Examiners' Report: Final Honour School of Mathematics Part C Trinity Term 2023

October 26, 2023

Part I

A. STATISTICS

- **Numbers and percentages in each class**

  See Table 1, page .

- **Numbers of vivas and effects of vivas on classes of result.**
  As in previous years there were no vivas conducted for the FHS of Mathematics Part C.

- **Marking of scripts.**
  The dissertations and mini-projects were double marked. The remaining scripts were all single marked according to a pre-agreed marking scheme which was very closely adhered to. For details of the extensive checking process, see Part II, Section A1.

  For information on steps taken in response to the Marking and Assessments Boycott (MAB) please see Part I, Section B.

- **Numbers taking each paper.**
  See Table 7 on page .

Table 1: Numbers in each class (post-2021 classification)

	Number			Percentages %		
	2023	(2022)	(2021)	2023	(2022)	(2021)
Distinction	61	(46)	(60)	64.89	(58.23)	(60)
Merit	21	(19)	(20)	22.34	(24.05)	(20)
Pass	12	(14)	(18)	12.77	(17.72)	(18)
Fail	0	(0)	(2)	0	(0)	(20)
Total	94	(79)	(100)	100	(100)	(100)

Table 2: Numbers in each class (pre-2021 classification)

	Number					Percentages %				
	2020	(2019)	(2018)	(2017)	(2016)	2020	(2019)	(2018)	(2017)	(2016)
I	63	(58)	(53)	(48)	(44)	67.74	(57.43)	(56.99)	(57.14)	(50.57)
II.1	30	(40)	(26)	(23)	(31)	32.26	(39.6)	(27.96)	(27.38)	(35.63)
II.2	0	(2)	(13)	(12)	(9)	0	(1.98)	(13.98)	(14.29)	(10.34)
III	0	(1)	(1)	(1)	(3)	0	(0.99)	(1.08)	(1.19)	(3.45)
F	0	(0)	(0)	(0)	(0)	0	(0)	(0)	(0)	(0)
Total	93	(101)	(93)	(84)	(87)	100	(100)	(100)	(100)	(100)

B. Strike action

As a result of the marking and assessment boycott (MAB) alternative marking arrangements were organized for three dissertation topics and the following three exams. - C2.3 Representation Theory of Semisimple Lie Algebras - C3.1 Algebraic Topology - C4.1 Further Functional Analysis. Substitute assessors were recruited for each exam papers, whilst this delayed marks-sharing all scripts were received and script checked in time for the final board. The papers were reviewed and scaled in full by the examiners during the meeting. All replacement assessors were experienced markers with a suitable level of expertise in the subject matter.

Dispensation was granted from EPS and the Proctors for the three affected dissertation topics to be single marked, a process which breaks from typical convention with regards to project marking but is acceptable under the university guidance on mitigating steps against the MAB. All dissertations were moderated by Prof Gaffney and Prof Lotay and the exam board were welcomed to review the agreed marks.

C. Changes in examining methods and procedures currently under discussion or contemplated for the future

None.

D. Notice of examination conventions for candidates

The first notice to candidates was issued on 22nd February 2023 and the second notice on 24th April 2023. These contain details of the examinations and assessments.

All notices and the examination conventions for 2023 examinations are on-line at http://www.maths.ox.ac.uk/members/students/undergraduate-courses/examinations-assessments.

Part II

A1. General Comments on the Examination

The examiners would like to convey their grateful thanks for their help and cooperation to all those who assisted with this year's examination, either as assessors or in an administrative capacity. The chair would like to thank Anwen Amos, Clare Sheppard, Charlotte Turner-Smith, Waldemar Schlackow, Matt Brechin and the rest of the academic administration team for their support of the Part C and OMMS examinations.

In addition the internal examiners would like to express their gratitude to Prof Alan Champneys and Prof James Robinson for carrying out their duties as external examiners in a constructive and supportive way during the year, and for their valuable input at the final examiners' meetings.

Timetable

The examinations began on Monday 29th May and finished on Friday 9th June.

Mitigating Circumstances Notice to Examiners and other special circumstances

A subset of the examiners (the 'Mitigating Circumstances Panel') attended a pre-board meeting to band the seriousness of the individual notices to examiners. The outcome of this meeting was relayed to the Examiners at the final exam board, who gave careful regard to each case, scrutinised the relevant candidates' marks and agreed actions as appropriate. See Section E for further details.

Setting and checking of papers and marks processing

Following established practice, the questions for each paper were initially set by the course lecturer, with the lecturer of a related course involved as checker before the first draft of the questions was presented to the examiners. The course lecturers also acted as assessors, marking the questions on their course(s).

The internal examiners met in early January to consider the questions on Michaelmas Term courses, and changes and corrections were agreed with the lecturers where necessary. The revised questions were then sent to the external examiners. Feedback from external examiners was given to examiners, and to the relevant assessor for each paper for a response. The internal examiners met a second time late in Hilary Term to consider the external examiners' comments and assessor responses. The cycle was repeated for the Hilary Term courses, with the examiners' meetings in the Easter Vacation. Before questions were submitted to the Examination Schools, setters were required to sign off a camera-ready copy of their questions.

Exams were held in-person in the Exams Schools. Papers were collected by the Academic Administration team and made available to assessors approximately half a day following the examination. Assessors were made aware of the marking deadlines ahead of time and all scripts and completed mark sheets were returned, if not by the agreed due dates, then at least in time for the script-checking process.

A team of graduate checkers, under the supervision of Anwen Amos, Clare Sheppard and Charlotte Turner-Smith, reviewed the mark sheets for each paper of this examination, carefully cross checking against the mark scheme to spot any unmarked questions or parts of questions, addition errors or wrongly recorded marks. Sub-totals for each part were also checked against the mark scheme, noting correct addition. In this way a number of errors were corrected, with each change approved by one of the examiners who were present throughout the process.

Determination of University Standardised Marks

The Mathematics Teaching Committee issued each examination board with broad guidelines on the proportion of candidates that might be expected in each class. This was based on the average in each class over the last four years, together with recent historic data for Part C, the MPLS Divisional averages, and the distribution of classifications achieved by the same group of students at Part B.

The examiners followed established practice in determining the University standardised marks (USMs) reported to candidates. This leads to classifications awarded at Part C broadly reflecting the overall distribution of classifications which had been achieved the previous year by the same students.

We outline the principles of the calibration method.

The Department's algorithm to assign USMs in Part C was used in the same way as last year for each unit assessed by means of a traditional written examination. Papers for which USMs are directly assigned by the markers or provided by another board of examiners are excluded from consideration. Calibration uses data on the Part B classification of candidates in Mathematics and Mathematics & Statistics (Mathematics & Computer Science and Mathematics & Philosophy students are excluded at this stage).

This year a handful of students were given permission by the Education Committee to take summary sheets in to certain examinations. This dispensation was only granted to students who were sitting exams for which they received teaching in the previous academic year where A4 summary sheets containing notes were permitted for the whole cohort as a mitigating step against the impact of the covid-19 pandemic. In response to student concerns, as this could be seen as an advantage against other students, the exam board did not include these students in scaling. The agreed scaling was then applied to all students, including those permitted to make use of summary sheets.

The algorithm converts raw marks to USMs for each paper separately (in each case, the raw marks are initially out of 50, but are scaled to marks out of 100). For each paper, the algorithm sets up a map $R \to U$ (R = raw, U = USM) which is piecewise linear and automatically contains the points $(0,0)$, $(100,100)$. While the previous scaling process used in 2021 assigned two vertices for the preliminary scaling map using Part B marks, the algorithm used since 2022 only uses the number of firsts achieved at Part B to assign one vertex. This vertex, P_3, is placed at 72 USM with an associated raw mark that ensures that the number of 1st class Part C on the paper after scaling is the same as the number of Part C candidates taking the paper with a 1st class in Part B. The vertex P_3 is then joined to

(0,40) by a line segment, with a further vertex, P_2, placed at 57 USM on this line segment. The vertex P_2 is then joined by a new line segment to (0,10), and an additional vertex, P_1, is placed at 37 USM on the new line segment. The points $(0,0)$, P_1, P_2, P_3, $(100,100)$ provide the piecewise linear map for each paper's preliminary scaling map.

The results of the algorithm with the above default settings of the parameters provide the starting point for the determination of USMs.

This year a preliminary meeting of the internal examiners was held two days ahead of the examiners' meeting to assess the results produced by the algorithm alongside the reports from assessors. It was agreed that only a selection of scaling maps would be further reviewed at the final exam board, and that externals examiners would be given an opportunity to review all maps prior to the meeting. The examiners reviewed each paper and assessors' reports, and discussed the preliminary scaling maps and the preliminary class percentage figures. The examiners have scope to make changes, usually by adjusting the position of the vertices P_1, P_2, P_3 by hand, so as to alter the map raw $\rightarrow$ USM, to remedy any perceived unfairness introduced by the algorithm, in particular in cases where the number of candidates is small. They also have the option to introduce additional vertices. Adjustments were made to the default settings as appropriate, paying particular attention to borderlines and to raw marks which were either very high or very low.

Table 3: Vertex positions for the piecewise linear scaling maps, $P_1,\ P_2, P_3$ with the raw marks rescaled to be out of 50. The entries N_3, N_2, N_1 respectively give the number of incoming firsts, II.1s, and II.2s and below from Part B for that paper.

