Examiners' Report: FHS Mathematics Part A; Trinity Term 2004

Part I

A Statistics

- Numbers and percentages in each range

  Candidates are not classified in this examination, rather the marks awarded are carried forward for the consideration of Examiners in subsequent years. What is tabulated in this section, in order to summarise performance in Part A, is the distribution of candidates by rounded average USM in the ranges associated with the different classes. For comparison, the final column is an average of performance in 2003 FHS Mathematics (3rd and 4th year).

Range	Number	Percentages %	2003 Finals %
70-100	54	33	33
60-69	69	43	45
50-59	28	17	17
40-49	7	4	3
30-39	3	2	2
0-29	1	1	0
Total	162	100	100

- There were no vivas and no double-marking. The same system of checking was used as in all parts of FHS Mathematics.
- All candidates take all four papers, namely AC1, AC2, AO1 and AO2. (Recall that AC1 and AC2 contain respectively short and long questions on the compulsory core, while AO1 and AO2 respectively contain short and long questions on the options).

B. New examining methods and procedures

- This was the first occasion on which this exam was set. All four Internal Examiners examined in FHS Mathematics in 2003. Where possible the same or similar procedures were followed in this examination as in other parts of FHS Mathematics.

- The Examiners calculate four USMs for each candidate, one for each paper. As directed by the Departmental Teaching Committee, the Examiners base the calculation of USMs linearly on the raw marks and not on sums-of-squares. The range in which a particular USM falls is required to have a definite meaning in terms of quality. Thus the range 60 to 69 must mean work of upper-second quality, which is typically where over 40% of candidates are placed, while the much larger range 70-100 must mean work of first-class quality, where around 30% of mathematics candidates are placed. (Of course, this issue faces examiners in all parts of FHS Mathematics.) The Examiners therefore adopted an algorithm based on a piece-wise linear graph to convert raw marks to a USM for each paper. The details of the graphs are contained in the second part of this report.
- To assist them in arriving at the conversion for each paper, the Examiners have a range of material to take account of. As well as the scripts themselves and the Examiners' own recent experience as finals examiners, they have statistical information on overall performance on the papers. They also have tables giving the distribution of candidates among classes in FHS Mathematics and related schools for each of the past five years. These are sent to the Examiners by the Division of Mathematical and Physical Sciences. Finally they have written descriptors of the various classes, as adopted by the Department of Mathematics.
- When constructing the conversion algorithm, the Examiners took particular care to follow the effect of adjustments to the algorithm on the USMs of candidates at the bottom end of the distribution. While it is desirable to have a simple rule, uniformly applied, for converting raw marks to USMs, this care is the counterpart of the individual consideration formerly given to classifying candidates at the lower end.
- Candidates are permitted to proceed to Part B provided they pass Part A. Thus the Examiners are required to produce a Pass List, which is an innovation. (In previous years, Part I Examiners have produced a list of those deserving of honours, which is not the same.)
- On paper AC1, ten marks were available for 'the assessment of mathematical presentation'. Since AC1 contains three sections, each marked by one of the Examiners, a presentation mark out of ten was assigned for each section and these were averaged to arrive at the actual presentation mark. These marks were regarded as USMs rather than raw marks and so were not subject to later rescaling.
- Papers AC2, AO1 and AO2 are marked out of 100 while, because of the marks available for presentation, paper AC1 is marked out of 90.

C. Changes in examining methods and procedures currently under discussion or contemplated for the future

Some changes in the detail of the relationship of papers in Part A of the joint schools Mathematics and Philosophy and Mathematics and Computation with those in Mathematics will be made next year.

D. Notice of examination conventions for candidates

The candidates were given details of the examination conventions both in a supplement to their handbooks and in the notices sent to them by the examiners and attached below.

