Part C Mathematics & Philosophy 2023-24

May 25, 2023

Contents

1 Foreword

This Supplement to the Course Handbook specifies the Mathematics courses available for Part C in Mathematics & Philosophy in the 2024 examination. It should be read in conjunction with the Handbook for Mathematics & Philosophy for the academic year 2023-2024 to be issued in Michaelmas Term. The Handbook contains in particular information on the format and rubrics for written examination papers in Mathematics, and the classification rules applicable to Part C.

See the current edition of the Examination Regulations (`https://examregs.admin.ox.ac.uk/`) for the full regulations governing the examinations.

Part C of the Honour School of Mathematics & Philosophy

The following is reproduced from the *Examination Regulations* applicable to the 2024 examinations.

In Part C each candidate shall offer one of the following:

1. (i) A minimum of eight units and a maximum of ten units in Mathematics;
2. (ii) A minimum of six units and a maximum of ten units in Mathematics and one unit in Philosophy;
3. (iii) A minimum of three units and a maximum of four units in Mathematics and two units in Philosophy;
4. (iv) Three units in Philosophy;

from the lists for Mathematics and for Philosophy.

The schedule of units in *Mathematics* shall be published on the Mathematical Institute website by the beginning of the Michaelmas Full Term in the academic year of the examination concerned. No unit in Mathematics, and no subject in Philosophy (apart from the thesis), may be offered in both Part B and Part C.

A unit in Philosophy consists of one of the following:

(a) One of the subjects 101-118, 120, 124, 125, and 128, as specified in the Regulations for Philosophy in all Honour Schools including Philosophy. For Part C, these subjects shall be examined by a three-hour written paper together with a Part C Philosophy Essay of at most 5000 words.

(b) A Special Subject 198, as specified in the Regulations for Philosophy in all Honour Schools including Philosophy.

(c) A Part C Philosophy Thesis.

(d) A Special Subject in Philosophy as approved by the Joint Committee for Mathematics and Philosophy by regulations published in the University Gazette and communicated to college tutors by the end of the fifth week of Trinity Term in the year before the Part C examination in which it will be examined. No candidate may offer more than one Special Subject in Philosophy in Part C. In approving a Special Subject in Philosophy for Part C, the Joint Committee for Mathematics and Philosophy may specify that candidates will not be permitted to offer certain Special Subjects in combination with certain other subjects, or will be permitted to do so only on condition that in the papers on the other subjects they will not be permitted to answer certain questions. Subject to these qualifications, any candidate may offer any approved Special Subject.

The Schedule of Mathematics units for Mathematics & Philosophy

All units in the Schedule below are drawn from the list of Mathematics Department units and "Other" units available in Mathematics Part C.

In addition you may apply for special approval to be examined in Mathematics Department units not included in the Schedule; any such subject approved will then be treated as falling under the Schedule. For the procedure for seeking approval see below.

For the 2024 examination, the Schedule is as follows. (N.B. All topics listed are units unless otherwise stated).

Schedule

1. C1.1 Model Theory (MT)
2. C1.2 Gödel's Incompleteness Theorems (HT)
3. C1.3 Analytic Topology (MT)
4. C1.4 Axiomatic Set Theory (HT)
5. C2.2 Homological Algebra (MT)
6. C2.3 Representation Theory of Semisimple Lie Algebras (HT)
7. C2.4 Infinite Groups (MT)
8. C2.5 Non-Commutative Rings (HT)
9. C2.6 Introduction to Schemes (HT)
10. C2.7 Category Theory (MT)
11. C3.1 Algebraic Topology (MT)
12. C3.2 Geometric Group Theory (HT)
13. C3.3 Differentiable Manifolds (MT)
14. C3.4 Algebraic Geometry (MT)
15. C3.5 Lie Groups (HT)
16. C3.6 Modular Forms (MT)
17. C3.7 Elliptic Curves (HT)
18. C3.8 Analytic Number Theory (HT)

19. C3.9 Computational Algebraic Topology (HT)
20. C3.10 Additive Combinatorics (MT)
21. C3.11 Riemannian Geometry (HT)
22. C3.12 Low-Dimensional Topology and Knot Theory (HT)
23. C4.1 Further Functional Analysis (MT)
24. C8.1 Stochastic Differential Equations (MT)
25. C8.3 Combinatorics (MT)
26. C8.4 Probabilistic Combinatorics (HT)
27. C8.6 Limit Theorems and Large Deviations in Probability (HT)
28. CCD Dissertations on a Mathematical Topic [double unit] (MT/HT)
29. COD Dissertations on the History of Mathematics [double unit] (MT/HT)

And also any other unit course from the list of Mathematics Department units for which special approval has been granted.

Procedure for seeking approval of additional options where this is required

You may, if you have the support of your Mathematics tutor, apply to the Chair of the Joint Committee for Mathematics and Philosophy for approval of one or more other options from the list of Mathematics Department units for Part C. This list can be found in the schedule of units for Mathematics Part C.

Applications for special approval must be made through the candidate's college and sent to the Chair of the Joint Committee for Mathematics and Philosophy, c/o Academic Administrator, Mathematical Institute, to arrive by **Friday of Week 5 of Michaelmas Term**. Be sure to consult your college tutors if you are considering asking for approval to offer one of these additional options.

Given that each of these additional options, which are all in applied mathematics, presume facility with some or other results and techniques covered in first or second year Mathematics courses not taken by Mathematics & Philosophy candidates, such applications will be exceptional.

You should also be aware that there may be a clash of lectures for specially approved options and those listed in the above Schedule and with lectures in Philosophy.

Registration for Part C Mathematics courses 2023-24

Students will be asked to register for the options they intend to take by the end of week 11, Trinity Term 2023. It is helpful if their registration is as accurate as possible as the data is used to make arrangements for teaching resources. Towards the start of the academic year students will be given the opportunity to make edits to their course registration. Students will then be asked to sign up for classes at the start of Michaelmas Term 2023. Further information about this will be sent via email before the start of term.

2 C1.1 Model Theory

2.1 General Prerequisites

This course presupposes basic knowledge of First Order Predicate Calculus up to and including the Soundness and Completeness Theorems. A familiarity with (at least the statement of) the Compactness Theorem would also be desirable.

2.2 Overview

The course deepens a student's understanding of the notion of a mathematical structure and of the logical formalism that underlies every mathematical theory, taking B1 Logic as a starting point. Various examples emphasise the connection between logical notions and practical mathematics.

The concepts of completeness and categoricity will be studied and some more advanced technical notions, up to elements of modern stability theory, will be introduced.

2.3 Learning Outcomes

Students will have developed an in depth knowledge of the notion of an algebraic mathematical structure and of its logical theory, taking B1 Logic as a starting point. They will have an understanding of the concepts of completeness and categoricity and more advanced technical notions.

2.4 Synopsis

Structures. The first-order language for structures. The Compactness Theorem for first-order logic. Elementary embeddings. Loewenheim-Skolem theorems. Preservation theorems for substructures. Model Completeness. Quantifier elimination.

Categoricity for first-order theories. Types and saturation. Omitting types. The Ryll Nardzewski theorem characterizing aleph-zero categorical theories. Theories with few types. Ultraproducts.

2.5 Reading List

1. C.C. Chang and H. Jerome Keisler, *Model Theory* (Third Edition (Dover Books on Mathematics)Paperback)

2. Tent and Ziegler, *A Course in Model Theory*, Cambridge University Press, April 2012

3 C1.2 Gödel's Incompleteness Theorems

3.1 General Prerequisites

This course presupposes knowledge of first-order predicate logic up to and including soundness and completeness theorems for a formal system of first-order predicate logic (B1.1 Logic).

3.2 Overview

The starting point is Gödel's mathematical sharpening of Hilbert's insight that manipulating symbols and expressions of a formal language has the same formal character as arithmetical operations on natural numbers. This allows the construction for any consistent formal system containing basic arithmetic of a 'diagonal' sentence in the language of that system which is true but not provable in the system. By further study we are able to establish the intrinsic meaning of such a sentence. These techniques lead to a mathematical theory of formal provability which generalizes the earlier results. We end with results that further sharpen understanding of formal provability.

3.3 Learning Outcomes

Understanding of arithmetization of formal syntax and its use to establish incompleteness of formal systems; the meaning of undecidable diagonal sentences; a mathematical theory of formal provability; precise limits to formal provability and ways of knowing that an unprovable sentence is true.

3.4 Synopsis

Gödel numbering of a formal language; the diagonal lemma. Expressibility in a formal language. The arithmetical undefinability of truth in arithmetic. Formal systems of arithmetic; arithmetical proof predicates. Σ_0-completeness and Σ_1-completeness. The arithmetical hierarchy. ω-consistency and 1-consistency; the first Gödel incompleteness theorem. Separability; the Rosser incompleteness theorem. Adequacy conditions for a provability predicate. The second Gödel incompleteness theorem; Löb's theorem. Provable Σ_1-completeness. The ω-rule. The system GL for provability logic. The fixed point theorem for GL. The Bernays arithmetized completeness theorem; undecidable Δ_2-sentences of arithmetic.

3.5 Reading List

Lecture notes for the course.

3.6 Further Reading

1. Raymond M. Smullyan, *Gödel's Incompleteness Theorems* (Oxford University Press, 1992).

2. George S. Boolos and Richard C. Jeffrey, *Computability and Logic* (3rd edition, Cambridge University Press, 1989), Chs 15, 16, 27 (pp 170-190, 268-284).

3. George Boolos, *The Logic of Provability* (Cambridge University Press, 1993).

4 C1.3 Analytic Topology

4.1 General Prerequisites

Part A Topology, including the notions of a topological space, a continuous function and a basis for a topology. A basic knowledge of Set Theory, including cardinal arithmetic, ordinals and the Axiom of Choice, will also be useful.

4.2 Overview

The aim of the course is to present a range of major theory and theorems, both important and elegant in themselves and with important applications within topology and to mathematics as a whole. Central to the course is the general theory of compactness, compactifications and Tychonoff's theorem, one of the most important in all mathematics (with applications across mathematics and in mathematical logic) and computer science.

