Examiners' Report:
Final Honour School of Mathematics Part A
Trinity Term 2024

November 12, 2024

Part I

A. STATISTICS

- **Numbers and percentages in each class.**
  See Table 1.

Table 1: Numbers in each class

Range	Numbers				Percentages %			
	2024	(2023)	(2022)	(2021)	2024	(2023)	(2022)	(2021)
70–100	44	(44)	(59)	(53)	34.59	(30.77)	(36.65)	(37.32)
60–69	64	(67)	(71)	(57)	48.12	(46.85)	(44.1)	(40.14)
50–59	22	(25)	(22)	(29)	16.54	(17.48)	(13.66)	(20.42)
40–49	-	(-)	(6)	(-)	-	(-)	(3.73)	(-)
30–39	-	(-)	(-)	(-)	-	(-)	(-)	(-)
0–29	-	(-)	(-)	(-)	-	(-)	(-)	(-)
Total	133	(143)	(161)	(142)	100	(100)	(100)	(100)

Where less than 5 students are in a category this has been redacted for confidentiality purposes so they cannot be identified.

- **Numbers of vivas and effects of vivas on classes of result.**
  Not applicable.
- **Marking of scripts.**
  All scripts were single marked according to a pre-agreed marking scheme which was strictly adhered to. The raw marks for paper A2 are out of 100, and for the other papers out of 50. For details of the extensive checking process, see Part II, Section A.
- **Numbers taking each paper.**
  All 133 candidates are required to offer the core papers A0, A1, A2 and ASO, and five of the optional papers A3-A11. Statistics for these papers are shown in Table 2 on page
  .

Table 2: Numbers taking each paper

Paper	Number of Candidates	Avg RAW	StDev RAW	Avg USM	StDev USM
A8 OS	-	-		-	
A0	133	30.09	8.79	67.73	11.77
A1	133	31.08	7.42	68.53	9.49
A2	133	64.05	15.33	67.68	9.46
A3	62	36.08	8.76	69.77	13.69
A4	110	27.53	8.39	65.95	11.56
A5	74	31.43	9.19	66.46	12.48
A6	88	36.6	8.35	68.45	12.2
A7	52	24.79	8.44	64.87	10.7
A8	116	32.1	8.56	67.12	11.23
A9	75	29.73	10.23	64.73	13.36
A10	49	30.47	7.8	65.43	9.3
A11	44	26.66	8.96	62.14	13.82
ASO	133	32.76	6.73	68.07	8.66

B. New examining methods and procedures

None.

C. Changes in examining methods and procedures currently under discussion or contemplated for the future

None.

D. Notice of examination conventions for candidates

The first notice to candidates was issued on the 12th February 2024 and the second notice on the 13th May 2024.

These can be found at `https://www.maths.ox.ac.uk/members/students/undergraduate-courses/ba-master-mathematics/examinations-assessments/examination-20`, and contain details of the examinations and assessments. The course handbook contains the link to the full examination conventions and all candidates are issued with this at induction in their first year. All notices and examination conventions are online at `https://www.maths.ox.ac.uk/members/students/undergraduate-courses/examinations-assessments/examination-conventions`.

Part II

A. General Comments on the Examination

Acknowledgements

- Haleigh Bellamy for her work in supporting the Part A examinations throughout the year, and for her help with various enquiries throughout the year.
- Waldemar Schlackow for running the database and the algorithms that generate the final marks, without which the process could not operate.
- Alice Jones, Anwen Amos, Charlotte Turner-Smith for their help and support, together with the Academic Administration Team, with marks entry, script checking, and much vital behind-the-scenes work.
- The assessors who set their questions promptly, provided clear model solutions, took care with checking and marking them, and met their deadlines, thus making the examiners' jobs that much easier.
- Several members of the Faculty who agreed to help the committee in the work of checking the papers set by the assessors.
- The internal examiners and assessors would like to thank the external examiners, Prof Neil Strickland and Prof Mark Blyth, for helpful feedback and much hard work throughout the year, and for the important work they did in Oxford in examining scripts and contributing to the decisions of the committee.

Timetable

The examinations began on Monday 10th June and ended on Friday 21st June.

Setting and checking of papers and marks processing

As is usual practice, questions for the core papers A0, A1 and A2, were set by the examiners and also marked by them with the assistance of assessors. The papers A3-A11, as well as each individual question on ASO, were set and marked by the course lecturers/assessors. The setters produced model answers and marking schemes led by instructions from Teaching Committee in order to minimize the need for recalibration.

The internal examiners met in December to consider the questions for Michaelmas Term courses (A0, A1, A2 and A11). The course lecturers for the core papers were invited to comment on the notation used and more generally on the appropriateness of the questions. Corrections and modifications were agreed by the internal examiners and the revised questions were sent to the external examiners.

In a second meeting the internal examiners discussed the comments of the external examiners and made further adjustments before finalising the questions. The same cycle was repeated in Hilary term for the Hilary term long option courses and at the end of Hilary and beginning of Trinity term for the short option courses. *Papers A8 and A9 are prepared by the Department of Statistics and jointly considered in Trinity term.* Before questions were submitted to the Examination Schools, setters were required to sign off on a camera-ready copy of their questions.

The whole process of setting and checking the papers was managed digitally on SharePoint. Examiners adopted specific and detailed conventions to help with version checking and record keeping.

Examination scripts were collected by the markers from Exam Schools or delivered to the Mathematical Institute for collection by the markers and returned there after marking. A team of graduate checkers under the supervision of Anwen Amos and Charlotte Turner-Smith sorted all the scripts for each paper, cross-checking against the mark scheme to spot any unmarked questions or part of questions, addition errors or wrongly recorded marks. Also sub-totals for each part were checked against the marks scheme, noting any incorrect addition.