Paper	P_1	P_2	P_3	Additional corners	N_3	N_2	N_1
C1.1	12.37;37	21.53;57	40.53;72	50;100	0	8	4
C1.2	16.28;37	28.33;57	47;72	50;100	0	8	5
C1.3	13.02;37	22.67;57	42.67;72	50;100	1	10	3
C1.4	10.42;37	18.13;57	34.13;72	50;100	0	9	4
C2.1	11.07;37	19.27;57	36.27;72	50;100	0	4	6
C2.2	10.74;37	18.7;57	35.2;72	50;100	0	2	15
C2.3	9.44;37	16.43;57	30.93;72	50;100	0	3	4
C2.4	13.02;37	22.67;57	35;70	50;100	0	6	3
C2.5	10.42;37	18.13;57	35;70	50;100	0	1	5
C2.6	5.21;37	16;55	37;70	50;100	0	0	6
C2.7	10.09;37	17.57;57	33.07;72	50;100	0	5	15
C3.1	14;50	18.7;57	35.2;72	50;100	1	3	12
C3.10	8.79;37	15.3;57	28.8;72	50;100	1	7	6
C3.11	11.39;37	22;56	37.33;72	50;100	0	1	4
C3.12	13;50	20.97;57	39.47;72	50;100	1	2	3
C3.2	11.07;37	19.27;57	36.27;72	50;100	1	4	3
C3.3	11.72;37	20.4;53	38.4;70	50;100	0	2	10
C3.4	11.07;37	19.27;57	36.27;72	50;100	0	1	9
C3.5	12.70;37	22.1;57	41.6;72	50;100	0	1	3
C3.6	12.37;37	21.53;57	36;70	50;100	0	1	4
C3.7	9.77;37	17;57	35;72	50;100	0	2	10
C3.8	10.09;37	17.57;57	33.07;72	50;100	0	8	9
C4.1	7.81;37	13.6;57	25.6;72	50;100	0	3	12
C4.3	8.14;37	14.17;57	26.67;72	50;100	1	0	10
C4.4	50;100				0	0	0
C4.6	13.02;37	22.67;57	38;70	50;100	1	0	3

Paper	P_1	P_2	P_3	Additional corners	N_1	N_2	N_3
C4.8	12.70;37	22.1;57	35;70	50;100	0	0	3
C4.9	14.32;37	24.93;57	38;70	50;100	0	1	4
C5.11	12.70;37	22.1;57	41.6;73	50;100	1	7	7
C5.12	14.65;37	30;50	43;70; 50;100	49;90	2	7	5
C5.2	12.37;37	21.53;57	40.53;72	50;100	0	4	4
C5.5	13.02;37	22.67;57	42.67;72	50;100	1	18	11
C5.6	13.35;37	23.23;57	43.73;72	50;100	0	11	6
C5.7	8.79;37	15.3;57	30;70	50;100	0	3	4
C5.9	14;50	23.8;57	38;70	50;100	0	1	2
C6.1	16;45	26.07;57	40;70; 50;100	49;85	2	13	4
C6.2	12.04;37	20.97;57	39.47;72	50;100	1	9	5
C6.3	15.3;37	26.63;57	38;68; 50;100	47;85	0	4	1
C6.4	11.72;37	20.4;57	38.4;72; 50;100	49;85	0	1	3
C7.4	13.02;37	22.67;57	42.67;72	50;100	1	1	3
C7.5	12.37;37	21.53;57	40.53;72	50;100	0	1	2
C7.6	30;55	50;100			0	0	0
C7.7	12.70;37	22.1;57	41.6;72	50;100	1	9	2
C8.1	7.49;37	13;52	29;70	50;100	2	5	17
C8.2	25;50	34.13;72	50;100		1	4	11
C8.3	11.39;37	20;50	37.33;72	50;100	5	18	11
C8.4	11.72;37	20.4;57	38.4;72	50;100	3	16	10
SC1	13.99;37	24.37;53	43;70	50;100	5	18	13
SC10	12.37;37	21.53;54	40.53;72	50;100	0	7	5
SC2	12.70;37	22.1;57	41.6;72	50;100	1	20	11
SC4	11.07;37	19.27;57	36.27;72	50;100	3	13	22
SC5	12.37;37	21.53;57	40.53;72	50;100	0	14	13
SC6	50;100	-	-	-	0	0	0
SC7	13.35;37	23.23;57	39;70	50;100	2	11	9
SC9	12;50	17.57;57	33.07;72	50;100	1	6	8

B. Equality and Diversity issues and breakdown of the results by gender

Table 5: Breakdown of results by gender

Class	Number								
	2023			2022			2021		
	Female	Male	Total	Female	Male	Total	Female	Male	Total
Distinction	7	54	61	10	36	46	15	45	60
Merit	8	13	21	9	10	19	8	12	20
Pass	3	9	12	5	9	14	5	13	18
Fail	0	0	0	0	0	0	1	1	2
Total	18	76	94	24	55	79	29	71	100

Class	Percentage								
	2023			2022			2021		
	Female	Male	Total	Female	Male	Total	Female	Male	Total
Distinction	38.89	71.05	64.89	40	65.45	58.23	51.72	63.38	60
Merit	44.44	17.11	22.34	36	18.18	24.05	27.59	16.9	20
Pass	16.67	11.84	12.77	20	16.36	23.73	17.24	18.31	18
Fail	0	0	0	0	0	0	3.45	1.41	2
Total	100	100	100	100	100	100	100	100	100

Table 6: Breakdown of results by gender (pre-2021 classification)

Class	Number								
	2020			2019			2018		
	Female	Male	Total	Female	Male	Total	Female	Male	Total
I	16	47	63	8	50	58	6	47	53
II.1	4	26	30	9	31	40	7	19	26
II.2	0	0	0	0	2	2	3	10	13
III	0	0	0	0	1	1	1	0	1
F	0	0	0	0	0	0	0	0	0
Total	20	73	93	17	84	101	17	76	93

Class	Percentage								
	2020			2019			2018		
	Female	Male	Total	Female	Male	Total	Female	Male	Total
I	80	64.38	72.19	47.06	59.52	57.43	35.29	61.84	56.99
II.1	20	35.62	27.81	52.94	36.9	39.6	41.18	25	27.96
II.2	0	0	0	0	2.38	1.98	17.65	13.16	13.98
III	0	0	0	0	1.19	0.99	5.88	0	1.08
F	0	0	0	0	0	0	0	0	0
Total	100	100	100	100	100	100	100	100	100

C. Detailed numbers on candidates' performance in each part of the exam

Data for papers with fewer than six candidates are not included in the public versions of the examiners' report.

Table 7: Numbers taking each paper

Paper	Number of Candidates	Avg RAW	StDev RAW	Avg USM	StDev USM
C1.1	12	34.5	6.43	68.33	8.25
C1.2	13	39.69	11.97	74.54	23.3
C1.3	14	31.07	9.86	61.14	14.41
C1.4	13	26.85	7.71	65.69	9.52
C2.1	10	35.7	6.53	74.4	9.8
C2.2	17	39.59	6.24	81.06	10.87
C2.3	7	30.29	6.05	72.43	7.44
C2.4	9	34.67	7.75	72.67	11.63
C2.5	6	39.5	7.31	80.83	11.84
C2.6	6	39	12.33	80.17	16.8
C2.7	20	34.85	6.67	76.35	8.62
C3.1	16	36.38	11.48	76.5	18.24
C3.2	8	31.38	8.91	69.62	12.39
C3.3	12	41.25	4.71	79	9.21
C3.4	10	41.1	7.69	83.6	13.46
C3.5	4	40.75	3.3	73.75	5.68
C3.6	5	38.8	7.69	78.4	11.44
C3.7	12	35.83	7.6	76.42	9.82
C3.8	17	31.47	10.2	71.18	16.45
C3.10	13	24.85	8.87	67	12.54
C3.11	5	36.2	5.81	72.6	8.05
C3.12	6	27.17	16.39	62.17	22.53
C4.1	15	32.2	10.06	79.47	11.92
C4.3	11	36.27	8.83	83.64	10.57
C4.6	4	37.75	8.66	74.75	11.27
C4.8	3	40	1	80	2
C4.9	5	41.6	9.61	83	15.8
C5.2	9	37.33	4.36	70.56	6.13
C5.5	23	33.43	10.59	67	15.56
C5.6	17	38.53	7.94	72.06	11.25
C5.7	7	29.43	7.18	71.14	9.41
C5.9	4	41	3.92	78	9.2
C5.11	12	39.33	5.23	74.25	8.27
C5.12	12	43.33	3.92	73.67	9.32
C6.1	11	29.82	6.35	60.09	6.44
C6.2	11	30.36	7.7	63.82	8.08
C6.3	4	36.5	4.43	67	5.03
C6.4	3	40	7.81	74.67	8.96

Paper	Number of Candidates	Avg RAW	StDev RAW	Avg USM	StDev USM
C7.4	4	40.25	5.19	72.25	5.85
C7.5	3	37.67	7.51	73	10.82
C7.7	12	32.33	7.38	65.75	8.23
C8.1	17	29.41	8.3	71.59	10.41
C8.2	11	33.55	4.5	69.45	9.63
C8.3	28	32.54	5.12	66.36	7.69
C8.4	25	31.04	6.28	66.32	6.96
SC1	18	38.83	7.49	70.94	14.63
SC2	11	34.09	5.65	66.55	5.13
SC4	10	36	7.1	75	9.96
SC5	1	39		71	
SC7	2	46	2.83	89.5	7.78
SC9	10	29.3	9.2	69.6	11.34
SC10	2	34.5	2.12	66.5	2.12
C3.9	2	-	-	76.5	3.54
C5.1	1	-	-	90	-
C5.4	19	-	-	72.05	5.71
C6.5	7	-	-	69	3.65
CCD	89	-	-	75.05	7.47
COD	5	-	-	69.2	6.83
SC8	8	-	-	79.385	4.719

The tables that follow give the question statistics for each paper for Mathematics candidates. Data for papers with fewer than six candidates are not included in the public version of the report.

