Imperial College London	**DEPARTAMENT OF MATEMATHICS** **IMPERIAL COLLEGE LONDON** **Academic Year 2017-2018**

Introduction to Optimal Transport and Gradient Flows

Lectures: Fridays 10-12 (Huxley 6M42).
Office Hours: Thursdays 18-19, Huxley 6M26, or by email appointment or by skype.
Instructor's e-mail: carrillo [at] imperial.ac.uk

1. Introduction & Tools

1.1. Granular Flow Models. Nonlinear Diffusion Models. Aggregation-diffusion Equations. (1 session)

1.2. Distances between measures: transport distances. Properties. Connection to optimal mass transport theory. Self-similar scalings. (1.5 session)

1.3. General structure. Main examples: nonlinear aggregation, nonlinear diffusion, and their competition. Entropies and Free Energies. Formal Gradient Flow structure. (0.5 session)

2. Aggregation Equation: Characteristics

2.1. The linear case: measures sliding down a confining potential. The nonlinear case: Well-posedness for smooth potentials and the mean-field limit. (1 session)

2.2. More singular potentials: a fluid mechanics viewpoint for initial data in $L^1 \cap L^p$. Finite versus infinite time blow-up: Osgood condition. (1.5 session)

3. Convergence and Properties of Variational Schemes

3.1. Gradient Flow approach: JKO for the confinement potential and smooth interactions. Connection to mean-field limits. Properties and convergence of the variational scheme. (1.5 session)

3.2. Gradient Flow approach: JKO for mildly singular interactions. Total finite time collapse versus finite time blow-up. (1 session)