REPORT ON EXAMINATIONS

M.Sc. in Mathematical Modelling and Scientific Computing 2023-24

Part I

A. Statistics

- **Numbers and percentages in each class**

	Number				Percentage			
	2023/24	2022/23	2021/22	2020/21	2023/24	2022/23	2021/22	2020/21
Distinction	11	9	7	8	30.5	38	39	32
Merit	9	6	7	7	25	25	39	28
Pass	14	8	2	10	38.9	33	11	40
Fail	1	1	1	0	2.8	4	5.5	0
Incomplete	1	0	1	0	2.8	0	5.5	0

- **Vivas**
  The 35 candidates who submitted dissertations were examined by *viva voce*.

- **Marking of scripts**
  Written examinations were sat in Weeks 0 of Hilary and Trinity Terms 2024. Scripts were single-marked by assessors followed by a script check carried out by the Course Director. Finalisation of marks by the examiners took place during an Examiners' Meeting in week 4 of Michaelmas Term and week 3 of Hilary Term. Special Topics and Case Studies were double-marked by assessors. In cases where marks varied over the pass/fail borderline, or the difference in marks was greater than ten, the assessors were asked to meet and reconcile their marks. All marks were approved by the examiners during the Examiners' Meetings held in week 7 of Hilary Term and week 7 of Trinity Term, before being released to the candidates. All dissertations were read and marked by at least two examiners; marks were approved by all examiners at the Final Examiners' Meeting and by confidential correspondence.

B. Changes in examining methods etc. which the examiners would wish the faculty/department and the divisional board to consider

C. How candidates are made aware of conventions

The conventions are posted on the course website and electronic copies are circulated to the students. The Course Director discusses the conventions with the candidates and the candidates are reminded of them by email on several occasions during the year. The candidates are notified via email about any changes to the examination conventions and amended conventions are uploaded to the course website.

Part II

A. General comments on the examination

The examiners would like to convey their grateful thanks for their help and cooperation to all those who assisted with this year's examination, either as assessors or in an administrative capacity. In addition, the internal examiners would like to express their gratitude to Prof

Katerina Kaouri for carrying out her duties as external examiner in a constructive and supportive way during the year, and for valuable input at the Final Examiners' Meeting.

Setting and checking of papers
Following established practice, the questions for each paper were initially set by the course lecturer, with a qualified person involved as checker before the first drafts of the questions were presented to the Chair of Examiners and the External Examiner. The course lecturers also acted as assessors, marking the questions on their course(s).

Determination of University Standardised Marks
The examiners followed established practice in determining the University standardized marks (USMs) reported to candidates for the written examinations. The algorithm converts raw marks to USMs for each paper separately. For each paper, the algorithm sets up a map R → U (R = raw, U = USM) which is piecewise linear. The graph of the map consists of three line segments which join the points (0,0), (P,50), (D,70) and (100,100). The values of P and D are chosen so that the resulting USMs are in line with the mark descriptors in the Examination Conventions. Particular attention is paid to the scripts that lie around class borderlines after the mapping has been applied. The values of P and D for each of the four written examinations in 2023-24 is given in the table below.

Paper	P	D
A1	50	74
A2	50	70
B1	41	65
B2	40	70

B. Equal opportunities issues and sex breakdown
The breakdown of results by sex is given in the tables below. This data is based on the sex recorded against students' records.

	Number							
	2023-2024		2022-23		2021-22		2020-21	
	Female	Male	Female	Male	Female	Male	Female	Male
Distinction	5	6	2	7	2	5	2	6
Merit	2	7	2	4	0	6	3	4
Pass	8	6	4	4	0	2	4	6
Fail	0	1	1	0	0	1	0	0
Incomplete	1	0	0	0	0	2	0	0
Total	16	20	9	15	2	16	9	16

	Percentage							
	2023-2024		2022-23		2021-22		2020-21	
	Female	Male	Female	Male	Female	Male	Female	Male
Distinction	31.25	30	22.2	46.7	100	31.25	22.2	37.5
Merit	12.5	35	22.2	26.7	0	37.5	33.3	25.0
Pass	50	30	44.4	26.7	0	12.5	44.4	37.5
Fail	0	5	11.1	0	0	6.25	0	0
Incomplete	6.25	0	0	0	0	12.5	0	0
Total	100	100	99.9	100.1	100	100	99.9	100

C. Candidates' performance in each part of the examination

This course administers examinations internally in January and April, with each student sitting four papers. Each of the two sets of examinations is split into Paper A (Mathematical Methods) and Paper B (Numerical Analysis). Both sets of examinations went smoothly this year, with a good distribution of marks between failure and distinction ranges.