4.3 Synopsis

Separation axioms. Subbases. Lindelöf and countably compact spaces. Seperable spaces. Filters and ultrafilters. Convergence in terms of filters. Tychonoff's theorem. Compactifications, in particular the Alexandroff One-Point Compactification and the Stone-Čech Compactification. Proper maps. Completeness, connectedness and local connectedness. Components and quasi-components. Urysohn's metrization theorem. Paracompactness. Stone's Theorem; that metric spaces are paracompact. Totally disconnected compact spaces and Stone duality.

4.4 Reading List

1. 1. S. Willard, *General Topology* (Addison-Wesley, 1970), Chs. 1-8.

2. R. Engelking, *General Topology* (Sigma Series in Pure Mathematics, Vol 6, 1989)

5 C1.4 Axiomatic Set Theory

5.1 General Prerequisites

This course presupposes basic knowledge of First Order Predicate Calculus up to and including the Soundness and Completeness Theorems, together with a course on basic set theory, including cardinals and ordinals, the Axiom of Choice and the Well Ordering Principle.

5.2 Overview

Inner models and consistency proofs lie at the heart of modern Set Theory, historically as well as in terms of importance. In this course we shall introduce the first and most important of inner models, Gödel's constructible universe, and use it to derive some fundamental consistency results.

5.3 Synopsis

A review of the axioms of ZF set theory. Absoluteness, the recursion theorem. The Cumulative Hierarchy of sets and the consistency of the Axiom of Foundation as an example of the method of inner models. Levy's Reflection Principle. Gödel's inner model of constructible sets and the consistency of the Axiom of Constructibility ($V = L$). $V = L$ is absolute. The fact that $V = L$ implies the Axiom of Choice. Some advanced cardinal arithmetic. The fact that $V = L$ implies the Generalized Continuum Hypothesis.

5.4 Reading List

For the review of ZF set theory and the prerequisites from Logic:

1. D. Goldrei, *Classic Set Theory* (Chapman and Hall, 1996).
2. K. Kunen, *The Foundations of Mathematics* (College Publications, 2009).

For course topics (and much more):

1. K. Kunen, *Set Theory* (College Publications, 2011) Chapters (I and II).

5.5 Further Reading

1. K. Hrbacek and T. Jech, *Introduction to Set Theory* (3rd edition, M Dekker, 1999).

6 C2.2 Homological Algebra

6.1 General Prerequisites

A3 Rings and Modules is essential: good understanding of modules over fields (aka vector spaces), polynomial rings, the ring of integers, and the ring of integers modulo n; good familiarity with module homomorphisms, submodules, and quotient modules.

6.2 Overview

Homological algebra is one of the most important tools in mathematics with application ranging from number theory and geometry to quantum physics. This course will introduce the basic concepts and tools of homological algebra with examples in module theory and group theory.

6.3 Learning Outcomes

Students will learn about abelian categories and derived functors and will be able to apply these notions in different contexts. They will learn to compute Tor, Ext, and group cohomology and homology.

6.4 Synopsis

Overview of category theory: adjoint functors, limits and colimits, Abelian categories (2 hours)

Chain complexes: complexes of R-modules and in an abelian category, operations on chain complexes, long exact sequences, chain homotopies, mapping cones and cylinders (3 hours)

Derived functors: delta functors, projective and injective resolutions, left and right derived functors, adjoint functors and exactness, balancing Tor and Ext (5 hours)

Tor and Ext: Tor and flatness, Ext and extensions, universal coefficients theorems, Kunneth formula, Koszul resolutions (3 hours)

Group homology and cohomology: definition, basic properties, cyclic groups, interpretation of H^1 and H^2, the Bar resolution (3 hours).

6.5 Reading List

1. Weibel, Charles An introduction to Homological algebra (see Google Books)

7 C2.3 Representation Theory of Semisimple Lie Algebras

7.1 General Prerequisites

C2.1 Lie algebras is recommended, but not required. Results from that course will be used but stated. B2.1 Introduction to Representation Theory is recommended also, by way of a first introduction to ideas of representation theory.

7.2 Overview

The representation theory of semisimple Lie algebras plays a central role in modern mathematics with motivation coming from many areas of mathematics and physics, for example, the Langlands program. The methods involved in the theory are diverse and include remarkable interactions with algebraic geometry, as in the proofs of the Kazhdan-Lusztig and Jantzen conjectures.

The course will cover the basics of finite dimensional representations of semisimple Lie algebras (e.g., the Cartan-Weyl highest weight classification) in the framework of the larger Bernstein-Gelfand-Gelfand category $\mathcal{O}$.

7.3 Learning Outcomes

The students will have developed a comprehensive understanding of the basic concepts and modern methods in the representation theory of semisimple Lie algebras, including the classification of finite dimensional modules, the classification of objects in category ◯, character formulas, Lie algebra cohomology and resolutions of finite dimensional modules.

7.4 Synopsis

Universal enveloping algebra of a Lie algebra, Poincaré-Birkhoff-Witt theorem, basic definitions and properties of representations of Lie algebras, tensor products.

The example of $sl(2)$: finite dimensional modules, highest weights.

Category $\mathcal{O}$: Verma modules, highest weight modules, infinitesimal characters and Harish-Chandra's isomorphism, formal characters, contravariant (Shapovalov) forms.

Finite dimensional modules of a semisimple Lie algebra: the Cartan-Weyl classification, Weyl character formula, dimension formula, Kostant's multiplicity formula, examples.

Homological algebra: Lie algebra cohomology, Bernstein-Gelfand-Gelfand resolution of finite dimensional modules, Ext groups in category $\mathcal{O}$.

Topics: applications, Bott's dimension formula for Lie algebra cohomology groups, characters of the symmetric group (via Zelevinsky's application of the BGG resolution to Schur-Weyl duality).

7.5 Reading List

1. Course Lecture Notes.
2. J. Bernstein "Lectures on Lie algebras", in *Representation Theory, Complex Analysis, and Integral Geometry* (Springer 2012).

7.6 Further Reading

1. J. Humphreys, *Representations of semisimple Lie algebras in the BGG category $\mathcal{O}$* (AMS, 2008).
2. J. Humphreys, *Introduction to Lie algebras and representation theory* (Springer, 1997).
3. W. Fulton, J. Harris, *Representation Theory* (Springer 1991).

8 C2.4 Infinite Groups

8.1 General Prerequisites

Knowledge of the first and second-year algebra courses is helpful but not mandatory; in particular Prelims *M1: Groups and Group Actions*, *A0: Linear Algebra*, and *ASO: Group*

Theory. Likewise, the course *B3.5 Topology and Groups* would bring more familiarity and a different viewpoint of the notions treated in this course.

8.2 Overview

The course introduces some natural families of groups, with an emphasis on those that generalize abelian groups, various questions that one can ask about them, and various methods used to answer these questions; these involve among other things questions of finite presentability, linearity, torsion and growth.

8.3 Synopsis

Free groups; ping-pong lemma. Finitely generated and finitely presented groups. Residual finiteness and linearity.

Nilpotency, lower and upper central series. Polycyclic groups, length, Hirsch length, Noetherian induction. Solvable groups, derived series, law. Structure of linear solvable and nilpotent groups.

Solvable versus polycyclic: examples, characterization of polycyclic groups as solvable Z-linear groups. Solvable versus nilpotent: the Milnor -Wolf theorem characterizing nilpotent groups as solvable groups with sub-exponential (and in fact polynomial) growth.

8.4 Reading List

1. C. Drutu, M. Kapovich, *Geometric Group Theory*, (AMS, 2018), Chapters 13 and 14.
2. D. Segal, *Polycyclic groups*, (CUP, 2005) Chapters 1 and 2.
3. D. J. S. Robinson, *A course in the theory of groups*, 2nd ed., Graduate texts in Mathematics, (Springer-Verlag, 1995). Chapters 2, 5, 6, 15.

9 C2.5 Non-Commutative Rings

9.1 General Prerequisites

General Prerequisites: All the material of A3 Rings and Modules is essential: Basic properties of rings and modules. Ideals, prime ideals. Principal ideal rings, unique factorization rings, Euclidean rings. Finite fields. Modules over Euclidean rings.

Recommended material: From B2.1 Introduction to Representation Theory: semisimple modules and algebras, the Artin - Wedderburn theorem. From B2.2 Commutative Algebra: Noetherian rings and modules. Hilbert's basis theorem. Krull dimension.

9.2 Overview

This course builds on Algebra 2 from the second year. We will look at several classes of non-commutative rings and try to explain the idea that they should be thought of as functions on "non-commutative spaces". Along the way, we will prove several beautiful structure theorems for Noetherian rings and their modules.

9.3 Learning Outcomes

Students will be able to appreciate powerful structure theorems, and be familiar with examples of non-commutative rings arising from various parts of mathematics.

9.4 Synopsis

Examples of non-commutative Noetherian rings: enveloping algebras, rings of differential operators, group rings of polycyclic groups. Filtered and graded rings. (3 hours)

Jacobson radical in general rings. Jacobson's density theorem. Artin-Wedderburn. (3 hours)

Ore localisation. Goldie's Theorem on Noetherian domains. (3 hours)

Minimal prime ideals and dimension functions. Rees rings and good filtrations. (3 hours)

Bernstein's Inequality and Gabber's Theorem on the integrability of the characteristic variety. (4 hours)

9.5 Reading List

1. K.R. Goodearl and R.B. Warfield, *An Introduction to Noncommutative Noetherian Rings* (CUP, 2004).

9.6 Further Reading

1. M. Atiyah and I. MacDonald, *Introduction to Commutative Algebra* (Westview Press, 1994).
2. S.C. Coutinho, *A Primer of Algebraic D-modules* (CUP, 1995).
3. J. Björk, *Analytic D-Modules and Applications* (Springer, 1993).

10 C2.6 Introduction to Schemes

10.1 General Prerequisites

B2.2 Commutative Algebra is essential. *C2.2 Homological Algebra* is highly recommended and *C2.7 Category Theory* is recommended but the necessary material from both courses can be learnt during the course. *C3.4 Algebraic Geometry* is strongly recommended but not technically necessary. *C3.1 Algebraic Topology* contains many homological techniques also used in this course.