Determination of University Standardised Marks

The examiners followed the standard procedure for converting raw marks to University Standardized Marks (USM). The raw marks are totals of marks on each question, the USMs are statements of the quality of marks on a standard scale. The Part A examination is not classified but notionally 70 corresponds to 'first class', 50 to 'second class' and 40 to 'third class'. In order to map the raw marks to USMs in a way that respects the qualitative descriptors of each class the standard procedure has been to use a piecewise linear map. It starts from the assumption that the majority of scripts for a paper will fall in the USM range 57-72, which is just below the II(i)/II(ii) borderline and just above the I/II(i) borderline respectively. In this range the map is taken to have a constant gradient and is determined by the corners C_1 and C_2, which encode the raw marks corresponding to a USM of 72 and 57 respectively. The guidance requires that the examiners should use the entire range of USMs. Our procedure interpolates the map linearly from C_1 to $(M, 100)$ where M is the maximum possible raw mark. In order to allow for judging the position of the II(i)/III borderline on each paper, which corresponds to a USM of 40, the map is interpolated linearly between C_3 and C_2 and then again between $(0, 0)$ and C_3. Thus, the conversion of raw marks to USMs is fixed by the choice of the three corners C_1, C_2 and C_3. While the default y-values for these corners were given above and are not on the class borderlines, the examiners may opt to change those default values, e.g., to avoid distorting marks around class boundaries. The final choice of the scaling parameters is made by the examiners, guided by the advice from the Teaching Committee, considering the distribution of the raw marks and examining individuals on each paper around the borderlines.

The final resulting values of the parameters that the examiners chose are listed in Table 3.

Table 4 gives the resulting final rank and percentage of candidates with this overall average USM (or greater).

Table 3: Parameter Values

Paper	C1	C2	C3
A8 OS	12.04;37	20.97;57	39.47;72
A0	11.07;37	19.27;57	36.27;72
A1	11.07;37	19.27;57	36.27;72
A2	23.44;37	40.8;57	76.8;72
A3	14;37	24.37;57	41;70
A4	10.42;37	18.13;57	34.13;72
A5	12.04;37	20.97;57	39.47;72
A6	13.67;37	23.8;57	42;70
A7	9.44;37	16.43;57	32;72
A8	12.04;37	20.97;57	39.47;72
A9	12.04;37	16;50	22;60
A10	11.72;37	20.4;57	37;70
A11	10.74;37	18.7;57	36;72
A12	13.35;37	17;50	23.23;57
ASO	11.72;37	20.4;57	39;72

Table 4: Rank and percentage of candidates with this overall average USM (or greater)

Av USM	Rank	Candidates with this USM or above	%
94.5	1	1	0.75
92.3	2	2	1.5
92.1	3	3	2.26
89.6	4	4	3.01
88.8	5	5	3.76
88.6	6	6	4.51
86.5	7	7	5.26
86.1	8	8	6.02
83.9	9	9	6.77
83.3	10	10	7.52
81.9	11	11	8.27
81.3	12	12	9.02
79.7	13	13	9.77
78.6	14	14	10.53
78	15	15	11.28
77.4	16	16	12.03
76.85	17	17	12.78
75.8	18	18	13.53
75.7	19	19	14.29
75.45	20	20	15.04
75.2	21	21	15.79
74.4	22	22	16.54
73.25	23	23	17.29

Table 4: Rank and percentage of candidates with this overall average USM (or greater) [continued]

Av USM	Rank	Candidates with this USM or above	%
73.1	24	24	18.05
72.25	25	25	18.8
72.2	26	27	20.3
71.4	28	28	21.05
71.3	29	30	22.56
71.2	31	32	24.06
71.1	33	33	24.81
71	34	34	25.56
70.8	35	35	26.32
70.7	36	36	27.07
70.3	37	37	27.82
70	38	39	29.32
69.9	40	40	30.08
69.8	41	44	33.08
69.5	45	46	34.59
69.3	47	47	35.34
68.9	48	50	37.59
68.7	51	52	39.1
68.6	53	53	39.85
68.4	54	54	40.6
68.2	55	55	41.35
68	56	56	42.11
67.8	57	57	42.86
67.7	58	58	43.61
67.6	59	59	44.36
67.4	60	60	45.11
67.3	61	61	45.86
67.1	62	62	46.62
67	63	63	47.37
66.9	64	64	48.12
66.8	65	66	49.62
66.6	67	67	50.38
66.5	68	68	51.13
66.4	69	69	51.88
66.3	70	70	52.63
66.2	71	72	54.14
66.1	73	75	56.39
66	76	76	57.14
65.6	77	77	57.89
65.5	78	78	58.65
65.2	79	80	60.15
65	81	81	60.9

Table 4: Rank and percentage of candidates with this overall average USM (or greater) [continued]

Av USM	Rank	Candidates with this USM or above	%
64.8	82	82	61.65
64.7	83	83	62.41
64.5	84	84	63.16
64.4	85	85	63.91
64.2	86	86	64.66
64.1	87	87	65.41
63.9	88	88	66.17
63.6	89	90	67.67
63.4	91	91	68.42
63.2	92	92	69.17
63	93	93	69.92
62.9	94	95	71.43
62.6	96	96	72.18
62.3	97	97	72.93
62.1	98	98	73.68
61.9	99	99	74.44
61.4	100	100	75.19
60.8	101	101	75.94
60.7	102	104	78.2
60.3	105	105	78.95
60	106	106	79.7
59.9	107	109	81.95
59.6	110	110	82.71
59.2	111	112	84.21
58.9	113	113	84.96
58.6	114	114	85.71
58.4	115	115	86.47
58.3	116	116	87.22
57.7	117	117	87.97
57.4	118	118	88.72
57.3	119	119	89.47
55.5	120	120	90.23
55.4	121	121	90.98
55.2	122	122	91.73
55	123	123	92.48
54.7	124	124	93.23
54.6	125	125	93.98
53	126	127	95.49
52.9	128	128	96.24
52.4	129	129	96.99
52	130	130	97.74
50.3	131	132	99.25

Table 4: Rank and percentage of candidates with this overall average USM (or greater) [continued]

Av USM	Rank	Candidates with this USM or above	%
41.9	133	133	100

B. Equality and Diversity issues and breakdown of the results by gender

Table 5, page shows percentages of male and female candidates for each class of the degree.