Paper	Number of Candidates	Avg RAW	StDev RAW	Avg USM	StDev USM
A1	34	68.79	17.55	66.94	16.93
A2	34	61.18	12.55	61.18	12.55
B1	33	53.64	15.02	59.79	14.08
B2	34	56.15	16.09	60.82	13.19

The tables that follow give the question statistics for each paper. Examiners' comments for all papers can be found at the end of this document.

Paper A1: Mathematical Methods I

Question	Mean mark	StDev	Number of Attempts
Q1	17.19	3.77	32
Q2	22.36	3.32	14
Q3	17.05	5.69	20
Q4	12.43	5.29	21
Q5	19.03	6.38	29
Q6	15.27	5.34	30

Paper A2: Mathematical Methods II

Question	Mean mark	StDev	Number of Attempts
Q1	9.85	3.12	26
Q2	14.91	4.40	23
Q3	15.90	4.51	31
Q4	11.92	3.25	12
Q5	15.48	4.83	23
Q6	19.71	4.02	28

Paper B1: Numerical Solution of Partial Differential Equations and Numerical Linear Algebra

Question	Mean mark	StDev	Number of Attempts

Q1	15.5	4.81	32
Q2	9.75	5.4	8
Q3	12.48	4.31	29
Q4	8.76	4.84	21
Q5	15.09	4.66	32
Q6	18.67	3.83	9

Paper B2: Numerical Linear Algebra and Continuous Optimisation

Question	Mean mark	StDev	Number of Attempts
Q1	11.81	4.17	16
Q2	13.71	6.81	24
Q3	15.22	3.25	32
Q4	13.77	4.92	26
Q5	14.37	5.06	30
Q6	10.89	4.83	19

Performances on the special topics and dissertations also ranged from fail to distinction level. No student failed the Case Studies in Mathematical Modelling or Scientific Computing. 26 of 33 (78.8%) of Mathematical Modelling case studies resulted in Distinction grades. 17 out of 33 (51.5%) of Scientific Computing case studies resulted in Distinction grades. Grades for the Special Topics ranged from pass through to distinction. Of the 66 special topics submitted this academic year, 37 (56%) attained a Distinction grade, and 12 (18.2%) attained a Merit.

D. Distribution of Special Topics

Of the 19 topics listed this year, five failed to attract any students. There were no failing grades in Special Topics.

Special Topic Course	Number of Students
Finite Element Methods for PDE	9
Further Mathematical Biology	6
Integer Programming	4
Mathematical Geoscience	3
Mathematical Models of Financial Derivatives	5
Mathematical Physiology	7
Networks	2
Optimisation for Data Science	5
Perturbation Methods	4
Python in Scientific Computing	2
Stochastic Modelling of Biological Processes	12
Theories of Deep Learning	2
Topics in Fluid Mechanics	2
Viscous Flow	3

E. Names of members of the board of examiners

Examiners:

Prof C. Cartis (Chair)
Prof R. Baker
Prof P. Howell
Prof Y. Nakatsukasa
Prof S. J. Chapman (Final Exam Board only)
Prof P. Farrell (Final Exam Board only)
Prof K. Kaouri (External Examiner)

Assessors:

Dr D. Allwright
Dr M. Banaji
Dr Y. Ben-Ami
Miss G. Brennan
Prof C. Breward
Prof H. Byrne
Prof A. Cartea
Prof S.J. Chapman
Prof S. Cohen
Prof P. Dellar
Prof R. Erban
Prof P. Farrell
Prof E. Gaffney
Prof M. Giles
Dr K. Gillow
Prof I. Griffiths
Prof P. Grindrod
Prof R. Hauser
Prof I. Hewitt
Prof R. Lambiotte
Dr G. Maierhofer
Prof D. Moulton
Prof A. Münch
Dr J. Panovska-Griffiths
Dr C. Parker
Prof C. Reisinger
Dr M. Shirley
Dr M. Sune Simon
Prof J. Tanner
Prof R. Thompson
Prof S. Waters

G. Assessors' Comments

Paper A1: Mathematical Methods I

Q1.
This was the most popular problem, and students did well with part (a), and with finding the initial data and the parametric solution in (b).
Students frequently used the 'determinant criterion' to find one important boundary of the domain of definition, but nobody noticed the other limitation through the finite limit for x,y for large tau.
Sketching was usually done badly.

