Examiners' Report: Final Honour School of Mathematics Part B Trinity Term 2024

November 5, 2024

Part I

A. STATISTICS

- **Numbers and percentages in each class.**

  See Table 1.

	Numbers					Percentages %				
	2024	(2023)	(2022)	(2021)	(2020)	2024	(2023)	(2022)	(2021)	(2020)
I	51	(54)	(55)	(51)	(73)	38.93	(36.24)	(41.04)	(39.84)	(46.5)
II.1	50	(72)	(53)	(58)	(66)	38.17	(48.32)	(39.55)	(45.31)	(42.04)
II.2	24	(18)	(24)	(18)	(13)	18.32	(12.08)	(17.91)	(14.06)	(8.28)
III	-	(-)	(-)	(-)	(-)	-	(-)	(-)	(-)	(-)
P	-	(-)	(-)	(-)	(-)	-	(-)	(-)	(-)	(-)
F	-	(-)	(-)	(-)	(-)	-	(-)	(-)	(-)	(-)
Total	130	(149)	(134)	(157)	(151)	100	(100)	(100)	(100)	(100)

Table 1: Numbers and percentages in each class

  Table data with less than 5 students has been removed so that individuals cannot be identified.

- **Numbers of vivas and effects of vivas on classes of result.**

  As in previous years there were no vivas conducted for the FHS of Mathematics Part B.

- **Marking of scripts.**

  BEE Extended Essays, BSP Mathematical Modelling and Numerical Computation Structured Projects and coursework submitted for the History of Mathematics course were double marked.

  The remaining scripts were all single marked according to a preagreed marking scheme which was strictly adhered to. For details of the extensive checking process, see Part II, Section A.

- **Numbers taking each paper.**

  See Table 5.

B. Changes in examining methods and procedures currently under discussion or contemplated for the future

None.

C. Notice of examination conventions for candidates

The Notice to Candidates Offering Coursework was issued on the 3 March 2023. The first Notice to Candidates was issued on 15 March 2023 and the second notice on 28 April 2023.

All notices and the examination conventions for 2023 are online at Examination conventions.

Part II

A. General Comments on the Examination

The examiners would like to record their heartfelt thanks to all those who helped in the preparation, administering, and assessing of this year's examinations. The chair would like to thank Rosalind Mitchell, Alice Jones, Charlotte Turner-Smith, Waldemar Schlackow, Matt Brechin and the rest of the academic administration team for their support of the Part B examinations.

In addition the internal examiners would like to express their gratitude to Professor John Hunton and Dr Ed Brambley for carrying out their duties as external examiners in such a constructive and supportive way during the year and for their thoughtful contributions during the final examiners' meetings.

For the most part, the examination process went smoothly this year. However, some issues did arise and these can and should be addressed during the 2024-25 cycle. These were brought to teaching committee to be actioned.

Standard of performance

The standard of performance was broadly in line with recent years. In setting the USMs, we took note of

- the Examiners' Report on the 2023 Part B examination, and in particular recommendations made by last year's examiners, and the Examiners' Report on the 2023 Part A examination, in which the 2024 Part B cohort were awarded their USMs for Part A;
- the guidelines provided by the Mathematics Teaching Committee, including its recommendations on the proportion of candidates that might be expected in each class.

Setting and checking of papers and marks processing

The internal examiners initially divided between them responsibility for the units of assessment (that is, the exam papers and projects).

Following established practice, the questions for each paper were initially set by the course lecturer, with the lecturer of a related course involved as checker before the first draft of the questions was presented to the examiners. The course lecturers also acted as assessors, marking the questions on their course(s).

Requests to course lecturers to act as assessors, and to act as checker of the questions of fellow lecturers, were sent out early in Michaelmas Term, with instructions and guidance on the setting and checking process, including a web link to the Examination Conventions.

The internal examiners met at the beginning of Hilary Term to consider those draft papers on Michaelmas Term courses, and changes and corrections were agreed with the lecturers where necessary. Where necessary, corrections and any proposed changes were agreed with the setters. The revised draft papers were then sent to the external examiners. Feedback from external examiners was given to examiners and to the relevant assessor for response. The internal examiners at their meeting in mid Hilary Term considered the external examiners' comments and the assessor responses, making further changes as necessary before finalising the questions. The process was repeated for the Hilary Term courses, but necessarily with a much tighter schedule. Before questions were submitted to the Examination Schools, setters were required to sign off a camera-ready copy of their questions.

Exams were held in-person in the Exams Schools. Papers were collected by the Academic Administration team and made available to assessors approximately half a day following the examination. Assessors were made aware of the marking deadlines ahead of time and all scripts and completed mark sheets were returned, if not by the agreed due dates, then at least in time for the script-checking process.

A team of graduate checkers, under the supervision of Alice Jones and Charlotte Turner-Smith, sorted all the marked scripts for each paper of this examination, cross checking against the mark scheme to spot any unmarked questions or parts of questions, addition errors or incorrectly recorded marks. Also sub-totals for each part were checked against the mark scheme, noting correct addition. In this way a number of errors were corrected,and each change was signed by one of the examiners who were present throughout the process.

Throughout the examination process, candidates were treated anonymously, identified only by a randomly-assigned candidate number.

Timetable

Examinations began on Tuesday 21 May and ended on Friday 14 June.

Consultation with assessors on written papers

Assessors were asked to submit suggested ranges for which raw marks should map to USMs of 60 and 70 along with their mark-sheets, and almost all did so. In most cases these were in line with the assignments given by the assessors.

Determination of University Standardised Marks

The Mathematics Teaching Committee issued each examination board with broad guidelines on the proportion of candidates that might be expected in each class. This was based on the average in each class over the last four years, together with recent historic data for Part B.

We followed the Department's established practice in determining the University standardised marks (USMs) reported to candidates. Papers for which USMs are directly assigned by the markers or provided by another board of examiners are excluded from consideration. Calibration uses data on the Part A performances of candidates in Mathematics and Mathematics & Statistics (Mathematics & Computer Science and Mathematics & Philosophy students are excluded at this stage). Working with the data for this population, numbers N_1, N_2 and N_3 are first computed for each paper: N_1, N_2 and N_3 are, respectively, the number of candidates taking the paper who achieved in Part A average USMs in the ranges $[69.5, 100]$, $[59.5, 69.5)$ and $[0, 59.5)$.

The algorithm converts raw marks to USMs for each paper separately. For each paper, the algorithm sets up a map $R \to U$ (R = raw, U = USM) which is piecewise linear. The graph of the map consists of four line segments: by default these join the points $(100, 100)$, $P_1 = (C_1, 72)$, $P_2 = (C_2, 57)$, $P_3 = (C_3, 37)$, and $(0, 0)$. The values of C_1 and C_2 are set by the requirement that the number of I and II.1 candidates in Part A, as given by N_1 and N_2, is the same as the I and II.1 number of USMs achieved on the paper. The value of C_3 is set by the requirement that P_2P_3 continued would intersect the U axis at $U_0 = 10$. Here the default choice of *corners* is given by U-values of 72, 57 and 37 to avoid distorting nonlinearity at the class borderlines.

The results of the algorithm with the default settings of the parameters provide the starting point for the determination of USMs, and the Examiners may then adjust them to take account of consultations with assessors (see above) and their own judgement. The examiners have scope to make changes, either globally by changing certain parameters, or on individual papers usually by adjusting the position of the corner points P_1, P_2, P_3 by hand, so as to alter the map raw $\to$ USM, to remedy any perceived unfairness introduced by the algorithm. They also have the option to introduce additional corners. For a well-set paper taken by a large number of candidates, the algorithm yields a piecewise linear map which is fairly close to linear, usually with somewhat steeper first and last segments. If the paper is too easy or too difficult, or is taken by only a few candidates, then the algorithm can yield anomalous results—very steep first or last sections, for instance, so that a small difference in raw mark can lead to a relatively large difference in USMs. For papers with small numbers of candidates, moderation may be carried out by hand rather than by applying the algorithm.

This year a preliminary meeting of the internal examiners was held in advance of the final exam board meeting to compare the default settings produced by the algorithm alongside the reports from assessors. It was agreed that only a selection of scaling maps would be further reviewed at the final exam board, and that external examiners would be given an opportunity to review all maps prior to the meeting. Adjustments were made to the default settings as appropriate, paying particular attention to borderlines and to raw marks which were either very high or very low. Where the examiners were in doubt as to the most appropriate scaling, the preliminary scalings were held over to the final exam board meeting, where the factors considered by those in the preliminary meeting were reviewed and weighed before a final decision was made.

Table 2 on page gives the final positions of the corners of the piecewise linear maps used to determine USMs.

In accordance with the agreement between the Mathematics Department and the Computer Science Department, the final USM maps were passed to the examiners in Mathematics & Computer Science. USM marks for Mathematics papers of candidates in Mathematics & Philosophy were calculated using the same final maps and passed to the examiners for that School.

Comments on use of Part A marks to set scaling boundaries

None.