Table 5: Breakdown of results by gender

Class	Number								
	2024			2023			2022		
	Female	Male	Total	Female	Male	Total	Female	Male	Total
70–100	8	38	46	4	40	44	7	52	59
60–69	16	48	64	19	48	67	23	48	71
50–59	10	12	22	11	14	25	8	14	22
40–49	-	0	0	-	-	-	-	-	6
30–39	-	0	0	-	-	-	-	-	-
0–29	-	0	0	-	-	-	-	-	-
Total	35	98	133	39	105	144	43	118	161
Class	Percentage								
	2024			2023			2022		
	Female	Male	Total	Female	Male	Total	Female	Male	Total
70–100	22.86	38.78	34.59	10.26	38.46	30.77	11.9	48	37.32
60–69	45.71	48.98	48.12	48.72	46.15	46.85	50	36	40.14
50–59	28.57	12.24	16.54	28.21	13.46	17.48	35.71	14	20.42
40–49	-	-	-	-	-	-	-	-	1.41
30–39	-	-	-	-	-	-	-	-	-
0–29	-	-	-	-	-	-	-	-	-
Total	100	100	100	100	100	100	100	100	100

C. Detailed numbers on candidates' performance in each part of the exam

Individual question statistics for Mathematics candidates are shown in the tables below.

Paper A0: Linear Algebra

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14.3	14.61	5.32	101	3
Q2	14.94	15.05	4.56	118	1
Q3	14.88	15.96	6.4	47	5

Paper A1: Differential Equations 1

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14.57	14.95	4.87	61	2
Q2	15.55	15.55	3.81	120	0
Q3	15.63	15.94	5.08	85	2

Paper A2: Metric Spaces and Complex Analysis

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.23	16.56	4.12	94	4
Q2	19.25	19.25	5.25	101	0
Q3	14.14	14.28	4.97	78	2
Q4	12.74	13.18	5.01	74	3
Q5	14.98	15.23	5.87	118	3
Q6	16.67	17.14	5.29	66	3

Paper A3: Rings and Modules

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	19.17	19.17	4.53	54	0
Q2	16.4	16.65	5.7	48	2
Q3	17.64	18.32	5.38	22	3

Paper A4: Integration

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	13.75	14.03	4.53	92	3
Q2	12.4	13.49	6.11	59	8
Q3	13.21	13.64	4.7	69	4

Paper A5: Topology

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.63	16.63	5.31	68	0
Q2	14.78	14.94	4.96	68	1
Q3	14.92	14.92	3.82	12	0

Paper A6: Differential Equations 2

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	20.33	20.33	4.44	88	0
Q2	16.77	16.97	4.8	38	1
Q3	15.08	15.74	5.73	50	3

Paper A7: Numerical Analysis

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	12.1	12.45	4.42	47	2
Q2	12.62	13.04	5.84	45	2
Q3	8.64	9.75	4.03	12	2

Paper A8: Probability

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.83	16.93	4.54	107	1
Q2	12.6	13	5.44	57	3
Q3	16.99	17.22	4.66	68	1

Paper A8: Probability (old syllabus)

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15	15		1	0
Q2	10	10		1	0
Q3	-	-	-	-	-

Paper A9: Statistics

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	10.62	12.96	6.9	26	8
Q2	17.27	17.27	5.32	71	0
Q3	12.29	12.58	5.92	53	2

Paper A10: Fluids and Waves

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	11.53	11.86	3.94	29	1
Q2	16.2	16.51	5.15	43	1
Q3	14.16	16.88	6.63	26	6

Paper A11: Quantum Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15	15.23	5.13	43	1
Q2	10.07	10.04	4.12	27	1
Q3	13.63	14.53	4.32	17	2

Paper ASO: Short Options

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.74	17.25	3.98	32	2
Q2	16.79	16.79	4.54	29	0
Q3	19.85	19.85	3.72	13	0
Q5	15.89	15.89	4.57	80	0
Q6	17.09	17.7	4.24	44	2
Q7	16.58	16.9	3.58	30	1
Q8	16	16	2.71	4	0
Q9	12.71	12.91	4.55	34	1

D. Comments on papers and on individual questions

The following comments were submitted by the assessors.

Core Papers

A0: Algebra 1

Overall this set of questions worked quite well, producing a good spread of marks and distinguishing between candidates. Perhaps overall the questions were a little bit on the long side, but only a small number of candidates seemed to suffer from a lack of time.

Question 1 was a popular question, attempted by a majority of students. Part (a) was bookwork and generally answered very well. Several students produced good answers to part (b) but forgot the possibility of there being a mixture of positive and negative eigenvalues of maximum absolute value. Part (c)(i) caused several difficulties (often it was assumed that the matrix was diagonal rather than diagonalisable) but (c)(ii) was answered well, and many students had the right basic ideas for (c)(iii). Part (d) was not attempted by many students despite having a simple solution when the question was parsed.

Question 2 was the most popular question. Part (a) was generally answered very well. Part (b) most students had the correct idea, but often there was an incomplete explanation or a slight slip in calculating the minimal polynomial. The bookwork verification of part (c) was answered well, but the more challenging final computation distinguished between stronger candidates and weaker ones.