There was an ambiguity in the model solution due to a mistake:
In the first line of 1(b) of the paper, the specification u(0,x)=x should have read u(x,0)=x [THIS SHOULD BE CHANGED ON THE PAPER THAT GOES TO THE ARCHIVE FOR NEXT YEAR'S REVISIONS.] This resulted in some students using different initial data, which gives very similar results.
Knock-on effects: except that the requirement q\geq 0 needed to be read as p\geq 0. Some students made the adjustment or just carried on with two signs. Both versions give sensible outcomes for the domain of definition.
Potential complications on the sketching were not considered as students did generally badly there, and few marks were awarded across the board.

Q2. This was not so popular relative to the other questions, though students who did do the question most did well over very well. Main source of error were mistakes in the signs.

Q3. (a) went mostly well, a few cases failed to consider the \dot s boundary terms explicitly.
(b) also worked well, typical errors involved not observing that the integral condition provides a non-trivial restriction for the scalings or algebraic errors.
(c) Many got the first part ok, again algebra was the main source of error. Also, students dropped marks at the end, where the question clearly asked for a third order i.e. integrated equation to be formulated, plus boundary conditions.

Q4.
(a) went very well, this was bookwork.
(b) It was surprising that many students did not come up with the rarefaction solution.
Perhaps the fact that the question explicitly asked for the solution to be provided by the method of characteristics threw some off. A typical incomplete answer was to propagate the constant states but to leave a gap for the rarefaction wave (giving an empty area in the sketch of the char. projections).
(c) The first part of (c) is an extension of (b) so errors in (b) were often propagated to (c). This was not penalised
(d) Few students got to this stage due to previous errors, for which no allowance was made in the marking as this was supposed to be the hard part of the question. Some struggled to understand how to merge the wave and the shock.

Question 5 (average mark: 17/25 = 68%)
This was a fairly straightforward question, based on Greens functions (parts (a) and (b)) and distributions (part (c)). Similar material was covered in the lecture notes and examples sheets. Most candidates who attempted this question performed well, with several gaining full marks. Unfortunately, several candidates submitted weak solutions, suggesting limited understanding of some basic concepts.

Question 6 (average mark: 13/25 = 52%)
This question was non-standard and more challenging for the students. Part (a) involved generalising concepts about adjoint operators for second order differential operators to third order

operators. Most students realised how to proceed and produced good solutions. Part (b, i) involved applying the results from part (a) to a particular third order differential and was done well by most students. Parts (b,ii) was more challenging, with few students able to determine the correct boundary conditions for the adjoint operator. For part (b,iii), very few students were able to calculate the eigenfunctions y_0 and w_0 associated with the zero eigenvalue and/or to correctly recall the orthogonality conditions on f(x) needed for the BVP to admit solutions.

Paper B1: Numerical Solution of Partial Differential Equations

Most, potentially all, students attempted question 1 and only about a third attempted question 2. There were two or three students who received nearly perfect scores on question 1, while a sizeable group received scores of approximately 15/25. There were also some students who clearly struggled with the topic and received low scores. This performance is similar to prior years when I taught this material for the MMSC cohort. I anticipate scores for the Part C students to be higher, as has typically been the case in prior years.

Q3. The question was concerned with the finite difference approximation of a boundary-value problem for a second-order linear differential equation subject to homogeneous Dirichlet boundary conditions. There were 29 attempts at the question, but only three were close to being complete. The majority of those who attempted the question had difficulties using the assumption on the coefficient *b* to show that certain coefficients arising in the proofs are positive. Many also provided a Taylor expansion with an incorrect remainder term for the term $u_{j\pm1}$. Others had a correct remainder term for $u_{j\pm1}$., but then provided a different remainder term (from the lecture notes) for the finite difference, which does not follow from their remainder for $u_{j\pm1}$. No student correctly used the barrier function in part (b) to prove the maximum principle.

Q4. The question was concerned with the stability analysis of finite difference approximation of a boundary-value problem for a second-order linear differential equation subject to homogeneous Dirichlet boundary conditions on the unit square

$$-\Delta u + b\frac{\partial u}{\partial y} + c(x,y)u = f,$$

where *c* does not have a sign. Twenty-two candidates attempted this question, but only one was close to being complete. A fair number of students were able to show that first part of (a), but then did not use Cauchy-Schwarz to proceed. Part (c) has similar issues. Most were able to answer parts (b) and (d), although the same issues with the remainder terms in the Taylor expansions as in Q3 persisted.