10.2 Overview

Scheme theory is the foundation of modern algebraic geometry, whose origins date back to the work from the 1950s and 1960s by Jean-Pierre Serre and Alexander Grothendieck. It unifies algebraic geometry with algebraic number theory. This unification has led to proofs of important conjectures in number theory such as the Weil conjecture by Deligne and the Mordell conjecture by Faltings.

This course will cover the basics of the theory of schemes.

10.3 Learning Outcomes

Students will have developed a thorough understanding of the basic concepts and methods of scheme theory. They will be able to work with affine and projective schemes, as well as with (quasi-)coherent sheaves and their cohomology groups.

10.4 Synopsis

The Spec of a ring, Zariski topology, comparison with classical algebraic geometry.

Pre-sheaves and stalks, sheaves, sheafification. The abelian category of sheaves of abelian groups on a topological space. Direct and inverse images of sheaves. Sheaves defined on a topological basis.

Ringed spaces and morphisms of ringed spaces. Affine schemes, construction of the structure sheaf, the equivalence of categories defined by Spec.

Schemes, closed subschemes. Global sections. The functor of points.

Properties of schemes: (locally) Noetherian, reduced, irreducible, and integral schemes. Properties of morphisms of schemes: finite type, open/closed immersions, flatness. Simple examples of flat families of schemes arising from deformations.

Gluing sheaves. Gluing schemes. Affine and projective n-space viewed as schemes.

Products, coproducts and fiber products in category theory. Existence of products of schemes. Fibers and pre-images of morphisms of schemes. Base change.

Further properties of morphisms of schemes: separated, universally closed, and proper morphisms. Projective n-space and projective morphisms. Abstract varieties. Complete varieties. Scheme structure on a closed subset of a scheme.

Sheaves of modules. Vector bundles and coherent sheaves. The abelian category of sheaves of modules over a scheme. Pull-backs.

Quasi-coherent sheaves. Gluing sheaves of modules. Classification of (quasi-)coherent sheaves on Spec of a ring.

Čech cohomology. Vanishing of higher cohomology groups of quasi-coherent sheaves on affine schemes. Independence of Čech cohomology on the choice of open cover. Line bundles, examples on projective n-space.

Sheaf cohomology. Acyclic resolutions. Comparison of sheaf cohomology and Čech cohomology.

Brief discussion of (quasi-)coherent sheaves on projective n-space, graded modules, and Proj of a graded ring.

10.5 Reading List

1. Robin Hartshorne, *Algebraic Geometry.*
2. Ravi Vakil, *Foundations of Algebraic Geometry*, online notes on the website of Stanford University (open access)
3. Geir Ellingsrud and John Christian Ottem, *Introduction to schemes*, Available online (notes in progress): `https://www.uio.no/studier/emner/matnat/math/MAT4215/data/masteragbook-2023.pdf`

10.6 Further Reading

1. David Mumford, *The Red Book of Varieties and Schemes.*
2. David Eisenbud and Joe Harris, *The Geometry of Schemes.*
3. George R. Kempf, *Algebraic Varieties.*
4. Qing Liu, *Algebraic geometry and arithmetic curves*, Oxford University Press, 2002.

11 C2.7 Category Theory

11.1 General Prerequisites

There are no essential prerequisites but familiarity with the basic theory of groups, rings, vector spaces, modules and topological spaces would be very useful, and other topics such as Algebraic Geometry, Algebraic Topology, Homological Algebra and Representation Theory are relevant. Category Theory also has links with Logic and Set Theory, but this course will not stress those links.

11.2 Overview

Category theory brings together many areas of pure mathematics (and also has close links to logic and to computer science). It is based on the observation that many mathematical topics can be unified and simplified by using descriptions in terms of diagrams of arrows; the arrows represent functions of suitable types. Moreover many constructions in pure mathematics can be described in terms of 'universal properties' of such diagrams.

The aim of this course is to provide an introduction to category theory using a host of familiar examples, to explain how these examples fit into a categorical framework and to use categorical ideas to make new constructions.

11.3 Learning Outcomes

Students will have developed a thorough understanding of the basic concepts and methods of category theory. They will be able to work with commutative diagrams, naturality and universality properties and adjoint functors, and to apply categorical ideas and methods in a wide range of areas of mathematics.

11.4 Synopsis

Introduction: universal properties in linear and multilinear algebra.

Categories, functors, natural transformations. Examples including categories of sets, groups, rings, vector spaces and modules, topological spaces. Groups, monoids and partially ordered sets as categories. Opposite categories and the principle of duality. Covariant, contravariant, faithful and full functors. Equivalences of categories.

Adjoints: definition and examples including free and forgetful functors and abelianisations of groups. Adjunctions via units and counits, adjunctions via initial objects.

Representables: definitions and examples including tensor products. The Yoneda lemma and applications.

Limits and colimits, including products, equalizers, pullbacks and pushouts. Monics and epics. Interaction between functors and limits.

Monads and comonads, algebras over a monad, Barr-Beck monadicity theorem (proof not examinable). The category of affine schemes as the opposite of the category of commutative rings.

11.5 Reading List

1. T. Leinster, *Basic category theory*, (CUP, 2014) Chapters 1-5, available online arXiv: 1612.09375
2. E. Riehl, *Category theory in context* (Dover, 2016) Chapters 1-5, available online www.math.jhu.edu/ eriehl/context.pdf

11.6 Further Reading

1. S. Awodey, *Category theory*, Oxford Logic Guides (OUP, 2010)
2. D. Eisenbud, J. Harris, *The geometry of schemes.*
3. S. Lang, *Linear algebra* 2nd edition, (Addison Wesley, 1971) Chapter XIII, out of print but may be available in college libraries.
4. S. Mac Lane, *Categories for the Working Mathematician*, 2nd ed., (Springer, 1998)
5. D.G. Northcott, *Multilinear algebra* (CUP, reissued 2009)
6. E. Riehl, *Categorical Homotopy theory* (CUP, 2014), available online at www.math.jhu.edu/ eriehl/cathtpy.p

12 C3.1 Algebraic Topology

12.1 General Prerequisites

A3 Rings and Modules is essential, in particular a solid understanding of groups, rings, fields, modules, homomorphisms of modules, kernels and cokernels, and classification of finitely generated abelian groups. A5 Topology is essential, in particular a solid understanding of topological spaces, connectedness, compactness, and classification of compact surfaces. B3.5 Topology and Groups is helpful but not necessary, in particular the notion of homotopic maps, homotopy equivalences, and fundamental groups will be recalled during the course. There will be little mention of homotopy theory in this course as the focus will be instead on homology and cohomology. It is recommended, but not required, that students take C2.2 Homological Algebra concurrently.

12.2 Overview

Homology theory is a subject that pervades much of modern mathematics. Its basic ideas are used in nearly every branch, pure and applied. In this course, the homology groups of topological spaces are studied. These powerful invariants have many attractive applications. For example we will prove that the dimension of a vector space is a topological invariant and the fact that 'a hairy ball cannot be combed'.

12.3 Learning Outcomes

At the end of the course, students are expected to understand the basic algebraic and geometric ideas that underpin homology and cohomology theory. These include the cup product and Poincaré Duality for manifolds. They should be able to choose between the different homology theories and to use calculational tools such as the Mayer-Vietoris sequence to compute the homology and cohomology of simple examples, including projective spaces, surfaces, certain simplicial spaces and cell complexes. At the end of the course, students should also have developed a sense of how the ideas of homology and cohomology may be applied to problems from other branches of mathematics.

12.4 Synopsis

Brief introduction to categories and functors. Applications of homology theory: Invariance of dimension, Brower fixed point theorem.

Chain complexes of free Abelian groups and their homology. Short exact sequences. of chain complexes, the induced long exact sequence in homology, and naturality. The snake lemma, the five lemma, splitting properties for short exact sequences.

Simplicial homology via Delta complexes.

Singular homology of topological spaces, and functoriality. Relative homology. Chain homotopies, homotopy equivalences. Homotopy invariance and excision (details of proofs not examinable). Retractions, deformation retractions, quotients.

Mayer-Vietoris Sequence. Wedge sums, cones, suspensions, connected sums.

Degree of a self-map of a sphere. Application: the hairy ball theorem.

Cell complexes and cellular homology. Equivalence of simplicial, cellular and singular homology.

Cochains and cohomology of spaces. Cup products.

Künneth Theorem (without proof). Euler characteristic. Ext and Tor groups via free resolutions. (Co)homology with different coefficients. The Universal Coefficient Theorem (proof not examinable).

Topological manifolds and orientability. The fundamental class of an orientable, closed manifold and the degree of a map between manifolds of the same dimension. Poincaré duality (proof not examinable). Manifolds with boundary and Poincaré-Lefschetz duality (proof not examinable). Brief discussion of locally finite homology, and cohomology with compact supports. Cap product.

Alexander duality. Applications: knot complements, Jordan curve theorem.

12.5 Reading List

1. A. Hatcher, *Algebraic Topology* (Cambridge University Press, 2001). Chapters 2 and 3.

2. G. Bredon, *Topology and Geometry* (Springer, 1997). Chapters 4 and 5.

3. J. Vick, *Homology Theory*, Graduate Texts in Mathematics 145 (Springer, 1973).

12.6 Further Reading

1. W.S. Massey, *A Basic Course in Algebraic Topology*, (Springer, GTM 127, 1991).

2. P. May, *A Concise Course in Algebraic Topology* (University of Chicago Press, 1999).

3. J. Davis and P. Kirk, *Lecture Notes in Algebraic Topology* (AMS, 2001).

13 C3.2 Geometric Group Theory

13.1 General Prerequisites

Some familiarity with Cayley graphs, fundamental group and covering spaces (as for example in the course B3.5 Topology & Groups) would be a helpful though not essential prerequisite.

13.2 Overview

The aim of this course is to introduce the fundamental methods and problems of geometric group theory and discuss their relationship to topology and geometry.