Question 3 was the least popular question, but generally answered well by those that attempted it. The bookwork of (a) was generally answered well, and most students had a good attempt at (b). Part (c) distinguished between students, with several incorrect attempts based on a faulty induction argument. It was pleasing that even students who struggled with the more challenging part (c) gave a good attempt at part (d) which was putting everything together.t

A1: Differential Equations 1

Question 1 Question 1 which revolves around Picard's theorem was solved by a bit less than half the candidates. Parts a) and b) were generally well solved, though quite a few marks were lost on bi) because students forgot to show that T maps the set X to itself. Many students realised that part c) i) was a simple application of the uniqueness part of Picard, though some solutions were incomplete as they only considered the case where the intersection point is in the rectangle $[-h, h] \times [0, 2]$ considered in b). The second part of c) was designed to be more challenging and only few students were able to give a full proof of global existence, though more student obtained partial points e.g. for using (i) to establish the claimed bounds or using the monotone behaviour of solutions to prove the claimed asymptotics. Quite a few candidates successfully tackled c)iii), either by exploiting that the equation is autonomous and hence $y_a(\cdot) = y_0(\cdot - a)$, or using a Gronwall-argument.

Question 2 This question was solved by the vast majority of students and most solved Part a) and the earlier parts of b) well. Surprisingly, a lot of students computed the eigenvalues via the characteristic polynomial and did not realise that they could have just read them off since the question was posed so that all relevant matrices were diagonal (and some even the identity matrix). Sketching the phase diagram proved more challenging and many students could not use the fact that the unit circle is a vertical nullcline correctly. While many students successfully used separation of variables in (iv), many claimed that the trajectories were straight lines through the origin, instead of half-lines which asymptote to 0 as $t \to -\infty$. Most students correctly identified that $(0, 0)$ is an unstable critical point in $c\alpha$) and there were some nice correct arguments on the more challenging part β), using e.g. the signs of X and Y within the circle $x^2 + y^2 = \frac{1}{3}$ to establish stability. There were only a few correct solutions for the final part of c), which was set so that it did not require much calculation, but could rather be solved either by observing that for $X = Y = 0$ every point is stationary and hence every point is a stable critical point, or that these equations represented the the same system in forwards and backwards time direction and hence that systems that had periodic solutions centred around $(1, 1)$ would work,

Question 3

Most students successfully solved the PDE using the methods of characteristics and explained correctly why the data is Cauchy using either the Jacobian or arguing that the characteristic projections intersect the data curve transversally since they are part of orthogonal lines. The description of the characteristic projections was often not precise enough, with many students not realising the key point that the characteristic projections are only half-lines with slope one, asymptoting to points on the y axis, rather than the full lines $y = x + B$, leading to many incorrect domains of definitions in (iii). Many students correctly solved (iv), but few realised that the solutions u computed in the earlier parts all satisfy the constraints in (v) hence establishing non-uniqueness. In the final part of the question quite a few students incorrectly claimed that the domain of definition is unchanged, failing to take into account that the change in right hand side leads to a constraint on the allowed parameters t and hence to a reduced domain of definition.

A2: Metric Spaces and Complex Analysis

Question 1 was the most popular, followed by Question 2. Common mistakes in Question 1(b) were not to check well-definedness of the norm and the metric, and in Question 1(b)(iii) to assume without justification that the limiting function for the different sized balls is the same. For Question 2, a surprising number of candidates confused a homeomorphism with an isomorphism in 1(a) which subsequently led to misakes in 1(b). Most candidates did well at 2(b) although some attempted very convoluted proofs for 2(b)(iii). In Question 6, many different proofs for 6(b) were given; a common loss of a point was to forget to check or simply assuming conformality without proving it. Another source of loss of points was that to apply the hint for 6(c) one must verify the conditions for the function specified in the hint.

Question 3: A not particularly popular question, and not well done, with many attempts not extending beyond part (a). Most attempts omitted to address the converse in (a)(ii), not showing that the polar equations implied the Cartesian ones. Some comment on a non-zero determinant/Jacobian or the invertibility of the linear system was sufficient. Whilst a majority of the attempts completed the rest of (a), a worrying number of scripts showed candidates misremembered the multivariate chain rule, whilst others wrote $u = r\cos\theta, v = r\sin\theta$ rather than x and y.

In (b)(i) there is a sign change in the limits across the cut, with the limit from above equalling $-i\sqrt{r^2-1}$. Then (b)(ii) is a matter of applying the binomial theorem to $\sqrt{1-z^2}$. Part (b)(iii) was attempted by very few and was generously marked. Partial credit was given for claiming $\Sigma = \Sigma_+ \cup \Sigma_-$, even though this isn't strictly true, and full credit for any further attempt suggesting 'gluing' the cuts together to form a cylinder without any greater detail needed.

Question 4: A not particularly popular question, again not well done beyond part (a). Many completed (a) well, though too many attempts lost marks through lack of care: most calculated the residue of f'/f correctly when f has a zero, but failed to note conversely that f'/f is singular if and only if f is zero; others failed to mention the estimation theorem or residue theorem whilst applying them and so lost a mark here or there. The occasional attemot simply quoted the argument principle, when the question is clearly expecting a proof of a special case of that principle.

The analysis in (b)(i) is essentially of first year difficulty, just applying the algebra of limits – many attempts demonstrated this concisely, but many other scripts were either laboured or even inconclusive. Applying the estimation theorem to the inequality from (b)(i) shows that $|N-n| \leqslant 1/2$. As N and n are integers they are then equal, thus providing an alternative proof of the fundamental theorem of algebra.