Paper C1.1: Model Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.67	16.67	3.93	6	0
Q2	14.5	14.5	3.89	6	0
Q3	17.92	17.82	4.74	12	0

Paper C1.2: Gödel's Incompleteness Theorems

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.38	18.38	7.25	8	0
Q2	16.73	19	7.79	9	2
Q3	17.83	22	8.9	9	3

Paper C1.3: Analytic Topology

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	12	14	6.22	8	2
Q2	18	18	6.56	9	0
Q3	14.67	14.64	4.84	11	0

Paper C1.4: Axiomatic Set Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14.46	14.46	3.82	13	0
Q2	10	12.25	5.29	4	2
Q3	11.6	12.44	5.1	9	1

Paper C2.1: Lie Algebras

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	19.13	19.13	5.99	8	0
Q2	17.38	17.38	4.84	8	0
Q3	16.25	16.25	2.06	4	0

Paper C2.2: Homological Algebra

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	20.53	20.53	2.48	17	0
Q2	21	21		1	0
Q3	18.94	18.94	4.71	16	0

Paper C2.3: Representation Theory of Semisimple Lie Algebras

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.57	16.57	3.69	7	0
Q2	9	9	7.07	2	0
Q3	13.67	15.6	5.82	5	1

Paper C2.4: Infinite Groups

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	20.2	20.2	4.09	5	0
Q2	20.5	20.5	4.04	4	0
Q3	14.33	14.33	5.1	9	0

Paper C2.5: Non-Commutative Rings

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.33	18.33	4.59	6	0
Q2	20.8	20.8	3.63	5	0
Q3	23	23		1	0

Paper C2.6: Introduction to Schemes

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.8	17.8	7.05	5	0
Q2	18.5	23.33	9.95	3	1
Q3	18.75	18.75	6.75	4	0

Paper C2.7: Category Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.16	16.56	3.42	18	1
Q2	18.7	18.7	3.59	10	0
Q3	17.67	17.67	4.89	12	0

Paper C3.1: Algebraic Topology

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.08	18.5	8.02	10	2
Q2	18.46	19.92	7.89	12	1
Q3	15.8	15.8	7	10	0

Paper C3.2: Geometric Group Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	13	13	0	2	0
Q2	16.14	16.14	4.53	7	0
Q3	16	16	5.97	7	0

Paper C3.3: Differentiable Manifolds

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	19.4	19.4	2.61	5	0
Q2	19.33	19.33	2	9	0
Q3	22.4	22.4	3.13	10	0

Paper C3.4: Algebraic Geometry

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	21.43	21.43	6.02	7	0
Q2	21.11	21.11	3.14	9	0
Q3	15.6	17.75	7.86	4	1

Paper C3.5: Lie Groups

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	21	21	1	3	0
Q2	21	21	2.65	3	0
Q3	18.5	18.5	3.54	2	0

Paper C3.6: Modular Forms

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.2	18.2	4.97	5	0
Q2	16	16		1	0
Q3	21.75	21.75	2.22	4	0

Paper C3.7: Elliptic Curves

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15	15	6.28	9	0
Q2	14.83	17.4	7.57	5	1
Q3	20.8	20.8	2.9	10	0

Paper C3.8: Analytic Number Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	9.33	11.33	5.05	6	3
Q2	15.67	16.82	6.85	11	1
Q3	16.59	16.59	5.04	17	0

Paper C3.10: Additive and Combinatorial Number Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	9.45	9.7	3.98	10	1
Q2	14.44	14.44	4.16	9	0
Q3	9.9	13.71	8.92	7	3

Paper C3.11: Riemannian Geometry

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.67	17.67	4.51	3	0
Q2	17.6	17.6	3.36	5	0
Q3	20	20	0	2	0

Paper C3.12: Low-Dimensional Topology

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	5.5	5.5	0.71	2	0
Q2	17.5	17.5	8.19	4	0
Q3	13.67	13.67	8.09	6	0

Paper C4.1: Further Functional Analysis

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.21	18.21	3.87	14	0
Q2	13.5	15	7.53	9	1
Q3	13.29	13.29	7.36	7	0

Paper C4.3: Functional Analytical Methods for PDEs

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	23.5	23.5	2.12	2	0
Q2	17.45	17.45	4.03	11	0
Q3	17.78	17.78	5.07	9	0

Paper C4.6: Fixed Point Methods for Nonlinear PDEs

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18	18	3.92	4	0
Q2	17.5	17.5	7.78	2	0
Q3	22	22	2.83	2	0

Paper C4.8: Complex Analysis: Conformal Maps and Geometry

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18	18	3.61	3	0
Q2	24.5	24.5	0.71	2	0
Q3	17	17		1	0

Paper C4.9: Optimal Transport and Partial Differential Equations

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	20.4	20.4	3.58	5	0
Q3	21.2	21.2	6.3	5	0

Paper C5.2: Elasticity and Plasticity

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.5	15.5	3.54	2	0
Q2	17.71	17.71	3.25	7	0
Q3	20.11	20.11	2.93	9	0

Paper C5.5: Perturbation Methods

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.36	16.36	5.59	11	0
Q2	17.94	16.62	6.12	13	3
Q3	16.95	16.95	6.01	22	0

Paper C5.6: Applied Complex Variables

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.23	16.33	6.39	12	1
Q2	18.86	18.86	2.91	7	0
Q3	21.8	21.8	3.73	15	0

Paper C5.7: Topics in Fluid Mechanics

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15	15	4.32	7	0
Q2	14.43	14.43	3.31	7	0

Paper C5.9: Mathematical Mechanical Biology

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16	16		1	0
Q2	20.33	20.33	2.08	3	0
Q3	21.75	21.75	2.22	4	0

Paper C5.11: Mathematical Geoscience

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	19	19	2.71	7	0
Q2	21.09	21.09	3.53	11	0
Q3	14	17.83	6.54	6	3

Paper C5.12: Mathematical Physiology

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.57	20.5	6.24	6	1
Q2	20	20	3.71	9	0
Q3	24.11	24.11	1.05	9	0

Paper C6.1: Numerical Linear Algebra

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.22	15.22	3.42	9	0
Q2	12.75	12.75	4.03	4	0
Q3	15.56	15.56	4.98	9	0

Paper C6.2: Continuous Optimization

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	13	13.86	4.6	7	1
Q2	13.9	15	5.69	9	1
Q3	14.57	17	6.8	6	1

Paper C6.3: Approximation of Functions

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.75	16.75	3.59	4	0
Q2	19.75	19.75	1.71	4	0
Q3	11			0	1

Paper C6.4: Finite Element Methods for Partial Differential Equations

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q2	17.67	17.67	6.03	3	0
Q3	22.33	22.33	3.06	3	0

Paper C7.4: Introduction to Quantum Information

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	19.5	19.5	4.2	4	0
Q2	19	19		1	0
Q3	21.33	21.33	1.53	3	0

Paper C7.5: General Relativity I

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.67	18.67	2.52	3	0
Q3	19	19	7	3	0

Paper C7.7: Random Matrix Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	19.75	20.91	4.77	11	1
Q2	8.42	12.38	8.66	8	4
Q3	4.92	11.8	6.35	5	7

Paper C8.1: Stochastic Differential Equations

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	13.75	13.75	5.4	12	0
Q2	13.65	14.06	4.65	16	1
Q3	18.33	18.33	3.93	6	0

Paper C8.2: Stochastic Analysis and PDEs

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	13.6	14	4.12	9	1
Q2	19.18	19.18	2.75	11	0
Q3	12.67	16	5.86	2	1

Paper C8.3: Combinatorics

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14.92	14.92	3.12	25	0
Q2	16.81	16.81	2.99	27	0
Q3	17.8	21	7.46	4	1

Paper C8.4: Probabilistic Combinatorics

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.45	17.45	3.28	20	0
Q2	12	13.91	5.12	11	2
Q3	14.42	14.42	4.36	19	0

Paper SC1: Stochastic Models in Mathematical Genetics

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.94	19.88	5.02	16	1
Q2	15.83	18.8	8.38	5	1
Q3	19.13	19.13	5.08	15	0

Paper SC2: Probability and Statistics for Network Analysis

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17	17	3.13	11	0
Q2	17.8	17.8	2.78	10	0
Q3	10	10		1	0

Paper SC4: Statistical Data Mining and Machine Learning

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.4	18.4	5.21	10	0
Q2	16.5	16.5	2.52	4	0
Q3	18.33	18.33	1.86	6	0

Paper SC5: Advanced Simulation Methods

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	24	24		1	0
Q3	15	15		1	0

Paper SC7: Bayes Methods

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	23.5	23.5	2.12	2	0
Q2	22.5	22.5	0.71	2	0

Paper SC9: Interacting Particle Systems

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.5	16.5	4.84	10	0
Q2	8.5	8.5	4.95	2	0
Q3	12.89	13.88	6.85	8	1

Paper SC10: Algorithmic Foundations of Learning

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.5	17.5	0.71	2	0
Q2	16	16		1	0
Q3	18	18		1	0

D. Recommendations for Next Year's Examiners and Teaching Committee

For Dissertations and projects where reconciliation is required between two assessors, it would be helpful for the form to clarify a requirement for a brief explanation of the reason for the reconciliation, and a system for chasing up any assessors which do not include such a comment about reconciliation.

E. Comments on papers and on individual questions

The following comments were submitted by the assessors.

C1.1: Model Theory

Question 1 The main difficulty in this question was the axiomatization of T in part (b)(i). Only two candidates gave a correct answer. Some attempted axiomatizations only worked for finite models of T, others only for infinite models. The performance on this question was the weakest among the three questions, possibly because it had less bookwork material (only part (a) was bookwork, worth 5/25 marks).

Question 2 Many candidates struggled with the proof of the non-trivial direction in the criterion for quantifier elimination in part (b)(ii), even though this is pure bookwork. Only one candidate gave a rigorous proof in part (c)(i) for the the fact that $<$ is not L_s-definable. Likewise, answers to part (c)(iv) tended to be rather philosophical.

Question 3 This was the most popular question. It received many good solutions. Even the most challenging part at the end of (c)(ii) got a few correct answers, though half of the candidates fell into the trap by not realizing that in passing from T to $T*$ the non-principal type $p(x)$ could become principal.

C1.2: Gödel's Incompleteness Theorems

Question 1.: There were many good answers to this question.

In part (b), the correct answer is that any such structure is a model of R. However some of the axioms of R are not explicitly stated to be true in these structures, and are not quite straightforward to prove. I was fairly generous to people who felt the need to add these axioms.
Note that a complete description of the structure N_A is not given, is not an error in the question.

In part (c), some people confused the syntactic complexity of a formula in a set such as PA, with the complexity of a formula required to de ne the set PA as a whole. Thus PA is Δ_1 definable, but PA contains elements of arbitrarily high levels of complexity. Some people thought that all instances of the induction scheme were Σ_1 1, neglecting the fact that an instance of induction has a formula $\phi \in \mathcal{L}$ as a parameter, which may be arbitrarily complex.

Many people spotted the relevance of R being Σ_1-complete. A substantial number, however, did not point out that if PA were Σ_1-axiomatisable, then it would be true in any model of R, while PA is clearly not true in all of the structures in part (b).

There were many good answers to part (d).

Question 2.: Again there were quite a few good answers, but quite a few candidates encountered significant difficulties.
In part (b), some people thought the system was Θ complete (which it can't be, by Tarski's Theorem), and thought in particular that Φ_n was Σ_n-complete. In fact, by applying the First Incompleteness Theorem to Θ, one finds that Θ is not Σ_2-complete (and so not complete). Part (iv) may be done by applying an appropriate instance of the Second Incompleteness Theorem. Since Θ is true in N, Θ is sound and therefore Σ_n-sound.