The first part of the course begins with an introduction to presentations and the list of problems of M. Dehn. It continues with the theory of group actions on trees and the structural study of fundamental groups of graphs of groups.

The second part of the course focuses on modern geometric techniques and it provides an introduction to the theory of Gromov hyperbolic groups.

13.3 Synopsis

Free groups. Group presentations. Dehn's problems. Residually finite groups.

Group actions on trees. Amalgams, HNN-extensions, graphs of groups, subgroup theorems for groups acting on trees.

Quasi-isometries. Hyperbolic groups. Solution of the word and conjugacy problem for hyperbolic groups.

If time allows: Small Cancellation Groups, Stallings Theorem, Boundaries.

13.4 Reading List

1. J.P. Serre, *Trees* (Springer Verlag 1978).
2. M. Bridson, A. Haefliger, *Metric Spaces of Non-positive Curvature, Part III* (Springer, 1999), Chapters I.8, III.H.1, III. Γ5.
3. H. Short *et al.*, 'Notes on word hyperbolic groups', Group Theory from a Geometrical Viewpoint, Proc. ICTP Trieste (eds E. Ghys, A. Haefliger, A. Verjovsky, World Scientific 1990), available online at: `http://www.cmi.univ-mrs.fr/~hamish/`
4. C.F. Miller, *Combinatorial Group Theory*, notes: `http://www.ms.unimelb.edu.au/~cfm/notes/cgt-notes.pdf`

13.5 Further Reading

1. G. Baumslag, *Topics in Combinatorial Group Theory* (Birkhauser, 1993).
2. O. Bogopolski, *Introduction to Group Theory* (EMS Textbooks in Mathematics, 2008).
3. R. Lyndon, P. Schupp, *Combinatorial Group Theory* (Springer, 2001).
4. W. Magnus, A. Karass, D. Solitar,*Combinatorial Group Theory: Presentations of Groups in Terms of Generators and Relations* (Dover Publications, 2004).
5. P. de la Harpe, *Topics in Geometric Group Theory*, (University of Chicago Press, 2000).

14 C3.3 Differentiable Manifolds

14.1 General Prerequisites

A5: Topology and *ASO: Multidimensional Analysis and Geometry* are strongly recommended. (Notions of Hausdorff, open covers, smooth functions on $\mathbf{R}^n$ will be used without further explanation.) Useful but not essential: *B3.2 Geometry of Surfaces.*

14.2 Overview

A manifold is a space such that small pieces of it look like small pieces of Euclidean space. Thus a smooth surface, the topic of the Geometry of Surfaces course, is an example of a (2-dimensional) manifold.

Manifolds are the natural setting for parts of classical applied mathematics such as mechanics, as well as general relativity. They are also central to areas of pure mathematics such as topology and certain aspects of analysis.

In this course we introduce the tools needed to do analysis on manifolds. We prove a very general form of Stokes' Theorem which includes as special cases the classical theorems of Gauss, Green and Stokes. We also introduce the theory of de Rham cohomology, which is central to many arguments in topology.

14.3 Learning Outcomes

The candidate will be able to manipulate with ease the basic operations on tangent vectors, differential forms and tensors both in a local coordinate description and a global coordinate-free one; have a knowledge of the basic theorems of de Rham cohomology and some simple examples of their use; know what a Riemannian manifold is and what geodesics are.

14.4 Synopsis

Smooth manifolds and smooth maps. Tangent vectors, the tangent bundle, induced maps. Vector fields and flows, the Lie bracket and Lie derivative.

Exterior algebra, differential forms, exterior derivative, Cartan formula in terms of Lie derivative. Orientability. Partitions of unity, integration on oriented manifolds.

Stokes' theorem. De Rham cohomology. Applications of de Rham theory including degree.

Riemannian metrics. Isometries. Geodesics.

14.5 Reading List

1. M. Spivak, *Calculus on Manifolds*, (W. A. Benjamin, 1965).
2. M. Spivak, *A Comprehensive Introduction to Differential Geometry*, Vol. 1, (1970).
3. W. Boothby, *An Introduction to Differentiable Manifolds and Riemannian Geometry*, 2nd edition, (Academic Press, 1986).
4. M. Berger and B. Gostiaux, *Differential Geometry: Manifolds, Curves and Surfaces.* Translated from the French by S. Levy, (Springer Graduate Texts in Mathematics, 115, Springer–Verlag (1988)) Chapters 0-3, 5-7.
5. F. Warner, *Foundations of Differentiable Manifolds and Lie Groups*, (Springer Graduate Texts in Mathematics, 1994).
6. D. Barden and C. Thomas, *An Introduction to Differential Manifolds.* (Imperial College Press, London, 2003.)

15 C3.4 Algebraic Geometry

15.1 General Prerequisites

A3 Rings and Modules and B2.2 Commutative Algebra are essential. Noetherian rings, the Noether normalisation lemma, integrality, the Hilbert Nullstellensatz and dimension theory will play an important role in the course. B3.3 Algebraic Curves is useful but not essential. Projective spaces and homogeneous coordinates will be defined in C3.4, but a working knowledge of them would be useful. There is some overlap of topics, as B3.3 studies the algebraic geometry of one-dimensional varieties. Courses closely related to C3.4 include C2.2 Homological Algebra, C2.7 Category Theory, C3.7 Elliptic Curves, C2.6 Introduction to Schemes; and partly related to: C3.1 Algebraic Topology, C3.3 Differentiable Manifolds, C3.5 Lie Groups.

15.2 Overview

Algebraic geometry is the study of algebraic varieties: an algebraic variety is roughly speaking, a locus defined by polynomial equations. One of the advantages of algebraic geometry is that it is purely algebraically defined and applied to any field, including fields of finite characteristic. It is geometry based on algebra rather than calculus, but over the real or complex numbers it provides a rich source of examples and inspiration to other areas of geometry.

15.3 Synopsis

Affine algebraic varieties, the Zariski topology, morphisms of affine varieties. Irreducible varieties.

Projective space. Projective varieties, affine cones over projective varieties. The Zariski topology on projective varieties. The projective closure of affine variety. Morphisms of projective varieties. Projective equivalence.

Veronese morphism: definition, examples. Veronese morphisms are isomorphisms onto their image; statement, and proof in simple cases. Subvarieties of Veronese varieties. Segre maps and products of varieties.

Coordinate rings. The geometric form of Hilbert's Nullstellensatz. Correspondence between affine varieties (and morphisms between them) and finitely generated reduced K-algebras (and morphisms between them). Graded rings and homogeneous ideals. Homogeneous coordinate rings.

Discrete invariants of projective varieties: degree, dimension, Hilbert function. Statement of theorem defining Hilbert polynomial.

Quasi-projective varieties, and morphisms between them. The Zariski topology has a basis of affine open subsets. Rings of regular functions on open subsets and points of quasi-projective varieties. The ring of regular functions on an affine variety is the coordinate ring. Localisation and relationship with rings of regular functions.

Tangent space and smooth points. The singular locus is a closed subvariety. Algebraic re-formulation of the tangent space. Differentiable maps between tangent spaces.

Function fields of irreducible quasi-projective varieties. Rational maps between irreducible varieties, and composition of rational maps. Birational equivalence. Correspondence between dominant rational maps and homomorphisms of function fields. Blow-ups: of affine space at a point, of subvarieties of affine space, and of general quasi-projective varieties along general subvarieties. Statement of Hironaka's Desingularisation Theorem. Every irreducible variety is birational to a hypersurface. Re-formulation of dimension. Smooth points are a dense open subset.

15.4 Reading List

1. Karen E. Smith, *An invitation to algebraic geometry*

15.5 Further Reading

1. M Reid, *Undergraduate Algebraic Geometry*
2. K Hulek, *Elementary Algebraic Geometry*
3. A Gathmann, *Algebraic Geometry lecture notes*, online: `www.mathematik.uni-kl.de/en/agag/members/professors/gathmann/notes/alggeom`
4. Shafarevich, *Basic Algebraic Geometry 1*
5. D Mumford, *The Red Book of Varieties and Schemes*

16 C3.5 Lie Groups

16.1 General Prerequisites

ASO: Group Theory, *A5: Topology* and *ASO: Multidimensional Analysis and Geometry* are all useful but not essential. It would be desirable to have seen notions of derivative of maps from $\mathbf{R}^n$ to $\mathbf{R}^m$, inverse and implicit function theorems, and submanifolds of $\mathbf{R}^n$. Acquaintance with the notion of an abstract manifold would be helpful but not really necessary.

16.2 Overview

The theory of Lie Groups is one of the most beautiful developments of pure mathematics in the twentieth century, with many applications to geometry, theoretical physics and mechanics. The subject is an interplay between geometry, analysis and algebra. Lie groups are groups which are simultaneously manifolds, that is geometric objects where the notion of differentiability makes sense, and the group multiplication and inversion are differentiable maps. The majority of examples of Lie groups are the familiar groups of matrices. The course does not require knowledge of differential geometry: the basic tools needed will be covered within the course.

16.3 Learning Outcomes

Students will have learnt the fundamental relationship between a Lie group and its Lie algebra, and the basics of representation theory for compact Lie groups. This will include a firm understanding of maximal tori and the Weyl group, and their role for representations.

16.4 Synopsis

Brief introduction to manifolds. Classical Lie groups. Left-invariant vector fields, Lie algebra of a Lie group. One-parameter subgroups, exponential map. Homomorphisms of Lie groups and Lie algebras. Ad and ad. Compact connected abelian Lie groups are tori. The Campbell-Baker-Hausdorff series (statement only).

Lie subgroups. Definition of embedded submanifolds. A subgroup is an embedded Lie subgroup if and only if it is closed. Continuous homomorphisms of Lie groups are smooth. Correspondence between Lie subalgebras and Lie subgroups (proved assuming the Frobenius theorem). Correspondence between Lie group homomorphisms and Lie algebra homomorphisms. Ado's theorem (statement only), Lie's third theorem.

