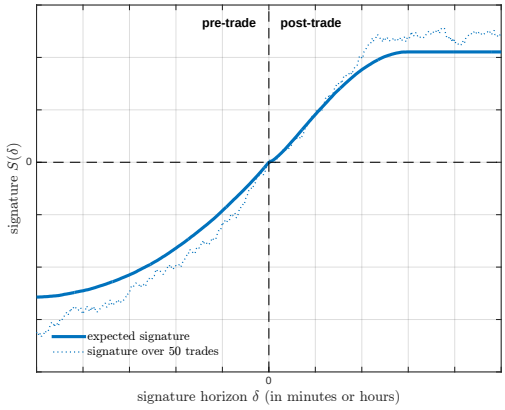

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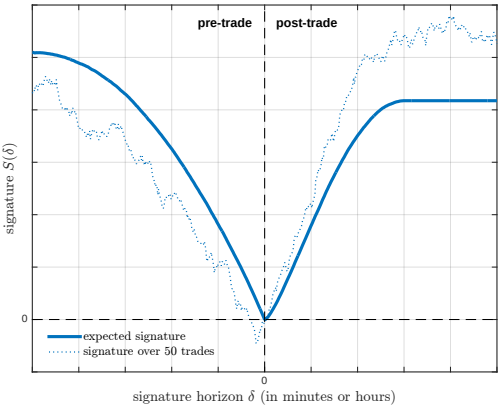
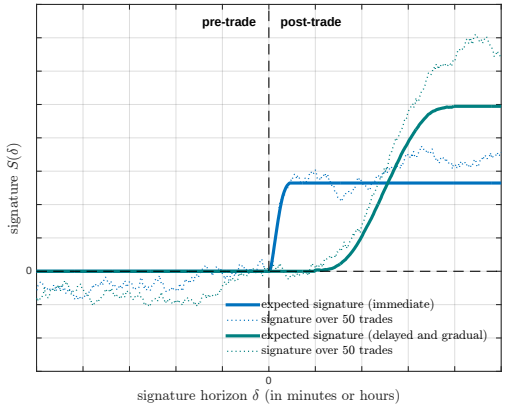
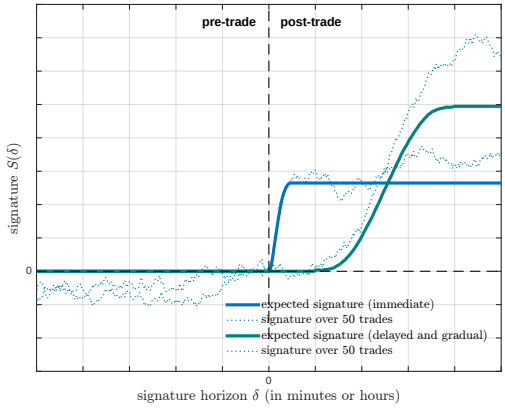


Reversal strategy

Signature examples at macroscopic level

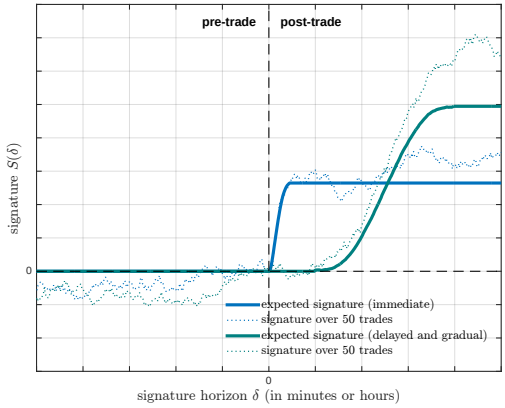


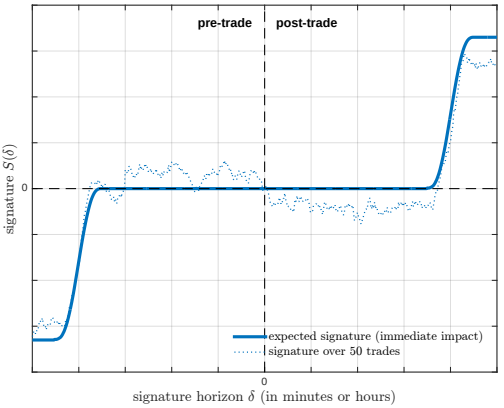
Signature examples at macroscopic level



Alpha / impact

Signature examples at macroscopic level





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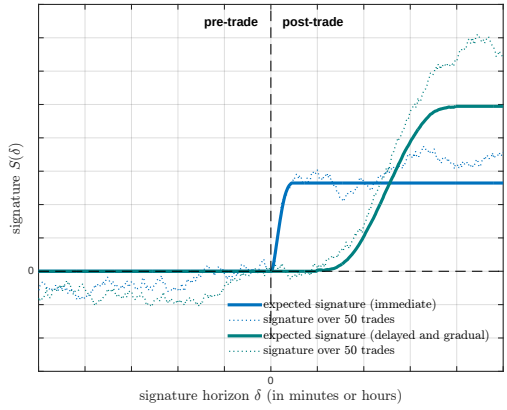

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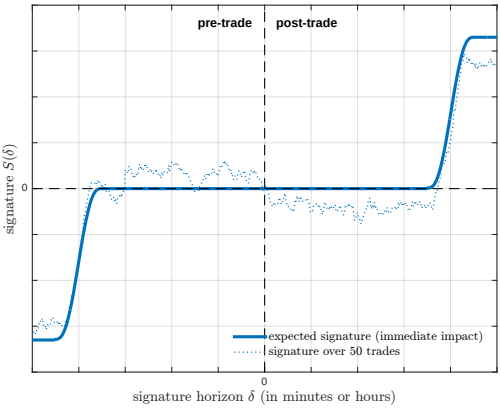
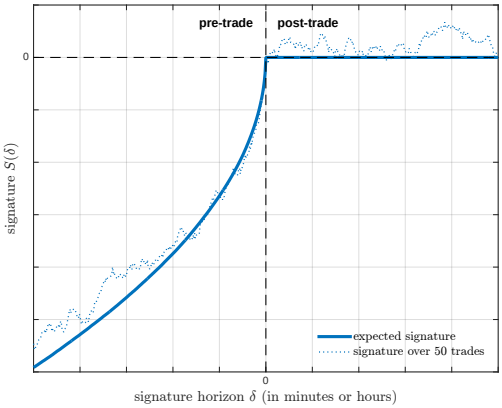
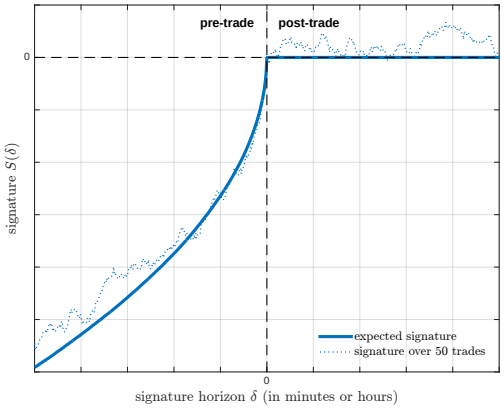


Sequential delayed impact

Signature examples at macroscopic level



Signature examples at macroscopic level



Stop-loss order

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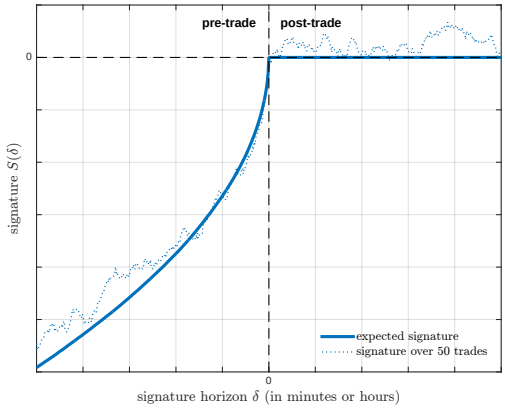