Part II

Section A. General Comments on the Examination

1. The papers were taken on Thursday morning (AC1) and afternoon (AC2), and Friday morning (AO1) and afternoon (AO2) of 9th week of Trinity Term, June 25th and 26th, at Ewert House in Summertown.
2. All questions on the Compulsory Core papers, AC1 and AC2, were composed and marked by the Internal Examiners. Questions, mark schemes and model answers were discussed initially with a second Internal Examiner, then with the whole panel of Internal Examiners and finally were sent for comments to the External Examiner. In arriving at the final form of these questions, the Examiners paid close attention to the published synopses and problem sheets and to the written guide-lines from the Departmental Teaching Committee on length and style of the short and long questions.
3. Draft questions on the Options papers, AO1 and AO2, were provided by the lecturers, following the usual pattern in other parts of maths finals. The drafts, again with mark schemes and model answers, and also with lecture synopses and problem sheets, were first discussed by the setter with one of the Internal Examiners, then by the whole panel, and then sent to the External Examiner. Again, close attention was paid to the Departmental guide-lines, particularly as these covered the desired character of the short questions.

   The lecturers also acted as assessors, marking the questions on their courses.
4. The following points were made in relation to the examining process:
   (a) Exam booklets rather than loose-leaf paper are now the rule. To facilitate the collection of scripts by sections (and indeed marking by sections), all rubrics specified that a new booklet be started for each new question. Consequently many booklets are used. The recurrent problem which arises is that occasionally candidates include answers from more than one section in the same booklet (despite repeated instructions not to).
   (b) Under the current rules governing conduct of examinations, the Senior Invigilator reads to the candidates a lengthy and general introduction before each paper. Possibly as a result of this, information specific to the paper, including a reiteration of the

instruction to start a new question in a new booklet, may be imperfectly retained by the candidates.

(c) As is often the case with FHS Mathematics, the process of collecting in scripts was quite intricate. It is important for the Chairman of Examiners to make his or her wishes for this process clear to the Senior Invigilator, who under the current rules is responsible for the conduct of this as of all other aspects of the examination.

(d) The checking of marks, in the sense of the detailed checking of scripts against printed marks lists produced from the data-base, is a crucial ingredient of the examining process. A small but significant number of errors was found, and confidence in the accuracy of the final result was thereby much increased.

(e) Production of the Options papers relies on the cooperation of the lecturers. The time-table is awkward, in that questions must be produced before the course has finished for Hilary Term lecturers, and before it has even started for Trinity Term lecturers. There were some minor delays, but overall this part of the process went very smoothly. The lateness of the exam (9th week) means that there is some slack after Easter. If the exam is moved forward, this timing may become more critical.

5. There is a single External Examiner for Part A and, with just four Internal Examiners, it would be unwieldy to have more. We were well-served by Professor Rees, our External, and are grateful to him. He found errors and ambiguities in the draft papers, and contributed in a helpful and constructive way at the Examiners' Meeting.

6. The introduction of the Part A examination required the writing of a new data-base, for which the Department engaged Alan Dyson. This was very successful. Alan was responsive, helpful and calm and, subject to the usual initial minor hiccups, the data-base seems to have worked well.

7. The Examiners are also grateful for administrative and secreterial support to Ana Fraser, Keith Gillow, Catherine Goodwin, Becci Love, Linda Mildenhall and Maria Moreno.

B. Equal opportunities issues and breakdown of the results by gender

The table below shows number and percentages of male and female candidates in the different ranges of average USM in Part A Mathematics.

	Male	Female	Total
70-100	37 (35%)	17 (30%)	54 (33%)
60-69	44 (41%)	25 (45%)	69 (43%)
50-59	16 (15%)	12 (21%)	28 (17%)
40-49	6 (6%)	1 (2%)	7 (4%)
30-39	2 (2%)	1 (2%)	3 (2%)
0-29	1 (1%)	0 (0%)	1 (1%)
Total	106	56	162

C. Detailed numbers on candidates performance in each part of the exam

1. It may be helpful, in the first year of this examination, to summarise what the rubrics for the different papers call for, and to note how far candidates followed these rubrics.

   For paper AC1, candidates are instructed to attempt all nine questions. Most (127 of 162) did so, the average number of attempts being 8.7. Of the ones who attempted fewer, two candidates attempted only 5 of 9.

   For paper AC2, candidates are instructed to attempt no more than five questions, of which the best four are credited. The average number attempted was 4.6, and all candidates received credit for four answers.

   For paper AO1, candidates are instructed to attempt ten questions. Only one attempted more (and was credited with the best ten), but the average number of attempts was only 9.3.

   For paper AO2, as for AC2, candidates are instructed to attempt no more than five questions, of which the best four are credited. The average number attempted was 4.5, and all but two candidates received credit for four answers.