(c) was completed by only a handful. Applying (a)(iii), the number of roots can be found by finding the winding number around the origin of the suggested quarter-circle's image under the quintic. The interval $[0, R]$ is mapped along the positive real axis, not changing in argument. For large enough R, the circular arc's image then wraps around the origin with argument changing by $5\pi/2$. Finally the imaginary axis has a linear image that decreases from an argument of $\pi/2$ back to 0. Overall a winding number of 1. No credit was given applying Rouché's theorem, as this trivializes the question and uses none of the earlier question, though credit would have been given if a proof of the theorem had been provided.

Question 5: A very popular question, reasonably well done. Barring arithmetic errors, candidates lost marks in (a) mainly from failing to quote the estimation theorem, the residue theorem or by bounding above complex integrals rather than their moduli. Quite a few candidates laboured to calculate the residue at $e^{\pi i/3}$ which was most easily found using the $g(a)/h'(a)$ formula. A surprising number of scripts had the contour entirely drawn within the first quadrant, but this had little to no impact on marks.

(b) was mainly done well, with most appreciating how to obtain the Laurent series for

$(\sin \pi z)^{-1}$ using the binomial theorem and most applying periodicity to find the residues in (ii). A significance for (c) is to appreciate this calculation, in principle, shows the coefficient of z^{2k-1} in $\varphi(z)$ is a rational multiple of π^{2k}. (c) then involves the standard square contour Γ_k with the function $z^{-2n}\varphi(z)$.

Question 6. Overall, the question was done okay by most candidates.
Part a) Here, i) and ii) were often correct when done at all.
Part b) i) Was done successfully by many people. ii) Was sometimes not attempted and sometimes people obtained 4/6 points as they did not calculate f^{-1}.
Part c) i) Was solved successfully by most people that attempted it. ii) Was rarely solved. Sometimes people just wrote yes or no without explanation.

Long Options

A3: Rings and Modules

Question 1 was on quadratic rings of integers $\zeta[\sqrt{d}]$ and in particular the use of a norm function to solve factorisation problems. It was a popular question with many good attempts. Many candidates however took an unnecessarily lengthy approach by not fully exploiting the properties of the norm. The last part, on showing the existence of infinitely many units in some examples with positive d, proved challenging as expected, but quite a few candidates did see the idea of finding a nontrivial unit and then taking powers to get an infinite sequence of units.

Question 2. This question, on polynomial rings, also proved popular and was generally well done. Most candidates did the bookwork in part (a) well. The factorisation examples in part (b) proved more diffcult (some candidates seemed unsure about the use of Gauss's Lemma), but there were many good attempts.

Question 3. This question, on modules, proved the least popular, but there were several good attempts. The first part, working out the analogue of the correspondence principle for modules, was done fairly well, though often some details were omitted. The last part, on modules with no proper nonzero submodule (ie simple modules), proved more challenging.However quite a few people did get the idea of first proving the cyclic property and then using the correspondence principle, and then looking at kernel and image for the last part.

A4: Integration

Question 1: This was a popular question, with the bookwork (a)(i) and (a)(iii) typically being very well answered, though there were some occasional mistatements of the MCT (requiring the f_n to be integrable, as opposed to measurable, or stating that the limit function f is integrable without imposing any condition on $\lim \int f_n$). (a)(ii) was less well done. Many candidates correctly observed that any simple ϕ with $0 \leq \phi \leq f$ must have $\phi = 0$ almost everywhere, and then stated without justification that $f = 0$ ae. (b)(i) was often well answered, and many students had the right idea in (b)(ii) (though not all took enough care to make it clear exactly what function was intended). (b)(iii) was found to be more tricky — quite a few answers tried to claim that uniformly continuous functions with $\liminf_{x\to\infty} f(x) = 0$ automatically satisfied $\limsup_{x\to\infty} f(x) = 0$ — but candidates who wrote down a negation of $\lim_{x\to\infty} f(x) = 0$, typically succeeded in making progress. (c) was found to be very challenging. Only a very small number of students made progress (or even correctly identified what the required function f would need to be); but there were a couple of excellent solutions.

Problem 2. Parts (a)(i) and (a)(ii) were done very well by students. Part (a)(iii) is obtained by induction, but many students failed to correctly prove it. For (b), parts (i) and (ii) seem to be basic material and the students knew it. Part (b)(iii) was certainly more challenging for students. Among the students who solved this, most of them preferred a solution in which $lim_n f_n$ was written as $inf_n sup_m \geq n f_m$. Part (c) also gave some difficulties to many students. In particular, among the students who successfully constructed the function $g : [0, 1] \to C$, only a few of them showed that is strictly increasing. Part (c)(ii) was attempted by most of the students and many of them pointed out that $g(A)$, where A is not Lebesque measurable, is the right candidate for the solution.

Problem 3. Students did pretty well parts (a)(i) and (a)(ii). Most of them knew how to apply Fatou's lemma in (a)(i) and derived (a)(ii) from the first part. For part (a)(iii), students knew how to show that $xsin(x)/(e^x$ - 1) is integrable on $(0, \infty)$, but many of them failed in computing the integral. Part (b) was certainly the hardest part of this problem. They had difficulties in bounding the tangent function (and work with it) in order to obtain the integrability characterization of f in terms of α and β.

A5: Topology

Question 1 was attempted by most candidates. Part a.i) was well done. In part a.ii) several candidates said that the compact subsets of $\mathbb{R}$ are intervals. Several correct proofs were given for a.iii) but when using sequences it was often claimed that the sequence defined converges (or that is is Cauchy) rather than passing to a subsequence and using compactness. Part a.iv) was trickier but there were many correct solutions. A common mistake was to alter the order of quantifiers.

Part b.i) was generally well done, even though some candidates missed the counterexample in the second part. For b.ii) in the first part some candidates assumed that X, Y were metric spaces. Several candidates gave correct counterexamples in the last part.

Question 2 was attempted by most candidates. Parts a.i), a.ii) were generally well done. A couple of different proofs were given for a.ii). Fewer candidates solved a.iii) but there were few mistakes when it was attempted.