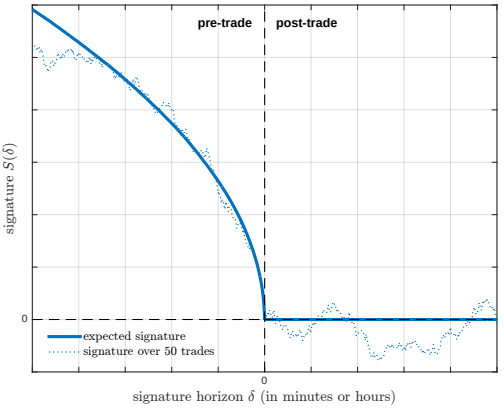

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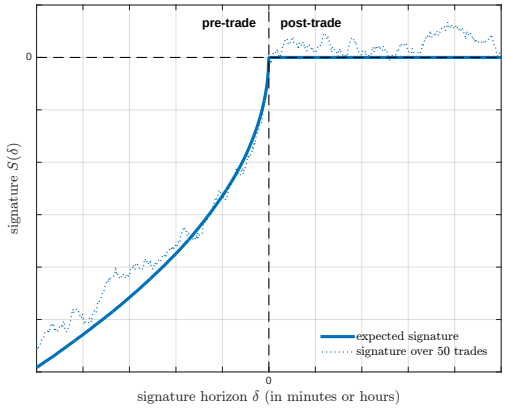

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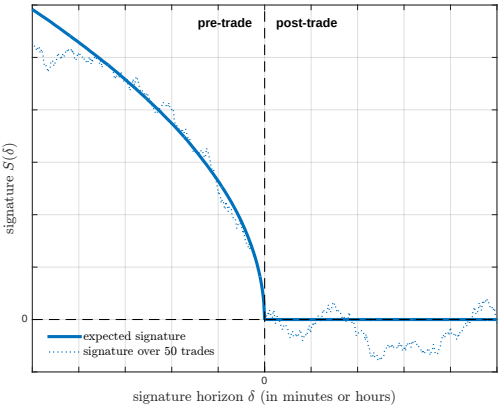

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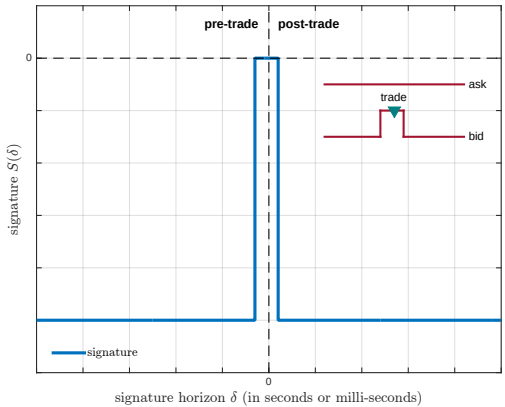

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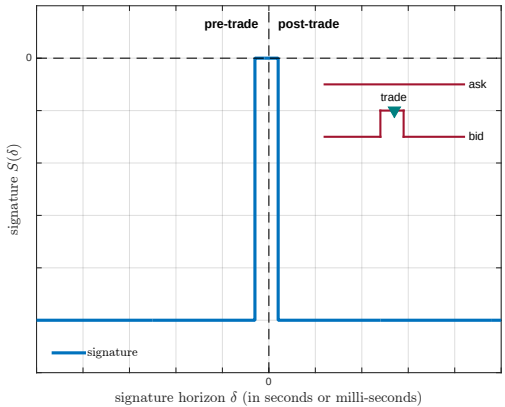

Adverse selection

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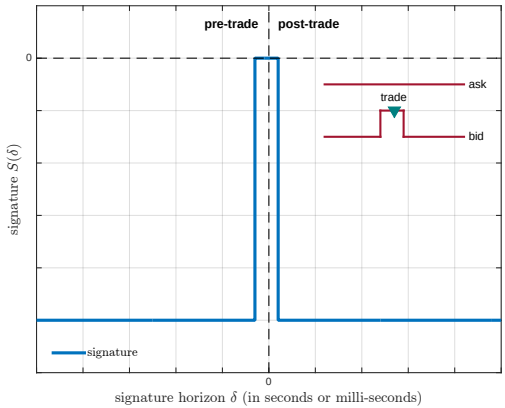

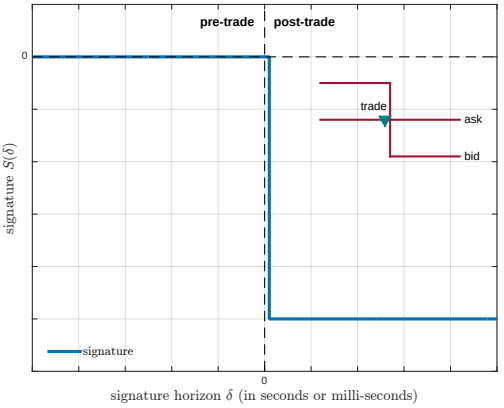

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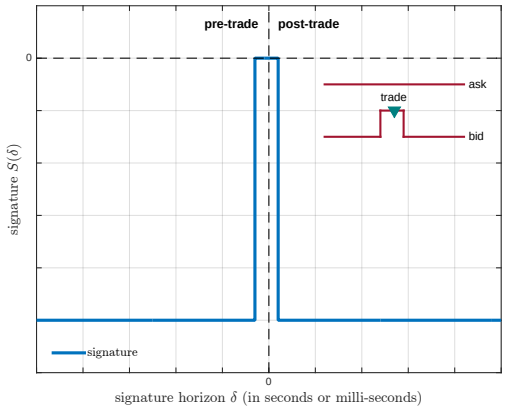

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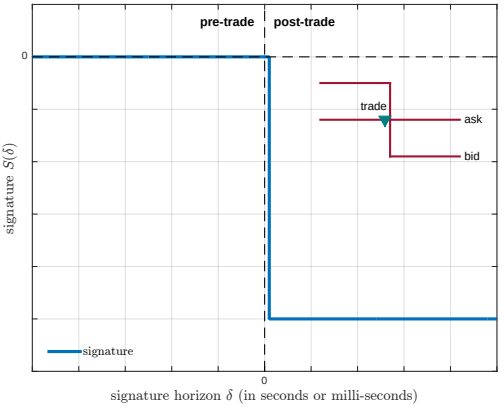
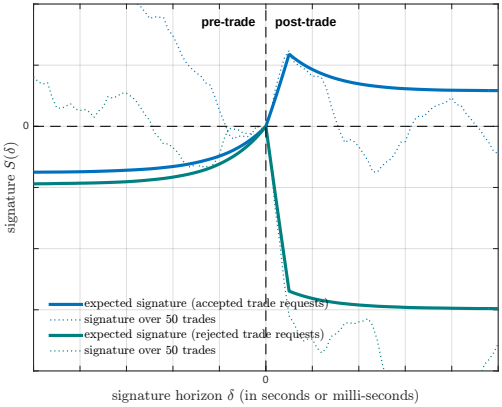


Latency arbitrage / run-over

Signature examples at microscopic level



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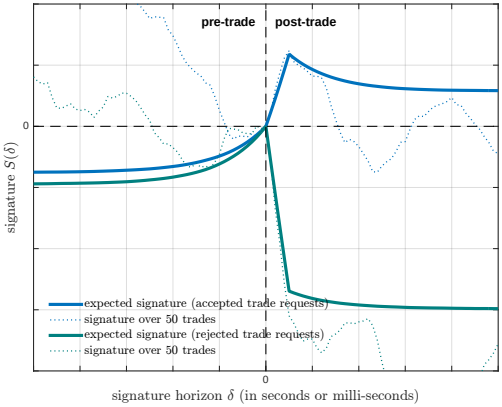