   In summary, the rubrics were followed quite closely except on AO1 where many candidates attempted nine rather than ten questions.

2. Means and standard deviations are given below in raw marks and USMs for each of the four papers.

Paper	aveRaw	sdRaw	aveUSM	sdUSM
AC1	61.1	12.3	66.8	10.7
AC2	70.1	15.8	67.8	12.9
AO1	55.2	18.6	64.8	14.1
AO2	65.8	18.9	67.4	12.9

   As noted above, the raw mark on AC1 is a mark out of 90, as there were ten marks available for presentation. The average presentation mark was 7.0 and the aveUSM on AC1 includes the presentation mark. Taking account of that, the average raw marks on AC1, AC2 and AO2 are close to where the Examiners were aiming. The striking figure is the low average of raw marks on AO1. Recall that this is the short-question option paper and, as noted in the previous paragraph, many candidates attempted fewer than the ten questions called for by the rubric. Despite fewer attempts, the average mark per attempt on this paper was 5.9, lower than the average of 7.0 on the short-question compulsory-core paper, AC1.

   It may be that the short options questions overall were too hard, or it may be that candidates are not used to being tested in this way. The Department might like to consider reducing the number of questions which a candidate is instructed to attempt on this paper.

3. The algorithm converting raw marks to USMs for each paper was based on a piece-wise linear function, interpolated between nodes specified as follows. For AC1:

USM	RawAC1
90	90
63	68
54	53
45	41
0	0

while for AC2, AO1 and AO2:

USM	RawAC2	RawAO1	RawAO2
100	100	100	100
70	79	66	77
60	54	43	50
50	47	33	34
0	0	0	0

In arriving at these functions, the Examiners have no discretion over the ends, which are fixed. The database allows for the possibility of more nodes, and different locations for them. We could not obtain a satisfactory result with just two internal nodes but were keen to keep the number of nodes as low as possible. We varied the positions of the nodes to check the robustness of the result to small changes. The choice made for the nodes for AO1 raises the average on that paper but it remains the lowest of the four, as the Examiners concluded that this was a fair reflection of performance on that paper.

4. Here we give means, standard deviations and number of attempts on individual problems. On papers AC2 and AO2 both the number of attempts for which credit was given and the number of 'unused' attempts is presented (recall that only the best 4 answers are credited to the total).

   For paper AC1, where each question is marked out of ten, and all questions should be attempted (by 162 candidates):

Subject	Question	rawAve	rawSD	Attempts
Algebra	1	8.22	1.71	160
	2	6.54	2.46	149
	3	7.57	2.17	154
Analysis	4	9.39	1.37	161
	5	8.43	2.15	159
	6	6.21	2.57	155
DiffEqns	7	6.36	1.97	159
	8	6.58	2.56	160
	9	3.80	1.93	148

For paper AC2, where each question is marked out of 25:

Subject	Question	rawAve	rawSD	Attempts	Unused
Algebra	1	20.71	5.39	73	6
	2	14.21	6.55	42	14
	3	19.77	3.35	73	2
Analysis	4	17.20	4.85	89	11
	5	16.60	5.44	63	25
	6	15.73	6.24	41	5
DiffEqns	7	19.11	3.42	134	5
	8	15.36	4.22	67	3
	9	15.02	5.68	66	27

Performance on the differential equations questions was markedly less good on AC1 and marginally less good on AC2, as compared to the other two sections, but these questions on AC2 attracted disproportionally many attempts.

For paper AO1, where each question is marked out of ten:

Subject	Question	rawAve	rawSD	Attempts
Groups in action	A1	8.21	1.72	68
Intro to Fields	B1	8.58	2.01	71
Number Theory	C1	6.51	2.52	73
Integration	D1	4.42	3.08	104
	D2	5.53	2.86	106
Topology	E1	8.22	2.25	76
	E2	5.25	3.14	71
Multivble Calc	F1	6.96	2.70	74
Calc. of Variations	G1	4.03	3.44	102
Class.Mech.	H1	4.38	3.28	105
Quant.Mech.	J1	6.16	2.11	98
Fluids &Waves (HT)	K1	8.05	1.24	76
Fluids &Waves(TT)	L1	5.45	2.79	69
Prob.(Laws)	M1	8.27	2.66	101
Prob.(Donnelly)	N1	5.12	3.22	91
Statistics	O1	4.58	2.24	50
	O2	3.24	2.75	41
Numerical Analysis	P1	4.90	3.29	59
	P2	5.77	2.89	56

As one should expect from the low average raw mark on this paper, some of these question averages are rather low - more than half are below 6. Future Part A Examiners would probably want to see higher question averages in future years.