Table 2: Position of corners of the piecewise linear maps

Paper	P_1	P_2	P_3	P_4	Additional Corners	N_1	N_2	N_3
B1.1	15.4;39	19;50	26.8;59	45;72	50;100	7	19	14
B1.2	13.33;39	23.2;59	44;72	50;100		7	22	15
B2.1	8.16;39	14.2;55	38.2;74	50;100		15	12	3
B2.2	6.43;32	13;52	17;60	23;70	50;100	12	9	5
B2.3	11.89;39	21;62	29;72	50;100		8	4	3
B3.1	10.63;39	18;52	40;72	50;100		19	16	10
B3.2	9.54;39	19;52	40;72	50;100		10	14	2
B3.3	14.02;39	24.4;59	42.4;74	50;100		10	12	2
B3.4	12.35;39	17;52	26;60	43;72	50;100	13	15	7
B3.5	10.4;39	16;50	22;60	37.6;74	50;100	16	21	7
B4.1	6.15;39	11;50	17;62	26;72	50;100	17	24	5
B4.2	6.95;39	19;62	31;72	50;100		12	17	3
B4.3	15;50	25;62	35;72	50;100		5	7	1
B4.4	10.05;39	22;60	33;72	50;100		5	5	0
B5.1	21;52	28;62	41;72	50;100		8	15	12
B5.2	14;42	25;62	37;72	50;100		15	24	15
B5.3	12.01;39	20.9;59	31.4;74	50;100		6	8	5
B5.4	9;32	20;50	29;62	36;72	50;100	5	8	5
B5.5	9;32	17;50	19;52	24;60	36;74 50;100	10	22	12
B5.6	12.24;39	21.3;59	35;72	50;100		9	18	8
B6.1	16;42	21;52	31;62	37;72	50;100	4	3	2
B6.2	11;35	19.1;59	38.6;74	50,100		7	12	6
B6.3	15;45	19;52	25;62	38;72	50;100	3	4	7
B7.1	14.94;39	28;62	41;72	50;100		6	9	4
B7.2	11;42	19;62	33;72	50;100		4	11	4
B7.3	16;62	32;72	50;100			4	6	2
B8.1	9;42	12;52	35.2;74	50;100		24	34	6
B8.2	10.23;39	15;52	21;62	38;72	50;100	16	19	3
B8.3	20;49	28;50	48;70	50;100		14	30	11
B8.4	13.61;39	17;50	23.7;59	40.2;74	50;100	7	26	14
B8.5	11;32	18;52	24.7;59	41.2;74	50;100	6	24	13
BSP	2000;100					1	4	12
SB1	20.8;39	39;62	56;72	66;100		7	27	10
SB1	34,100					7	27	10
SB2.1	13;40	24;62	39;72	50;100		12	32	12
SB2.2	11.95;39	20.8;59	39;72	50;100		15	33	13
SB3.1	9.25;39	13;50	20;62	33;72	50;100	15	38	16

B. Equality and Diversity issues and breakdown of the results by gender

Table 3: Breakdown of results by gender

Class	Number								
	2024			2023			2022		
	Female	Male	Total	Female	Male	Total	Female	Male	Total
I	7	44	51	5	49	54	5	50	55
II.1	14	36	50	23	49	72	19	34	53
II.2	9	15	24	7	11	18	15	9	24
III	-	-	-	-	-	-	-	-	-
P	-	-	-	-	-	-	-	-	-
F	-	-	-	-	-	-	-	-	-
Total	34	97	131	37	112	149	40	93	134
Class	Percentage								
	2024			2023			2022		
	Female	Male	Total	Female	Male	Total	Female	Male	Total
I	20.59	45.36	38.93	13.51	43.75	36.24	12.5	53.19	41.04
II.1	41.18	37.11	38.16	62.16	43.75	48.32	47.5	36.17	39.56
II.2	26.47	15.46	18.32	18.92	9.82	12.08	37.5	9.57	18.32
III	-	-	-	-	-	-	-	-	-
P	-	-	-	-	-	-	-	-	-
F	-	-	-	-	-	-	-	-	-
Total	100	100	100	100	100	100	100	100	100

Table 4: Rank and percentage of candidates with this or greater overall USMs

Av USM	Rank	Candidates with this USM and above	%
92	1	1	0.76
91	2	2	1.53
88	3	4	3.05
87	5	5	3.82
85	6	6	4.58
84	7	8	6.11
84	7	8	6.11
83	9	9	6.87
81	10	10	7.63
80	11	16	12.21
79	17	21	16.03
78	22	24	18.32
77	25	26	19.85
76	27	28	21.37
75	29	31	23.66
74	32	32	24.43
73	33	37	28.24
72	38	40	30.53
71	41	42	32.06
70	43	49	37.4
69	50	57	43.51
68	58	66	50.38
67	67	69	52.67
66	70	78	59.54
65	79	85	64.89
64	86	87	66.41
63	88	88	67.18
62	89	93	70.99
61	94	101	77.1
59	102	106	80.92
58	107	107	81.68
57	108	108	82.44
56	109	111	84.73
55	112	114	87.02
54	115	118	90.08
53	119	120	91.6
52	121	121	92.37
51	122	123	93.89
50	124	125	95.42
48	126	126	96.18
46	127	128	97.71
40	129	130	99.24
32	131	131	100

C. Detailed numbers on candidates' performance in each part of the examination

The number of candidates taking each paper is shown in Table 5.

Table 5: Numbers taking each paper

Paper	Number of Candidates	Avg RAW	StDev RAW	Avg USM	StDev USM
B1.1	40	33.78	10.82	62.98	14.37
B1.2	43	32.44	10.43	65.09	13.14
B2.1	29	34.17	10.72	74	13.03
B2.2	26	20.19	11.66	61.35	18.56
B2.3	14	25.07	12.68	62.36	25.48
B3.1	44	33.61	11.05	69.43	15.62
B3.2	25	36.04	7.26	70.16	9.7
B3.3	23	37.48	7.96	72.65	11.96
B3.4	34	36.18	10.82	71.47	15.55
B3.5	42	31.55	8.49	69.57	10.62
B4.1	45	24.16	10.06	69.2	13.02
B4.2	32	26.75	8.66	68.91	10.23
B4.3	13	30	9.89	67	14.74
B4.4	10	32.2	7.91	72.9	10.66
B5.1	27	31.7	10.03	64.74	15.57
B5.2	47	30.98	10.35	67.32	15.59
B5.3	20	27.75	7.33	68.45	10.79
B5.4	19	29.63	11.86	63.21	20.3
B5.5	33	28.21	9.08	64.79	13.75
B5.6	35	27.6	8.26	65.4	11.18
B6.1	10	30.3	13.01	62.4	22.52
B6.2	19	29	9.59	66.05	12.62
B6.3	15	25.33	12.1	57.27	19.87
B7.1	19	33.74	9.12	66.37	14.18
B7.2	20	24.35	9.62	62.15	17.43
B7.3	13	24.92	11.06	65.77	16.47
B8.1	55	27.33	11.61	66.22	16.89
B8.2	33	32.3	10.67	69.85	15.29
B8.3	34	43.68	6.53	70.56	13.69
B8.4	27	27.78	9.13	61.3	12.99
B8.5	35	30.63	8.59	64.03	11.27
BSP	11	1332.18	275.89	68.36	13.03
SB1	6	40.17	17.36	75	4.1
SB2.1	18	30.17	10.35	64.33	13.03
SB2.2	24	34	9.87	70.5	13.01
SB3.1	43	26.65	8.55	66.16	12.5
BO1.1	12	-	-	67.92	15.85
BO1.1X	12	-	-	67.92	10.36
BEE	10	-	-	81.1	10.31

Individual question statistics for Mathematics candidates are shown below for those papers offered by no fewer than six candidates.

Paper B1.1: Logic

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.55	16.6	7.13	35	3
Q2	15.76	16.06	5.72	32	1
Q3	18.07	19.69	5.46	13	2

Paper B1.2: Set Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.23	16.24	5.15	34	1
Q2	16.5	17.41	6.83	34	2
Q3	12.24	13.94	7.35	18	3

Paper B2.1: Introduction to Representation Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.52	18.52	6.5	27	0
Q2	15.79	15.79	6.16	24	0
Q3	16	16	6.03	7	0

Paper B2.2: Commutative Algebra

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	12.2	12.2	7.19	20	0
Q2	8.79	9.46	5.51	13	1
Q3	7.71	8.32	6.27	19	2

Paper B2.3: Lie Algebras

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	9.18	9.18	5.83	11	0
Q2	13.2	13.2	5.69	10	0
Q3	16.86	16.86	8.34	7	0

Paper B3.1: Galois Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14.87	15.5	6.96	22	1
Q2	16.97	16.97	5.63	31	0
Q3	17.49	17.49	6.19	35	0

Paper B3.2: Geometry of Surfaces

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.43	17.43	4.07	21	0
Q2	16.9	17.47	4.87	19	1
Q3	18.64	20.3	7.07	10	1

Paper B3.3: Algebraic Curves

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.32	17.32	4.15	19	0
Q2	18.86	18.86	5.54	21	0
Q3	20.29	22.83	6.99	6	1

Paper B3.4: Algebraic Number Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	11.63	15.55	8.1	11	5
Q2	17.67	17.67	5.68	24	0
Q3	19.24	19.24	6.13	33	0

Paper B3.5: Topology and Groups

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14.91	14.88	5.04	33	1
Q2	15.7	16.06	4.9	32	1
Q3	15.76	16.84	6.05	19	2

Paper B4.1: Functional Analysis I

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	11.48	12.08	5.63	25	2
Q2	12.97	13.21	6.17	28	1
Q3	10.97	11.22	5.55	37	1

Paper B4.2: Functional Analysis II

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	11.62	11.62	4.27	21	0
Q2	13	13	4.41	26	0
Q3	13.86	16.12	6.26	17	4

Paper B4.3: Distribution Theory and Fourier Analysis: An Introduction

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.3	15.3	5.23	10	0
Q2	16.1	16.1	5.86	10	0
Q3	12.67	12.67	5.32	6	0

Paper B4.4: Fourier Analysis and PDEs

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.8	17.8	2.39	10	0
Q2	7	7	0	1	0
Q3	15.22	15.22	5.7	9	0

Paper B5.1: Stochastic Modelling and Biological Processes

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.45	16.45	4.84	22	0
Q2	15	15	8.8	12	0
Q3	14.95	15.7	6.44	20	1

Paper B5.2: Applied PDEs

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.03	15.83	6.53	29	2
Q2	16.16	16.43	6.06	42	1
Q3	12.73	13.35	5.55	23	3

Paper B5.3: Viscous Flow

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	12.94	12.94	5.08	18	0
Q2	15.47	15.47	3.29	19	0
Q3	9.33	9.33	2.08	3	0

Paper B5.4: Waves and Compressible Flow

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	12.92	12.92	7.89	13	0
Q2	16.65	16.65	5.2	17	0
Q3	14	14	5.83	8	0

Paper B5.5: Further Mathematical Biology

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.45	15.45	6.73	11	0
Q2	14.88	14.88	4.54	32	0
Q3	11.64	12.39	5.79	23	2

Paper B5.6: Nonlinear Systems

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.08	15.32	4.92	25	1
Q2	12.42	12.65	4.46	23	1
Q3	11.59	13.27	5.87	22	5