Asymmetric last look

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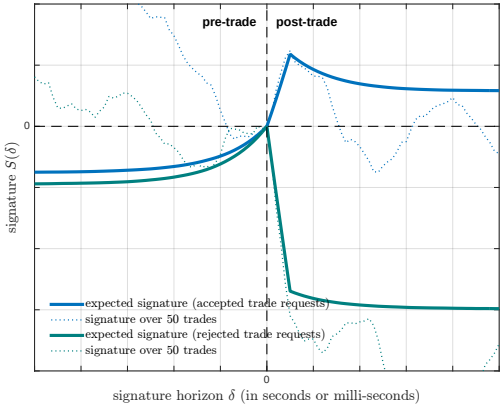

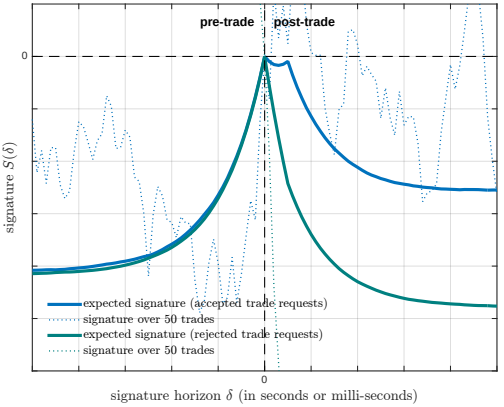

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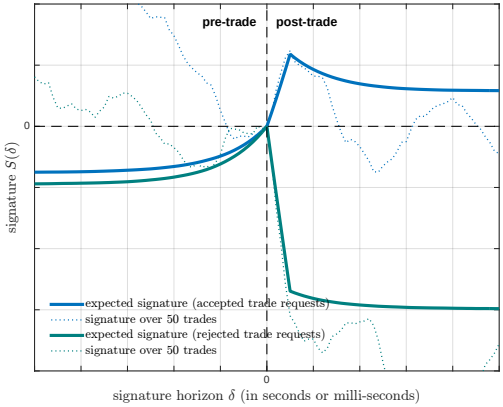

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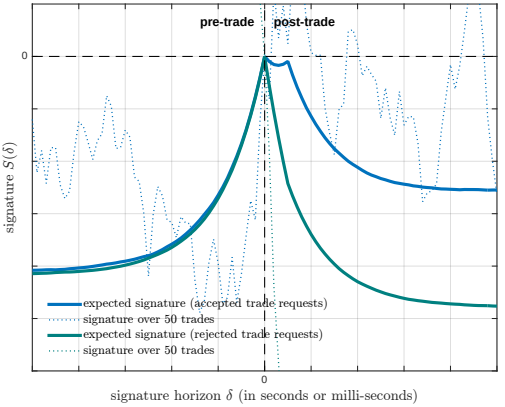

Symmetric last look

Statistical properties of signatures

Assume a simple model (a) price follows a random walk with variance σ^2, (b) periodic trades at frequency Δ, (c) stochastic trade sign and size

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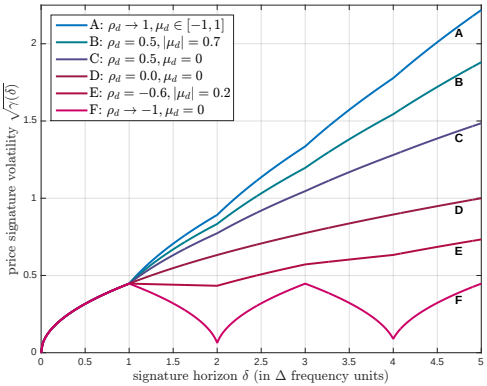
where $M=\lfloor\delta/\Delta\rfloor\wedge N$, and

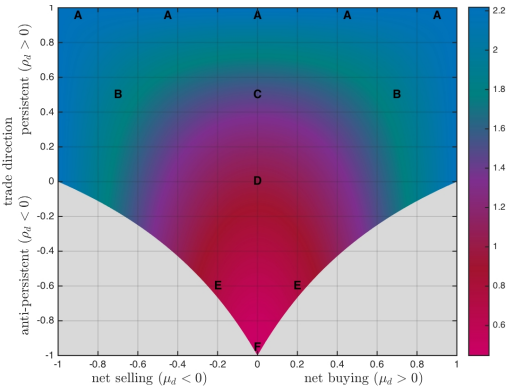
$$\psi_\rho(M,\delta)=\begin{cases}\frac{1}{2}NM(2\delta-\Delta(M+1))+\frac{1}{6}M(M+1)(2M\Delta+\Delta-3\delta) & \rho=1\\ \rho(1-\rho^M)\left(\frac{N\delta}{1-\rho}-\frac{\delta+N\Delta}{(1-\rho)^2}-\frac{(\rho+1)\Delta}{(\rho-1)^3}\right)+M\rho^{M+1}\left(\frac{\Delta(N-M)+\delta}{1-\rho}-\frac{2\Delta}{(1-\rho)^2}\right) & \rho\neq 1\end{cases}.$$

Statistical properties of signatures

signature volatility



strategy parameter regions



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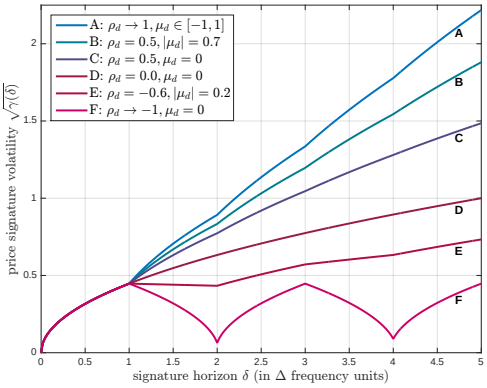

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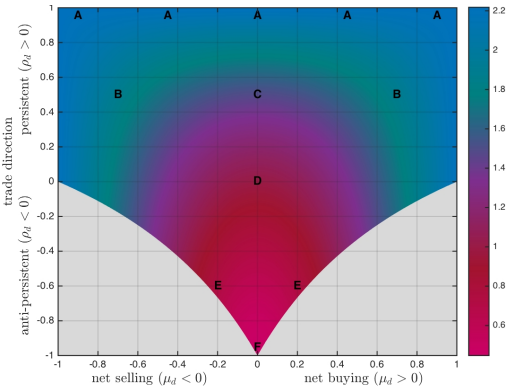

Under the "noise trader" null-hypothesis, the signature variance is a function of

- signature horizon vs trading frequency (up to $\delta < \Delta$ the simple "$\sqrt{T}$" rule applies)
- properties of the trading strategy (i.e. average and serial correlation of trade sign; amounts)

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Ramsay and Dalzell (1991), Ramsay and Silverman (1997) pioneers of the contemporary literature.

Functional data analysis

To illustrate – in its simplest form – FDA calculates quantities like:

$$SSH(\delta) = \sum_{k=1}^{K} N_k (S_k(\delta) - S(\delta))^2, \qquad \text{(between group variation)},$$

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$$SSE(\delta) = \sum_{k=1}^{K} \sum_{n=1}^{N_k} (s_n^{(k)}(\delta) - S_k(\delta))^2 \qquad \text{(within group variation)}.$$

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An example of a test for equality of functions (i.e. signatures), is the globalised F test:

$$\mathcal{T}_{FG} = \frac{N-K}{K-1} \int \frac{SSH(\delta)}{SSE(\delta)} d\delta \sim a\chi_b^2 \text{ approximately},$$

(developed by Zhang and Liang, 2014).