As a guide to the relative popularity of the sections, with 162 candidates each attempting 10 of the 19 questions on AO1, one might expect each question to attract about 85 attempts.

For paper AO2, where each question is marked out of 25:

Subject	Question	rawAve	rawSD	Attempts	Unused
Groups in action	A2	16.83	5.14	58	1
Intro to Fields	B2	15.57	6.53	28	14
Number Theory	C2	13.71	6.47	28	4
Integration	D3	14.57	6.61	58	9
	D4	18.10	6.66	10	3
Topology	E3	15.72	7.62	29	6
	E4	15.67	6.98	6	4
Multivble Calc	F2	12.67	5.69	3	1
Calc. of Variations	G2	19.37	3.81	131	2
Class.Mech.	H2	15.09	6.55	11	1
Quant.Mech.	J2	16.49	6.05	41	4
Fluids & Waves (HT)	K2	19.95	5.05	44	4
Fluids & Waves(TT)	L2	16.20	4.82	60	7
Prob.(Laws)	M3	15.43	6.13	68	6
Prob.(Donnelly)	N2	8.71	3.00	14	8
Statistics	O4	-	-	0	0
	O5	8.90	4.46	10	3
Numerical An.	P3	13.00	5.65	20	1
	P4	18.30	6.74	27	1

Again, as a guide to the relative popularity of the sections, with 162 candidates each attempting 5 of the 19 questions on AO2, one might expect each question to attract about 43 attempts.

Section D. Comments on papers and individual questions

(i) AC1

Algebra

1. Almost all candidates were able to state that dim V=4 and to show that the conditions for an inner product were satisfied. A substantial number of candidates were not sure of the definition of an antisymmetric matrix and some of those that were did not realise that the diagonal elements must be zero. Otherwise most knew how to approach this part of the question, although some made things more difficult than necessary by considering $n \times n$ matrices rather than 2×2 matrices. Most were able to do the last part, often without using the previous parts of the question, but some made mistakes in normalising.

2. Most candidates answered the first three parts correctly. Most of those who attempted the final part knew how to approch the proof but some were unsure how to incorporate the

hints and missed the crucial point that if dim $V = n+1$, then the orthogonal complement of the eigenvectors of V has dimension $\leq n$.

3. The bookwork at the start of the question proved surprisingly difficult for a few candidates. Obviously answering parts (i) to (iv) correctly depends strongly on knowing this bookwork. Most of those who answered the bookwork correctly were able to answer the majority of (i)-(iv) correctly. In (i) several candidates overlooked the word 'positive' in the question and conseqently answered incorrectly.

Analysis

4. This was generally well done. A few candidates mistakenly thought they were being asked to give a derivation rather than just a statement of the Cauchy-Riemann equations.

   Most candidates showed that the real and imaginary parts were harmonic, although some tried an unnecessarily complicated approach based on the derivation of the Cauchy-Riemann equations. A few candidates got into difficulties taking partial derivatives, and some seemed to think that the CR equations held at the origin (possibly they were thinking of the modulus squared function). Some showed directly (by considering limits) that the function were not holomorphic, rather than the easier method of checking the CR equations failed.

5. Again this was fairly well done, but a few candidates did badly. Many candidates didn't get the definitions exactly right. Several seemed confused about the definition of an essential singularity; some thought it meant non-isolated, or even isolated. A few were confused about the definition of removable singularity; some thought it meant zero residue.

   In the rider, a common mistake, as usual, was to think that the zeroes of the *sin* function in the denominator gave rise to essential singularities. Many candidates resorted to unnecessarily complicated series calculations.

6. There was a great deal of confusion about regular singular points. The great majority of candidates thought that to show a singularity at z_0 was regular one needed to consider $\lim_{z\to 0} zp(z)$ rather than $\lim_{z\to z_0}(z-z_0)p(z)$ etc. This meant that almost everyone failed to see that -2 was regular. Quite a few candidates said that every point EXCEPT 0 and -2 was a regular singular point. Some thought -1 was singular.