Paper B6.1: Numerical Solution of Differential Equations I

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.6	18.6	8.38	5	0
Q2	13.63	13.63	5.73	8	0
Q3	14.43	14.43	7.11	7	0

Paper B6.2: Numerical Solution of Differential Equations II

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	11.78	12.29	6.6	17	1
Q2	16.18	16.18	4.2	17	0
Q3	13.6	16.75	7.2	4	1

Paper B6.3: Integer Programming

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	13	14.5	7.86	8	1
Q2	13	13.86	6.52	14	1
Q3	8.75	8.75	7.03	8	0

Paper B7.1: Classical Mechanics

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	12.89	12.89	4.62	9	0
Q2	18.4	19.64	6.65	14	1
Q3	15.75	16.67	6.08	15	1

Paper B7.2: Electromagnetism

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	12.53	12.53	5.41	19	0
Q2	9.55	9.55	4.59	11	0
Q3	12.42	14.4	6.99	10	2

Paper B7.3: Further Quantum Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	12.55	12.55	6.11	11	0
Q2	12.82	13.5	7.01	10	1
Q3	10.2	10.2	2.17	5	0

Paper B8.1: Martingales through Measure Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	12.56	15.49	8.26	43	12
Q2	12.06	13	7.94	28	3
Q3	12.78	12.78	5.5	37	0

Paper B8.2: Continuous Martingales and Stochastic Calculus

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.21	15.21	5.79	33	0
Q2	16.25	16.25	3.01	20	0
Q3	18.38	18.38	8.08	13	0

Paper B8.3: Mathematical Models of Financial Derivatives

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	21.37	22.28	4.74	25	2
Q2	23.38	23.35	3.35	23	1
Q3	17	19.55	7.59	20	8

Paper B8.4: Communication Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14.63	14.63	5.58	27	0
Q2	9.22	12.67	6.06	6	3
Q3	13.29	13.29	4.93	21	0

Paper B8.5: Graph Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14.63	14.63	3.3	30	0
Q2	14.03	14.85	6.38	27	2
Q3	17.85	17.85	5.27	13	0

Paper SB1.1/1.2: Applied Statistics/Computational Statistics

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	19	19	3.46	3	0
Q2	18	18	2.65	3	0
Q3	17.5	17.5	0.71	2	0
Q4	20	20	0	1	0
PR	25	25	4.36	3	0

Paper SB2.1: Foundations of Statistical Inference

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.06	16.06	5.16	18	0
Q2	13.94	14.67	6.93	15	1
Q3	7.4	11.33	7.16	3	2

Paper SB2.2: Statistical Machine Learning

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.25	18.25	5.6	16	0
Q2	17.88	18.75	6.02	16	1
Q3	13.65	14	4.62	16	1

Paper SB3.1: Applied Probability

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	11.31	11.31	4.02	29	0
Q2	13.08	13.36	4.95	36	1
Q3	16.05	16.05	4.95	21	0

Assessors' comments on sections and on individual questions

The comments which follow were submitted by the assessors, and have been reproduced with only minimal editing. The examiners have not included assessors' statements suggesting where possible borderlines might lie; they did take note of this guidance when determining the USM maps. Some statistical data which can be found in Section C above has also been removed.

B1.1: Logic

Question 1: The definition of a proof in the system inadvertantly omitted the case of an assumption. Some followed the definition given, while most used the correct definition; both were allowed. One student successfully exploited the error to give a trivial proof of (b). Parts (a), (c), and (d) posed few serious problems. A fair few struggled with (b), with some attempting to induct on formula complexity. Part (f) was generally easy for those who'd managed (e), which was most. This question lacked a truly challenging part, and many got a perfect score.

Question 2: Some did not know how to get started with (a), while those who did gave good solutions. Part (b) was fine. In (c), many just claimed the existence of a counterexample structure rather than exhibiting one. Parts (d) and (e) were close to bookwork and were done well, though some failed to argue that what they constructed in (e) was consistent. Most got the idea for (f) of considering universal quantifiers, but many then argued informally and only a few gave an actual proof. Part (g) was done quite well.

Question 3: f was referred to on the exam as a predicate symbol, while it could of course only be a function symbol; almost all corrected the error. Parts (a) and (b) were almost always well done. Part (c) generated quite a few confused solutions, and quite a few proved compactness via completeness rather than appealing to (b), but generally it was done well. Part (d) was straightforward, but many wrote long-winded explanations. Most had the right idea for (e), but the solutions varied in completeness, and quite a few tried an induction applying only to sentences rather than arbitrary formulas. In (f), many neglected to give an infinite model, and only a few successfully proved countable categoricity – though most correctly argued that this would suffice.

B1.2: Set Theory

Question 1 (a) Mostly fine.

(b) Surprisingly many failed to write a correct formula for (i). Parts (ii) and (iv) were mostly well done. Part (iii) stumped some, and many who answered it correctly used an overcomplicated class function in the recursion rather than just taking power set. It seems that by the time it got to (iii) and (iv), many had ceased to pay careful attention to the requirement to observe uses of the axioms, and most failed to note the use of Infinity in (iii) and of Comprehension/Foundation in (iv).

(c) Most answered (i) correctly, but quite a few tried to define the subsets of size n using a formula depending on n then "take the union". Parts (ii) and (iii) were exercises in formalisation which were rarely done well as such; many gave more or less informal constructions and arguments rather than explaining how to apply the recursion theorem and giving a corresponding induction. A creative solution seen twice was to take the cardinality of the disjoint union.

Question 2 (a) In (i), some seemed to ignore the "precisely" in the question and showed only one direction. In (ii) and (iii), a common mistake was to cite totality of the ϵ-order from lectures, ignoring the prescription to give proofs "directly from the relevant definitions". (b) Quite standard and generally very well done. In (ii), a fair number noticed the solution $3^2 = 1 + 2^3$ in finite ordinals – the question should have required α and β to be infinite. (c) Most could answer (i) fairly well. A surprisingly common mistake in (ii) was to claim that if $\beta < \epsilon = \bigcup_{n \epsilon \omega} f(n)$ then $\beta = f(n)$for some n. For (iii), most could construct a new element following (i), but many took a base smaller than ϵ or failed to justify why the result is not ϵ. A few thought to give ω_1 as an example.

Question 3 (a) In (i), many ignored or didn't properly address showing that $\aleph$ is a cardinal. Part (ii) was mostly bookwork. The direction (WO) $\Rightarrow$ (AC) was mostly fine. Many reproduced the argument for the converse verbatim from notes, but then the deduction that $\alpha < \aleph(X)$ was frequently incomplete; only a few successfully worked this into the proof as intended.

(b) This was mostly done well, though only a few took appropriate care over 0 in (ii).

(c) Proving this from (AC) was mostly done well, though there were some overcomplicated proofs using $\aleph^*$. The converse stumped many, but most who had the idea of considering a surjection $\aleph^*(X) \to X$ could conclude from there.

B2.1 Introduction to Representation Theory

Question 1 gives an alternative proof of the fact that the character table of a finite group is square using the Row Orthogonality Theorem. It was the most popular question on the 2024 B2.1 exam, and was attempted by everyone with perhaps one exception. It was also done very well by most people. Several candidates didn't realise that one cannot use the "standard result" that the irreducible characters form a basis for the space of class functions in (c) as this would make the question trivial; thus they did not earn many marks in part (c). Instead they should have used Q1(b) together with an application of Maschke's Theorem to the regular representation of G on $\mathbb{C}G$.

Question 2 was also very popular, seeing that it consisted nearly entirely of 'seen' material. However it was also quite tricky and only very few people got marks over 20/25 for this question. Part (a) was done very well; many people lost one or two marks in part (b) for failing to explain why $|\lambda_1 + \cdots + \lambda_n| = n$ for complex numbers $\lambda_1, \cdots, \lambda_n$ with $|\lambda_1| = \cdots = |\lambda_n|$ implies that they are all equal. The majority of the correct solutions for part (c) again used an application of Maschke's Theorem to the regular representation of G on $\mathbb{C}G$: it is also possible to deduce it from the Column Orthogonality Theorem. Although part (d) was entirely bookwork, it was not done well at all.

Question 3 was the least popular question, with less than ten attempts overall. Most people really struggled with it, but a couple of candidates did manage to get through to the end.

B2.2: Commutative Algebra

Question 1 Part (e) was the most difficult part of this question. This required remembering that a finitely generated R_S-algebra T has (by definition) the property that there is a surjection of R_S-algebras $\phi : R_S[x_1, \ldots, x_d] \to T$ for some $d \geq 0$. If $I := \ker(\phi)$, the one can define

$$U := R[x_1, \ldots, x_d]/(I \cap R[x_1, \ldots, x_d]).$$

If T is a domain then I is prime, and then so is $I \cap R[x_1, \ldots, x_d]$, implying that U is then a domain if T is. An equivalent approach is to define U as the R-subalgebra of T generated by a finite number of generators of T as a R_S-algebra.

Question 2 Part (c) (i) was the most difficult part of this question. One defines $Q_n(y) = P((x - y^n) + y^n, y)$ and one has to verify that the polynomial $P(u + y^n, y)$ viewed as a polynomial in y with coefficients in $k[u]$, has a constant dominant term for n sufficiently large. This follows from a short computation. Many students also had difficulties with Part (d). Here the argument is that if one had a flag

$$\mathbf{p}_0 \supsetneq \mathbf{p}_1 \supsetneq \mathbf{p}_2 \supsetneq (0)$$

of prime ideals in $k[x, y]$, then one could choose a non zero polynomial $P(x, y)$ in $\mathbf{p}_2$, which would lead to the lower bound $\dim(k[x, y]/(P(x, y))) \geq 2$. This however contradicts (c) (ii), because by (c) (ii) we have

$$\dim(k[x, y]/(P(x, y))) \leq \dim(R_n) = 1,$$

since integral extensions preserve dimension.