Functional data analysis

. . . where

$$a = \overline{\delta}\frac{N-K-2}{(K-1)(N-K)}\mathrm{tr}(\gamma_c^{\otimes 2}) \qquad \text{and} \qquad b = \overline{\delta}^2\frac{(K-1)(N-K)^2}{(N-K-2)^2}\mathrm{tr}(\gamma_c^{\otimes 2}),$$

and $\gamma_c(\delta_1,\delta_2) = \gamma(\delta_1,\delta_2)/\sqrt{\gamma(\delta_1,\delta_1)\gamma(\delta_2,\delta_2)}$, $\gamma^{\otimes 2}(\delta_1,\delta_2) = \int \gamma(\delta_1,u)\gamma(u,\delta_2)du$, and estimates for γ functions can be obtained as:

$$\begin{aligned}
\widehat{\gamma}(\delta_1,\delta_2) &= \frac{1}{N-K}\sum_{k=1}^{K}\sum_{n=1}^{N_k}(s_n^{(k)}(\delta_1) - S_k(\delta_1))(s_n^{(k)}(\delta_2) - S_k(\delta_2)),\\
\widehat{\mathrm{tr}^2(\gamma)} &= \frac{(N-K)(N-K+1)}{(N-K-1)(N-K+2)}\left(\mathrm{tr}^2(\widehat{\gamma}) - \frac{2\mathrm{tr}(\widehat{\gamma}^{\otimes 2})}{N-K+1}\right),\\
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\end{aligned}$$

where $N = \sum_k N_k$. See, e.g., Horváth and Kokoszka (2012), Zhang (2014) for further details.

Non-parametric resampling approach

Standard FDA does not apply straight "out of the box" to signature analysis ...

$$S(\delta) = \frac{1}{q'\iota} \sum_n q_n d_n (P_{t_n+\delta} - P_{t_n}), \qquad \text{for } \delta \in [-\underline{\delta}, \overline{\delta}].$$

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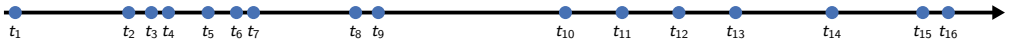
- stationary bootstrap (Politis and Romano, 1994) works well ...
- but it doesn't make full use of known signature dependence structure

Adaptive Block Bootstrap

Short signature horizon, i.e. $\delta = \longmapsto\!\shortmid$



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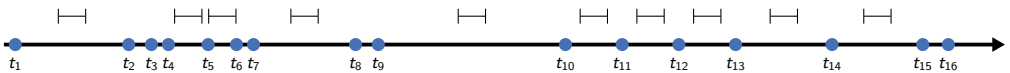

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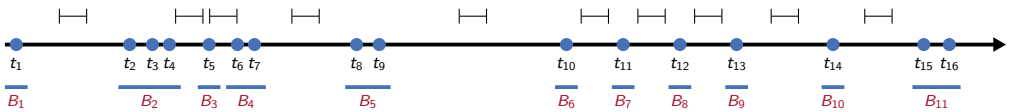
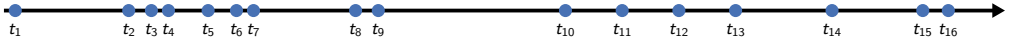


Adaptive Block Bootstrap

Medium signature horizon, i.e. $\delta =$ ⊢——⊣



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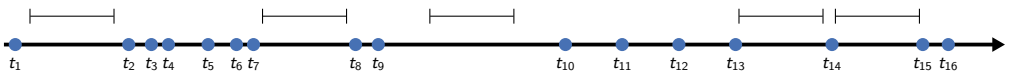

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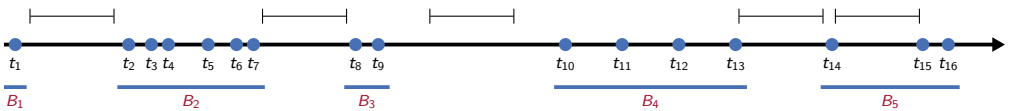

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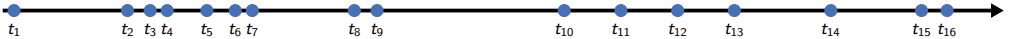

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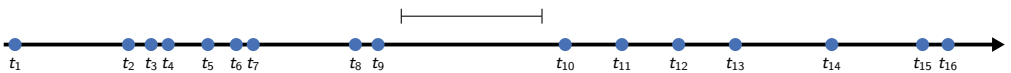

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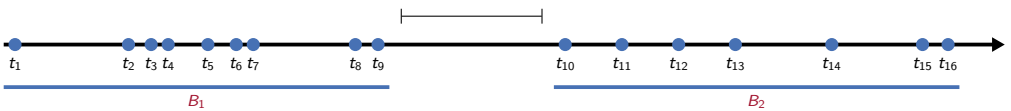
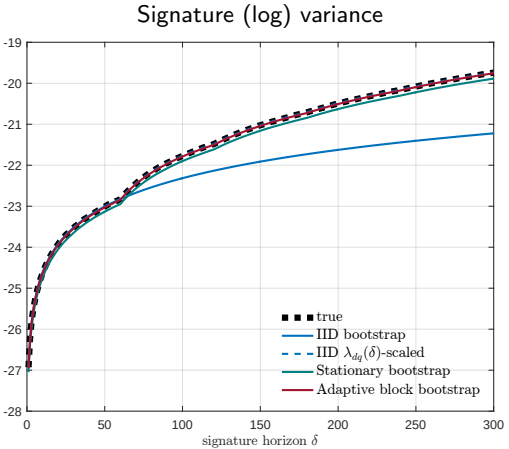
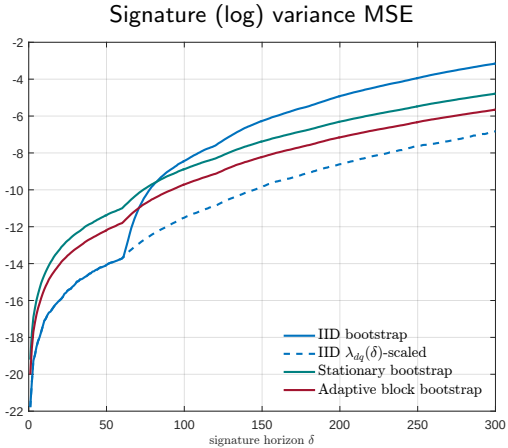


Bootstrap performance





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- Prisoner's dilemma. Externalising LP creates impact that adversely impacts internalising LP. When mixing them in aggregation process, all LPs may externalise and making everyone worse off.
- Efficient execution. Select a moderate number of LPs (say 5, not 50), trade full amount, and *do not mix internalisers and externalisers*.

Game theory context

- Aggregation. Traders in the FX market routinely place liquidity providers (LPs) in competition for their flow
- Winner's curse. Because "true" price is unobserved and the LPs are unaware of competitors' prices, the more LPs in the aggregator, the stronger the adverse selection on deals won
- Last look. Protective mechanism used by the LP to mitigate adverse selection / avoid outright arbitrage by rejecting trade requests on stale or otherwise inaccurate quotes.
- Prisoner's dilemma. Externalising LP creates impact that adversely impacts internalising LP. When mixing them in aggregation process, all LPs may externalise and making everyone worse off.
- Efficient execution. Select a moderate number of LPs (say 5, not 50), trade full amount, and *do not mix internalisers and externalisers*.

See Oomen (2017a,b), and Butz and Oomen (2018) for further details.

Signature case studies

Case-study I : Aggregation versus LP exclusivity

A trader executes using an aggregator with multiple LPs

Case-study I : Aggregation versus LP exclusivity

A trader executes using an aggregator with multiple LPs but ...

- trade request rejects complicate the workflow
- addition of LPs has meant spreads are gradually widening out

They are open to a radical change or experiment to improve matters.