   The actual series calculations in the example were done reasonably well.

Differential Equations

7. The question in Picard iteration was answered by almost all candidates, and was answered well, although too many candidates thought that the 'box' condition was more important than the Lipschitz condition in determining convergence of the iteration.

8. In contrast, this question on boundary conditions and uniqueness of solutions was not answered with any knowledge - the majority of answers achieved 3 marks out of 10.

9. This, the Green's function question, was reasonably answered, the main difficulties arising from inabilities to get the homogeneous solutions satisfying boundary conditions correct, and then failing to ensure the 'Wronskian' based method yielded a solution that satisfied the boundary conditions. Overall candidates tended to write a lot, when their time might have been better employed thinking a little more; perhaps the short question format encouraged 'haste', and a non-stop writing mode.

(ii) AC2

Algebra

1. A question on annihilators and transposes, this was generally popular and well-done with many good answers. More than half the candiates attempted it and more than half of them scored 21 or more (67 from 130 attempts).

2. A question on minimum polynomial and diagonalisation, this was much less popular than the previous. Most attempts (but not all) gave a form of the Primary Decomposition Theorem appropriate to a complex vector space (when everything factorises). There were some very garbled answers and this part of the course seems to be less well understood, with just 8 'alphas' from 78 attempts.

3. A question on rings and ideals, this was generally popular and well-done with 48 'alphas' from 103 attempts. The most common error was to show that $\theta^{-1}(J)$ is prime but not that it is an ideal. Many failed to give an example for the last part, but two found examples different from that proposed by the setter.

Analysis

4. This was the most popular analysis question in AC2. The bookwork was done fairly well. Many candidates used the Cauchy estimates/Cauchy for derivatives to derive Liouville; I only gave full credit if there was a full treatment of how to prove the estimates for the derivatives. A fairly large minority got the right idea for how to do the rider.

5. The first integral was quite well done. Some candidates were too sketchy in showing that the integral around the semicircle tended to zero. Many unnecessarily resorted to Jordan's Lemma. Several candidates tried to use $\cos z/z^2+1$ rather than e^{iz}/z^2+1 and hence were unable to get a correct solution.A few candidates considered residues at $-i$ as well as i.

   The second integral proved less easy, but several candidates got it out. One or two got the integral by a recursion formula rather than by a contour integral as asked.

6. This was the least popular of the three questions, but there were some very good answers. The earlier part of the question was found the more difficult. Many candidates thought the coefficients of a Möbius map had to be real, or even integral. Many failed to realise that composition of maps was the group operation (although such candidates often did write down the inverse of a Mobius map correctly). Finding the required transformation in the later part of the question was generally well done.

DiffEqns

7. The phase plane question was by far the most popular, and was answered well. Early parts of answers were marred by candidates not answering the actual question, while candidates found the latter part of the question challenging to get correct, so that there were few alpha answers.

8,9 Candidates answered the bookwork on characteristics (Q8) and Laplace transforms (Q9) well, but found the second part of each question very difficult. Not many candidates showed confidence in solving the heat equation using Laplace tranforms, or in determining where the solution of the equation in Q8 became singular, or was uniquely defined.

Overall, answers showed considerable variety, and that much had been learnt.

(iii) AO1 and AO2

A and B: Groups in Action and Introduction to Fields

The short questions in both Groups in Action (A1, A1(P)) and Introduction to Fields (B1, B1(P)) were well done on the whole, with a good number of 10's, and the vast majority of candidates getting at least 6 on the questions they attempted. Clearly the candidates had learned up the appropriate definitions and the obvious pieces of bookwork. However when they did come unglued they tended to write such complete nonsense that it was clear that they had absolutley no understanding of the proofs they had learned. And I got a strong impression that that was also the case for lots of candidates who had still managed to learn the bookwork accurately enough to get the marks. Significant numbers did the definitions and bookwork correctly, but went completely off the rails on the more or less trivial applications at the ends of the questions.

The longer questions (A2 and B2) attracted a good spread of marks, but rather a low proportion of alphas, and very few 25's. They were marginally easier than the corresponding A3 questions I marked in Part I this year, but were not nearly so well done.