Question 3 The most difficult part of this question seems to have been Part (d). Here the argument is that a subring T of $\mathbf{Q}$ must be of the form $\mathbf{Z}_S$, where $S \subseteq \mathbf{Z}$ is a multiplicative set. To see this, note that if $a/b \in T$, where a and b are coprime, then there are $n, m \in \mathbf{Z}$ such that $na + mb = 1$. Hence $n(a/b) + m = 1/b \in T$. Thus T must be the ring obtained by localising $\mathbf{Z}$ at the denominators of all the elements of T. This implies that T is a PID, because by standard properties of localisations, all the ideals I of T are generated in T by $I \cap \mathbf{Z}$, and $I \cap \mathbf{Z}$ is principal since $\mathbf{Z}$ is a PID. Hence T is a UFD, and hence integrally closed.

B2.3: Lie Algebras

Question 1 This question was attempted by the majority of candidates. Part *a*) was answered well, though the question of which nilpotent Lie algebras possess faithful semisimple representations was poorly addressed. In part *b*) showing that the semisimplicity of $V \otimes W$ implies that of V and W was successfully done by only a small number of candidates though the converse was established by most. In part c) many candidates did not see how part *b*) was relevant.

Question 2 This was the most popular question, with almost all candidates attempting it. Answers to part *a*) were overall good, though some candidates failed to see why finite-dimensionality was necessary to their argument. Most candidates made substantial progress in part *b*) in that they applied the hypothesis to the adjoint representation, and correctly deduced that $\mathfrak{g}$ was the direct sum of a semisimple Lie algebra and an abelian one (or something close to this). Deducing from this that $\mathfrak{g}$ is indeed semisimple requires using the hypothesis in a different way, and fewer candidates saw how to do this.

Question 3 This question was slightly less popular than the other two, but most candidates who attempted it achieved good marks on it. Parts *a*) and *b*) were well-answered, with parts *c*) and *d*) causing more difficulties. A number of candidates used the correct strategy in part *c*) but fell foul of calculation errors, and a number of candidates who correctly answered *c*) did not seem to appreciate how to use it in answering part *d*). Many candidates, in answering part *d*), computed a number of coroots in $\mathfrak{h}$, and while the coroots can be used in answering that part, no such calculation was necessary if one had answered the previous parts of the question.

B3.1: Galois Theory

Question 1 This was the least popular question. Part (a) was purely bookwork, so it was disappointing to see more than half of the students struggle so much with it. The reverse inclusion in part (b) caught out many candidates, despite being a straightforward application of the main theorem of Galois Theory. It was pleasing to see a variety of correct solutions for part (c): one used the characterisation given in part (a) of the question for (i), another one used the Primitive Element Theorem, but only one or two persons found the intended solution that uses part (b) to identify $N := \mathrm{Gal}(K/M)$ as the largest normal subgroup of $G := \mathrm{Gal}(K/F)$ contained in $H := \mathrm{Gal}(K/L)$ and then considers the permutation action of G on G/H with kernel N to conclude.

Question 2 was a popular question, and mostly done well. Several people lost marks in part (b) for not explaining how to use the fact that the cyclotomic field extension is Galois. A significant number of candidates proposed to use $\alpha = \zeta + \zeta^4 + \zeta^7$ in part (d). Whilst this orbit sum is indeed invariant by the correct Galois automorphism, unfortunately it evaluates to zero and thus does not generate a quadratic extension of $\mathbb{Q}$. Part (e) was done well by most people who did well in part (d).

Question 3 was the most popular question, and was done very well overall. For part (c), several candidates found interesting counterexamples to (b)(i)$\Rightarrow$(ii), such as $\mathbb{F}_{p^p}/\mathbb{F}_p$ for a general prime p, and $\mathbb{Q}(\cos(2\pi/7))$ for $p = 3$. The simplest counterexample to (b)(ii)$\Rightarrow$(i) was $\mathbb{Q}(e^{\frac{2\pi i}{3}})$ with $p = 3$.

B3.2: Geometry of Surfaces

Question 1 : A popular question, largely well done. In (c) most candidates recognized that a regular octagon, with internal angles of $\pi/4$, was needed, but few explained (via a continuity argument) why such an appropriately sized octagon existed. Few achieved full marks for (d). Many discounted the possibility of a simple, smooth closed geodesic bounding a region homeomorphic to a disc and received partial credit for this. In fact, such a geodesic does exist which splits the surface into two regions with zero Euler characteristic. Given a regular hyperbolic octagon, centred on the origin in $\mathbb{D}$, then any diametric arc between two vertices of the octagon is such a geodesic. It remains to show the ends of the arc meet smoothly, which can be done with the Gauss-Bonnet theorem or by considering how the octagon's edges meet at a single vertex.

Question 2 : This question was done well, with many high marks and one perfect mark achieved. (b)(ii) can be done by adding a meridian to S or correctly using its Euler characteristic of zero. Mistakes at this part either resulted from using the wrong Euler characteristic or applying the global Gauss-Bonnet theorem despite S not being a closed surface. Another common error was to calculate $\iint K \mathrm{d}A$ with $\mathrm{d}A = \mathrm{d}u\,\mathrm{d}v$ rather than the correct $\mathrm{d}A = \sqrt{EG - F^2}\,\mathrm{d}u\,\mathrm{d}v$.

There was an error in 2(c)(ii), which was more evident in the solution than in the question. The solution had the answer of $f(v) = \sqrt{1+v^2}$ rather than the correct answer of $f(v) = 1 + v^2$. This unfortunately meant that the hint was not helpful, and in fact the surface S can no longer be isometrically embedded in $\mathbb{R}^3$ as X, at least not as far as $v = 1$, but the first fundamental forms can still be made to agree which many candidates did successfully, gaining full marks for this part. The later parts were independent of this error.

(d) made use of the Theorema Egregium, which demonstrates here that any isometry must fix v. Consequently an isometry of S translates u and/or reflects in u.

Question 3 : A relatively unpopular question, but one that was done exceptionally well. Occasionally marks were lost in (c)(iii), explaining how a local co-ordinate could be assigned to the point at infinity.

B3.3 Algebraic Curves

Question 1 For (b), I expected students to reproduce the Gram–Schmidt type proof in the lectures on diagonalizing quadratic forms, in the special case of dimension 3. But most students either gave an ad hoc argument, with varying success, or quoted the result I wanted them to prove, for which I tended to give half marks.

Part (e), and to a lesser extent (d), were found hard. In (d) I expected the answer that $\pi(p)$ lies in $C^* \cap L$ iff $[r, s, t]$ lies in T_pC. Then in (e) I expected students to say that

$$\begin{aligned}\deg C^* &= \deg C^* \cdot \deg L = \#\{C^* \cap L\} = \#\{p \in C : [r,s,t] \in T_pC\}\\ &= \#\big(C \cap \{[a,b,c] \in \mathbb{CP}^2 : rP_x(a,b,c) + sP_y(a,b,c) + tP_z(a,b,c) = 0\}\big)\\ &= d(d-1),\end{aligned}$$

using Bézout and all intersection multiplicities 1. Only a few saw this.

Question 2 Quite a lot of good answers. For full marks in the bookwork in (a), ideally I wanted mention that $\mathbb{C}[x, y, z]$ is a UFD, so factorization $P = Q_1 \cdots Q_m$ in $\mathbb{C}[x, y, z]$ is unique up to order and multiplication by units, but multiplying by units doesn't change the corresponding curves $D_1, \ldots, D_m$. Many people did not mention units.

Question 3 Fewer than a third of students answered this question, maybe because they find the Riemann–Roch material difficult, and (c) looked scary. But nearly everyone who made a serious attempt scored very highly. For those who knew the bookwork, this was a better bet than question 1.

B3.4: Algebraic Number Theory

Question 1 Questions similar to this one have been asked before. The early parts were done fairly well though a lot (perhaps even the majority) of candidates made a calculational error in part (b).

Many candidates asserted without proper proof that we do not have $N\mathfrak{a}_i = 1$ in (f).

A good number of candidates got the correct argument in (g).

Question 2 Part (a) is a standard and easy class group calculation which, as expected, most candidates did well. Quite a few did not explicitly rule out the possibility that the prime ideal dividing 2 is principal.

The rest of the question is about a simple example of a genus character (though this term was not used in the question) which, so far as I know, is a new type of question for this course. It seemed to work quite well, with a good spread of marks.

Question 3 This was a standard type of question and similar questions featured in lectures, and have also been seen on past exams, though the specific field $\mathbf{Q}(\sqrt{-46})$ does not seem to have come up before.

(a) This kind of class group computation is very standard and features every year. Somewhat unusually, the Minkowski constant was not given to the candidate - however, they could have got it from Q2 (a). A small handful of candidates did write down the wrong Minkowski constant, which caused them serious issues.

(b) was generally found very straightforward.

(c) is a standard type of question and many candidates were able to proceed up to the equation $5u^4 - 460u^2 + 46^2 = \pm 1$. A number of candidates struggled to justify the fact that this has no solutions, but a number of other candidates did manage this, by a variety of methods, some longer than others.

B3.5 Topology and Groups

Question 1 (46 attempts): This question tested the understanding of homotopy theory. The general level of solutions was good, though this turned out to be the most difficult question. Even though (a)(i) was bookwork central to the material (existence of inverses in the fundamental group), there were few completely correct solutions, and most homotopies from $u \cdot u^{-1}$ to c_x given fixed the image of $1/2$ at y, or contained some other similar issue. Most solutions to (a)(ii), (a)(iii), (b)(i), and (b)(ii) were correct. However, there were almost no complete solutions to (b)(iii), though there were several solutions that had the key ideas in place. A nice alternative approach to the official solution that appeared in some scripts was to homotope the map f on edges not lying in a spanning tree T into $f(T)$ using (a)(iii), then use the fact that T is contractible and hence that $f|_T$ is 0-homotopic.

Question 2 (46 attempts): This question tested knowledge of the Seifert–van Kampen theorem. Solutions to part (a) were generally satisfactory, with occasionally some parts of the statement of the Seifert–van Kampen theorem missing. In (b)(i) and (ii), the majority of the candidates got the right idea about the homotopy retraction, though the application of the Seifert–van Kampen theorem sometimes lacked detail. Part (b)(iii) proved to be difficult, with just a few candidates noticing that the space homotopy retracts onto $S^2 \vee T^2$.