Basics of representation theory: sums and tensor products of representations, irreducibility, Schur's lemma. Compact Lie groups: left-invariant integration, complete reducibility. Representations of the circle and of tori. Characters, orthogonality relations. Peter-Weyl theorem (statement only).

Maximal tori. Roots. Conjugates of a maximal torus cover a compact connected Lie group. Weyl group. Reflections. Weyl group of *U(n)*. Representations of a compact connected Lie group are the Weyl-invariant representations of a maximal torus (proof of inclusion only). Representation ring of maximal tori and *U(n)*.

Killing form. Remarks about the classification of compact Lie groups.

16.5 Reading List

1. J. F. Adams, *Lectures on Lie Groups* (University of Chicago Press, 1982).

2. T. Bröcker and T. tom Dieck, *Representations of Compact Lie Groups* (Graduate Texts in Mathematics, Springer, 1985).

16.6 Further Reading

1. R. Carter, G. Segal and I. MacDonald, *Lectures on Lie Groups and Lie Algebras* (LMS Student Texts, Cambridge, 1995).

2. W. Fulton, J. Harris, *Representation Theory: A First Course* (Graduate Texts in Mathematics, Springer, 1991).

3. F. W. Warner, *Foundations of Differentiable Manifolds and Lie Groups* (Graduate Texts in Mathematics, 1983).

17 C3.6 Modular Forms

17.1 General Prerequisites

Part A Number Theory, Topology and Part B Geometry of Surfaces, Algebraic Curves (or courses covering similar material) are useful but not essential.

17.2 Overview

The course aims to introduce students to the beautiful theory of modular forms, one of the cornerstones of modern number theory. This theory is a rich and challenging blend of methods from complex analysis and linear algebra, and an explicit application of group actions.

17.3 Learning Outcomes

The student will learn about modular curves and spaces of modular forms, and understand in special cases how to compute their genus and dimension, respectively. They will see that modular forms can be described explicitly via their q-expansions, and they will be familiar with explicit examples of modular forms. They will learn about the rich algebraic structure on spaces of modular forms, given by Hecke operators and the Petersson inner product.

17.4 Synopsis

Overview and examples of modular forms. Definition and basic properties of modular forms. Topology of modular curves: a fundamental domain for the full modular group; fundamental domains for subgroups Γ of finite index in the modular group; the compact surfaces X_Γ; explicit triangulations of X_Γ and the computation of the genus using the Euler characteristic formula; the congruence subgroups $\Gamma(\mathrm{N})$,$\Gamma_0(N)$ and $\Gamma_1(N)$; examples of genus computations. Dimensions of spaces of modular forms: general dimension formula (proof non-examinable); the valence formula (proof non-examinable). Examples of modular forms: Eisenstein series in level 1; Ramanujan's Δ-function; some arithmetic applications. The Petersson inner product. Modular forms as functions on lattices: modular forms of level 1 as functions on lattices; Eisenstein series revisited. Hecke operators in level 1: Hecke operators on lattices; Hecke operators on modular forms and their q-expansions; Hecke operators are Hermitian; multiplicity one.

17.5 Reading List

1. F. Diamond and J. Shurman, *A First Course in Modular Forms*, Graduate Texts in Mathematics 228, Springer-Verlag, 2005

2. R.C. Gunning, *Lectures on Modular Forms*, Annals of mathematical studies 48, Princeton University Press, 1962.

3. J.S. Milne, *Modular Functions and Modular Forms*, www.jmilne.org/math/CourseNotes/mf.html

4. J.-P. Serre, *A Course in Arithmetic*, Chapter VII, Graduate Texts in Mathematics 7, Springer-Verlag, 1973.

18 C3.7 Elliptic Curves

18.1 General Prerequisites

It is helpful, but not essential, if students have already taken a standard introduction to algebraic curves and algebraic number theory. For those students who may have gaps in their background, I have placed the file "Preliminary Reading" permanently on the Elliptic Curves webpage, which gives in detail (about 30 pages) the main prerequisite knowledge for the course.

18.2 Overview

Elliptic curves give the simplest examples of many of the most interesting phenomena which can occur in algebraic curves; they have an incredibly rich structure and have been the testing ground for many developments in algebraic geometry whilst the theory is still full of deep unsolved conjectures, some of which are amongst the oldest unsolved problems in mathematics. The course will concentrate on arithmetic aspects of elliptic curves defined over the rationals, with the study of the group of rational points, and explicit determination of the rank, being the primary focus. Using elliptic curves over the rationals as an example, we will be able to introduce many of the basic tools for studying arithmetic properties of algebraic varieties.

18.3 Learning Outcomes

On completing the course, students should be able to understand and use properties of elliptic curves, such as the group law, the torsion group of rational points, and 2-isogenies between elliptic curves. They should be able to understand and apply the theory of fields with valuations, emphasising the p-adic numbers, and be able to prove and apply Hensel's Lemma in problem solving. They should be able to understand the proof of the Mordell-Weil Theorem for the case when an elliptic curve has a rational point of order 2, and compute ranks in such cases, for examples where all homogeneous spaces for descent-via-2-isogeny satisfy the Hasse principle. They should also be able to apply the elliptic curve method for the factorisation of integers.

18.4 Synopsis

Non-singular cubics and the group law; Weierstrass equations.

Elliptic curves over finite fields; Hasse estimate (stated without proof).

p-adic fields (basic definitions and properties).

1-dimensional formal groups (basic definitions and properties).

Curves over p-adic fields and reduction mod p.

Computation of torsion groups over $\mathbb{Q}$; the Nagell-Lutz theorem.

2-isogenies on elliptic curves defined over $\mathbb{Q}$, with a $\mathbb{Q}$-rational point of order 2.

Weak Mordell-Weil Theorem for elliptic curves defined over $\mathbb{Q}$, with a $\mathbb{Q}$-rational point of order 2.

Height functions on Abelian groups and basic properties.

Heights of points on elliptic curves defined over $\mathbb{Q}$; statement (without proof) that this gives a height function on the Mordell-Weil group.

Mordell-Weil Theorem for elliptic curves defined over $\mathbb{Q}$, with a $\mathbb{Q}$-rational point of order 2.

Explicit computation of rank using descent via 2-isogeny.

Public keys in cryptography; Pollard's $(p-1)$ method and the elliptic curve method of factorisation.

18.5 Reading List

1. J.W.S. Cassels, *Lectures on Elliptic Curves*, LMS Student Texts 24 (Cambridge University Press, 1991).

2. N. Koblitz, *A Course in Number Theory and Cryptography*, Graduate Texts in Mathematics 114 (Springer, 1987).

3. J.H. Silverman and J. Tate, *Rational Points on Elliptic Curves*, Undergraduate Texts in Mathematics (Springer, 1992).

4. J.H. Silverman, *The Arithmetic of Elliptic Curves*, Graduate Texts in Mathematics 106 (Springer, 1986).

18.6 Further Reading

1. A. Knapp, *Elliptic Curves, Mathematical Notes 40* (Princeton University Press, 1992).

2. G, Cornell, J.H. Silverman and G. Stevans (editors), *Modular Forms and Fermat's Last Theorem* (Springer, 1997).

3. J.H. Silverman, *Advanced Topics in the Arithmetic of Elliptic Curves*, Graduate Texts in Mathematics 151 (Springer, 1994).

19 C3.8 Analytic Number Theory

19.1 General Prerequisites

Basic ideas of complex analysis. Elementary number theory. Some familiarity with Fourier series will be helpful but not essential.

19.2 Overview

The aim of this course is to study the prime numbers using the famous Riemann ζ-function. In particular, we will study the connection between the primes and the zeros of the ζ-function. We will state the Riemann hypothesis, perhaps the most famous unsolved problem in mathematics, and examine its implication for the distribution of primes. We will prove the prime number theorem, which states that the number of primes less than X is asymptotic to $X/\log X$.

19.3 Learning Outcomes

In addition to the highlights mentioned above, students will gain experience with different types of Fourier transform and with the use of complex analysis.

19.4 Synopsis

Introductory material on primes. Arithmetic functions: Möbius function, Euler's ϕ-function, the divisor function, the σ-function. Multiplicativity. Dirichlet series and Euler products. The von Mangoldt function.

The Riemann ζ-function for $\Re(s) > 1$. Euler's proof of the infinitude of primes. ζ and the von Mongoldt function.

Schwarz functions on $\mathbf{R}$, $\mathbf{Z}$, $\mathbf{R}/\mathbf{Z}$ and their Fourier transforms. *Inversion formulas and uniqueness*. The Poisson summation formula. The meromorphic continuation and functional equation of the ζ-function. Poles and zeros of ζ and statement of the Riemann hypothesis. Basic estimates for ζ.

The classical zero-free region. Proof of the prime number theorem. Implications of the Riemann hypothesis for the distribution of primes.

19.5 Reading List

Full printed notes will be provided for the course, including the non-examinable topics (marked with asterisks above). The following books are relevant to the course.

1. G. H. Hardy and E. M. Wright, *An introduction to the Theory of Numbers* (Sixth edition, OUP 2008). Chapters 16, 17, 18.

2. H. Davenport, *Multiplicative number theory* (Third Edition, Springer Graduate texts 74), selected parts of the first half.

3. M. du Sautoy, *Music of the primes* (this is a popular book which could be useful background reading for the course).

20 C3.9 Computational Algebraic Topology

20.1 General Prerequisites

Some familiarity with the main concepts from algebraic topology, homological algebra and category theory will be helpful.

20.2 Overview

Ideas and tools from algebraic topology have become more and more important in computational and applied areas of mathematics. This course will provide at the masters level an introduction to the main concepts of (co)homology theory, and explore areas of applications in data analysis and in foundations of quantum mechanics and quantum information.

20.3 Learning Outcomes

Students should gain a working knowledge of homology and cohomology of simplicial sets and sheaves, and improve their geometric intuition. Furthermore, they should gain an awareness of a variety of application in rather different, research active fields of applications with an emphasis on data analysis and contextuality.

20.4 Synopsis

The course has two parts. The first part will introduce students to the basic concepts and results of (co)homology, including sheaf cohomology. In the second part applied topics are introduced and explored.