In part b.i) several candidates did not give the topology on the quotient set, so they received no marks. Part b.ii) was done by most candidates. There were many successful attempts for b.iii) but some students tried to use only that X is Hausdorff rather than metric space and got nowhere.

Many different, generally correct, solutions were given for c.i). A common mistake was to not verify that the topology on the quotient space was indeed the indiscrete topology. In part c.ii) many candidates realized that one should use connectedness. Some missed a point when they gave an incomplete argument. Part c.iii) was the most challenging but many candidates gave a correct construction.

Question 3 was attempted by fewer candidates, but the candidates who attempted it did generally well. Parts a.i), a.ii) were generally well done. In part a.iii) several candidates did not show that the map defined is a homeomorphism as it was required. In part a.iv) several candidates gave the right answer but without a proof, or with an incorrect or incomplete proof and marks were taken off for this. Part a.v) was well done.

There were a few mistakes in the definition of the link in b.i). Several candidates gave correct and different constructions for b.ii). Many candidates managed to construct a complex with the required property in b.iii). However several claimed that Δ^n and S^n are homeomorphic, which is obviously false.

A6: Differential Equations 2

Question 1 - This was a very popular question attempted by almost every candidate. It was perhaps slightly on the easy side, and it was generally done well, although many candidates made very heavy weather of their algebra, with incredibly lengthy calculations for things that could be done in a few steps with a bit more considered thought. Some quite common slips were to get the sign of the discontinuity in the gradient of the Green's function wrong, and to add both Ax and Bx^{-1} for the general solution in the final part (the latter solves the homogenous equation but not the boundary conditions).

Question 2 - Part (a) was straightforward and generally done well, although a surprising number of candidates got themselves confused about the sign of λ and the associated solutions of this equation (many said that λ ought to be negative in order to have trigonometric solutions). Some misread the boundary conditions. Part (b) was done well on the whole; a common error was to say that one could 'choose' to take $a_1 = 0$, rather than to show how this is required from the coefficient of x in the expansion, and quite a lot of attempts to write down the general formula for a_{2k} went wrong. For Part (c), a surprising number of candidates seemed to have forgotten how to look for separable solutions, most noticed the connection with the earlier parts appropriately, though some got themselves quite confused about the rescaling required to convert the radial equation to the Bessel equation, and it was quite common to try to apply the $r = a$ boundary condition at an inappropriate stage of the working.

Question 3 - Part (a) proved to be surprisingly challenging - particular the root at large negative x, where one has to recognise that the exponential term becomes negligibly small. Those who did better were generally the ones who started by sketching a graph (as repeatedly suggested during the lectures) to help work out roughly where the solutions should be expected. A very common error was to replace the exponential with its Taylor series regardless whether x was small or not (such an expansion of course converges, but is not an asymptotic series - as needed here - unless x is small). For Part (b), which was mostly done well, the most common error was not to consider or explain what boundary conditions need to be applied for this inner problem. For part (c), a pleasing number of candidates realised how similar this was to the earlier part, but many were confused where/when to apply the initial conditions - only some candidates realised that the problem as posed is effectively already the boundary layer scaling (relative to the longer timescale $t \sim 1/\delta$), and wanted to introduce an extra boundary layer near $t = 0$.

A7: Numerical Analysis

Question 1 was attempted by most candidates. (a-i) is bookwork, although perhaps not the easiest one. Most candidates apparently found Part (a-iii) to be very challenging, although once one observe that H_1, H_2 are orthogonal and so do not change singular values, and for $I - \alpha ww^T$ it is not hard to find the eigenvalues and eigenvectors using the process of (a-ii) (which also seemed more challenging than intended), it is relatively straightforward.

(b)-(i) Many failed to see that a column of the inverse can be computed by solving a linear system with right-hand side e_i. (iii) was explained in lecture, but many did not discuss the delicate interaction between L (lower-triangular matrices) and P (permutation). The final part of part (c) was intended to be challenging, and this was confirmed when marking.

Question 2 was also attempted by most candidates. (a-i,b-i) are bookwork. Many thought (a-ii) was related to the double-degree exactness of Gauss quadrature; that is not really the point here. In (b-ii) it is important to note the exactness of Gauss quadrature for degree up until $2n + 1$. I was pleased to see that quite a few candidates got (b-iv) correctly; this was perhaps easier than I thought (unlike most of the other problems, which appears to have been harder than intended).

Question 3 was attempted by far fewer ($\approx$25) candidates. (a-i) several candidates answered the method is multistep. This would be a misnomer as it does not use previous solutions, e.g. y_{n-1} to get y_{n+1}; however it is not strictly incorrect, so such solutions received partial marks. Working out the Taylor series in (a-ii) was challenging to many. Using the theorems given in lecture received half the mark. (a-iii) Some candidates seemed to be confusing A-stability with zero-stability.

(b) was intended to be a nice blend of the IVP and NLA components of the course, but few made serious progress in (iii), (iv); perhaps they were exhausted at this point. (i), (ii) were meant to be a relatively straightforward application of Gerschgorin, however even these appeared to be not so easy in an exam.

A8: Probability

See Mathematics and Statistics report.

A9: Statistics

See Mathematics and Statistics report.

A10: Fluids and Waves

Q1: This question was found quite challenging on the whole. While part (a) was generally done very well, part (b) proved more difficult than expected. Relatively few candidates were able to correctly derive the boundary condition in (b)(i) and recognized that this boundary condition ensures that the spatial dependence $f(r)$ in the separable solution for ϕ has to be $f(r) \propto r^2$ (part (b)(ii)). Combined with Laplace's equation, this $f(r)$ determines $g(\theta) = A\cos 2\theta + B\sin 2\theta$. Very few candidates were able to determine the particle paths in (b)(iv), meaning that many candidates incorrectly guessed the answer to part (c)(ii) without first calculating the streamlines in part (c)(i).