Question 3 (29 attempts): This question tested knowledge of covering spaces, and was on the easier side. In (a)(i), candidates sometimes failed to argue why a covering map is surjective. Most candidates had a correct example for (a)(ii). Part (a)(iii) has a simple unintended example, where the domain is disconnected and the map is a covering map on each component. This was also awarded full marks. It is worth trying to construct an example where the domain is connected. Solutions to part (b)(i) were typically correct. However, there was only one complete solution to part (b)(ii), as candidates usually missed the possibility that, if $U_1, \dots, U_n$ are open neighbourhoods of the points of $f^{-1}(p)$ such that $f|_{U_i} \colon U_i \to f(U_i)$ is a homeomorphism, and $V := \bigcap_{i=1}^n f(U_i)$, then there might be points of $f^{-1}(V)$ not in $\bigcup_{i=1}^n U_i$. Solutions for (b)(iii) and (c) were typically correct.

B4.1: Functional Analysis I

Question 1 More than half the candidates attempted this question with variable degrees of success. Surprisingly, a large proportion of the candidates could not apply/quote correctly triangle inequalities on Euclidean space, and thus failed to complete the bookwork component of this question. There are three different modes of convergence in this question (pointwise convergence, uniform convergence and L^2-convergence) and only the candidates who recognised their difference could successfully complete the question.

Question 2 70% of the candidates attempted this question. Few candidates noted that the statement $(c_{00})^* = \ell^1$ entails an explicit description of the implicit isometric isomorphism (rather than an abstract existence of such isomorphism). In (b), about half of the candidates who attempted to show $(\star\star) \Rightarrow (\star)$ forgot to show that the functional they would like to define is well-defined.

Question 3 80% of the candidates attempted this question. The overall performance of the candidates was generally ok. Typical errors and omissions include: positivity of $\langle \cdot, \cdot \rangle_A$, completeness of $\langle \cdot, \cdot \rangle_A$, boundedness of the algebraic inverse of AA^*, the norm estimate obtained from the Riesz representation theorem was with respect to $\| \cdot \|_A$. A few candidates attempted (c)(ii), and most of them had good ideas.

B4.2: Functional Analysis II

Question 1 The early parts of 1a) on nowhere dense sets were well solved, with many correct examples in (ii), iii) constructed using (subsets) of the rational numbers in the real numbers. The analogue of part iv) for sequence spaces was a question on a problem sheet so it was surprising that relatively few students tried to follow the strategy seen there of proving that the sets A_k have non-empty interior and are closed, where the later part requires a careful argument via a subsequence that converges a.e. Part b) on Fourier series was solved well, as was the proof in ci) that boundedness of T ensures boundedness of S_n. The other direction of cii) was as expected challenging as it brings together different parts of the course, with the closed graph theorem providing a good route to proving continuity of T. In the final part of b) partial marks were scored by quite a few students by pointing out that the set cannot be both dense and closed since it cannot be the whole space as it does not contain the identity. Only a few candidates took this idea further to show that the set can't be dense as e.g. any ball with radius $r < 1$ around the identity is disjoint from it and there were only a few correct proofs that the set is also not closed.

Question 2 Question 2 was the most popular question. Part a) was mostly seen material, including iv) which was on a problem sheet, but proved to be quite challenging. All students provided the correct definition of a compact operator in b) and quite a few realised that the easiest way of constructing an example in ii) is to try and choose operators so that $T \circ S = 0$. The third part of b) was designed to be challenging, and only a few students realised that they could apply the open mapping theorem when they viewed T as a surjective map into its image, as seen in related proofs in the lecture. A key point of part c) is that basic properties such as boundedness and closedness encountered in the course depend on the choice of norm. This caused a lot of problems not only in the later parts, but already in the first part which can be deduced from Arzela-Ascoli using that the sup norm is controlled by the 1 norm and that maps which are in the unit ball with respect to the 1 norm are 1-Lipschitz, so equicontinuous.

Question 3 The first half of the question on spectral theory caused more difficulty than expected. While part (ii) was designed to be quite challenging, the other parts of a) could be solved by using well known techniques. As such it was surprising to see that while essentially all students successfully used the result on Neumann series to solve (i), very few thought to use this in (iv), even though similar arguments were seen in the lecture. There were also only few correct examples of operators with $\|T\| > r_\sigma(T)$, though students could have used triangular 2×2 matrices to obtain simple examples.

The second half of Q3 on weak convergence was solved better than the first, with the bookwork parts (i) and (ii) correctly solved by nearly all students and many good attempts at solutions of (iii) using uniform continuity of g and density of the continuous functions in L^2. The last part of the question, which was similar to a question on a recent exam, was correctly solved only by very few students, though partial marks were scored for observing that weak limits of sequences in the unit sphere will always be in the closed unit ball thanks to Mazur's theorem.

B4.3: Distribution Theory

Question 1 For question 1 the general level of answers was good and while no candidate got all the 25 marks some got very close. Most candidates got full marks for part (a), which is a combination of book work and routine examples. It was a little surprising that many didn't do well on part (b), which asks to prove a version of the generalized Riemann-Lebesgue lemma. This was an exercise on a problem sheet and we did examples of a very similar nature on a number of occasions in lectures. The more elaborate version of the Riemann-Lebesgue lemma appearing in part (c) was expected to cause some difficulties and many also struggled to make progress on its last part.

Question 2 The general level of answers to question 2 was good, though on a slightly lower level than for question 1. No candidate got full marks, where in particular marks were lost in the last part of the question. Question 2concerns aspects of localization and restrictions of a distribution and its order. Most candidates got close to full marks on part (a), which is a combination of book work and routine examples. Part (b) is similar to an exercise on a problem sheet and was generally well done. Part (c) is a new example and caused some difficulties.

Question 3 Except for a few cases the quality of answers to question 3 was somewhat more diverse than in questions 1 and 2. The question concerns distributional derivatives and aspects of the chain rule. The book work part of (a) was generally well done, but the last part, supposed to be a routine example using mollification, caused some difficulties. Part (b) of the question is more challenging, but many candidates who attempted it did the first parts(i) and (ii) quite well. Only very few did the last part (iii).

B4.4: Fourier Analysis and PDE's

Question 1 All candidates attempted question 1 and the general level of answers was good, though no candidate got all the 25 marks. Most candidates got close to full marks for the book work in part (a). The differentiation rule in part (b), which amounts to a variant of book work, also attracted many good answers. The new question in part (c) was in some cases attempted solved using the Fourier inversion formula for L1 functions from the course. However, without further work and assumptions, it doesn't give the required result, and consequently those answers didn't receive many marks. Most other candidates who attempted this part got full marks for its first part but struggled on its last part.

Question 2 Only one candidate attempted question 2.

Question 3

Most candidates attempted question 3. The quality of answers was somewhat more diverse than in question 1. Most candidates provided good answers on the book work in part (a). Surprisingly few candidates got full marks on the book work variant in part (b), whereas almost all candidates who attempted the part (c) got at least half of the available marks.

B5.1: Stochastic Modelling and Biological Processes

Question 1 This question was the most popular attempted by 86% of the students. Students did well on the whole.

In part (a) a few students tried (successfully or unsuccessfully) to derive the given time-dependent distribution, which is do-able but quite a lot more work than simply *confirming* that it satisfies the chemical master equation with the given initial conditions. Many forgot to check that the distribution satisfied the initial conditions. The calculations were a bit awkward, but on the whole well done.

Part (b)(i) required an understanding of the jump chain and was largely well done, with some errors in calculation, but relatively few conceptual errors.

Part (b)(ii) was where most marks were lost, although a reasonable number of students completed it. Some failed to correctly write down the recurrence relation for $\tau(n, 1)$, and some were unsure how to use the recurrence relation to get a sequence of improving estimates of the expected time to extinction.

Question 2 This question was the least popular attempted by only 37% of the students. The success rate was somewhat lower than for Question 1.

Part (a) was bookwork and was generally well done. Students who realised that they needed to look for radial solutions to the Fokker-Planck equation generally progressed well after this.

Part (b)(i) required setting up the system of PDEs and boundary conditions which was essential to progress further. Students largely managed this.

In part b(ii) most students were able to write down the general solutions in the cytoplasm and outside the cell; but the calculations involving matching up solutions at the common boundary, key to eventually getting the flux, were a bit challenging and sometimes went wrong. This is where most marks were lost.

Part (b)(iii) involved interpreting the limiting behaviour and students who got this far mostly seemed to understand the physical/biological meanings behind the calculations.

Question 3 This question was very popular: attempted by 80% of the students. It also had the lowest success rate of the three questions.

Part (a) was bookwork and largely well done.

Although part (b) was also largely bookwork, correctly choosing the limits of integration for each of the two integrations in order to arrive at the required double-integral posed some challenges and was where most errors were made. Sometimes the answer was presented without adequate justification.

Part (c) relied on having done and understood part (a); and then involved understanding the biological meaning of the invariant distribution in order to decide what needed to be calculated. Those who managed part (a) well on the whole did part (c) well too.

Part (d) relied on part (b) and success in (d) tended to follow that in (b). (Because part (b) posed some problems, these reappeared in part (d).) Where the concept was understood, errors in calculation led to some loss of marks, and some students left out the part involving comparison with the case of no chemotaxis.

B5.2: Applied PDEs

Question 1 was the second most popular question. Students generally did well in the (a). A few made computational mistakes upon solving for the scaling coefficients, others set up the wrong equations for these. Marks were also dropped by not answering the question in full, e.g. writing the scaled version of all conditions as requested in the question. Part (iii) was not always completed or algebraic mistakes were made upon solving the ODE boundary vaulue problem. In (b), most students formulated the BVP defining the Green's function correctly. However, quite a few failed to obtain the Green's function for the sphere, either by not coming up with a reasonable way of attacking the problem (despite the hints) or due to algebraic errors.

Question 2 was the most popular question, done by all students. Students generally did well. Almost everyone solved (a) correctly for the characteristics. In (b), marks were lost (rarely) for not providing the envelope or (more frequently) for not stating the domain of definition correctly or providing deficient sketches, or not stating $u(x,t)$. (c) was mostly answered correctly, though some students forgot to state the answer for the speed s or the causality in terms of u_-^+. Many students also attacked (d) and found the ODE/IVP for ξ. There was some subtlety for stating all pieces of u correctly, which some students missed.