Case-study I : Aggregation versus LP exclusivity

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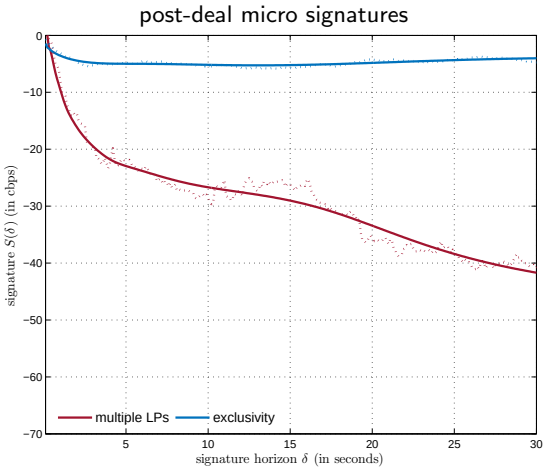
DB proposes a "firm" feed and tighter spreads than what the trader receives in aggregate across all LPs, on the basis that they become the trader's exclusive liquidity partner.

... trader believes the flow at source is latency sensitive and directional

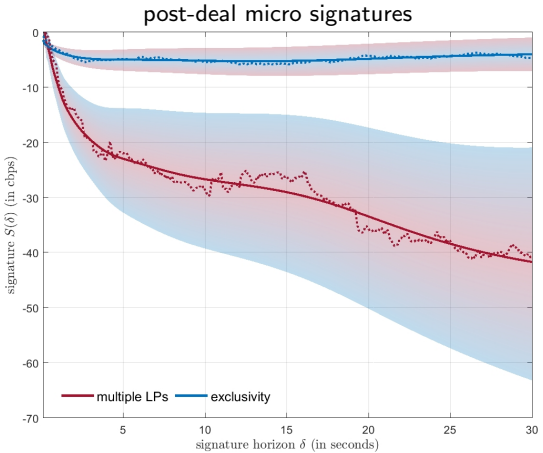
... DB believes the flow is benign at source, but that the aggregator design is the issue

Case-study I : Aggregation versus LP exclusivity



- Trader tries out exclusivity arrangement for one main currency pair
- It appears to radically lower post-deal impact (i.e. aggregator design explains the difference)
- But is it significant?

Case-study I : Aggregation versus LP exclusivity



- Trader tries out exclusivity arrangement for one main currency pair
- It appears to radically lower post-deal impact (i.e. aggregator design explains the difference)
- But is it significant?
- FDA + resampling $\rightarrow$ yes, it is highly significant!

Epilogue

Trader adopts the exclusive feed

(with backup LP for resilience)

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✓ improved trader experience

- ... response time ↓
- ... rejects ×
- ... spreads ↓
- ... costs ↓
- ... workflow simplification ↑

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✓ improved LP experience

- ... volume ↑
- ... winner's curse ×
- ... prisoner's dilemma ×

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- ... prisoner's dilemma ×

	aggregator	exclusivity
Trader's execution setup		
# LPs	> 5	**1**
externalisers	probably	no
stack sweep	yes	N/A

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	aggregator	exclusivity
Trader's execution setup		
# LPs	> 5	**1**
externalisers	probably	no
stack sweep	yes	N/A
DB liquidity configuration		
nominal spread	1.2	0.3
response time	100ms	1ms
reject rate	$\approx 10\%$	0.0%

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- ... response time ↓
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- ... costs ↓
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	aggregator	exclusivity
Trader's execution setup		
# LPs	> 5	**1**
externalisers	probably	no
stack sweep	yes	N/A
DB liquidity configuration		
nominal spread	1.2	0.3
response time	100ms	1ms
reject rate	$\approx 10\%$	0.0%
Trader's transaction costs		
observed spread	0.5	0.3
effective spread	> 0.5	0.3

Note: figures are for illustrative purposes only.

Case-study II : Consistency of LP risk management style

A trader executes using an aggregator with 7 LPs

Case-study II : Consistency of LP risk management style

A trader executes using an aggregator with 7 LPs but is unsure it's working well.

- mixed experience on selected execution (impact, reject rates)
- regularly speaks with LPs' sales representatives about the liquidity offering, but can't quite identify (whether there is) an issue

Case-study II : Consistency of LP risk management style

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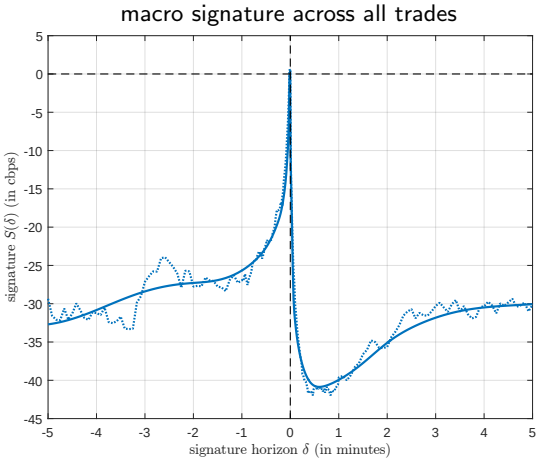
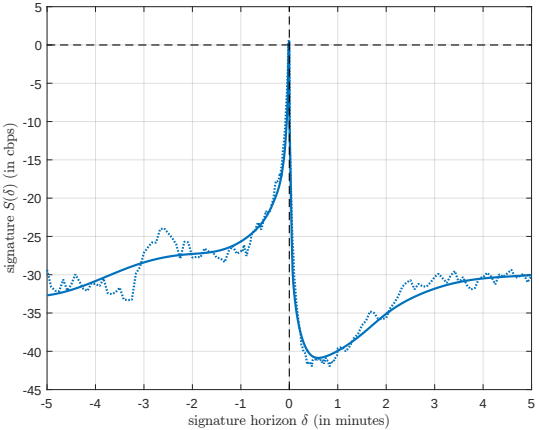
- mixed experience on selected execution (impact, reject rates)
- regularly speaks with LPs' sales representatives about the liquidity offering, but can't quite identify (whether there is) an issue

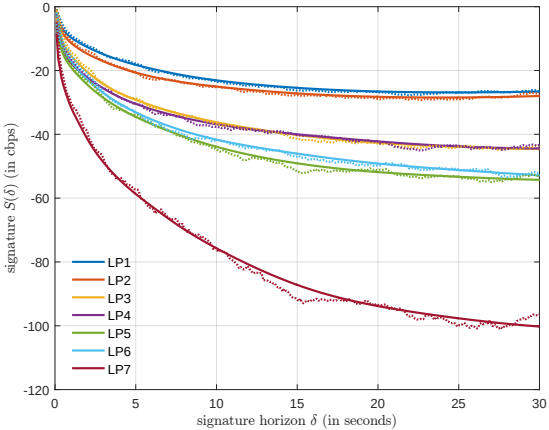
A quantitative data-driven analysis is conducted using an anonymised trade set

Case-study II : Consistency of LP risk management style



Case-study II : Consistency of LP risk management style





Case-study II : Consistency of LP risk management style

Apply FDA on the pair-wise micro signatures . . . does post-deal impact vary by LP?

LP 1
LP 2
LP 3
LP 4
LP 5
LP 6
LP 7

Case-study II : Consistency of LP risk management style

Apply FDA on the pair-wise micro signatures . . . does post-deal impact vary by LP?

	LP 1
LP 1	
LP 2	40.8%
LP 3	0.0%
LP 4	0.1%
LP 5	0.0%
LP 6	0.0%
LP 7	0.0%

Case-study II : Consistency of LP risk management style

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	LP 1	LP 2
LP 1		$\approx$
LP 2	40.8%	
LP 3	0.0%	0.0%
LP 4	0.1%	0.2%
LP 5	0.0%	0.0%
LP 6	0.0%	0.0%
LP 7	0.0%	0.0%

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	LP 1	LP 2	LP 3
LP 1		$\approx$	$\neq$
LP 2	40.8%		$\neq$
LP 3	0.0%	0.0%	
LP 4	0.1%	0.2%	73.6%
LP 5	0.0%	0.0%	9.8%
LP 6	0.0%	0.0%	28.7%
LP 7	0.0%	0.0%	0.0%

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Apply FDA on the pair-wise micro signatures . . . does post-deal impact vary by LP?