Core: Homology and cohomology of chain complexes. Algorithmic computation of boundary maps (with a view of the classification theorm for finitely generated modules over a PID). Chain homotopy. Snake Lemma. Simplicial complexes. Other complexes (Delaunay, Cech). Mayer-Vietoris sequence. Poincare duality. Alexander duality. Acyclic carriers. Discrete Morse theory. (6 lectures)

Topic A: Persistent homology: barcodes and stability, applications todata analysis, generalisations. (4 lectures)

Topic B: Sheaf cohomology and applications to quantum non-locality and contextuality.Sheaf-theoretic representation of quantum non-locality and contextuality asobstructions to global sections. Cohomological characterizations and proofs of contextuality.(6 lectures)

20.5 Reading List

H. Edelsbrunner and J.L. Harer, *Computational Topology - An Introduction*,AMS (2010).

See also, U. Tillmann, Lecture notes for CAT 2012, in `http://people.maths.ox.ac.uk/tillmann/CAT.html`

Topic A:

1. G. Carlsson, *Topology and data*, Bulletin A.M.S.46 (2009), 255-308.

2. H. Edelsbrunner, J.L. Harer, *Persistent homology: A survey*, Contemporary Mathematics 452 A.M.S. (2008), 257-282.

3. S. Weinberger, *What is ... Persistent Homology?*, Notices A.M.S. 58 (2011), 36-39.

4. P. Bubenik, J. Scott, *Categorification of Persistent Homology*, Discrete Comput. Geom. (2014), 600-627.

Topic B:

1. S. Abramsky and Adam Brandenburger, The Sheaf-Theoretic Structure Of Non-Locality and Contextuality. In *New Journal of Physics*, 13(2011), 113036, 2011.

2. S. Abramsky and L. Hardy, Logical Bell Inequalities, Phys. Rev. A 85, 062114 (2012).

3. S. Abramsky, S. Mansfield and R. Soares Barbosa, The Cohomology of Non-Locality and Contextuality, in *Proceedings of Quantum Physics and Logic 2011*, Electronic Proceedings in Theoretical Computer Science, vol. 95, pages 1-15, 2012.

21 C3.10 Additive Combinatorics

21.1 General Prerequisites

Students should be familiar with elementary number theory such as modular arithmetic and prime factorisations. The first half of ASO Number Theory (not including the material on quadratic residues) will be quite sufficient. At one point we will state and use Euler's formula for planar graphs. This is covered in B8.5 Graph Theory, but the statement may be understood independently of that course. Early in the course we will use some simple facts about the distribution of prime numbers. This topic is explored in far greater detail in C3.8 Analytic Number Theory, but we will not assume attendance at that course.

21.2 Overview

The aim of this course is to present classic results in additive and combinatorial number theory, showing how tools from a variety of mathematical areas may be used to solve number-theoretical problems.

We will begin by looking at classical theorems about writing natural numbers as the sums of squares and primes. For instance, we will prove Lagrange's theorem that every number is the sum of four squares, and we will show that every large integer is the sum of a bounded number of primes.

Next we will look at more general sets of integers, proving a famous theorem of Roth: every set of integers with positive density contains three distinct elements in arithmetic progression.

We will also look at the structure of finite sets A of integers which are almost closed under addition in the sense that their sumset $A + A := \{a_1 + a_2 : a_1, a_2 \in A\}$ is relatively small. The highlight here is Freiman's theorem, which states that any such set has a precise combinatorial structure known as a generalised progression.

Finally, we will look at instances of the sum-product phenomenon, which says that it is impossible for a finite set of integers to be simultaneously additively- and multiplicatively structured. This section draws from a particularly rich set of other mathematical areas, including graph theory, geometry and analysis. Nonetheless, prerequisites will be minimal and we will develop what we need from scratch.

21.3 Synopsis

The classical bases. Every prime congruent to 1 modulo 4 is a sum of two squares. Every natural number is the sum of four squares. *Discussion of sums of three squares*. Schnirelman density. Application of Selberg's sieve to show that every large number is the sum of at most C primes for some fixed C.

Progressions of length 3. Basic properties of Fourier transforms. Roth's theorem that every subset of $\{1, \dots, N\}$ of size at least δN contains three elements in arithmetic progression, provided N is sufficiently large in terms of δ.

Sumsets and Freiman's theorem. Basic sumset estimates. Additive energy and its relation to sumsets: statement (but not proof) of the Balog-Szemerédi-Gowers theorem. Bohr sets and Bogolyubov's theorem. Minkowski's second theorem (statement only). Freiman's theorem on sets with small doubling constant.

Sum-product theorems. The crossing number inequality for graphs. The Szemerédi-Trotter theorem on point-line incidences, and application to prove that either $|A+A|$ or $|A \cdot A|$ has size at least $c|A|^{5/4}$. The Prékopa-Leindler inequality, quasicubes and sumsets. Proof of Bourgain and Chang's result that either the m-fold sumset $A + A + \cdots + A$ or the m-fold product set $A \cdot A \cdot \cdots \cdot A$ has size at least $|A|^{f(m)}$, where $f(m) \to \infty$.

If time allows the course will conclude with a brief non-examinable discussion of Gowers's work on Szemerédi's theorem for progressions of length 4 and longer, which ties together several earlier strands in the course.

21.4 Reading List

Full printed notes will be produced for the course and these will be the primary resource. M. Nathanson's two books *Additive Number Theory* cover much of the material in the course. T. Tao and V. Vu *Additive Combinatorics* is also useful.

22 C3.11 Riemannian Geometry

22.1 General Prerequisites

Differentiable Manifolds is required. An understanding of covering spaces will be strongly recommended.

22.2 Overview

Riemannian Geometry is the study of curved spaces and provides an important tool with diverse applications from group theory to general relativity. The surprising power of Riemannian Geometry is that we can use local information to derive global results.

This course will study the key notions in Riemannian Geometry: geodesics and curvature. Building on the theory of surfaces in R^3 in the Geometry of Surfaces course, we will describe the notion of Riemannian submanifolds, and study Jacobi fields, which exhibit the interaction between geodesics and curvature.

We will prove the Hopf–Rinow theorem, which shows that various notions of completeness are equivalent on Riemannian manifolds, and classify the spaces with constant curvature.

The highlight of the course will be to see how curvature influences topology. We will see this by proving the Cartan–Hadamard theorem, Bonnet–Myers theorem and Synge's theorem.

22.3 Learning Outcomes

The candidate will have great familiarity working with Riemannian metrics, the Levi-Civita connection, geodesics and curvature, both in a local coordinate description and using coordinate-free expressions. The candidate will gain understanding of Riemannian submanifolds, Jacobi fields, completeness, and be able to prove and apply fundamental results in the subject, including the theorems of Hopf–Rinow, Cartan–Hadamard, Bonnet–Myers and Synge.

22.4 Synopsis

Riemannian manifolds: basic examples of Riemannian metrics, Levi-Civita connection.

Geodesics: definition, first variation formula, exponential map, minimizing properties of geodesics.

Curvature: Riemann curvature tensor, sectional curvature, Ricci curvature, scalar curvature.

Riemannian submanifolds: examples, second fundamental form, Gauss–Codazzi equations.

Jacobi fields: Jacobi equation, conjugate points.

Completeness: Hopf–Rinow and Cartan–Hadamard theorems

Constant curvature: classification of complete manifolds with constant curvature.

Second variation and applications: second variation formula, Bonnet–Myers and Synge's theorems.

22.5 Reading List

1. M.P. do Carmo, Riemannian Geometry, (Birkhauser, 1992).
2. J.M. Lee, Riemannian Manifolds: An Introduction to Curvature, (Springer, 1997).

3. S. Gallot, D. Hulin and J. Lafontaine, Riemannian Geometry, (Springer, 1987).

4. W. Boothby, An Introduction to Differentiable Manifolds and Riemannian Geometry, 2nd edition, (Academic Press, 1986).

23 C3.12 Low-Dimensional Topology and Knot Theory

23.1 General Prerequisites

B3.5 Topology and Groups (MT) and C3.1 Algebraic Topology (MT) are essential. We will assume working knowledge of the fundamental group, covering spaces, homotopy, homology, and cohomology. B3.2 Geometry of Surfaces (MT) and C3.3 Differentiable Manifolds (MT) are useful but not essential, though some prior knowledge of smooth manifolds and bundles should make the material more accessible.

23.2 Overview

Low-dimensional topology is the study of 3- and 4-manifolds and knots. The classification of manifolds in higher dimensions can be reduced to algebraic topology. These methods fail in dimensions 3 and 4. Dimension 3 is geometric in nature, and techniques from group theory have also been very successful. In dimension 4, gauge-theoretic techniques dominate. This course provides an overview of the rich world of low-dimensional topology that draws on many areas of mathematics. We will explain why higher dimensions are in some sense easier to understand, and review some basic results in 3- and 4-manifold topology and knot theory.

23.3 Learning Outcomes

The students will become acquanted with topological and smooth manifolds. They will master important techniques from Morse theory and learn how to manipulate handle decompositions of manifolds. They will get an idea about the role of the h-cobordism theorem and the Whitney trick in higher-dimensional topology. They will learn a variety of techniques in knot theory, including how to manipulate diagrams using Reidemeister moves, how to derive knot invariants from Seifert surfaces, and how some of these are related to 4-dimensional quantities. They will be able to represent 3-manifolds using Heegaard decompositions, how to write them as sums of prime pieces using normal surface theory, and how to construct 3-manifolds via Dehn surgery and branched double covers along links. Finally, they will be able to represent 4-manifolds using Kirby diagrams and how to determine their homeomorphism type using the intersection form.