A1. The definitions and orbit stabilizer theorem were well done on the whole, but large numbers of candidates were unable to compute the conjugacy classes of S_3.

A2. The bookwork was not as well done as in the short questions, probably because there are several different things that have to be checked, and this needs to be organized somewhat.

An alarming number of candidates wrote things like

$$ug = vg \Rightarrow (u - v)g = 0 \Rightarrow u - v = 0 \Rightarrow u = v,$$

and wanted to check that $\rho(g + h) = \rho(g) + \rho(h)$.

Most candidates who attempted the last bit had a pretty clear idea about what the rotation group of the tetrahedron was, but a good number lost marks for just listing the two possible types of rotation, without giving any reason as to why there aren't any more. (This was a bit tough on them, but the question did say "Show that the group has order 12, ...", and in the lectures I did give two PROOFS that the order really is 12. The marking scheme approved by the examiners also made it clear that I intended to take 3 marks off out of a total of 9 on this part of the question for candidates who did no more than list the rotations. The simplest way to get order 12 is to apply the orbit stabilizer theorem to a vertex, and a good number of candidates did just that. Alternatively, any sort of analysis of what possibilities there might be for an axis of rotation of a tetrahedron got credit, and several candidates went that way.)

A surprising number gave the order of a rotation through $(2\pi)/3$ as 2. I really don't know what is going on here.

B1. The tower lemma was well done, but I got a very strong feeling in a lot of the answers that the candidate had absolutely no idea what the proof was all about.

The application sorted candidates out, I suppose. An alarming number gave the degree of $Q[\sqrt{2}, \mathrm{i}]$ over Q as 3.

B2. This question got a good spread of marks, with a good number of candidates getting all the way through. Most candidates essentially knew what to do both for the bookwork and the application, but many still couldn't quite get it right! One common mistake on the first part of the bookwork was to assume that there was already an element α pre-existing in some extension field which was a root of $f(x)$, whereas the object of the exercise is to construct a brand new field containing a root.

C: Number Theory

C1. I thought this (short) question worked rather well. Perhaps (b)ii was a little challenging, since there were several students who got full marks for the rest of the problem, but didn't know where to start here. Consequently I awarded points to students who seemed to have understood the sort of proof by contradiction argument needed, but had failed to come up with the correct starting point. For future reference, it might have been fairer to instead write something like:

> By considering numbers of the form $6X - 1$ for suitable $X \in \mathbb{N}$, deduce that there are infinitely many primes of the form $6n + 5$.

C2. The first two parts of the long question were certainly well within the reach of all the candidates. Unfortunately, the structure of part (a) and part (b) meant that if they failed to remember the statements correctly in (a), then they necessarily struggled with (b). I therefore decided to award points for correct method here, even if they didn't correctly remember the statement of Gauss' Law of Quadratic Reciprocity. In addition to this, one or two students chose to tackle this particular problem entirely differently (and hence entirely outside the mark scheme) via the use of Gauss' Lemma. I awarded marks to these students as I thought best.

Several people made mistakes in the first part of (c): the most common was the false equality "$2^{2^n} = 2 \times 2^{2^{n-1}}$"!

D: Integration

D1, D2. For the short questions, the intention of the examiner was to set very straightforward applications of important standard theorems, rather than to ask for definitions or book-work. On the whole, the questions seemed to be popular and well-answered. The marking scheme rewarded careful verification of all the hypotheses of the theorems involved.

D3. (long) This was a conventional 'statement/application' problem, and solutions were generally of a satisfactory quality.

D4. (long) This question probed theory rather than applications, and was less popular. It was clear, however, that those who had enjoyed this aspect of the course coped very well with the question.

E: Topology

For questions E1 and E2, on paper AO1, a slightly modified marking scheme was used. The resulting total marks for each question were little changed with slightly higher totals for a few candidates under the modified schemes.

E1. Well answered by many candidates. Scores less than 10 usually came from partial answers.

E2. Less well answered than the previous question. A number of candidates gave an incorrect definition for the product topology. For the second part many students wrote incorrectly that $i_y^{-1}(\cup_{i\in I}(U_i \times V_i)) = \cup_{i\in I} U_i$ without considering whether $y \in V_i$. Answers to the last part were often muddled or incomplete.