	LP 1	LP 2	LP 3	LP 4
LP 1		$\approx$	$\neq$	$\neq$
LP 2	40.8%		$\neq$	$\neq$
LP 3	0.0%	0.0%		$\approx$
LP 4	0.1%	0.2%	73.6%	
LP 5	0.0%	0.0%	9.8%	17.5%
LP 6	0.0%	0.0%	28.7%	39.4%
LP 7	0.0%	0.0%	0.0%	0.0%

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LP 1		$\approx$	$\neq$	$\neq$	$\neq$
LP 2	40.8%		$\neq$	$\neq$	$\neq$
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	LP 1	LP 2	LP 3	LP 4	LP 5	LP 6
LP 1		$\approx$	$\neq$	$\neq$	$\neq$	$\neq$
LP 2	40.8%		$\neq$	$\neq$	$\neq$	$\neq$
LP 3	0.0%	0.0%		$\approx$	$\approx$	$\approx$
LP 4	0.1%	0.2%	73.6%		$\approx$	$\approx$
LP 5	0.0%	0.0%	9.8%	17.5%		$\approx$
LP 6	0.0%	0.0%	28.7%	39.4%	79.2%	
LP 7	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%

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	LP 1	LP 2	LP 3	LP 4	LP 5	LP 6	LP 7
LP 1		$\approx$	$\neq$	$\neq$	$\neq$	$\neq$	$\neq$
LP 2	40.8%		$\neq$	$\neq$	$\neq$	$\neq$	$\neq$
LP 3	0.0%	0.0%		$\approx$	$\approx$	$\approx$	$\neq$
LP 4	0.1%	0.2%	73.6%		$\approx$	$\approx$	$\neq$
LP 5	0.0%	0.0%	9.8%	17.5%		$\approx$	$\neq$
LP 6	0.0%	0.0%	28.7%	39.4%	79.2%		$\neq$
LP 7	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	

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Apply FDA on the pair-wise micro signatures . . . does post-deal impact vary by LP?

	LP 1	LP 2	LP 3	LP 4	LP 5	LP 6	LP 7
LP 1		$\approx$	$\neq$	$\neq$	$\neq$	$\neq$	$\neq$
LP 2	40.8%		$\neq$	$\neq$	$\neq$	$\neq$	$\neq$
LP 3	0.0%	0.0%		$\approx$	$\approx$	$\approx$	$\neq$
LP 4	0.1%	0.2%	73.6%		$\approx$	$\approx$	$\neq$
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LP 7	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	

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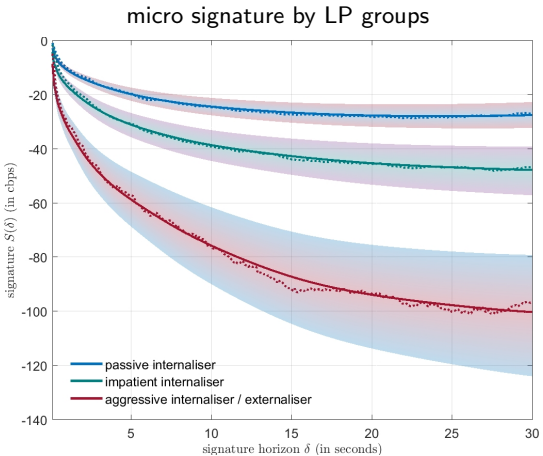


Natural classification into:

a) passive internalisers,

b) impatient internalisers,

c) aggressive internalisers or externalisers

(as discussed in Butz and Oomen, 2018)

Case-study II : Consistency of LP risk management style

Is the LP classification stable over time?

Case-study II : Consistency of LP risk management style

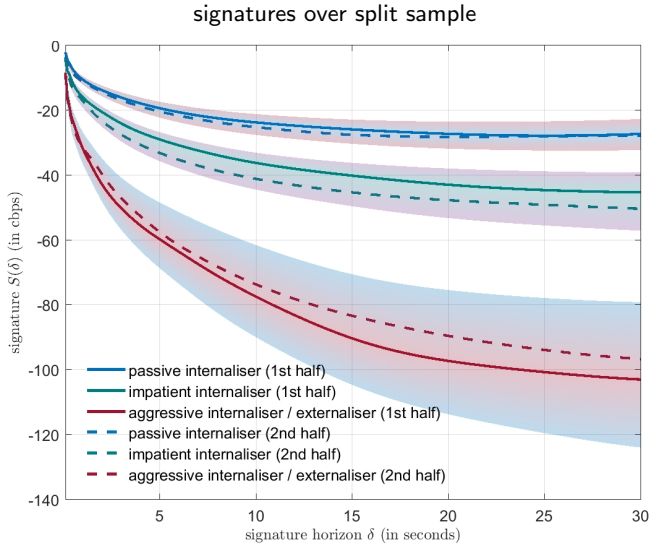
Is the LP classification stable over time? Apply FDA on LP group signatures across a split sample.

Case-study II : Consistency of LP risk management style

Is the LP classification stable over time? Apply FDA on LP group signatures across a split sample.

		1st half of sample			2nd half of sample		
		LP 1-2	LP 3-6	LP 7	LP 1-2	LP 3-6	LP 7
1st half	LP 1-2		$\neq$	$\neq$	$\approx$	$\neq$	$\neq$
	LP 3-6	0.6%		$\neq$	$\neq$	$\approx$	$\neq$
	LP 7	0.0%	0.0%		$\neq$	$\neq$	$\approx$
2nd half	LP 1-2	73.9%	0.9%	0.0%		$\neq$	$\neq$
	LP 3-6	0.0%	45.5%	0.1%	0.1%		$\neq$
	LP 7	0.0%	0.0%	84.4%	0.0%	0.1%	

Case-study II : Consistency of LP risk management style



Epilogue

Trader reduces # of LPs and intensifies relationship with passive internalisers

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Trader reduces # of LPs and intensifies relationship with passive internalisers

- ✓ reducing post-deal impact
- ✓ reducing direct and indirect execution costs
- ✓ simplifying the liquidity pool, reducing overheads

Case-study III : Monitoring of aggregator performance

A trader runs the following 9-week experiment:

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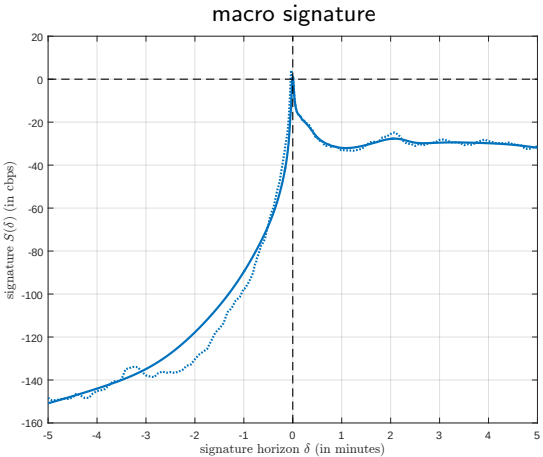
Let's calculate the price signatures week-by-week ...