Many people spotted that the given set of statements is consistent and not 1-consistent, and at least approximately why. Some candidates proved that for each Θ,$PA \cup \{Pr_P A\Theta\}$ is consistent, which isn't strong enough. There were quite a few ingenious uses of compactness-type ideas.

Question 3. was found to be more challenging than the other two questions. There were, all the same, some very good solutions given to it.
A common error in part (c) was simply not following the algorithm for finding fixed points precisely, and ending in finding an incorrect fixed point.
Many people saw that (d) was an induction on the length of a formal proof in GL. There were quite a few complete answers given. Some, however, omitted to deal with the case where a step in a formal proof used a rule of inference, or made other errors.

C1.3: Analytic Topology

Question 1.: There were many good answers to this question.

A common problem in this question, as in others, was incomplete recall of bookwork.

In parts (a) and (b), some people confused components with quasi-components. Others assumed, implicitly or explicitly, that quasi-components were clopen.

In part (c), some missed the use of functional separation and of the Stone-Čech property in part (i). The flow of logic in part (c) was pretty direct; some failed to spot this. Quite a few people didn't spot that a space X that has components and quasi-components different, provides a counterexample in part (iv).

Question 2.: There were quite a few good answers to this question.

In part (b)(i), a number of candidates got the idea of showing that the simple functions were dense, by showing that there was a simple function in every basic open set, but some detail or other went wrong, and the function they chose ended up not being simple because it took irrational values, or had a discontinuity at some irrational point.

As for (b)(ii), there were some good answers to this part. Some seemed to have a sense of roughly what went wrong with any proposed countable basis but couldn't pin it down firmly enough.

Given the results of the previous parts, many found it straightforward to deduce in (b)(iii) that the given product couldn't be metrisable.

Question 3.: There was quite a lot of chasing of commutative diagrams in this question, which some people found easier than others.

Part (a)(iii) proved difficult. Relatively few spotted that if all the maps π_λ are continuous, then there is a continuous bijection from the product with topology $\mathcal{T}$, to the Tychonoff product, which must therefore be a homeomorphism if $\mathcal{T}$ is compact.

The last part of (b)(ii) was intended to be a reasonably direct application of (b)(i). It was found to be difficult, on the whole.

Part (c)(iii) was found to be very difficult. Few people pinned down the fact that the given function f cannot be extended continuously to the portion $(\beta X \setminus h'[X]) \times (\beta X \setminus h'[X])$ of $\gamma(\mathbb{N} \times \mathbb{N})$.

C1.4: Axiomatic Set Theory

Overall candidates did worse than expected with numerous omissions of important checks costing marks in otherwise good answers. In particular often absoluteness (or transitivity of a class) was not mentioned when required, despite the note at the top of the exam.

Question 1 Part (a) was done worse than expected. Often students did not mention the required absoluteness, did not show the properties of V they used or claimed that 'being an ordinal' is absolute for transitive classes which may fail in the absence of **Foundation**. Only few candidates realised that the fact that $\forall x \in V\ \mathcal{P}^V(x) = \mathcal{P}(x)$ gives upwards absoluteness of being an ordinal.

In part (b) again a lot of candidates failed to mention explicitly that W is transitive and hence 'being the unordered pair of x, y', 'being the union of x' and 's having a minimal element' were absolute between W and U and hence that it is enough to check whether the U-unordered pair (resp. U-union) belongs to W.

Checking whether this was the case or not often went wrong and so few candidates constructed counterexamples. A lot of candidates were incorrectly convinced that because U does not satisfy **Foundation**, W also cannot satisfy **Foundation**.

Question 2 Part (a) was well-done although not all candidates who attempted this question mentioned why the uniqueness bit of a function satisfying the conditions (up to some α) was important.

In part (b) almost no candidate realized that without **Powerset** they should not be using the von Neumann hierarchy V_α to define the rank, but have to do so by what is an easy case of general recursion.

Only one candidate made any progress on part (c), trying to define an R-rank as in (b).

Question 3 In part (a) a lot of candidates did not mention let alone explain why the unbounded function from $cf(\alpha)$ into α could be taken to be increasing or if they did why their definition of the increasing function still maps into α (where special cases might need to be considered).

Part (b) was generally well done, although only few candidates showed that the unbounded function $f : cf(\kappa) \to \kappa$ belongs to H_κ and hence can be taken as a parameter in the formula witnessing the failure of **Replacement**. Again, the fact that 'being a function' is absolute for H_κ, U (or V), and using this to claim that the antecedent of **Replacement** holds, was rarely done.

In part (c) surprisingly few candidates gave a correct recursive construction of arbitrarily large elements of i_β or d although very similar results had been proven numerous times in the course and problem sheets. Generally candidates found it difficult to check whether some β belongs to d although blindly following the definition of d would have worked easily. In the very last part most candidates realized that you should consider $d = \bigcap c_\alpha$ but did not see that $\beta \in d$ implies that $f(\beta) \notin \beta$.

C2.1: Lie Algebras

Overall candidates did well on this paper, showing real insight into the subject.

1. Question 1: The was the most popular question and also the best-answered. Some candidates failed to give enough detail in describing how a central extension corresponded to a cocyle, but most did well on parts (a) and (b). In part (c) a number of candidates failed to consider all possibilities when checking that an alternating bilinear map $\alpha\colon \mathfrak{s} \times \mathfrak{s} \to \mathsf{k}$ automatically satisfies $\alpha(x, [y.z]) + \alpha(y, [z, x]) + \alpha(z, [x, y]) = 0$ (for any $x, y, z \in \mathfrak{s}$): most noticed that it sufficed to check this on a basis $\{a, b\}$ where $[a, b] = b$, but then only considered the triple (a, a, b) say, without commenting on the other possibilities. Many candidates gave a complete answer to part (d), often after realising how it related to part (b).
2. Question 2: This question was also well-answered. A number of different arguments were given for part (a): for example, some candidates noted that if $D\mathfrak{s} = \mathsf{s}$ and $\dim(\mathfrak{s}) = 3$ then $\mathfrak{s}$ must be simple. (At this point, the classification of semisimple Lie algebras shows $\mathfrak{s}$ must be isomorphic to $\mathfrak{sl}_2$, but candidates it seems all had better taste than to crack this part of the question with that particular sledgehammer.) Whether or not one notices that $\mathfrak{s}$ must be simple, it is relatively straight-forward to argue that a Cartan subalgebra $\mathfrak{h}$ of $\mathfrak{s}$ must be one-dimensional and that the associated Cartan decomposition is then $\mathfrak{s} = \mathfrak{h} \oplus \mathfrak{s}_\alpha \oplus \mathfrak{s}_{-\alpha}$, whence choosing a basis for each of the three lines and scaling appropriately one obtains an $\mathfrak{sl}_2$-triple so that $\mathfrak{s} \cong \mathfrak{sl}_2$. Part (b) was the most challenging part, with a number of candidates considering a semidirect product coming from an action of $\mathfrak{sl}_2$ by derivations on $\mathfrak{gl}_1$, which however will always split into a direct sum. The final part was well-answered overall.

3. This was the least popular question, though most of those who offered it submitted answers showing a good understanding of the material. Few candidates sought to use part (b) in answering part (c), which allows you to explicitly describe $C_{\mathfrak{g}}(\mathfrak{a})$ and hence show that $C_{\mathfrak{g}}(\mathfrak{a})$ is abelian precisely when $0 \notin \Psi(\mathfrak{a})$ and $|\Psi(\mathfrak{a})| = \dim(V)$. Without that fact, candidates were unable to make much progress on (d).

C2.2: Homological Algebra

Question 1 was attempted by all the candidates. Part (a) was bookwork. Part (b) was similar to calculations done in the course and on problem sheets and most candidates did quite well on it. Many used the Koszul resolution of R/I to calculate and some used a resolution of I. Some of the candidates did not calculate the homology of the resulting complex correctly. Part (c) was new. Several students were successful but many did not give full answers to the parts. That required using the injectivity of Q/Z, the fact that the duality sends kernels to cokernels, and using the hom-tensor adjunction.

Question 2 was not attempted by any of the candidates.

Question 3 was attempted by by all candidates. a(i) was straightforward bookwork. In a(2) most candidates used the hint, but not all used the fact that any group is a filtered colimit of its finitely generated subgroups. Many candidates did not include the fact that tensor products commute with colimits (which follows from the hom-tensor adjunction) in their argument, so the solution was not complete. In b(i) some candidates used a flat resolution of Q/Z and some the fact that Q/Z is a filtered colimit of Z/nZ. In b(ii) many candidates showed the vanishing for free abelian groups of finite rank but did not argue that any torsion free abelian group is a filtered colimit of free groups of finite rank. Given parts b(i) and b(ii) most candidates did well on part b(iii). The equivalence of 1 and 3 using the symmetry of the tensor product was not explained by some of the candidates. Part (c) was new and was answered successfully by some of the candidates.

C2.3: Representation Theory of Semisimple Lie Algebras

Question 1 was attempted by all candidates. Parts (a.i), (c.i) (bookwork) and parts (c.ii), (c.iii) (straightforward inductions) were generally answered very well. A few candidates had difficulties completing part (b), and all candidates struggled with part (a.ii). Those who attempted part (c.iv) generally produced a correct example.

Question 2 was attempted by only three candidates. Part (a) was straight forward and solutions were generally correct, albeit sometimes overly complicated. Parts (b) and (c) were more difficult, but candidates made good progress. Part (d) was attempted by only one candidate.

Question 3 was attempted by all candidates. Candidates generally did well on part (a), spotting the link between parts (a.i) and (a.ii). For part (b), most candidates managed to find the desired example in part (b.i), but some candidates struggled with completing part (b.ii). Many candidates had difficulties describing the set $\psi(V)$ in part (c.i), but succesfully identified the desired conditions in part (c.ii).

C2.4: Infinite Groups

Question 1. This question was attempted by half of the candidates, most of the time successfully. For part (a) the argument was not always entirely rigorous (using an induction, for instance). In (c) the arguments showing that G' is not finitely generated were, at times, rather convoluted.

Question 2. This question was the least popular, probably because students are less comfortable with the notion of wreath product. In the formulae about the Hirsch length, some students stated and proved only one inequality.