23.4 Synopsis

The definition of topological and smooth manifolds. Morse theory, handle decompositions, surgery. Every group can be the fundamental group of a manifold in dimension greater than three. The h-cobordism theorem, (without proof) and the Whitney trick. Application: The generalized Poincaré conjecture. Knots and links: Reidemeister moves, Seifert surface and genus, Alexander polynomial, fibred knots, Jones polynomial, prime decomposition, 4-ball genus 3-manifolds: Heegaard decompositions, unique prime decomposition, loop theorem (without proof), lens spaces, Dehn surgery, branched double cover 4-manifolds: Kirby calculus, the intersection form, Freedman's and Donaldson's theorems (without proof)

23.5 Reading List

1. W. B. Raymond Lickorish, *An introduction to knot theory*
2. Dale Rolfsen; American Mathematical Society *Knots and links*
3. Alexandru Scorpan, *The wild world of 4-manifolds*
4. John W. Milnor; Michael Spivak; R. O. Wells, *Morse theory*
5. John W. Milnor; Laurence C. Siebenmann; J. Sondow, *Lectures on the h-cobordism theorem*

24 C4.1 Further Functional Analysis

24.1 General Prerequisites

Students wishing to take this course are expected to have a thorough understanding of the basic theory of normed vector spaces (including properties and standard examples of Banach and Hilbert spaces, dual spaces, and the Hahn-Banach theorem) and of bounded linear operators (ideally including the Open Mapping Theorem, the Inverse Mapping Theorem and the Closed Graph Theorem). Some fluency with topological notions such as (sequential) compactness and bases of topological spaces will also be assumed, as will be basic familiarity with the Lebesgue integral. A number of these prerequisites will be reviewed (briefly) during the course, and there will be a document available on the course webpage summarising most of the relevant background material.

24.2 Overview

This course builds on what is covered in introductory courses on Functional Analysis, by extending the theory of Banach spaces and operators. As well as developing general methods that are useful in operator theory, we shall look in more detail at the structure and special properties of "classical" sequence spaces and function spaces.

24.3 Learning Outcomes

By the end of this course, students will be able to:

1. Establish and use both extension and separation versions of the Hahn Banach Theorem, and geometric properties of the norm, to obtain dualities between embeddings and quotients and establish reflexivity both abstractly and in important examples, such as Lebesgue spaces.

2. work with the weak and weak*-topologies on Banach spaces, establish and use the Banach-Alaoglu theorem, relating this to characterisations of reflexivity, and describe closures in both norm and weaker topologies using annihilators, and preannhilators.

3. Manipulate properties of compact and Fredholm operators on Banach and Hilbert spaces, to establish and use the Fredholm alternative, and obtain spectral theorems for compact operators both in abstract and concrete settings.

24.4 Synopsis

Normed vector spaces and Banach spaces. Dual spaces. Direct sums and complemented subspaces. Quotient spaces and quotient operators.

The Baire Category Theorem and its consequences (review).

Hahn-Banach extension and separation theorems. The bidual space. Reflexivity. Completion of a normed vector space.

Convexity and smoothness of norms. Lebesgue spaces and their duals.

Weak and weak* topologies. The Banach-Alaoglu theorem. Goldstine's theorem. Equivalence of reflexivity and weak compactness of the closed unit ball. The Schur property of ℓ^1 Weakly compact operators.

Compactness in normed vector spaces. Compact operators. Schauder's theorem on compactness of dual operators. Completely continuous operators.

The Closed Range Theorem. Fredholm theory: Fredholm operators; the Fredholm index; perturbation results; the Fredholm Alternative. Spectral theory of compact operators. The Spectral Theorem for compact self-adjoint operators.

24.5 Reading List

1. M. Fabian et al., *Functional Analysis and Infinite-Dimensional Geometry* (Canadian Math. Soc, Springer 2001)

2. N.L. Carothers, *A Short Course on Banach Space Theory* (LMS Student Text, CUP 2004).

24.6 Further Reading

1. J. Conway, *A course in Functional Analysis* (Springer 2007)

2. B. Bollobas, *Linear Analysis: An Introductory Course* (CUP 1999)

25 C8.1 Stochastic Differential Equations

25.1 General Prerequisites

Integration theory: Riemann-Stieljes and Lebesgue integral and their basic properties Probability and measure theory: σ-algebras, Fatou lemma, Borel-Cantelli, Radon-Nikodym,

L^p-spaces, basic properties of random variables and conditional expectation, Martingales in discrete and continuous time: construction and basic properties of Brownian motion, uniform integrability of stochastic processes, stopping times, filtrations, Doob's theorems (maximal and L^p-inequalities, optimal stopping, upcrossing, martingale decomposition), martingale (backward) convergence theorem, L^2-bounded martingales, quadratic variation; Stochastic Integration: Ito's construction of stochastic integral, Ito's formula.

25.2 Overview

Stochastic differential equations (SDEs) model evolution of systems affected by randomness. They offer a beautiful and powerful mathematical language in analogy to what ordinary differential equations (ODEs) do for deterministic systems. From the modelling point of view, the randomness could be an intrinsic feature of the system or just a way to capture small complex perturbations which are not modelled explicitly. As such, SDEs have found many applications in diverse disciplines such as biology, physics, chemistry and the management of risk. Classic well-posedness theory for ODEs does not apply to SDEs. However, when we replace the classical Newton-Leibnitz calculus with the (Ito) stochastic calculus, we are able to build a new and complete theory of existence and uniqueness of solutions to SDEs. Ito formula proves to be a powerful tool to solve SDEs. This leads to many new and often surprising insights about quantities that evolve under randomness. This course is an introduction to SDEs. It covers the basic theory but also offers glimpses into many of the advanced and nuanced topics.

25.3 Learning Outcomes

By the end of this course, students will be able to analyse if a given SDEs admits a solution, characterise the nature of solution and explain if it is unique or not. The students will also be able to solve basic SDEs and state basic properties of the diffusive systems described by these equations.

25.4 Synopsis

Recap on martingale theory in continuous time, quadratic variation, stochastic integration and Ito's calculus.

Levy's characterisation of Brownian motion, stochastic exponential, Girsanov theorem and change of measure, Burkholder-Davis-Gundy, Martingale represenation, Dambis-Dubins-Schwarz.

Strong and weak solutions of stochastic differential equations, existence and uniqueness.

Examples of stochastic differential equations. Bessel processes.

Local times, Tanaka formula, Tanaka-Ito-Meyer formula.

25.5 Reading List

1. J. Obloj, *Continuous martingales and stochastic calculus* online notes. Students are encouraged to study all the material up to and including the Ito formula prior to the course.

2. D. Revuz and M. Yor, *Continuous martingales and Brownian motion* (3rd edition, Springer).

25.6 Further Reading

1. I. Karatzas and S. E. Shreve, *Brownian Motion and Stochastic Calculus*, Graduate Texts in Mathematics 113 (Springer-Verlag, 1988).

2. L. C. G. Rogers & D. Williams, *Diffusions, Markov Processes and Martingales Vol 1 (Foundations) and Vol 2 (Ito Calculus)* (Cambridge University Press, 1987 and 1994).

3. R. Durrett, *Stochastic Calculus* (CRC Press).

4. B. Oksendal, *Stochastic Differential Equations: An introduction with applications* (Universitext, Springer, 6th edition).

5. N. Ikeda & S. Watanabe, *Stochastic Differential Equations and Diffusion Processes* (North–Holland Publishing Company, 1989).

6. H. P. McKean, *Stochastic Integrals* (Academic Press, New York and London, 1969).

26 C8.3 Combinatorics

26.1 General Prerequisites

B8.5 Graph Theory is helpful, but not required.

26.2 Overview

An important branch of discrete mathematics concerns properties of collections of subsets of a finite set. There are many beautiful and fundamental results, and there are still many basic open questions. The aim of the course is to introduce this very active area of mathematics, with many connections to other fields.

26.3 Learning Outcomes

The student will have developed an appreciation of the combinatorics of finite sets.

26.4 Synopsis

Chains and antichains. Sperner's Lemma. LYM inequality. Dilworth's Theorem.

Shadows. Kruskal-Katona Theorem.

Intersecting families. Erdos-Ko-Rado Theorem. Cross-intersecting families.

VC-dimension. Sauer-Shelah Theorem.

t-intersecting families. Fisher's Inequality. Frankl-Wilson Theorem. Application to Borsuk's Conjecture.

Combinatorial Nullstellensatz.

26.5 Reading List

1. Bela Bollobás, *Combinatorics*, CUP, 1986.
2. Stasys Jukna, *Extremal Combinatorics*, Springer, 2007

27 C8.4 Probabilistic Combinatorics

27.1 General Prerequisites

B8.5 Graph Theory and *A8: Probability. C8.3 Combinatorics* is not as essential prerequisite for this course, though it is a natural companion for it.

27.2 Overview

Probabilistic combinatorics is a very active field of mathematics, with connections to other areas such as computer science and statistical physics. Probabilistic methods are essential for the study of random discrete structures and for the analysis of algorithms, but they can also provide a powerful and beautiful approach for answering deterministic questions. The aim of this course is to introduce some fundamental probabilistic tools and present a few applications.

27.3 Learning Outcomes

The student will have developed an appreciation of probabilistic methods in discrete mathematics.

27.4 Synopsis

First-moment method, with applications to Ramsey numbers, and to graphs of high girth and high chromatic number.

Second-moment method, threshold functions for random graphs.

Lovász Local Lemma, with applications to two-colourings of hypergraphs, and to Ramsey numbers.

Chernoff bounds, concentration of measure, Janson's inequality.

Branching processes and the phase transition in random graphs.

Clique and chromatic numbers of random graphs.

27.5 Reading List

1. N. Alon and J.H. Spencer, *The Probabilistic Method* (third edition, Wiley, 2008).

27.6 Further Reading

1. B. Bollobás, *Random Graphs* (second edition, Cambridge University Press, 2001).
2. M. Habib, C. McDiarmid, J. Ramirez-Alfonsin, B. Reed, ed., *Probabilistic Methods for Algorithmic Discrete Mathematics* (Springer, 1998).
3. S. Janson, T. Luczak and A. Rucinski, *Random Graphs* (John Wiley and Sons, 2000).
4. M. Mitzenmacher and E. Upfal, *Probability and Computing: Randomized Algorithms and Probabilistic Analysis* (Cambridge University Press, New York (NY), 2005).
5. M. Molloy and B. Reed, *Graph Colouring and the Probabilistic Method* (Springer, 2002).
6. R. Motwani and P. Raghavan, *Randomized Algorithms* (Cambridge University Press, 1995).