Case-study III : Monitoring of aggregator performance



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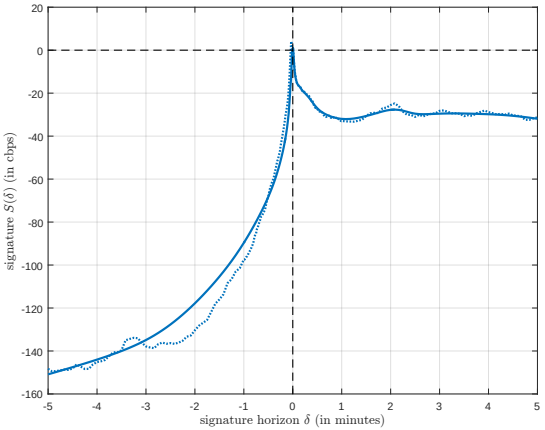
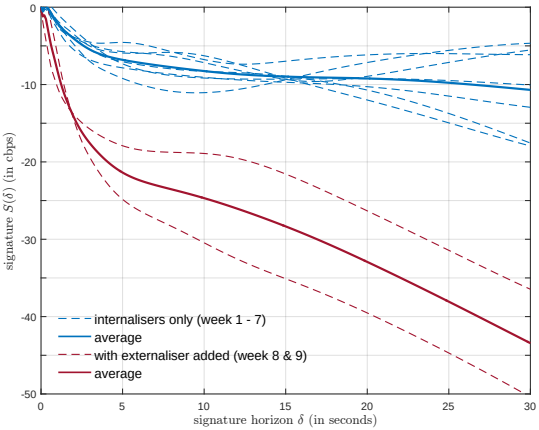




Case-study III : Monitoring of aggregator performance

Apply FDA on the pair-wise micro signatures . . . does post-deal impact vary by week?

Week 1
Week 2
Week 3
Week 4
Week 5
Week 6
Week 7

Case-study III : Monitoring of aggregator performance

Apply FDA on the pair-wise micro signatures . . . does post-deal impact vary by week?

	Week 1
Week 1	
Week 2	70.6%
Week 3	41.2%
Week 4	93.6%
Week 5	59.1%
Week 6	90.1%
Week 7	63.7%

Case-study III : Monitoring of aggregator performance

Apply FDA on the pair-wise micro signatures . . . does post-deal impact vary by week?

	Week 1	Week 2
Week 1		$\approx$
Week 2	70.6%	
Week 3	41.2%	38.0%
Week 4	93.6%	94.2%
Week 5	59.1%	48.0%
Week 6	90.1%	69.9%
Week 7	63.7%	31.3%

Case-study III : Monitoring of aggregator performance

Apply FDA on the pair-wise micro signatures . . . does post-deal impact vary by week?

	Week 1	Week 2	Week 3
Week 1		$\approx$	$\approx$
Week 2	70.6%		$\approx$
Week 3	41.2%	38.0%	
Week 4	93.6%	94.2%	43.5%
Week 5	59.1%	48.0%	85.6%
Week 6	90.1%	69.9%	68.4%
Week 7	63.7%	31.3%	47.6%

Case-study III : Monitoring of aggregator performance

Apply FDA on the pair-wise micro signatures . . . does post-deal impact vary by week?

	Week 1	Week 2	Week 3	Week 4
Week 1		$\approx$	$\approx$	$\approx$
Week 2	70.6%		$\approx$	$\approx$
Week 3	41.2%	38.0%		$\approx$
Week 4	93.6%	94.2%	43.5%	
Week 5	59.1%	48.0%	85.6%	51.8%
Week 6	90.1%	69.9%	68.4%	88.2%
Week 7	63.7%	31.3%	47.6%	49.5%

Case-study III : Monitoring of aggregator performance

Apply FDA on the pair-wise micro signatures . . . does post-deal impact vary by week?

	Week 1	Week 2	Week 3	Week 4	Week 5
Week 1		≈	≈	≈	≈
Week 2	70.6%		≈	≈	≈
Week 3	41.2%	38.0%		≈	≈
Week 4	93.6%	94.2%	43.5%		≈
Week 5	59.1%	48.0%	85.6%	51.8%	
Week 6	90.1%	69.9%	68.4%	88.2%	88.5%
Week 7	63.7%	31.3%	47.6%	49.5%	62.3%

Case-study III : Monitoring of aggregator performance

Apply FDA on the pair-wise micro signatures . . . does post-deal impact vary by week?

	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6
Week 1		≈	≈	≈	≈	≈
Week 2	70.6%		≈	≈	≈	≈
Week 3	41.2%	38.0%		≈	≈	≈
Week 4	93.6%	94.2%	43.5%		≈	≈
Week 5	59.1%	48.0%	85.6%	51.8%		≈
Week 6	90.1%	69.9%	68.4%	88.2%	88.5%	
Week 7	63.7%	31.3%	47.6%	49.5%	62.3%	87.1%

Case-study III : Monitoring of aggregator performance

Apply FDA on the pair-wise micro signatures . . . does post-deal impact vary by week?

	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7
Week 1		≈	≈	≈	≈	≈	≈
Week 2	70.6%		≈	≈	≈	≈	≈
Week 3	41.2%	38.0%		≈	≈	≈	≈
Week 4	93.6%	94.2%	43.5%		≈	≈	≈
Week 5	59.1%	48.0%	85.6%	51.8%		≈	≈
Week 6	90.1%	69.9%	68.4%	88.2%	88.5%		≈
Week 7	63.7%	31.3%	47.6%	49.5%	62.3%	87.1%	

Case-study III : Monitoring of aggregator performance

Apply FDA on the pair-wise micro signatures . . . does post-deal impact vary by week?

	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8
Week 1		$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\neq$
Week 2	70.6%		$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\neq$
Week 3	41.2%	38.0%		$\approx$	$\approx$	$\approx$	$\approx$	$\neq$
Week 4	93.6%	94.2%	43.5%		$\approx$	$\approx$	$\approx$	$\neq$
Week 5	59.1%	48.0%	85.6%	51.8%		$\approx$	$\approx$	$\neq$
Week 6	90.1%	69.9%	68.4%	88.2%	88.5%		$\approx$	$\neq$
Week 7	63.7%	31.3%	47.6%	49.5%	62.3%	87.1%		$\neq$
Week 8	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	

Case-study III : Monitoring of aggregator performance

Apply FDA on the pair-wise micro signatures . . . does post-deal impact vary by week?

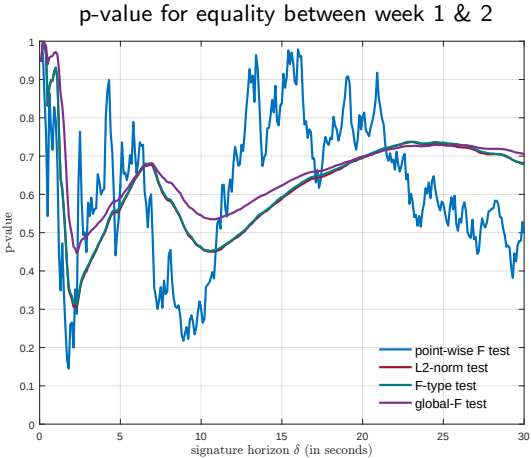
	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9
Week 1		$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\neq$	$\neq$
Week 2	70.6%		$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\neq$	$\neq$
Week 3	41.2%	38.0%		$\approx$	$\approx$	$\approx$	$\approx$	$\neq$	$\neq$
Week 4	93.6%	94.2%	43.5%		$\approx$	$\approx$	$\approx$	$\neq$	$\neq$
Week 5	59.1%	48.0%	85.6%	51.8%		$\approx$	$\approx$	$\neq$	$\neq$
Week 6	90.1%	69.9%	68.4%	88.2%	88.5%		$\approx$	$\neq$	$\neq$
Week 7	63.7%	31.3%	47.6%	49.5%	62.3%	87.1%		$\neq$	$\neq$
Week 8	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%		$\approx$
Week 9	1.1%	1.8%	0.1%	1.5%	0.3%	1.2%	0.6%	9.4%	

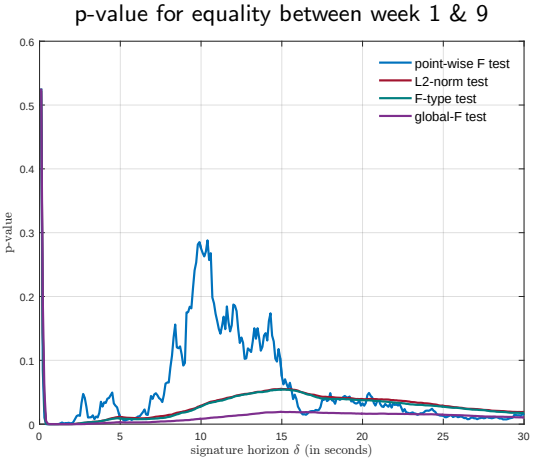
Case-study III : Monitoring of aggregator performance

Apply FDA on the pair-wise micro signatures . . . does post-deal impact vary by week?