Question 3. This question was attempted by all candidates, but only few obtained high marks. Many candidates could not prove the validity of the noetherian induction. In question (c) many candidates did not realize that the inductive definition of the upper central series must be formulated simultaneously with an inductive proof that it is composed of characteristic subgroups (or, at least, of normal subgroups), since the definition uses the fact that the previous group in the series is normal. In (c) (iv) an unexpectedly high number of candidates encountered difficulties, because they attempted to prove that quotients of nilpotent groups are nilpotent using the upper central series. The noetherian induction in the end was attempted by very few.

C2.5: Non-Commutative Rings

The answers were generally of very good standard and the few errors were mostly in the unseen (final) parts of the questions.

Question 1 was attempted by all candidates with good results. The last part (c) had frequent errors in the application of the Jacobson density theorem to a subring of R/I.

Question 2 was attempted by all except one candidate, with good results overall. Few candidates justified carefully the injectivity of the homomorphism f in part (d).

Question 3 was attempted by one candidate with a nearly complete solution. In part (c,ii) the equality $R = J$ can be proved avoiding all computations by using the Poisson bracket on $gr(R)$.

C2.6 Introduction to Schemes

Question 1 This was the most popular question with parts (b), (c.ii), and (c.iii) being the main points of differentiation amongst attempts. While (b) was mostly handled successfully, the students who answered it more succintly also made further progress on (c). For (c.ii), while students mostly recognized the r in (r, e) as corresponding to the composition of a given $f : \mathrm{Spec}(R) \to X$ with the canonical $i : X \to \mathbb{A}^1_{\mathbb{Z}}$, they tended not to notice that the fiber of $i \circ f$ over the origin is $\mathrm{Spec}(R/r)$ which halted progress. There was one complete attempt.

Question 2 This was the second most popular question with parts (b.i), (d), and (e) being the differentiators amongst solutions. There were some troubles with (b.i) stemming from not drawing the correct diagram or perhaps from not noticing that the universal property of base-changes completely settles the question. For (d) and (e), the difficulty students encountered seems to be summarized in first noticing/articulating that a morphism $f : A \to B$ factors through a subring $C \subseteq B$ iff the image of f is contained in C, and then in using that finite maps of rings are integral for (d) and the given hint for (e). There were two complete attempts.

Question 3 This was slightly less popular than Q2, although the solution attempts were very strong. For part (f.ii), some students did not note that X and X_{red} have the same underlying topological space and the cohomology groups in question are coming from cohomology of sheaves of abelian groups. There was one complete attempt.

C2.7 Category Theory

Question 1. Almost all of the candidates attempted this question. The more difficult parts were (a)(iii) (the direction η monomorphism implies η_x is a monomorphism), where using Yoneda Lemma was useful, and (b)(iii).

Question 2. Some candidates preferred to prove the dual statements in (a)(iii),(iv) in terms of limits in the opposite category. This was an acceptable solution. In (a)(v), some candidates forgot to check that if C has pushouts then so does A
C. The more challenging part of the question was (b)(ii); a proof of the independence of choices in the definition of the colimit maps was expected.

Question 3. In (b)(ii), the candidates were expected to provide details about the identification of T-algebras with k-vector spaces, including how the vector space structure is obtained from the T-algebra structure. The most challenging part was (c), particularly the definition of the multiplication in (ii).

C3.1: Algebraic Topology

Question 1.

Many candidates took an explicit simplical approach to part (a.i), though a few did quote Kunneth. Part (a.ii) was generally fine, though with some confusions about the generating 1-cocycles. Parts (b) and (c.i) were fine for the most part. The final part (c.ii) saw many reasonable answers, though there was variation in level of thoroughness and what elements were taken as clear.

Question 2.

Most serious answers were alright for parts (a) and (c), and also for part (b) though with some differences of opinion about what was too obvious to require mentioning. Though some candidates did not get part (d.iii), a fair number did work through to a correct answer to this more difficult part.

Question 3.

Most answers to part (a) were reasonable, though with some errors in low degrees. In part (b) some candidates reduced the question to that of the degree of a reflection on the sphere but either presumed the the answer there (which amounted to presuming the core of the question) or did not sufficiently justify that degree. Most answers to part (c) were reasonable, though with some issues about the quantifications in the definition. Though there were some correct answers to part (d), also a number of candidates tried (mostly with errors) to use a global homological characterization of orientability, expressly against the indication to use the local notion from part (c).

C3.2 Geometric Group Theory

Question 1. This question was attempted by few candidates, and none managed to answer all the new sub-questions in it. Very few saw for instance that the question (b) was an immediate application of the last part of (a), (iv). It is likewise rather disappointing that very few remembered the proof of (c), (ii), and that there were almost no attempts to answer (c), (iii).

Question 2. This question was attempted by most candidates. Quite a few though were unable to describe the tree on which an amalgamated product acts with fundamental domain an edge. There were a number of inaccurate definitions of reduced words for amalgamated products and HNN extensions. There were very few complete answers to (c), (iv), even if this was simply an application of the writing of elements in HNN extensions as reduced words.

Question 3. This question was also popular with candidates. In many answers to (a), (ii), there was confusion between equalities in the group G and equalities in the free group $F(S)$. In (a), (iii), many students forgot to add words that become trivial in G. Surprisingly few managed to answer (b), (iii), considering that it was a direct application of the defining property of a Dehn presentation. There were more good answers to question (c), *a priory* a more difficult question because both more novel and more complex.

C3.3: Differentiable Manifolds

There was a small error in question 3(b): the question should also have said that X, Y are oriented, for degrees to be defined. I do not think any candidates were adversely affected, as this was a bookwork part; in fact, only one candidate appeared to notice the omission.

The MMath candidates did, on average, significantly better than the OMMS candidates (average raw marks 41.3 versus 31.1); I thought the MMath cohort was very strong, mostly deserving of distinctions.

Question 1. Attempted by 7 out of 20 candidates, and mostly done well. In (a), many candidates did not say that a chart (U, φ) should have *open* image $\varphi(U) \subseteq X$; some forgot to say the charts in an atlas must cover X.

Question 2. The most difficult question, attempted by 15 out of 20 candidates. (The strategic move would have been to answer questions 1 and 3.) Most candidates lost a mark by not describing the domains of the maximal integral curves in (b).

Nobody gave complete, correct answers to (c),(d). For (c), less than half the candidates managed to reduce the equation $\mathcal{L}_u\alpha = 0$ to the form

$$x_1\frac{\partial \alpha_i}{\partial x_1} + x_2\frac{\partial \alpha_i}{\partial x_2} + x_3\frac{\partial \alpha_i}{\partial x_3} + 2\alpha_i = 0.$$

What I hoped they would do next is to say that on a ray (tx_1, tx_2, tx_3) for $t \in [0, \infty)$ the solutions must grow like t^{-2}, homogeneous in t, and so must be zero to be continuous at $(0, 0, 0)$, but noone saw this. For (d) we get

$$x_1\frac{\partial w_i}{\partial x_1} + x_2\frac{\partial w_i}{\partial x_2} + x_3\frac{\partial w_i}{\partial x_3} - w_i = 0.$$

Now the solutions must grow like t^{+1}, homogeneous in t, and so to be smooth at $(0, 0, 0)$ must be of the form $w_i = \sum_{j=1}^{3} A_{ij}x_j$. Several candidates saw these solved the equation, but none justified that these were all the solutions.

Question 3. Attempted by 18 out of 20 candidates, and answered very well by many, some with full marks. Part (a) says 'Standard results about integration of exterior forms may be used if clearly stated'. Many candidates lost a mark by using Stokes' Theorem without stating it.

C3.4: Algebraic Geometry

Q1 and Q2 were the most popular. Q3 was also done by a few candidates but almost nobody applied (b) (i) when solving (b) (ii) and (c) (ii), which is strange. In Q2 (b), several candidates showed that that $\nu(\mathbf{P}^2(k))$ is closed by providing explicit equations for the set $\nu(\mathbf{P}^2(k))$. It can be done in this way but it also follows from the fact that $\mathbf{P}^2(k)$ is complete, a fact used by very few.

C3.5: Lie Groups

Question 1. This was mostly fine. There have been a few small mistakes or omissions, for example, using implicitly that $GL(n, R)$ is a Lie group in (a)(i), or in (b)(ii) not realising that there are no two-dimensional Lie subalgebras. In (b)(iii), the candidates were expected to provide justification for the claim that $SO(3)$ was not simply connected.

Question 2. Some solutions to (a)(iii) were a bit involved, a simpler solution would have been applying (a)(ii) to the abelian Lie subalgebra generated by X and Y . For (b)(iii), it was expected to use the decomposition of G given by some version of the QR decomposition ("Iwasawa decomposition"). Some candidates did not give details for the proof. There are (at least) two possible solutions for this: one to use the action on the upper half plane as in the hint, the other to use Gram-Schmidt orthogonalisation.

Question 3. The difficulty encountered here was in (c), not realising how to use characters to decompose the tensor product. There have also been a few omissions in (b)(i) (not arguing how to realise S_n in $SU(n)$ rather than $U(n)$) and in (b)(ii), in the explicit realisation of $R(T)^W$ in terms of symmetric polynomials.

C3.6 Modular Forms

Question 1: This was done by all the students. It was based quite closely on material from the lectures and problem sheets, and also similar in part to something on the 2018 examination. The method used therefore should have been fairly clear. Indeed most students did very well on this question, in particular in applying the method for computing the genus in part (c) given all the information found earlier in the question.

Question 2: Not a very popular question. It had the most amount of original material, though part (a) was just modified bookwork. The question was not done well, though the sample of students taking it was very small.

Question 3: This was done by most of the students. It seemed to be a good mix of bookwork and similar material, and original aspects of varying difficulty. I was very pleased that many students did very well on this, some giving a quite concise and correct deduction for the final part of the question.

C3.7: Elliptic Curves

Question 1: This was a fairly popular question. The first parts were bookwork and some fairly routine applications of Hensel's lemma. The latter was not always done that accurately. A good number of students got the point of the hint for the final part (that certain sets had to overlap), and presented nice solutions. One found a completely different method based upon setting $z = 1$ and associating to the given surface an elliptic curve, and then applying the Hasse bound.

Question 2: Again a fairly popular question. I was pleased that this was done very nicely overall, even the parts on formal groups. A few students scored highly, but none were able to see how to transform their isomorphism into one involving a rational function.

Question 3: Almost everyone attempted this question. It was the most routine of the three and done very well overall. A couple of students did go awry in (b) by not computing b_1 correctly.