For paper AO2:

E3. Popular question with all but the last part well answered.

E4. Less successful question with many low marks. The marking scheme was changed, reducing the proportion of marks for the last part when it was found that no student answered it successfully.

In retrospect, these AO2 questions were a little too demanding for students to answer at the end of the second year. Students found the ‘unseen’ ends too difficult.

F: Multivariable Calculus

F1. The short question, well answered though many stumbled on the last part.

F2. The long question had very few answers. No-one answered the last part successfully.

My comments at the end on section E also hold here.

G: Calculus of Variations

G1. This short question was a standard piece of bookwork with a simple calculation. Unfortunately it was very poorly done. It is alarming that several of our students cannot solve $d^2y/dx^2 = -\lambda y, y(0) = y(1) = 0$ where λ is a constant.

G2. In contrast to G1, this long question was very well done indeed. The only part that proved a real problem was integrating $y/\sqrt{(1-y^2)}$ with respect to y - most candidates could not do this. Several candidates did not properly explain, in the proof of the first part, why $\int \eta f(x)dx = 0$ for all appropriate functions η implies that f is identically zero.

H: Classical Mechanics

H1. Virtually everyone remembered the general formula for the inertial matrix but a significant number of them could not integrate $(x_i^2 + x_j^2)$ over a hemisphere (several integrated over a cube). Very few candidates remembered correctly the parallel axes theorem.

H2. (Long question - different marker) A fairly straightforward small-oscillation problem which gave the frequencies and didn’t ask for the normal modes, which made it shorter than an old a6 problem would have been. However, it attracted fewer attempts than I would have expected, just 11 with 2 ’alphas’. Most got the picture correct. Quite a few knew that they were supposed to use the length constraints but got into muddles with Pythagoras, and quite a lot of arithmetic errors too.

As you see from the remarks made on the three questions that I marked (G1, G2 and H1), candidates by and large struggled over very standard calculations that we would expect them to be able to do very easily.

J: Quantum Mechanics

There were 101 attempts at the short question (J1) and 46 attempts at the long question (J2). The short question did not work well, and relatively few candidates scored more than 7. I knew already from talking to my own students after the exam that it was too long compared with other subjects (except for the small number who spotted the shortcuts). However, the main reason why few candidates scored 10 was that, in response to the instruction "Write down the time-dependent Schrödinger equation …", the majority of candidates started their answer "The time-*independent* Schrödinger equation is …", losing 1 of the 2 marks by giving the wrong equation. There was also a sign error in the hint, which fortunately caused few problems, partly because most ignored the hint, and anyway it made no difference to the final answer.

By contrast, the long question worked well. Most attempts managed a good β standard, and there was a reasonable tally of α answers. The majority of candidates fell into the trap of calculating the expected value of x rather than $|x|$ in the last part.

I suppose that it was inevitable that there would be some problems with consistency of standard with the short questions, when the setters were working independently with no precedents to guide them. That will presumably adjust with experience in future years. Of more concern, I was surprised that there was not better correlation between the short and the long question marks for those who did both. The correlation coefficient was 0.43, but more striking was the fact that two of the four people scoring 25 on the long question managed only 5 and 7 on the short question (the others getting 9 and 10).

K: Fluid Dynamics and Waves (HT)

Both the short (K1) and long (K2) questions were, as instructed, straightforward. The short question, which was entirely bookwork, attracted about 75 attempts of which the vast majority scored 7–9. The material appeared to have been well understood and marks were primarily lost for failure to distinguish between fixed and material ('dyed') volumes. The long question, bookwork plus a standard but nontrivial calculation, attracted about 50 attempts, of which about 30 were of alpha standard. The bookwork appeared to have been well understood and was accurately remembered; most candidates handled the contour integral well. Small numbers of marks were lost for minor arithmetical slips.

Overall I was encouraged by the high standard. There were very few negligible attempts and not many bad ones.

L: Fluid Dynamics and Waves (TT)

The short question (L1) was a deliberately straightforward question on vortex motion, which nevertheless produced a range of answers. More candidates than expected had difficulty in finding the velocity components (in polar coordinates) from the complex potential. The last part was generally well done.

The long question (L2) on water waves was extremely straightforward, and on the whole well done. The most demanding part was explaining how the linearised boundary conditions are obtained, and this is where many candidates lost some marks.