28 C8.6 Limit Theorems and Large Deviations in Probability

28.1 General Prerequisites

Part A Probability and Part A Integration are required. B8.1 (Measure, Probability and Martingales), B8.2 (Continuous Martingales and Stochastic Calculus) and C8.1 (Stochastic Differential Equations) are desirable, but not essential.

28.2 Overview

The convergence theory of probability distributions on path space is an essential part of modern probability and stochastic analysis allowing the development of diffusion approximations and the study of scaling limits in many settings. The theory of large deviation is an important aspect of limit theory in probability as it enables a description of the probabilities of rare events. The emphasis of the course will be on the development of the necessary tools for proving various limit results and the analysis of large deviations which have universal value. These topics are fundamental within probability and stochastic analysis and have extensive applications in current research in the study of random systems, statistical mechanics, functional analysis, PDEs, quantum mechanics, quantitative finance and other applications.

28.3 Learning Outcomes

The students will understand the notions of convergence of probability laws, and the tools for proving associated limit theorems. They will have developed the basic techniques for the establishing large deviation principles and be able to analyze some fundamental examples.

28.4 Synopsis

1) (2 lectures) We will recall metric spaces, and introduce Polish spaces, and probability measures on metric spaces. Weak convergence of probability measures and tightness, Prohorov's theorem on tightness of probability measures, Skorohod's representation theorem for weak convergence.

2) (2 lectures) The criterion of pre-compactness for distributions on continuous path spaces, martingales and compactness.

3) (4 hours) Skorohod's topology and metric on the space $D[0,\infty)$ of right-continuous paths with left limits, basic properties such as completeness and separability, weak convergence and pre-compacness of distributions on $D[0,\infty)$. D. Aldous' pre-compactness criterion via stopping times.

4) (4 lectures) First examples - Cramér's theorem for finite dimensional distributions, Sanov's theorem. Schilder's theorem for the large deviation principle for Brownian motion in small time, law of the iterated logarithm for Brownian motion.

5) (4 lectures) General tools in large deviations. Rate functions, good rate functions, large deviation principles, weak large deviation principles and exponential tightness. Varadhan's contraction principle, functional limit theorems.

28.5 Reading List

1. J.-D. Deuschel and D. W. Stroock, *Large Deviations* (AMS).

2. A. Dembo and O. Zeitouni, *Large Deviations Techniques and Applications*, Stochastic Modelling and Applied Probability, (Springer).

3. S. N. Ethier and T. G. Kurtz: *Markov Processes: Characterization and Convergence*, Wiley Series in Probability and Statistics, (Wiley-Interscience).

4. D. W. Stroock and S.R.S Varadhan, *Multidimensional Diffusion Processes*, (Springer, 1979), Chapter 1.

28.6 Further Reading

1. P. Billingsley, *Convergence of Probability Measures*, Wiley Series in Probability and Statistics, (Wiley-Blackwell; 2nd Edition edition, 23 Aug 1999).

2. R. S. Ellis, *Entropy, Large Deviations, and Statistical Mechanics*, Classics in Mathematics, (Springer)

29 CCD Dissertations on a Mathematical Topic

29.1 Overview

STUDENTS MUST OFFER A DOUBLE-UNIT CCD DISSERTATION ON A MATHEMATICAL TOPIC OR A COD DISSERTATION IN THE HISTORY OF MATHEMATICS.

Students may offer a double-unit dissertation on a Mathematical topic for examination at Part C. A double-unit is equivalent to a 32-hour lecture course. Students will have a supervisor for their dissertation and will meet with them 4-5 times during Michaelmas and Hilary terms, together with the other students offering that dissertation topic. The group size will be between 2 and 5 students. The first meeting will take place in week 7 or 8 of Michaelmas term to provide them with the information needed to start work on their dissertation over the Christmas vacation and to agree the pattern of project supervision in Hilary term.

Students considering offering a dissertation should read the *Guidance Notes on Extended Essays and Dissertations in Mathematics* available at:

`https://www.maths.ox.ac.uk/members/students/undergraduate-courses/teaching-and-learning/projects`.

Application The list of potential dissertation topics will be published on Friday of week 0 of Michaelmas term, following the Dissertation Information Session. For each potential topic there will be a short abstract outlining the topic, details of prerequisite knowledge, suggested references and possible avenues of investigation. There will be a limit on the number of students each supervisor is able to supervise and this information will also be provided.

You will be asked to submit a ranked list of dissertation choices via an online form by 12 noon on Friday of week 3. You will need to submit 6 choices, and will be given the opportunity to explain if there is a particular reason why you would like to do a specific topic. For example, you may like to undertake a dissertation in an area in which you are hoping to go on to further study.

You are not expected to make contact with the dissertation supervisor(s) before submitting your choices but if you have a question about a dissertation topic you should feel free to email the supervisor for further information.

Projects Committee will meet in week 4 to decide upon the allocation of dissertation topics and will seek to ensure that students receive one of their top choices as far as possible. You will be notified of which project you have been allocated at the start of week 5.

Assessment Dissertations are independently double-marked, normally by the dissertation supervisor and one other assessor. The two marks are then reconciled to give the overall mark awarded. The reconciliation of marks is overseen by the examiners and follows the department's reconciliation procedure (see `https://www.maths.ox.ac.uk/members/`

students/undergraduate-courses/teaching-and-learning/projects).

Submission An electronic copy of your dissertation should be submitted via the Mathematical Institute website to arrive no later than **12 noon on Monday of week 1, Trinity term 2024**. Further details may be found in the *Guidance Notes on Extended Essays and Dissertations in Mathematics*.

30 COD Dissertations on the History of Mathematics

30.1 Overview

STUDENTS MUST OFFER A DOUBLE-UNIT CCD DISSERTATION ON A MATHEMATICAL TOPIC OR A COD DISSERTATION IN THE HISTORY OF MATHEMATICS.

Students may offer a double-unit dissertation on a topic related to the History of Mathematics for examination at Part C. A double-unit is equivalent to a 32-hour lecture course. Students will have a supervisor for their dissertation and will meet with them 4-5 times during Michaelmas and Hilary terms, together with the other students offering that dissertation topic. The group size will be between 2 and 5 students. The first meeting will take place in week 7 or 8 of Michaelmas term to provide them with the information needed to start work on their dissertation over the Christmas vacation and to agree the pattern of project supervision in Hilary term. In addition there are lectures on writing mathematics and using LaTeX in Michaelmas and Hilary terms. See the lecture list for details.

Candidates considering offering a dissertation should read the *Guidance Notes on Extended Essays and Dissertations in Mathematics* available at:

https://www.maths.ox.ac.uk/members/students/undergraduate-courses/teaching-and-learning/projects.

Application Students wishing to do a dissertation based on the History of Mathematics should contact Dr Christopher Hollings at christopher.hollings@maths.ox.ac.uk by Wednesday of week 1 with a short draft proposal. Dr Hollings will contact you to arrange a short informal interview to discuss the proposal further. All decisions made by Dr Hollings will be communicated to students by the end of week 2.

All proposals supported by Dr Hollings will then be referred to Projects Committee who meet in week 4 for final approval. With the support of Dr Hollings students must submit a COD Dissertation Proposal Form to Projects Committee by the end of week 3. The form can be found in the Dissertation Guidance - https://www.maths.ox.ac.uk/members/students/undergraduate-courses/teaching-and-learning/part-c-students/teaching-and-learning/dissertations.

Students whose proposal is not supported by Dr Hollings will be given the option to to submit a ranked list of dissertation choices via an online form by 12 noon on Friday of week 3. You will need to submit 6 choices, and will be given the opportunity to explain if there is a particular reason why you would like to do a specific topic. This is optional for Part C students and compulsory for OMMS students.

Projects Committee will meet in week 4 to decide upon the allocation of dissertation topics and will seek to ensure that students receive one of their top choices as far as possible. You will be notified of which project you have been allocated at the start of week 5.

Assessment Dissertations are independently double-marked, normally by the dissertation supervisor and one other assessor. The two marks are then reconciled to give the overall mark awarded. The reconciliation of marks is overseen by the examiners and follows the department's reconciliation procedure (see `https://www.maths.ox.ac.uk/members/students/undergraduate-courses/teaching-and-learning/projects`).

Submission An electronic copy of your dissertation should be submitted via the Mathematical Institute website to arrive no later than **12 noon on Monday of week 1, Trinity term 2024**. Further details may be found in the *Guidance Notes on Extended Essays and Dissertations in Mathematics*.

31 An Introduction to LaTeX

31.1 General Prerequisites

There are no prerequisites. The course is mainly intended for students writing a Part B Extended Essay or a Part C Dissertation but any students are welcome to attend the two lectures given in Michaelmas Term. Note that there is no assessment associated with this course, nor credit for attending the course.

31.2 Overview

This short lecture series provides an introduction to LaTeX.

LaTeX is a markup language, released by Donald Knuth in 1984 and freely sourced, for the professional typesetting of mathematics. (It is based on the earlier TeX released in 1978.) A markup language provides the means for rendering text in various ways - such as bold, italicized or Greek symbols - with the main focus of LaTeX being the rendering of mathematics so that even complicated expressions involving equations, integrals and matrices and images can be professionally typeset.

31.3 Learning Outcomes

Following these introductory lectures, a student should feel comfortable writing their own LaTeX documents, and producing professionally typeset mathematics. The learning curve to producing a valid LaTeX document is shallow, and students will further become familiar will some of the principal features of LaTeX such as chapters, item lists, typesetting mathematics, including equations, tables, bibliographies and images. Then, with the aid of a good reference manual, a student should feel comfortable researching out for themselves further features and expanding their LaTeX vocabulary

31.4 Reading List

The Department has a page of LaTeX resources at https://www.maths.ox.ac.uk/members/it/faqs/latex which has various free introductory guides to LaTeX.