C3.8 Analytic Number Theory

Overall questions 2 and 3 were successful, but question 1 was not popular with the candidates (a majority answered questions 2 and 3) and seems to have been pitched a bit too high. A couple of candidates seemed to suffer from time-issues, but this did not seem to be a widespread problem. In general the questions were able to distinguish between candidates, and almost all candidates put in a good overall attempt showing clear basic understanding of the course.

Question 1 was attempted by a minority of the candidates, and generally the answers were moderately incomplete. In retrospect this question was probably both too long and too difficult. Although the first parts were only fairly small variations on content covered in lectures and example sheets, many candidates didn't fully see the strategy. Some stronger candidates answered the earlier parts well, but then appeared to struggle in the later parts.

Question 2 was generally answered pretty well, particularly considering the overall question was a bit different to questions from previous years. Most candidates did well on (a) and (b) which helped verify that they had understood the basics, but several struggled a bit more with the later parts, particularly (d). Although many candidates were on the right track, few answered (d) completely and this helped to distinguish between the strongest candidates.

Question 3(a)(b) and (c) were generally all answered well (as one would expect, since they were a combination of bookwork and small variations on things seen before. This helped to verify those who had understood the basics of the course, although perhaps there were a few too many marks veryeasily available. It was pleasing to see that most candidates had the right strategy in mind for answering the hardest part (d), even if only a small number of candidates were able to fully complete this part of the question. This part did well at distinguishing the strongest candidates.

C3.10: Additive and Combinatorial Number Theory

Question 1. a) There was a good range of attempts towards part a), with some managing to solve this completely successfully while some not being able to make much progress.

b) Surprisingly, only one student answered part i) of this question correctly which required a reasonably straightforward application of the pigeonhole principle, although there were a lot of partial attempts towards this. There were more completely correct attempts towards part ii) though most students seem to find this slightly tricky as well.

c) There were many partial attempts towards part i) with few completely correct solutions, despite this being a very close variant of a similar question from last year's exam. No candidate got anywhere close to solving part ii).

Question 2. Parts of this question required adapting bookwork material in a slightly different setting while some parts were quite new. For instance, most candidates answered part b) i) and ii) completely correctly while only one candidate managed to completely correctly solve part b) iii). In general, there was a good range of attempts towards this question.

Question 3. a) Students did quite well on part a) of Question 3, with most scoring close to full marks.

b) This required combining results from two different sections of the course; first solving this for the case when $A_1, \ldots, A_4$ are sets of integers by considering the Fourier transform and then applying a Freiman isomorphism from $A_1 \cup \cdots \cup A_4 \subseteq \mathbb{R}$ to sets of integers. Most students did the first step but did not realise they had to do the second step, with only one student getting full marks via this method. Two students followed a slightly different strategy and solved this by applying the Cauchy-Schwarz inequality iteratively, without an application of the Fourier transform.

c) There was a wide range of attempts towards both part c) i) and ii), despite part c) i) being very closely related to an exam question from last year.

d) This was a new and tricky question with only one student managing to solve this completely correctly, while a few others provided some partial attempts.

C3.11: Riemannian Geometry

Question 1. Part (a) was bookwork or seen on a problem sheet and was usually done well, with marks only lost for not giving sufficient justification for the orientation-preserving property. Part (b)(i) was usually done well, with marks only lost for not quoting uniqueness of geodesics sufficiently clearly. Part (b)(ii) proved to be more challenging, though most students understood the idea of the question even if they were not able to give a sufficiently formal argument. Part (c) proved to be the most difficult part of the question, with few students answering it correctly or at all. Students that did answer (c) usually the understood the idea of the question, i.e. using (b)(ii) to extend the geodesic, but the answers were typically lacking in some details. This was a popular question, attempted by all but two students, and had a range of marks from very low to very high.

Question 2. Part (a) was bookwork and done correctly by all. Part (b) was also bookwork and usually done well, with marks only lost for lack of justification in the proof, using the choice of orthonormal basis. Part (c) was usually done well, with students mostly understanding exactly how to do the calculation quickly. Part (d) produced a mixed response. Not all students used the hint to show that the metric was Einstein, but those that did were able to do see quickly. Most students knew how to calculate the sectional curvatures in (d), but often made errors in the calculation, particularly in computing the Lie bracket. Part (e) was not attempted by all students, and those that did attempt it were often able to find the minimality condition, but were unable to show that the minimal hypersurface was a maximum for the volume. This was the most popular question and was attempted by all students, with a spread of marks from medium to very high.

Question 3. Part (a) was bookwork and was done well. Part (b) was also bookwork and was also done well, with almost all of the relevant justifications given. Part (c)(i) was similar to an example seen on a problem sheet and was done well. Part (c)(ii) was done well, with marks only lost for not arguing correctly why the example was Ricci positive. Part (c)(iii) was an easy application of bookwork and was done well. Part (c)(iv) proved to be more challenging, with students unable to find and/or prove that there was a suitable counterexample. Part (c)(v) produced a mixed response, with one student spotting the right answer immediately, and the other wrongly attempting to produce a counterexample. This was not a popular question, with only two students attempting it, but both did very well.

Question 3. Part (a) was bookwork and usually fine. Most students understood well what to do in (b). The only common issues in (b) were not explaining why the exponential map is surjective (using Hopf–Rinow) and why the metric defined in (ii) is complete (again, using Hopf–Rinow). Part (c) proved challenging, with most students not realizing that they had to look at Jacobi fields on the round sphere and relate them to Jacobi fields on the product.

C3.12: Low-Dimensional Topology and Knot Theory

Question 1 (2 attempts): This question tested knowledge of immersions, embeddings, and the Whitney trick. Problems even with the definitions. No substantive progress on most parts.

Question 2 (8 attempts): This question tested knowledge of Heegaard diagrams and 3-

manifolds. Several good solutions. Some missed details in part (a)(i). Part (a)(ii) was bookwork and usually most had the right idea, but some missed details. In part (b)(i), only few explained how to write down the relations explicitly. In part (b)(iv), several candidates correctly chose $\#_g(S^1 \times S^2)$ but some did not justify their answer.

Question 3 (10 attempts): This questions tested knowledge of knot theory, and solutions were overall the best for this question. In part (a)(i), most definitions missed that a slice surface has to be compact, connected, and oriented. In part (a)(ii), some solutions missed the part that the signature is well-defined. This is very similar to the proof that the Alexander polynomial is well-defined, given in the lectures. Part (b)(i) and (ii) proved to be harder, but there were some correct solutions. Part (b)(iii) was generally fine.

C4.1: Further Functional Analysis

Question 1 Q1: This question was by far the most popular and generally very well done.

Q2: This question was slightly less popular but there is a grater variation of marks. There were not too many common mistakes. Parts that were done were mostly done correctly, but many candidates could not do the second part of the question. Parts (b)(iii) and (b)(iv) were particularly challenging.

Q3: This question was attempted by only few candidates. Most of the candidates found (b)(ii-ii) and (c) to be hard.

C4.3: Functional Analytic Methods for PDEs

Q1. This question was attempted by only a small number of candidates, and was handled mostly well.

Q2. This question was attempted by all candidates. The bookwork parts were handled mostly well, though a few candidates forgot to mention that the function in the definition of weak derivatives need to be locally integrable and the ellipticity constants need to be positive. In (a)(ii), a few candidates proved the product rule by approximation without realising that it would be shorter to argue directly. In (b)(iii), most candidates had a feeling how to do it, but about half struggled to see how the balls can be kept as $B_{1/2}$ and B_1 as in the statement. In (b)(iv), those who tried this leg quoted correctly, with minor exceptions, the formula for the weak derivative of $(u - k)^+$ but may or may not see how to use it to reach the conclusion.

Q3. This question was attempted by almost all candidates. The bookwork parts were handled well. In (a)(iii), most candidates saw immediately the first inequality in the chain, but only about half managed the rest. In (b)(iii), almost all had the right feeling that they needed to show that the limit function satisfies the Laplace equation and must thus be zero, but only about half managed correctly through all turns. Fewer candidates attempted (b)(iv), and only those who correctly related the norm of v_m to that of u_m could manage it in the end.

C4.6 Fixed Point Methods for Nonlinear PDEs

6 students took the exam of the course.

Question 1 was about the first part of the course regarding Banach Contraction Mapping principle and Brower's fixed point theorem. The question was chosen by 5 students. The solutions ranged from very good to almost perfect, with only one exception just above the sufficiency level.

Question 2 was about Schauder's fixed point theorem and applications to nonlinear PDEs, with the use of weak maximum principle. The question needed a good handling knowledge of preliminary material such as Sobolev embeddings and standard properties of the Laplacian. The question was chosen by 4 students. The solutions ranged from very good to almost perfect, with only one exception just above the sufficiency level.

Question 3 was about variational inequalities and applications to non-linear PDEs of p-Laplacian type. As in question 2, also here it was necessary to have a good handling knowledge of preliminary material such as Sobolev embeddings and standard properties of the Laplacian. The question was chosen by 3 students. The solutions ranged from very good to almost perfect.

Overall, the exam had a very good outcome.

C4.8 Complex Analysis: Conformal Maps and Geometry

1 All the students answered the first problem. The second part of (iv) and (v) are relatively harder. One missed the second part of (iii). The students did fine. They all know what they are supposed to prove. The parts they missed need a bit of trick. It is understandable that they could lose the point in the exam.

2 The initial parts of the questions are seen. The last two parts are not covered in the lecture. The two students who did problem achieve nearly full marks. They know the conformal geometry part well. Their proofs are very clear and thorough.

3 Most of the problems are unseen, but manageable by extending our Part A2 techniques. One student answered this question.

C4.9: Optimal Transport & Partial Differential Equations

Question 1 was very popular and attempted by most of the candidates. Parts (a) and (b) were straightforward and were in general answered quite well. Some candidates found part (e) more challenging. It seems that most of the students understood the difference between push-forward of measures and composition with inverse maps but not all.

Question 2 was attempted by much fewer candidates. Parts (a) and (b) were reasonably straightforward bookwork. Candidates found part (c) more difficult. Part (d) was a simple consequence of part (c) and spotted well, however part (e) was not attempted by the candidates at all.