Q2: This was the most popular question, and was answered well on the whole. Parts (a) and (b) were generally well done. In part (c) some candidates attempted to use Milne–Thomson's circle theorem in the z-plane (where the cylinder is an ellipse), rather than in the ζ-plane (where the cylinder is a circle); as well as being incorrect, this led to difficulties in calculating the stagnation points. Finally, many candidates assumed that the fluid flow is unbounded for the elliptical cylinder, when in fact the velocity singularity (and hence the need for the Kutta condition) only emerges in the limit $R \to a$, which is the case covered in lectures.

Q3: The bookwork in this question was generally well done, with clear descriptions of the linearization process in (a). However, the logic for the separation of variables in part (b) was often muddled, with very few students showing that the no-penetration boundary conditions $\partial\phi/\partial x = 0$ at $x = 0, a$ leads to $\phi \propto \cos(m\pi x/a)$, and similarly for the y-direction. Instead, this functional form (or the analogue with sin) was made as an *ansatz*. In part (c) very few students managed to calculate the interface shape driven by the given pressure profile and, in particular, to derive the denominator $\Omega - \omega$ that gives resonance as $\Omega \to \omega$.

A11: Quantum Theory

All candidates but one chose Question 1. Approximately the same number of candidates chose Questions 2 and 3. The average marks on Question 1 and 3 were similar, whereas on Question 2 the average was considerably lower. Only the strongest candidates have made progress on the challenging c) parts of questions and for them time seems to have been an issue, and led to a cutoff

Question 1

In part a) only about half of the candidates found the trick to shift the coordinate $x = y - a$, and some of those who did not perform the shift struggled with trigonometric identities. Part b) was generally done well by reproducing the abstract proof of the continuity equation (rather than by showing that the continuity equation holds for the concrete wave function $\Psi(t, x)$ obtained in b) ii)). In part c) due to missing some equations or typos, only a few candidates managed to correctly derive $f(k) = -\sqrt{\kappa_0^2 - k^2}\tan(3k)$ with $\kappa_0^2 = \frac{2ma^2}{\hbar^2}(V_1 - V_0)$, and nobody gave the correct argument for the existence of a real solution for $V_1 > V_1^{(\mathrm{crit})}$ (equivalent to $\kappa_0 > \pi/6$).

Question 2

Part a) was generally done well, but surprisingly only very few candidates knew what to do with L^2 in the expression of the Laplacian, and hence have failed to reduce the problem to a radial Schrödinger equation. No candidate managed to obtain the correct recursion relation in c) i), but a few still gave the map asked for in c) ii).

Question 3

In contrast to Question 2, candidates choosing Question 3 all knew the spectrum of J^2. They also showed very good command of manipulating commutators. In part b) ii) about half of the candidates forgot about the $\sin\theta$ factor in the sphere surface element $\sin\theta d\theta d\phi$, but have otherwise done well. Even the very unfamiliar part c) was navigated well, although no candidate managed to determine γ and μ correctly. A common mistake was to forget about operator ordering in taking the adjoint to find an expression for J_-, which has led to issues in the computation of the product J_-J_+.

Short Options

ASO: Q1. Number Theory

b(i) and b(ii) had a typo: (x/p) should have been (n/p). This small typo did not cause any problems and there was no need to change the marking scheme. Part a was B/S and candidates answered it mostly correctly. Part b was S/N and many candidates gave good answers to (i), (ii) and (iii). (iv) was a little more challenging and not many candidates gave a full proof of it. Part c was N but had similarities to proofs in lectures as the hint indicated. Many candidates gave good solutions for it.

ASO: Q2. Group Theory

Apart from a handul of candidates everyone was able to define Sylow p-subgroups correctly and state the three Sylow theorems. This gave a generous 8/25 points to almost everyone.

Question (b) was simple booking-keeping and well done by the vast majority of students. Still, this simple question allowed to distinguish between candidates who had trouble with the basic concepts of subgroup and normal subgroup and the others. Those that did not get the 2 marks allocated to this question did not get more than 14/25, which is what I suggest should be scaled to 60/100.

Question (c) was the core of the paper. It allowed students to show their skill and resulted in a nice spread of marks. Only 8 out of 33 students got the full 8 marks allocated to this question. Many made the same error in recalling the definition of the derived subgroup (seeing it as the set of commutators rather than the subgroup they generate).

Question (d)(i) was very close to one in the example sheet and was done well by virtually everyone, yielding 3 marks.

Question (d)(ii) on the other hand was meant to be more difficult and indeed only 1 student (out of 33) got it (congratulations to them!). Most others either did not attempt it or provided incorrect and confused answers. A couple of them were on the right lead though. In retrospect, I think I could have added a hint saying that one could use the result of part (c) and (d)(i) to conclude.

To conclude, I think that the paper was at the appropriate level of difficulty. Had I had the opportunity, I would have changed the mark scheme by giving fewer points to (a) (which was quick bookwork) and more to (c) (where the main work and the most thought need to be put), so as to get a better spread of marks. I suggested to rescale the marks as above 12 to 50, 14 to 60 and 18 to 70, to reflect this.

ASO: Q3. Projective Geometry

Question 3: In part (a), only (iii) caused some trouble. Most candidates found a bijection of the type described in the problem by choose coordinates on V . Only one candidate solved this part using duality, which was the most natural and elegant solution. In part (bii), some students overlooked one or two degenerate cases, but this did not seem to reflect any lack of understanding. Most candidates who attempted part (biii) correctly identified the condition on the matrix form of T. But few exhibited an infinite family of transformations satisfying this condition.

ASO: Q4. Multidimensional Analysis and Geometry

Not attempted by any candidates.