Question 3 (a) and (b) were bookwork or straightforward and done by most students who attempted this question. In (c), some students failed to see that the term proportional to s emerged by correctly applying d/dt to the integral limits (which depend on on time). Part

(d) was only done by very few students, and only very few got a result that allowed them to identify $H = 1$ as the correct lower boundary for h_1.

B5.3: Viscous Flow

Question 1

This question was attempted by all bar two of the candidates. The bookwork contained in part (a) was generally fine; the most common mistake was to not define n as the normal to the surface element. Numerous candidates didn't justify why the shear stress at the surface was given by $\sigma_1 3$. Candidates were not penalised for using standard vector decompositions without defining them. In part (b), numerous students went straight to 2D rather than starting with the 3D version of Navier–Stokes as instructed to. Many incorrectly interpreted "There is no imposed pressure gradient" as meaning that they could *apriori* cross out ∇p . All bar one candidate were able to find the Couette flow for $u_s(z)$, but relatively few candidates managed to solve the resulting problem for û using a Fourier series approach. Some solutions were marred by alegebraic manipulation errors. Sketching the graphs proved difficult for many candidates that got that far. Those that made it to part (c) generally got somewhere with both subparts, although their ranking of strategies often missed out the crucial information that all the strategies altered the log term in the same way.

Question 2

This question was attempted by all bar one of the candidates, and was generally well done. There was a minor typo in the question (the boundary conditions were listed as applying on $z = 0$ rather than on $\zeta = 0$). This did not trip up any of the candidates and no one was penalised for writing z rather than ζ to define the boundary anywhere in part (a). Some candidates stated that $W \to 0$ as $\zeta \to \infty$ as a necessary boundary condition and were penalised accordingly. Some candidates did not adequately explain why the pressure gradient was given by $-U_s U_s'$. In part (b), some candidates struggled to get the algebra right to obtain the ode for f. Unfortunately, no candidates were able to solve the two coupled ODEs for g and U_s. Instead, they opted to substitute the results into the ODEs to find g. Often these attempts were defeated by algebraic manipulation errors. Only a few candidates managed to provide convincing arguments for the second part of (b)(iii).

Question 3 Only three candidates attempted this question, and the average mark was lower than for the other two questions. Solutions to part (a) lacked essential detail in places. Only one of the candidates correctly found u'. None of the candidates integrated conservation of mass across the layer in the standard way to arrive at the thin film equation. In part (b), candidates struggled to use the right boundary conditions on the right regions, and none of the candidates realised that they needed to apply p' and dp'/dx' continuous at $x' = 1$ to obtain the solution. Only one candidate attempted part (c); they integrated in x rather than z and thus didn't make much progress.

B5.4: Waves and Compressible Flow

Question 1 This question was relatively popular but was the least well answered: it seemed to attract the weaker candidates. Many candidates just hadn't learned the basic bookwork required for parts (a) and (b), often mistakenly treating ρ as a constant. In part (c), one could obtain the required radially symmetric steady mass conservation result $2\pi r\rho(r)\phi'(r) = q$ either from $\boldsymbol{\nabla}\cdot(\rho\boldsymbol{\nabla}\phi) = 0$ or from the given statement about mass flux across a circle, but few candidates made significant progress with either approach. The straightforward but fiddly algebraic manipulations required to find the maximum of $f(\xi)$ also caused a lot of problems.

Question 2 This was the most popular question and attracted the highest marks. The submitted solutions to parts (a) and (b) were generally ok, though often over-complicated. There were a lot of problems with the calculations required for part (c), with many candidates again over-complicating their solutions and then getting lost in the detail, as well as inconsistency in use of "±". Only the very strongest candidates had any idea how to approach part (d).

Question 3 This was the least popular question but attracted some good solutions. In part (a), candidates often tried to expand everything out rather than observing immediately that $c^2/\rho^{\gamma-1}$ must be constant. Part (b) was a basic piston withdrawal problem, but still the careful arguments needed to convincingly derive the given expressions were often lacking. In part (c), all candidates identified Newton's second law, but many struggled to accurately eliminate ρ to obtain the given ODE. In part (d), the sketching was generally ok, but there were not many convincing explanations of the behaviour as $t \to \infty$.

B5.5: Further Mathematical Biology

Question 1 Only a minority of candidates attempted this question. In (a) several marks were lost by many candidates for not fully explaining the biological meaning of each term in the equation (note that there were 10 marks allocated to (a) in total, so it should have been clear that a detailed explanation was required). In (b) most candidates could do (i) correctly, but in (ii) many failed to realise that the equations for $0 < t < \tau$ are not relevant for finding the steady state. Part (c) was relatively well answered by those that attempted it.

Question 2 Parts (a)–(c)(i) were well answered by the majority of those that attempted it. Most did not identify the number of cases in (ii). Note there was a small typo in the paper – in the equation for $L(T)$, lowercase variables x and t were used (rather than X and T).

Question 3 The majority of candidates attempted this question. Parts (a) and (b) were well answered. In part (c), many candidates struggled to draw the two different forms of the null clines and illustrate how these can be used to determine the potential for patterning via a diffusion-driven instability. In part (d) few candidates could write down the form of the solution to the linearised problem in order to determine the additional constraints.

B5.6: Nonlinear Systems

Most of the candidates demonstrated good understanding of the bookwork material. The exam was well-balanced with all three questions having the similar level of difficulty. While some candidates submitted some incomplete solutions, they often achieved at least 40% of raw marks. Other candidates also made successful attempts at more advanced parts of each question. In fact, each question received one complete solution (getting the perfect raw mark of 25), illustrating the solvability of each question under the exam conditions.

Question 1

covered the material on discrete-time dynamical systems (maps), with most candidates demonstrating that they were able to find fixed points and their stability. Some marks were lost when candidates investigated the existence and stability of 2-cycles and 3-cycles.

Question 2

covered some material on continuous-time dynamical systems (planar ODEs), with most candidates demonstrating that they can identify their steady states and investigate their stability. Some marks were lost when candidates investigated the Hopf bifurcation.

Question 3

many candidates were able to calculate the eigenvalues of matrix M. This 3×3 matrix has only two eigenvalues, with one of the eigenvalues having algebraic multiplicity 2 and geometric multiplicity 1. Some candidates made errors in finding the stable, unstable and center subspaces, because they did not find the corresponding generalized eigenvector.

B6.1: Numerical Solution of Differential Equations I

Question 1 Nearly all candidates who attempted this question received most of the marks. The question was similar to a question on the MMSC exam, which was provided to the students along with the solutions. There were a few small mistakes in applying the Cauchy-Schwarz inequality or forgetting the factor of $\sqrt{2}$ in the inequality $a + b \leq \sqrt{2}\sqrt{a^2 + b^2}$. Otherwise, they were able to correctly handle the new material and the quality of answers was generally high.

Question 2 Most candidates attempted this question and generally did the bookwork correctly. However, there were a number of candidates who did not correctly define the θ-scheme for the zero-th order term and an inhomogeneous forcing function (even though part (b) hinted at the correct form). No candidate optimally estimated $|\lambda(k)|$, leading to a suboptimal bound depending on T. The most common mistake was not bounding λ from above and from below separately. As a result, most candidates did not complete part (d) or do it correctly. Only a few candidates set up the contradiction in (e) correctly, while most tried to copy verbatim the argument from lecture, which does not lead to a strict inequality. Overall, about half of the candidates were able to complete some of the new material, while the other half earned close to zero marks on new material.

Question 3 Most candidates attempted this question. The most common mistakes were not using an explicit remainder in Taylor's theorem in parts (a-b) and not justifying that the quadratic formula provided both values of r in part (d). Only about half of the students were able to successfully complete parts (a-b) despite the hints. As with the previous question, about half of the candidates were able to complete some to most of the new material, while the other half earned close to zero marks on new material.

B6.2 Optimisation for Data Science

Comments awaited.

B6.3: Integer Programming

Question 1 was answered by 65% of the candidates, the highest mark obtained being 24 and the lowest 1. The distribution of marks showed a good spread across the middle to high range, but there was of a small cluster of disappointingly low marks. All in all the question worked well, offering a range of book work, adaptation to book work, and parts that required detailed understanding to achieve complete marks.

Question 2 was the most popular and was selected by 94% of candidates. The highest mark was 22 and the lowest 1. The spread of marks was good, and the question served as a good differentiator between different levels of understanding of the course materials.

Question 3 saw the lowest uptake, with 53% of candidates choosing this problem. The highest mark was 25 and the lowest 3. The marks came out generally low, indicating that candidates found it quite hard in comparison with the other two problems. Candidates who had revised the problem sheets, where related similar material had been discussed in some detail, should have found this problem familiar, but the problem was probably too hard for candidates who had revised only with the course notes.

B7.1: Classical Mechanics

Question 1 This question was somewhat nonstandard and few candidates were able to make significant progress beyond the routine material. Few candidates were able to eliminate the constants by rescaling the time and space variables or the Lagrangian, and few found the Lagrange points at $(0, \pm\sqrt{2^2/3 - a^2})$.

Question 2 This was a largely routine computational question and there were many good discussions of the definition of Euler angles. Candidates who didnt get lost in the later calculations were able get good marks.

Question 3 Most candidates were able to perform the computations concerning the Poisson brackets of conserved quantities for the Kepler problem, although a good number got lost in the calculations. Few knew the geometric meaning of the the conserved quantities for the last part, in particular that the angular momentum is perpendicular to the plane of motion, and the Runge-Lenz vector pointing along the major axis.

B7.2: Electromagnetism

Question 1 This was the most popular question (attempted by 18 out of 19), and the answers given were generally rather good (13 marks on average), but often incomplete. Points were often lost in part (a) for not explaining why $\mathbf{E} = 0$ in a conductor, and for not proving (though correctly recalling from lectures) the formula $\sigma/\varepsilon_0 = (\mathbf{E}^+ - \mathbf{E}^-) \cdot \mathbf{n}$ for the discontinuity of the electric field at a charged surface. Several solutions of part (c.i) had the correct idea, but did not prove that the boundary conditions are fulfilled, or had errors. The last parts, (c.ii) and (d), were attempted by only 1 and 5 students, respectively.