Question 3 was attempted by most of the candidates. Parts (a) and (b) were generally well answered, although some students found difficulties in writing the good definition of weak solutions for the PDE. Part (c) was satisfactorily answered by half of the candidates. Some of them did more work than needed. Actually, instead of using directly the course content, they reproved certain parts making their answers longer than needed.

Summary: This year's exam seems to have worked quite well to distinguish between the achieved knowledge of the students in the course. Overall, I am quite happy with the spread of marks with respect to pandemic years, showing the high level of achievement of part of the cohort in the first time the course is taught fully in person with normal interactions.

C5.1: Solid Mechanics

Q1: This question was very well done, though the integration in part (d) proved more tricky than anticipated.

Q2: This question was very well done, with only a minor algebraic error.

Q3: This question was not attempted.

C5.2: Elasticity and Plasticity

Question 1: This question attracted few attempts. The bookwork in part (a) was well done. The solutions for the similar material in part (b) were largely incomplete: while all solutions correctly deduced that exponential decay in the far field is only possible if $c < c_2$, none verified that there are no nontrivial solutions if $c < c_1$ or $c = c_1$. The first half of the tail in part (c) was well done, but there were no complete solutions to the second half.

Question 2: This question attracted many attempts. The bookwork in part (a) was well done. The similar material in part (b) was poorly answered overall, though several scripts stated correctly that the conditions arose from continuity of the beam as well as a force and moment balance at the contact point. There were many good attempts to the similar material in parts (c)(i)–(iii), though quite a few scripts failed to exploit efficiently the symmetry or the form of the boundary conditions. There were several excellent attempts but no complete solutions to the tail in part (c)(iv). Some candidates integrated the reaction force over the domain without taking care of the jump discontinuities in the third derivative of w at the contact points.

Question 3: This question was attempted by all candidates. The bookwork in part (a) was well done overall though only a minority took the most efficient route. The similar material in parts (b) and (c) was extremely well done overall, with nearly all candidates scoring close to full marks. The first half of the tail in part (c) was well done overall modulo some algebraic slips, while there were several good attempts but no complete solutions to the second half.

C5.4: Networks

C5.5: Perturbation Methods

Question 1. This question was attempted by about half the candidates. In part (a) many candidates lost marks for not fully justifying all assumptions made, nor showing complete working for the contour integral. Few candidates could successfully extend the results from (a) to arrive at the correct solution in (b).

Question 2. This question was attempted by about half the candidates. The majority scored close to full marks in parts (a) and (b). In part (c) most candidates spotted the need for a secularity condition, but many struggled with the algebra and so could not deduce the correct ordinary differential equation.

Question 3. Almost all candidates attempted this question. Part (a) was generally well answered, though none spotted that a (simpler) ansatz of the form $y = \exp(S(x)/\epsilon)$ could be used. Part (b) was also generally well answered, though some candidates lost marks for not correctly identifying the position of the boundary layer (at $x = 0$) and some made algebraic mistakes in the expansions.

C5.6: Applied Complex Variables

Question 1 The part which caused the most trouble on this question was part (a), perhaps because it was a little unusual. The rest of the question was handled very well on the whole.

Some candidates made mistakes identifying the velocity and the potential at the point A, as the sink is approached.

Question 2 Most of this question was handled very well. In part (c) a common mistake was to inadvertently give a definition for $\frac{(z-1)^{1/2}}{z^{1/2}}$ rather than $\frac{(1-z)^{1/2}}{z^{1/2}}$. No candidate managed to get the answer out in part (d), which required care with branches of square roots when evaluating the extra pole contributions. However, a good many managed to solve part (e), given the answer which had been provided in (d). There was one small mistake in Q2(e) which should have asked candidates to find a solution bounded at infinity (the solution is unbounded at the origin).

Question 3 This was a very popular question and there were a lot of very good answers. The bookwork was handled well, and very few candidates had trouble despite the additive decomposition having a double pole.

C5.7: Topics in Fluid Mechanics

Question 1 This was a very popular question. Part (a)(iv) was quite a lot of work for just 4 marks, and most candidates did not manage the complete derivation. There were relatively easy marks available at the end of the question for those candidates that carried on.

Question 2 This was another popular question. The parts of the question which caused the most trouble were (b)(iii) and (c)(iii). Most candidates could not write down the drainage flux, or evaluate it in terms of f. In part (b)(iii) it was perhaps a little confusing for the question to ask for the flux in terms of $f(0)$ and f'(0), since $f(0)$ is zero and f'(0) is infinite, while the flux depends on the product ff' evaluated at zero which is finite.

In (b)(ii) no candidate identified that $f(0) = 0$ is an atmospheric pressure condition, despite writing down exactly the same condiution in (c)(i) when asked for a condition imposing atmospheric pressure.

Question 3 This was an extremely unpopular question.

C5.9: Mechanical Mathematical Biology

Q1. This question had the lowest average mark, and also had the fewest attempts . In part (a), most candidates did well with (i)-(iii), which constituted pretty straightforward bookwork. Part (iv) caused more issues. The solution can be found with very little work, but some candidates got tangled up in lengthy calculations, losing time and never reaching the right answer. Very few attempts were given for part (b). Part (b)(i) involved a geometric calculation, but no candidates seemed aware how to begin. Part (ii) could largely be completed without having done (i), and was done correctly by one or two candidates.

Q2. This question was quite hit or miss, with a wide variance in marks. Part (a) was bookwork, involving careful setting up up the geometric relations, which threw a few candidates off. Part (b)(i) was mostly done well, though a common mistake was writing $\lambda_s = 0$ instead of 1. Nobody completed part (b)(ii) perfectly, but several candidates made a very good effort, only struggling to obtain a complete system. A common mistake made by those who did well on this part was thinking that $O(\epsilon^3)$ was needed to obtain a closed system.

Q3. This question was attempted by most candidates, and with a good average mark. A few nearly perfect answers were given.Part (a) was a small adaptation on an analysis done in lectures. Most candidates got the general idea, but only one correctly saw that at the point of contact, $\lambda = L_1/L_0 \neq 1$. In part (b), (i) and (ii) were straightforward and done well, while (iii) and (iv) required careful attention to boundary conditions and the different stress components. Part (iv) required first showing that $\alpha \to 0$ as $\gamma \to \infty$, and then secondly showing that as $\alpha \to 0$, $t_3 \to -\infty$.

C5.11: Mathematical Geoscience

qn 1:

1(a) was well answered, although no-one got the percentages of the 3 main inorganic carbon sources.

1(b) was well answered.

1(c)(i) This was well answered; (ii) most students got the first 2 approximations, fewer got the third and the fourth equation; (iii) no-one was able to answer this final part of the question.

qn 2:

This was the most popular question on the paper.

2(a) This was answered well by nearly all students.

2(b)(i) This was well answered; (ii) some students had problems with finding the order ϵ term in the equation for A; (iii) this was well answered by nearly all the students.

2(c) There was a typo in the rhs of the equation for σ: the f^2 should have been F^2. This was pointed out by a few students. Most wrote the correct expression for σ. Those who wrote f^2 were not penalised. The stability criterion was then correctly identified for the given $f(\xi)$.

Comments on C5.11 Question 3

This question attracted quite a broad spread of marks. Most candidates managed part (a) well, although the explanation to arrive at the limiting expression for h in (a)(iii) was often unnecessarily complicated (conservation of mass should not need to be appealed to). A common mistake in (b) was not multiplying by r before integrating, and then being stuck with how to deal with the flux term. Part (c) caused a surprising amount of confusion, with many candidates bogged down in algebra to analyse stability of the steady states, which could more easily be reasoned graphically. One or two candidates understood that the last part - including hysteresis - was best tackled by constructing the bifurcation diagram.

C5.12: Mathematical Physiology

There was an even spread of candidates attempting questions 1, 2 and 3.

Question 1
Parts (a)–(d) were generally well completed by all. In part (a), many candidates missed the

p and q that appear as multipliers in the ordinary differential equations arising from the Law of Mass Action for the generalized system. Algebraic mistakes were the most common reason for losing marks in part (e). Part (f) was more challenging.

Question 2
Part (a) was well completed by all. In part (b), some candidates did not solve the ordinary differential equation for $n + h$. Parts (c)–(d) were well completed. Some candidates did not obtain the correct form for g in part (e). Many found part (f) challenging and did not complete this.

Question 3
This was the best completed question by the candidates with most scoring highly. Some candidates struggled with the non-dimensionalization in part (b), forgetting to non-dimensionalize the $J_{\pm}$ terms. Some candidates did not manage to complete part (e).

C6.1: Numerical Linear Algebra

Q1 was attempted by most candidates, and most attempts made good progress. Most questions in (a) were answered satisfactorily, but attempts based on direct calculations are likely to be messy. For the case of orthonormal rows, saying the proof does not hold is not a complete argument.

(b) (iii) was meant to not require much calculations (the matrix is rank-1), but most attempts seemed to have taken long time with lengthy calculations. The answer for (ii) was intended to be a hint for the final (iv), but few candidates seem to have noticed that.

Q2 was not very popular. The bookwork problem (a-i) always seem to receive some answers that confuse full QR vs. thin QR. The problem on stability was probably the least popular among all, with not many serious attempts. (b) was also not handled very well, especially the sufficient condition for GMRES requiring a full n steps.

Q3 was fairly popular, and many attempts received good marks. Aside from the basic setup the questions are reasonably aligned with the materials in the lecture notes. Courant-Fischer should be used with clarity.

C6.2: Continuous Optimisation

Q1 was attempted by most candidates, and most attempts made good progress. (a) is bookwork and was solved by most. In (b-i), it is the ability to handle the inverse (not just the matrix itself) efficiently that makes low rank updates so attractive. It was nice to see many good solutions in (c), which the setter thought would be challenging.

Q2 was not popular. (a) requires serious calculations; tiny slips were however not penalised severely. (b) was perhaps the least popular and not many attempts received near-full marks. Despite its look, (c) is not so difficult once one sees the connection to the previous problem.

Q3 was quite popular. It is important to note convexity in (a-i); otherwise a KKT point cannot be claimed to be a global minimiser. (ii) is straight forward once one sees the trick. The expectation in (b-ii) was not to simply say that for a convex optimisation problem, any local minimiser is a global minimiser (which received partial credit); either a general proof or an argument specifically for this problem was expected. Despite the challenging appearance of (iii), a number of attempts made good progress, which was a pleasant surprise.

C6.3 Approximation of Functions

Everything went smoothly and no problems arose. All the students did Q1, and most did Q2. Only a couple did Q3. The answers were quite good in most cases.