On the whole, however, the performances on questions L1 and L2 Fluid Dynamics were pleasing.

M: Probability (Laws)

M1. This (short) question was essentially the same as the first couple of parts of an old Finals question. Most candidates attempting it got 8 or more marks out of 10, and there were lots of 10s - the candidates who got good marks confirmed that they understood the basic material that the question was about. A surprising number of candidates got poor marks (say 3 or fewer) and I think it is reasonable to be confident that these candidates did not have much understanding of this part of the syllabus.

M3. The marks obtained on this long question covered the full range. Many candidates were able to do about half of it well, and there seemed to be a good number of attempts that scored over 20 marks, but only a few that got full marks. There were also quite a lot of fairly low marks. The part most candidates found hardest, and where quite a few attempts were aborted, was the calculation of the mgf of $\frac{1}{n}Y_n$ (and, to some extent, taking its limit).

N: Probability (Donnelly)

N1. This short question was straightforward and many candidates achieved high marks. Some candidates stumbled on the bookwork, and some solved the wrong equation for the stationary distribution: $\pi = P\pi$, rather than the correct $\pi = \pi P$.

N2. In this long question, part (a) was straightforward and was completed successfully by most of the candidates who attempted it. Virtually all candidates who attempted the question gained at least substantial part marks.

About a third of the candidates who attempted part (a) made no attempt at the more difficult part (b), and over half made no serious attempt at part (b). Of those who did, most had difficulty in correctly formulating the transition probabilities of the Markov chain. Candidates who nonetheless correctly calculated the stationary distribution of their (incorrect) chain had the opportunity to gain (up to the full 6) marks for this section of the question. Some candidates who had the wrong chain also successfully used the answer given to (iii) to correctly answer part (iv).

That so few candidates were able to successfully translate the text in the question into the correct mathematical formulation was disappointing. Their concerns about this aspect may have dissuaded others from attempting this part of the question.

O: Statistics

Maths&Stats candidates tended to do a little better than Maths FHS candidates on all questions. Unsurprisingly there were no answers from other joint schools candidates.

O1. (Short) The description of how to obtain the estimate and the expected number was surprisingly often wrong, although there was a general understanding of what a chisquare test is. Many students did not describe the reasons for binning clearly. The degrees of freedom were mostly fine.

O2. (Short) The estimator was typically found, but not always was the distribution completely determined. A typical error was to use a z-test instead of a t-test in the last part of the question.

O4. (Long) The Neyman-Pearson Lemma was usually stated well, but not often proved. The simple likelihood ratio test worked generally satisfactorily; not everyone picked up on there only being one observation. For the UMP test, quite a few students tried a generalized LRT approach, which does not yield a UMP test.

O5. (Long) The bookwork was mostly fine, although some students did not mention regularity conditions for the Cramer-Rao lower bound to hold. The estimator was generally found; a surprisingly common mistake was to claim $E(1/X) = 1/E(X)$, which is not true in general. The Fisher information was usually found.

P: Numerical Analysis

P1,P2. The short questions on this course appear to have been reasonably successful with a number of full marks and many serious attempts. A few very low marks may be due to candidates running out of time, or more likely candidates who had not followed this lecture course attempting answers merely because the mathematics is recognisable.

P3,P4. Because students in the past have found the relevant material difficult, I set a relatively straightforward question P4 on orthogonal factorisation and the Q R Algorithm: the result was a large proportion of very high marks amongst those candidates who attempted the question. Question P3 had a broader range of marks, with one candidate scoring full marks.

Section E. Comments on performance of identifiable individuals

There are at present no prizes available in Part A, and nothing to report here.

F. Names of members of the Board of Examiners

Prof K P Tod (Chairman), Dr A Dancer, Dr P Northrop, Dr J Norbury, Prof E Rees (external).

Assessors for Papers AO1 and AO2:

Dr D J Acheson, Dr T Browning, Prof P Donnelly, Dr K Hannabuss, Prof R Haydon, Dr S D Howison, Dr N Laws, Dr G Luke, Prof P K Maini, Dr G Reinert, Prof M Vaughan-Lee, Dr A Wathen.

The Examiners are grateful to all the assessors for their help and cooperation.

Paul Tod

Chairman of Part A Examiners

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