ASO: Q5. Integral Transforms

Q5(a) was done correctly by the vast majority of students who tried Q5. Errors were failing to make it clear that F is a general distribution (by using integrals instead of inner products) or missing out on the continuity requirement in the definition of a distribution or simple miscalculation in (a,ii). Many students struggled with Q5(b): In (i), they often tried integration by parts rather than variable substitution, and in (b,ii), they struggled with differentiating t^{z-1} with respect to t correctly. Q5(c) had in principle three steps: Transforming the ODE+BCs with the Laplace transform, identifying the integrating factor, decomposing the RHS into partial fractions and inverting these into the spatial domain. Some, but only very few, got all three of these right. Quite a number got the equation in p right (first step) and a smaller but still significant fraction managed to do the second step correctly (correct integrating factor). Overall, the difficult parts where Q5(b) and getting to the last step in 5(c) without algebra errors.

ASO: Q6. Calculus of Variations

Mostly candidates achieved nearly full marks on parts (a) to (c), while parts (d) and (e) proved more challenging. It is important to distinguish between partial and total derivatives in part (a).

For (d) almost all candidates tried to substitute the given parametric solution rather than deriving it. The expression in the square root is the square of the numerator divided by h^2. Many candidates were unable to simplify the denominator into the required form. A few included a stray contribution from the lower integration limit when calculating.

There was an unfortunate typo. The last line of part (d) should have read $\sqrt{c-h}f(x)$ to match the displayed equation above, not $\sqrt{\lambda-h}f(x)$. Almost all candidates answered the question as intended. One commented that λ should be c. Very few candidates realised that $k \ll 1$ also implies $h + c = 2h + \mathcal{O}(k^2)$ so $f(x) = \sin(\sqrt{2/h}\,x)$.

For part (e) several candidates asserted that H_G is constant along solutions, but did not show that the constant is zero. There are two Euler–Lagrange equations, one for x and one for y, but only one Beltrami identity. The function G does not depend explicitly upon x.

This system does not conserve energy. An external torque is required to keep the string rotating with constant angular velocity ω.

ASO: Q7. Graph Theory

Question 7: This question was largely well done, but with (a)(v) proving the most difficult part. There are various ways to demostrate $n \leqslant 2^k$, with the following perhaps being the most direct. Without loss of generality we may assume that $V = V(K_n) = V(G_i)$ for each bipartite graph. Then for each i there is a map $p_i\colon V \to \{1, 2\}$ assigning each vertex to its part of G_i. As K_n is complete, then no two vertices can be in the same part of each graph G_i. Consequently $(p_1, \ldots, p_k)$ is an injective map from V to $\{1, 2\}^k$.

(b)(iii) Note that the two 8 cycles will include each vertex, and the one cycle determines the edge set of the other. There are 144 oriented 8-cycles. This figure needs dividing by 2 to discount orientation and again needs dividing by 2 and 'first cycle, second cycle' leads to the same decomposition as 'second cycle, first cycle'. So the answer is 36.

ASO: Q8. Special Relativity

(a) and (b) are book material. Most of the students did well. Students may lose marks for skipping too much of the details of the proof.

(c): most of the students could do (i)-(iii), with minor issues, a few students completed part of (iv) and (v). (v) seems the hardest for all the students.

ASO: Q9. Modelling in Mathematical Biology

(a) (i) Most candidates struggled with giving the correct biological motivation for the terms in the model. (ii) Done very well by most candidates, although some candidates stated the carrying capacity to be $\frac{1}{k}$ when here it is $\frac{a-b}{k}$.

(b) (i) While most candidates knew what they needed to do for this question many got bogged down in the algebra. Most candidates realised that they needed to show that the steady state equation had real roots, but few stated (and showed) that they should be non-negative to be biologically realistic. Many did not notice that for the quadratic equation for the non-zero steady state the discriminant is clearly positive for small n (hence solutions are real) and also less than minus the coefficient of N, which is positive for small n, so that the result follows immediately.

Few candidates saw that the initial condition was to the right of the largest steady state.

To find the critical value of n many candidates realised that the discriminant needed to be zero but then really struggled with the algebra.

Fewer than 5 candidates got the ecological significance part, namely, that controlling a pest species by releasing more pests (sterile) is only going to be an effective control strategy if the number of pests released is small. In this case, the pest population will decrease to less than a quarter of its normal carrying capacity.

(c) (i) Most candidates did well on explaining the biological meaning of these equations.

(ii) Many candidates struggled to sketch the nullclines and only a few were able to correctly calculate the signs of the derivatives either side of the nullclines.

(iii) Very few candidates attempted this part of the question. Only 2 candidates answered it correctly.

E. Comments on performance of identifiable individuals

1. Prizes

The Gibbs Prizes for Mathematics Part A were awarded to: Yunhao Lou, St. Catherine's College
Zhenyu Yang, Corpus Christi College

F. Names of members of the Board of Examiners

- **Examiners:**
  Dr. Richard Earl (Chair)
  Prof. James Maynard
  Prof. Harald Oberhauser
  Prof. Melanie Rupflin
  Dr. Neil Laws
  Prof. Julien Berestycki
  Prof. Mark Blyth (External Examiner)
  Prof. Neil Strickland (External Examiner)
  Prof. Owen Jones (External Examiner)

- **Assessors:**
  Prof Emmanuel Breuillard
  Prof Paul Dellar
  Dr Daniel Drimbe
  Prof Ian Hewitt
  Prof Kobi Kremnitzer
  Prof Yuji Nakatsukasa
  Prof Philip Maini
  Prof Kevin Mcgerty
  Prof Mark Mezei
  Prof Andreas Muench
  Prof Panagiotis Papazoglou
  Prof Qian Wang
  Prof Stuart White
  Prof Dominic Vella