	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9
Week 1		$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\neq$	$\neq$
Week 2	70.6%		$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\neq$	$\neq$
Week 3	41.2%	38.0%		$\approx$	$\approx$	$\approx$	$\approx$	$\neq$	$\neq$
Week 4	93.6%	94.2%	43.5%		$\approx$	$\approx$	$\approx$	$\neq$	$\neq$
Week 5	59.1%	48.0%	85.6%	51.8%		$\approx$	$\approx$	$\neq$	$\neq$
Week 6	90.1%	69.9%	68.4%	88.2%	88.5%		$\approx$	$\neq$	$\neq$
Week 7	63.7%	31.3%	47.6%	49.5%	62.3%	87.1%		$\neq$	$\neq$
Week 8	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%		$\approx$
Week 9	1.1%	1.8%	0.1%	1.5%	0.3%	1.2%	0.6%	9.4%	

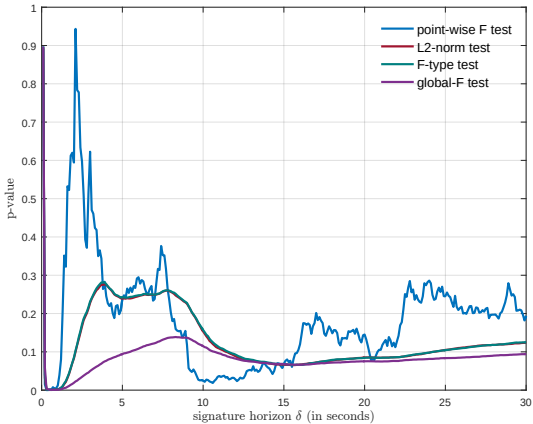
Case-study III : Monitoring of aggregator performance



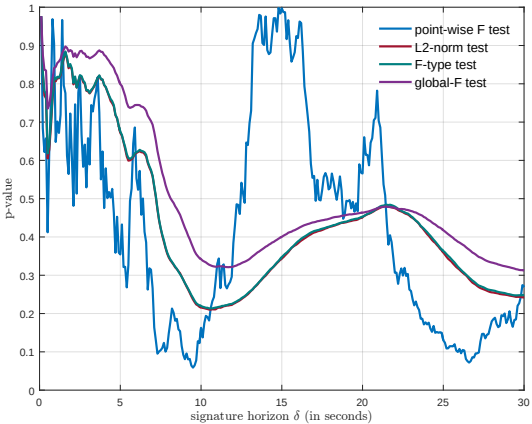


Case-study III : Monitoring of aggregator performance

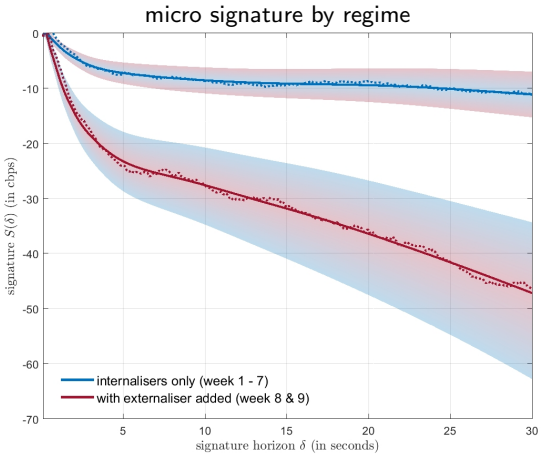
p-value for equality between week 8 & 9



p-value for equality between week 2 & 7

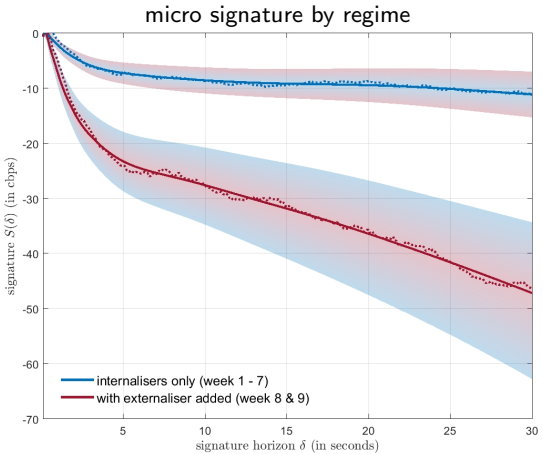


Case-study III : Monitoring of aggregator performance



- FDA indicates large & statistically significant impact on post-deal impacts associated with introduction of externaliser

Case-study III : Monitoring of aggregator performance



- FDA indicates large & statistically significant impact on post-deal impacts associated with introduction of externaliser
- if maintained, would give rise to "prisoner's dilemma" where both the trader and the LPs are worse off (see Oomen, 2017a, for more details)

Case-study III : Monitoring of aggregator performance

What if the trader had not informed the LPs about the experiment?

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What if an LP doesn't inform the trader that they'll switch their risk management style from internalisation to externalisation?

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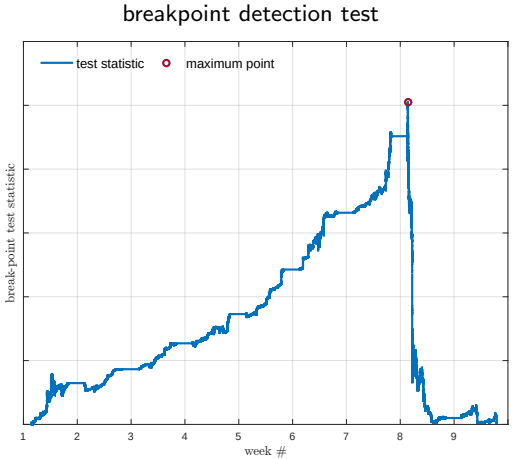
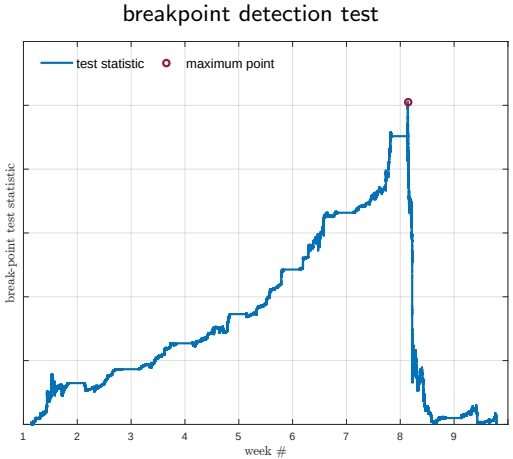
FDA can be used to check for structural breaks in the signatures.

Case-study III : Monitoring of aggregator performance



- The trader executes >15,000 trades with LPs over a 9 week period.

Case-study III : Monitoring of aggregator performance



- The trader executes >15,000 trades with LPs over a 9 week period.
- A break is identified 83 trades or $23\frac{1}{2}$ minutes after the actual break!

Epilogue

Candidate externaliser LP was not admitted to the pool, and everyone lived happily ever after ...

Thank you for your attention!

Note: the paper is now published in *Quantitative Finance*, 19 (5), 733 – 761

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