Question 2 This was attempted by 10 out of 19 and scored fewer marks on average (10) than questions 1 and 3. Parts (a.i) and (a.ii) were handled very well, but no solution scored more than 1 (out of 2) points for (a.iii). For example, many argued that $\mathbf{B}(\mathbf{r})$ is independent of φ, whereas it is only the *coefficients* of the (φ-dependent!) vectors $\mathbf{e}_\varphi$ and $\mathbf{e}_\rho$ that are independent of φ. In (b.i), many students could not define the magnetic dipole moment correctly, and struggled to identify the current density using delta distributions. Noone made substantial progress on part (b.iii), despite the clear hint. Part (c.i) was done very well, but only 4 students attempted the remaining parts. Two struggled with the integration in (c.ii) and only one solution got the idea (and full marks) for (c.iii).

Question 3 This was attempted by 12 out of 19 and scored on average 12 marks. Part (a) was answered almost perfectly. In (b.i) not everyone argued convincingly through the orthogonality of the Fourier basis. Part (b.ii) was generally well done, apart from some algebra errors; but noone mentioned (and checked) the boundary conditions for the magnetic flux density $\mathbf{B}$. Only 3 students attempted (c). Part (c.i) was done well, but noone explained the choice of the square-root (the other choice leads to an unphysically exponentially increasing field in $z < 0$). Part (c.ii) was easy and perfectly done. In part (c.iii) however, all answers failed to recognize that $\mathbf{E}' \times \mathbf{B}' = \mathrm{Re}(\mathbf{E}'_{\mathbb{C}}) \times \mathrm{Re}(\mathbf{B}'_{\mathbb{C}})$ is not the same as $\mathrm{Re}(\mathbf{E}'_{\mathbb{C}} \times \mathbf{B}'_{\mathbb{C}})$.

Summary. The range of marks achieved cover a wide spread, and all questions were attempted by more than half of the candidates. The average marks for question 2 were slightly lower than for questions 1 and 3. Very few students attempted all parts of a question, suggesting that the exam was slightly too long overall.

B7.3: Further Quantum Theory

Question 1 This was a popular question. Almost all candidates did a fine job with the bookwork at the start describing irreducible representations of angular momentum operators. Part (b) was meant to be straightforward and similar to work in lectures and homework exercises, but many candidates failed to produce the full expressions for the basis elements in the combined-spin basis. Part (c) resisted most attempts, with only a few candidates correctly identifying which irreps consisted of symmetric versus anti-symmetric states. (A repeating confusion involved a faulty invocation of the spin-statistics theorem, which was irrelevant here as there was no mention of intrinsic spin as opposed to some total angular momentum representation of unspecified origin.) Part (d) proved too difficult, with no candidates realising they had to simultaneously work in a basis that diagonalised total intrinsic spin, total orbital angular momentum, and total overall angular momentum while also tracking the subset of anti-symmetric states.

Question 2 This was another very popular question. Almost all candidates were able to re-derive the stationarity property of the Rayleigh quotient at eigenvectors. In part (b) essentially all candidates recalled the statement of the (homogeneous) virial theorem, but many were unable to summarise the derivation. Part (c) could be solved by direct integration without too much trouble, but many candidates ran into computational difficulties. A clever alternative trick to determine the expectation value of the kinetic energy operator was to relate the trial wave function to the ground state wave function of the simple harmonic oscillator. A number of candidates noticed this, but a number incorrectly generalised the method to the potential energy operator as well (or implicitly assumed that the trial wave function was an exact ground state wave function for the anharmonic Hamiltonian). The most important aspect of part (d) went un-noticed by all candidates: that the proportionality of expectation values of kinetic and potential energies derived in the virial theorem holds irrespective of whether the approximate stationary wave function is a true energy eigenstate. Without this caveat, an argument based on the virial theorem in this question was incorrect. Part (e) could be attempted independently of the previous parts and was meant to be a straightforward application of the Bohr–Sommerfeld quantisation condition. Candidates who attempted it mostly did well.

Question 3 This was attempted by only a minority of candidates. Part (a) was a recounting of a bookwork derivation, but many candidates failed to treat degeneracy appropriately (which requires a specified choice of basis as well as the introduction of ambiguities in the first-order corrections to the eigenstates). Part (b) required the computation of several matrix elements of $X_1 X_2^3$ between harmonic oscillator energy eigenstates. In practice, inspection could reduce this to a computation of one non-zero matrix element, simplifying the calculation appreciably. No candidates identified this simplification correctly, but some accomplished the calculation reasonably well. Part (c) was generally not done well as it required a careful treatment of degeneracy and the aforementioned ambiguities at first order. A few candidates had the right idea, however. Part (d) did not see much in the way of attempts, though it could be attempted without solving part (c) just by using the given equation.

B8.1: Probability, Measure and Martingales

Question 1 Part (a) seemed to be handled quite well. Most students obtained full marks for (i), with a small minority struggling with basic set theory. Most students successfully used the hint for (ii) (although some lost a mark for not making a predictable choice of V), while a minority tried to show the submartingale property directly, with variable success. (iii) and (iv) similarly caused few problems; most students used the hint of applying the conditional Jensen inequality, while others argued more directly, expanding the square. A large minority of students lost a mark in (v) for not accurately distinguishing between boundedness in and membership to L^1, or by applying the submartingale convergence theorem to S_n^2 and trying to argue that this implies convergence of S_n. Answers in part (b) were more variable, possibly in part due to time constraints. In (i), most students understood how to use (II) and (III), but many did not think of using the first Borel-Cantelli lemma to make use of (I). The application of the second Borel-Cantelli lemma in (ii) caused fewer problems. Many students seemed to be caught off guard by (iii), and were not prepared to have to re-use previous arguments to conclude. Overall in (b), some students lost marks for attempting $\varepsilon - \delta$ proofs that did not use probability theory, or for attempting to use Doob's maximal inequality (from (a)) in ways that were unhelpful.

Question 2 showed a wide range of quality of answers. Some students struggled with recalling the definition of the conditional expectation and were confusing Part A probability conditional probability with conditional expectation used here. However most students got the bookwork correct, even if many did not justify the simple properties in (a)(iii). (a)(iv) witnessed many false attempts with either incomplete answers or with MCT being used where its conditional version was required. (a)(v) typically either got full marks or no marks at all. Part (b)(i)-(iii) of this question was typically done well by those who made a serious attempt at it, but some scripts dis-played completely false arguments. (b)(iv) was mostly done well but, again, some students were confused about some basic bookwork notions.

Question 3 In Question 3 (a)(i) a mark or two were often lost either by a simple mistake with set operations or by forgetting to argue one (simpler) inclusion (or just saying it is obvious or clear). A surprising number of students struggled with (a)(ii) and were not able to compute conditional expectation using joint density, despite this being on a problem sheet. In part (b) most students advanced well but often with mistakes or lack of details on the way. Some were confused about how to define the objects introduced in the question. Common reasons for marks lost where the essence of the argument was presented well was forgetting to verify integrability or adaptness condition for a martingale (which were very easy here), or quoting a version of the stopping theorem which was not applicable.

B8.2: Continuous Martingales and Stochastic Calculus

Question 1 This was the most popular question attempted by all but one candidate. There was a wide range in the quality of answers with a few candidates scoring full marks and some that were not able to reproduce the bookwork. For part (a) marks were lost for not being precise enough in the definition of a local martingale and for omitting the checking of integrability in the second subpart. For part (b) almost all could state the optimal stopping theorem and many could do the second part. The careful showing that the expectation of the martingale at the stopping time was zero proved more difficult for many. In part (c) Ito's formula was correctly used by most and many could deduce the normality. One person successfully used Dubins-Dambanis-Schwarz for this part. The final part proved challenging for most with only a couple of complete and correctly argued solutions.

Question 2 This was the second most popular question, again with a wide spread of marks. It was the most difficult to score very highly with only one person able to get the final probability density. In part (a) marks were lost again for not showing integrability when giving a direct proof that M was a martingale. A few candidates tried to use Ito's formula but did not show that the local martingale was a martingale. In the second part of (a) many candidates did not use the optimal stopping theorem correctly often assuming that M was uniformly integrable. Part (b) was well done. For part (c) many candidates could start this, showing some ideas about how to proceed but getting right the way to the end was challenging.

Question 3 There were not so many attempts at this question and candidates either got it almost all out or could not get started. Many of the strongest candidates scored very well on this question as it was reasonably straightforward manipulation of stochastic integrals against Brownian motion. The only tricky part was the final result in (d). Here only a couple of people realised that the first Borel-Cantelli lemma was the best way to attack it.

B8.3: Mathematical Models of Financial Derivatives

Generally, the students did extremely well in the three questions.

Question 1 Most students were able to do this question to a high standard. Many students' answers were not clear althoug the thrust of the answer was OK (only in extrem cases of lack of clarity did the student lose some marks).

Question 2 Many students scored 25/25

Question 3 Generally, students did very well. Some of the material (deriving the pricing PDE, for example) was not seen and most students were able to to this successfully. As expected, the last two parts of the question were the ones where students missed marks. Part (d) involved a bit of carpetntry to take an expected value and some students were not able to get all the details correct. The last part could be derived from basic principles or from the equation provided in the paper and some students struggled, but overall very good performance

B8.4: Information Theory

Question 1 was attempted by almost all students. Some identified a slight error (in bii, p can be optimal even if $p(m) > p(m-1)$), but this did not appear to lead to significant variation in performance. Students struggled slightly with the proof of optimality in biii, and with giving a clear argument in cii.

Question 2 was attempted by a few students. Students generally did well at showing that the Elias code cannot be optimal, but struggled more with the calculations in part c.

Question 3 was widely attempted. The data processing inequality in aiii proved difficult for some, and the calculation of the capacity in bi (which is not a simple example) also caused some confusion.

Overall the exam was well done, with a range of performance and understanding demonstrated.