Q5. The question was concerned with the stability analysis of the implicit Euler finite difference approximations of the initial-value problem

$$\frac{\partial u}{\partial t} + a\frac{\partial u}{\partial x} + u = \kappa\frac{\partial^2 u}{\partial x^2}, \quad -\infty < x < \infty, \quad t > 0,$$

subject to the initial condition $u(x, 0) = u_0(x)$, in the discrete ℓ^2 norm via Fourier analysis. This was a popular question and was attempted by all but one candidate. The first part of the question were generally very well done by all candidates. Most had almost complete answers to the second part; the most common mistake was saying that a complex number is ≥ 0. The final part of the question had similar mistakes to Q3 and Q4 with respect to Taylor expansions. Only a handful of students correctly identified replacing the backwards difference operator with a central differing operator to obtain second order accuracy in space. About the same number of students were close to receiving full marks.

Q6. The question was concerned with the finite difference approximation of an initial-boundary-value problem for the first-order hyperbolic PDE

$$\frac{\partial u}{\partial t} + a\frac{\partial u}{\partial x} = 0$$

posed on the $x \in (-\infty, \infty)$. There were nine attempts at the question, and most were very close to being complete. The algebra in part (c) was probably too long for the time allotted.

Paper A2: Mathematical Method II

Q1: A number of incomplete answers were submitted by candidates who also submitted Question 2. In particular, some candidates attempted the first half of Question 1 and then decided to focus on Question 2, which counted as their best answer. Most of the candidates (who took Question 1 for assessment) were able to correctly find the stable, unstable and center subspaces in part (a). Some candidates forgot to use that the center manifold is tangent to the center subspace in part (b).

Q2: The candidates showed good understanding of maps, by finding fixed points, 2-cycles and their stability in part (a). The beginning of Question 2 did not cause many difficulties and it was often submitted by candidates who ended up with Question 1 as their best question. There were also some very good attempts in finding and classifying bifurcations in part (c) of Question 2.

Q3: overall the students did good in this question.
Q3(a) – A common mistake was missing one of the two constants in the solution of the second order ODE.
Q3(b)(i) – The majority of candidates successfully completed this.
Q3(b)(ii) – Idem.
Q3(b)(iii) – Many students did mistakes in the algebraic manipulations, even though they took the right path. Only a few got to the final result.
Q3(b)(iv) – Only a few (yet unsuccessful) attempts to this question.

Q4: a few students went for it and the results were not very successful.
Q4(a)(i) – They had a quite good intuition on the physics of the heat equation but they usually lacked of clarity and concision in articulating their ideas.
Q4(a)(ii) – Most of them integrated well the heat equation and applied the boundary conditions to find the temperature T(x,t). Fewer succeeded in obtaining the position of the interface s(t).

Q4(b)(i) – The same comment as for Q4(a)(i) would generally apply here. None paid attention to the peculiarities of the diffusion coefficients involved and their implications. Also, they were inclined to omit the conservation of mass (5d), or refer to it differently.
Q4(b)(ii) – The majority had troubles in implementing the change of variables.
Q4(b)(iii) – Very few attempts to this question and no success.

Q5 and Q6: In general the students did the book work parts well. Even that was enough to pass.
The de question required asymptotic which a few did very well. I think some on the course struggled with such assymptotic expansions.
The integral equation question was easier and I would have expected most students to score 70+%

Paper B2: Numerical Linear Algebra and Continuous Optimisation

Q1: Question 1 focused on the action of unitary matrices to pre-condition a matrix so that it is approximately diagonal. Part a) considered upper Hessenberg form and Part b) creating the bidiagonal structure for Gobub Kahn initialization before computing singular values.

Q2: Question 2 considered iterative methods for approximately solving linear systems of equations. Part a) involved the analysis of steepest descent, Part b) discussed conditioning of GMRES, and Part c) considered the use of upper Hessenberg form for an efficient and stable calculation of GMRES.
Students struggles with both questions, despite the material having all been presented in lectures and/or lecture notes. This struggling may be due to the nature of the material being challenging for students at this stage of their studies, or it might be the breaking up of the NLA material into exams B1 and B2 with the B2 material lectured in November of 2023 and examined in April 2024. This struggling with the NLA material, especially in the B2 exam, has occurred regularly and similar questions are generally answered much better by Part C students who are presumably more familiar with the Oxford exam system.

Q3,4,5,6: Questions 3 and 5 were most popular and most accessible. Still some students struggled with basics of classifying local stationary points.

Overall, I felt the students did better than in some previous years. There were many students who attempted 3 to 4 questions (so all of them) and I assume they did less on NLA. There were also 2–3 stellar performances.