B8.5: Graph Theory

Question 1 Most students answered (a)–(c) well, but very few managed to do part (d). A common error in (a) was omitting the condition $|G| \geq k+1$ for k-connectivity (or some equivalent statement implying $\kappa(K_n) = n-1$). In part (b) most students realised that we can remove one endvertex of each edge of a disconnecting set of edges to prove $\kappa(G) \leq \lambda(G)$. However care must be taken to ensure that we don't remove all the vertices on one side of the edge-cut (e.g., a tree can be disconnected by the removal of any edge, but we need to remove a non-leaf end-vertex of that edge to deduce that $\kappa(G) \leq 1$.) Students had serious difficulties with (d). Common errors included trying to show $\kappa(G) = \delta(G)$ (despite the graph in (c)(ii) being a counterexample). Others assumed that *all* paths between pairs of vertices are of length ≤ 2 and tried to deduce the structure of the graph from this.

Question 2 Performance on this question was rather variable, with some students doing very well, others very badly. In part (b), a number of students noted that one can add a new vertex in the exterior face and join it to all the original vertices. The result then follows from the $3n-6$ bound on the number of edges in a planar graph, which was allowed to be quoted. For students following the model solution, a common error was failing to deal with the case of disconnected or acyclic graphs. A similar issue occurred in part (e) where a number of students failed to consider the case when G is not 2-connected. Quite a few students did poorly on this question, often using vague and non-rigourous topological ideas, or unjustified statements about what an outerplanar graph 'should' look like to deduce the results.

Question 3 Relatively few students attempted this question, although those that did generally did it well. The main issues were with part (c), where some students were not able to formalise why the graph on pairs ij with $|z_i - z_j| > 1$ is K_6-free. In (d), once having identified what components could exist, quite a few students lost some marks by just stating that it was obvious that one needed all but one component a K_4, rather than writing out a suitable induction argument.

Summary: The average marks were similar across the three questions, with question 3 typically having a slightly higher average despite few students attempting it. The spread of marks was good, with strong correlation between marks in both (or all) questions attempted.

BO1.1: History of Mathematics

Both the extended coursework essays and the exam scripts were blind double-marked. The marks for essays and exam were reconciled separately. The two carry equal weight when determining a candidate's final mark. The first half of the exam paper (Section A) consists of six extracts from historical mathematical texts, from which candidates must choose two on which to comment; the second half (Section B) gives candidates a choice of three essay topics, from which they must choose one. The Section B essay accounts for 50% of the overall exam mark; the answers to each of the Section A questions count for 25%.

Throughout the course, candidates were invited to analyse historical mathematical materials from the points of view of their 'context', 'content', and 'significance', and these were the three aspects that candidates were asked to consider when looking at the extracts provided in Section A of the exam paper. A number of candidates chose to use these as subheadings within their answers. The word 'significance' was used consistently throughout the course to capture a broad sense of where a given source sits within the historical development of mathematics. This usage was repeatedly stressed. Some candidates were penalised however for considering this only in the narrow sense of 'importance'.

The Section A questions 1–6 were attempted by 9, 3, 7, 2, 3, and 4 candidates, respectively. The answers to the most popular question, question 1, were quite varied. In several cases, marks were lost for failing to expand adequately upon the analysis/synthesis distinction that lies at the heart of this question. Some candidates chose to respond to the question by giving an account of the changing use of the word 'analysis', but this is not what was required. It was important here not simply to describe mathematical developments, but also to discuss the contemporaneous attitudes towards them — the better answers to the question did just that.

For the extract in question 2, it was fairly easy to discuss the 'content' aspect, which meant that it was all the more important for candidates to address the other parts as thoroughly as possible. Question 3 was in many respects a tricky question in that it invited candidates to discuss a specific idea (indivisibles) within the much broader context of the development of calculus and analysis, a topic that was covered in considerable depth in the course. It was therefore quite easy for candidates to wander off the immediate topic of the extract. Pitfalls here included a failure to address the subtle distinction between the often conflated notions of infinitesimals and indivisibles — a distinction though that Wallis himself did not always make clear. Some candidates were rather vague on the contributions of Cavalieri and Torricelli, while few made any comment on the reference to the method of exhaustion that is implicit in the extract. Democritus, the originator of the notion of an indivisible, was universally absent from responses to this question.

Answers to questions 4 and 5 each called for additional points that were easily missed: in the former that the extract makes reference to the method of finite differences, which was the basis for Euler's (and indeed Leibniz's) development of the calculus, and in the latter that it wasn't just pure group theory that grew from Galois's work, but also of course Galois theory.

Question 6 was a good example of how the references given for the extracts may contain pointers towards things that ought to be included in answers, something that some candidates overlooked: we see that Riemann was very specifically developing a theory of functions of a complex variable, something that Cauchy before him had not quite made explicit. Otherwise, this question was generally well done.

The essay questions 7 and 9 were attempted by 5 and 9 candidates, respectively; no candidate attempted question 8. A common problem that arose with responses to question 7 was that some candidates did not carry their use of specific examples through to the 'challenges' part of their essay. In answers to question 9, some candidates lost sight of the fact that they were supposed to be writing about attitudes *towards* geometry, and instead wrote short narrative histories *of* geometry.

The extended coursework essays were of a decent standard overall, though marks were lost in places for too great a reliance on secondary sources — the use of primary sources was a central part of the reading course upon which this work was based, and so this should have been reflected in the submitted essays. Similarly, a lack of decent referencing and proper bibliographies was penalised in a number of cases. The better essays were those that took a particular question or point of view as their central thread, rather than simply providing a narrative account of the writings of Bolzano, Cantor, Dedekind, and others.

Statistics Options

Reports of the following courses may be found in the Mathematics & Statistics Examiners' Report.

SB1.1/1.2: Applied and Computational Statistics

SB2.1: Foundations of Statistical Inference

SB2.2: Statistical Machine Learning

SB3.1: Applied Probability

Computer Science Options

Reports on the following courses may be found in the Mathematics & Computer Science Examiners' Reports.

CS3a: Lambda Calculus & Types

CS4b: Computational Complexity

Philosophy Options

The report on the following courses may be found in the Philosophy Examiners' Report.

102: Knowledge and Reality

127: Philosophical Logic

D. Comments and Recommendations from the Examination Board

Chair comments brought to teaching committee to improve on exam process.

E. Comments on performance of identifiable individuals

1.Aggregation of marks for the award of the classification on the successful completion of Parts A and B

Classification for a candidate was determined through the following method:

- 10 units at Part A (counting A2 as a double-unit and, for candidates offering 6 long options, two of the long option papers as half-units)
- 6 units (or equivalent) at Part B.

The two average USMs will be:

1. The relative weightings of the Parts is as follows:
   (a) The weighting of Part A is 40%.
   (b) The weighting of Part B is 60%.
2. The relative weightings of the Parts is as follows:
   (a) The weighting of Part A is 100%.
   (b) The weighting of Part B is 0%.

The first class Strong Paper Rule says that to get a first class degree the candidate must have:

(a) average USM ≥ 69.5;

(b) at least 6 units in Parts A and B with USMs ≥ 70;

(c) at least 2 units in Part B with USMs ≥ 70.

The analogous rules apply for II.1 and II.2 degrees. The examiners considered all candidates near each borderline who had been caught by the Strong Paper Rule, that is, who satisfied (a) but failed (b) or (c), and so were due to receive the lower degree class. For two such candidates at the I/II.1 borderline the examiners decided to suspend the examination conventions, and placed the candidates in the first class.

2. Prizes

Prizes were awarded as follows.

Gibbs Prize £500: Franciszek Knyszewski, St Catherine's College
Gibbs Prize £200: Henry Saunders, Mansfield College

Part B Junior Mathematical Prize £200: Xingyu Nie, St John's College
Part B Junior Mathematical Prize £200: Yutong Chen, Oriel College

IMA Prize: Henry Saunders, Mansfield College

F. Names of members of the Board of Examiners

- **Examiners:**

  Prof. Ben Green (Chair)
  Dr Neil Laws
  Prof. Jochen Koenigsmann
  Prof. Radek Erban
  Prof. Raphael Hauser
  Prof. Xenia De La Ossa

  Prof John Hunton (External)
  Dr Ed Brambley (External)

- **Assessors:**
  Prof. Alex Scott
  Prof. Alvaro Cartea
  Prof. Andras Juhasz
  Prof. Andrea Mondino
  Prof. Andreas Muench
  Dr Andrei Constantin
  Prof. Andrew Dancer
  Dr Carmen Constantin
  Dr Charles Parker
  Prof. Christopher Beem
  Prof. Christopher Breward
  Prof. Christopher Hollings
  Prof. Damian Rössler
  Prof. Dmitry Belyaev
  Prof. Dominic Joyce
  Dr Emilio Rossi Ferrucci
  Dr Erik Panzer
  Prof. François Caron
  Prof. Geoff Nicholls
  Prof. Ian Hewitt
  Prof. James Newton
  Prof. Jan Kristensen
  Prof. Jan Obloj
  Dr. Jasmina Panovska-Griffiths
  Prof. Jason Lotay
  Prof. Jochen Koenigsmann
  Prof. José Antonio Carrillo de la Plata
  Prof. Konstantin Ardakov
  Prof. Luc Nguyen
  Prof. Paul Balister
  Prof. Peter Howell
  Prof. Radek Erban
  Prof. Raphael Hauser
  Dr Robert Hinch
  Prof. Ruth Baker
  Prof. Yuji Nakatsukasa
  Dr Martin Bays
  Dr Jinhe Ye
  Dr Antonio Esposito
  Dr Antonio Girao
  Dr Catherine Wilkins
  Dr Chuanxia Zheng
  Dr Mark Van Der Wilk
  Dr Matthew Hoban
  Dr Murad Banaji
  Dr Richard Earl
  Dr Shiwei Liu
  Dr Aleksander Horawa
  Dr Carlos Outeiral Rubiera
  Mr Edgar Sucar
  Mr Harrison Waldon
  Mr Jakub Skrezeczkowski
  Mr Jori Merikoski
  Prof. Alain Goriely
  Prof. Benjamin Hambly
  Prof. Christop Reisinger
  Prof. Coralia Cartis
  Prof. Damian Rossler
  Prof. Derek Moulton
  Prof. Ehud Hrushovski
  Prof. Harald Oberhauser
  Prof. James Maynard
  Prof. Kevin McGerty