C6.4: Finite Element Methods for Partial Differential Equations

Q1. The question was concerned with the finite element approximation of a fourth-order elliptic boundary-value problem restated as a system of two second-order linear elliptic PDEs, and the convergence analysis of the finite element approximation of this elliptic system. Only one candidate attempted the question, but did not manage to get beyond the initial bit of part (a).

Q2. The question was concerned with the construction of a continuous piecewise quadratic finite element approximation of a mixed (homogeneous) Dirichlet–Neumann boundary-value problem on a triangular domain Ω, and the derivation of an optimal-order convergence result in the $H^1(\Omega)$ norm via Galerkin orthogonality in part (d) of the question, and in the $L^2(\Omega)$ norm by the Aubin–Nitsche duality argument in part (e) of the question. This was a popular question; it was attempted by all candidates who took the exam.

Q3. The question was concerned with the finite element approximation of the Lamé system of partial differential equations of linear elasticity, and the analysis and approximation of the associated quadratic (and therefore convex) energy minimization problem. This question, too, was popular and was attempted by all candidates who took the exam.

C6.5: Theories of Deep Learning

C7.4: Introduction to Quantum Information

Question 1. This was the most popular question. Nearly all students handled parts (a) and (b) well, although some mixed up 'superposition' and 'interference'. Unexpectedly, in part (c), many students incorrectly claimed that, in the worst case, four oracle calls are needed, not three. Part (d) was mostly accurate except for a small group who erred in notation, writing $f_0 1$ instead of $f_a b(01)$, which resulted in incorrect calculations. While parts (e) and (f) are interconnected, students found quantum circuit design in part (f) much more difficult than defining the unitary transformation in part (e). Part (g) was generally well answered, its success not being contingent on the correctness of the preceding parts.

Question 2. Overall, the question was well answered and students scored well. Although parts (a) and (b) were straightforward, some students struggled with succinctly justifying their answer to part (b). Responses to part (c) were generally satisfactory, yet some found difficulty describing the impact of the dephasing channel on the off-diagonal elements of the density operator. Part (d), while conceptually more challenging, yielded largely satisfactory answers. In part (e), a majority of students missed some marks due to difficulties defining the probability of success. Various methods were employed to address part (f), and these were, in almost all cases, successful.

Question 3. While this question was the least popular, those who attempted it performed notably well. Part (a), being book-work, saw most students providing accurate answers. Students adeptly handled part (b), which isn't surprising given that stepping through quantum circuits was a recurring theme in many exercises. That said, some solutions lacked good explanations. What proved surprising and slightly disheartening was that in part (c) the majority of students failed to correctly identify the post-measurement states. Part (d) posed little challenge, with very few students answering incorrectly. The final part, (e), was generally well-answered, with only a few, typically trivial, calculation errors.

C7.5: General Relativity I

Question 1 This was attempted by all candidates and was generally the question in which the candidate scored the most marks in. Part a was on the whole well done and most marks were usually awarded, including the new material a (iv). One small issue was with a (i) where it was frequently stated that for an affine parameter the derivative of the Lagrangian vanished, which is true, but then explaining why this means that we can use either Lagrangian was then missing. Part b was well done on the whole, however there were instances where for b (ii) the candidate merely stated where the singularities of the metric were and did not give a range for the r coordinate. They had clearly thought about the answer, and probably knew it correctly but neglected to write the range down. Almost everyone obtained the full 3 marks for c (i). For c(ii) a couple of candidates took the Lagrangian, inputted $\theta = \frac{\pi}{2}$, $\dot{\theta} = 0$, and then said that one can find solutions to the equations of motion obtained from the Euler–Lagrange equations. This is not correct since they must show that the equation of motion for θ is also satisfied. This requires them to impose the correct boundary condition for the θ EOM and then show that $\ddot{\theta} = \dot{\theta} = 0$ hence planar motion. Part d was well done, candidates lost marks on d (ii) for one of two reasons. The took the plus sign when taking the square root rather than the minus sign as required for an infalling geodesic, or they failed to compute the final integral despite it being quite standard.

Question 2 Question 2 was only attempted by 3 candidates, and of those only one completed the problem scoring high marks. The new material in part d, which probably put most other candidates off from trying the exam was completed perfectly.

Question 3 For question a(i) a number of candidates got almost to the correct answer however needed to use that $d(A^-1A) = 0$ for an invertible matrix A. They obtained 5/6 generally for this book work question. Most people got full marks on part b and all but one got full marks for c i). Part c ii) despite having been seen in lectures, albeit in coordinate free notation, was not completed correctly by many. They understood to use both the conditions from part c i) but were unable to use them in the correct combination to reach the answer. For part d candidates typically obtained half the available marks, though there were a number of candidates who obtained full marks here.

C7.6: General Relativity II

Question 1

This was the least attempted question of the three. part a was very standard and almost everyone obtained full marks for this book work part. For part b both (i) and (ii) were generally completed well, however for part (iii) there were varying qualities of answer.

Part c was where most of the candidates lost their marks. Drawing the Penrose diagram for the spacetime was not completed by any of the candidates. Typical problems were to get the range of R incorrect, or U and V from the previous question. Candidates often put the singularity in the wrong place trying to mimic previously seen Penrose diagrams despite this being a substantially different metric to ones they have considered in the course.

Question 2

Question 2 was the most popular question. Part a (i), (ii), and (iv) were all completed to a high standard. On the other hand very few candidates managed to get full marks on part a (iii). Some just quoted that there are $\frac{n(n+1)}{2}$ independent Killing vectors rather than showing anything while others said from (5) that there are 2 indices in the equation and therefore $16 - 6$ independent Killing vectors.

For part b a number of candidates did not compute the conserved quantity correctly. They computed the Q in part a (iv) which is not conserved for the given Lagrangian and is only valid without external forces, i.e. electromagnetism. There were also a worrying number of candidates who wrote that $\mathcal{L} = -1$ rather than $g_{\mu\nu}\dot{x}^{\mu}\dot{x}^{\nu} = -1$. For part b (iii) a few candidates took a further derivative of the equation defining the trajectory to compute the acceleration $\ddot{r}$, however this does not imply the result.

Question 3

In general people scored the best on this question. Most could use the hint in part a (ii) to correctly solve the question. To further distinguish the candidates this hint should probably not have been given. Part b was book work and was completed perfectly by most candidates as was part c. Part d had mixed answers. Some managed to get the question perfectly while others did not manage to set up the problem in the correct way by using conservation of energy. For those who ignored conservation of energy they did not get the correct answer. A little thought about what was being computed and by playing around with setting one of the masses to zero would have allowed those candidates to see that their answer could not have been correct.

C7.7: Random Matrix Theory

Question 1 was attempted by most of the candidates. Parts (a), (b) and (c) were straightforward, and were in general answered well. Some candidates found parts (d) and (e) more difficult. Several of those who successfully completed part (d) failed to spot that the result can be used immediately in part (e); that is, they failed to use the hint and took a less direct route (in some cases successfully).

Question 2 was attempted by relatively few candidates. Part (a) was reasonably straightforward bookwork. Many candidates found part (b) difficult; only a few completed it successfully. Many candidates started part (b) correctly, but then failed to make use of the orthogonality of the Hermite polynomials.

Question 3 was attempted by quite a few candidates. Most of those who did attempt it gained relatively high marks. Parts (a) and (b) were straightforward. Most of the candidates who attempted part (c) were able to complete it. Several candidates gained high marks on part (d), but a few struggled to apply Gaudin's lemma in this setting. Part (e) proved difficult. Most candidates who attempted it failed to see how to use the hint.

Overall, most candidates did well, but a small number found it difficult to get started, despite there being a sizeable bookwork component.

C8.1: Stochastic Differential Equations

Question 1 was attempted by most of the candidates. Parts (a), (b) were straightforward, and were in general answered well. Some candidates didn't noticed that the equation was vector-valued and used the one-dimensional expression for the generator, leading some some serious mistakes in (b)(iv). Part (b)(i) required a good understanding of local martingales and time change and was answered in reasonable ways only by few candidates, none remarked that the localizing sequence must be adapted to the new filtration. Part (c) was attempted by very few candidates in not-very successful ways and failing to note the connections with the previous discussion of time change.

Question 2 was attempted by most of the candidates.. Part (a) was reasonably straightforward bookwork and was answered correctly by the majority. Part (b)(i) was attempted more or less successfully by most of the candidates, with various degrees of precision. Part (b)(ii) was computationally elementary but required a bit of maturity in considering the situation and representing the stopping time via a drift, only few managed to get this point. Part (c) was attempted by very few candidates in confused ways, many understood the use of dominated convergence for (ii) but none got the idea of (i).

Question 3 was attempted by few candidates. Most of those who did attempt it gained relatively high marks. Parts (a) was straightforward and part (b) required some computation with the generator and the use of martingales to estimate stopping times. Most candidates which attempted part (b) were able to perform the computaiton but not all were able to get the consequences right. Part (c) was computationally easy but only few candidates tried it, those who tried managed to get a good mark.

Overall, most candidates did well, but a small number found it difficult to get started, despite there being a sizeable bookwork component.

C8.2: Stochastic Analysis and PDEs

Question 1 and 2 were the most popular, although a decent amount of candidates also attempted Question 3. Overall, most candidates made much progress on parts a) and b), and a large number also made progress on part c) of each question. The sufficient condition of question 1b) was solved by nearly all candidates but some trouble arose with the induction step needed for the reverse implication and candidates tried to use martingale arguments for genuine Markov processes. For Question 2 many candidates failed to give a rigorous argument for 2c). In Question 3, most candidates managed to solve 3b) but a common mistake was to not rigorously verify that A^α is a CAF. 3c) was more technical but a few candidates were very successful here as well.

C8.3: Combinatorics

Q1: A very popular question. Almost everyone gave a correct construction of a symmetric chain decomposition in part (a); surprisingly, many candidates then proved Sperner's Lemma in part (b) by writing out another proof that there is a suitable chain decomposition (rather than just using the decomposition from (a)). There are several methods to answer (c), but one common mistake was to take a symmetric chain decomposition and then assume that a random element would be equally likely to lie in any of the chains (which is not the case, as they are not all the same size). Part (d) was perhaps a bit tricky, as there were very few correct attempts.

Q2: Another popular question. Part (a) was mostly done well, as was the first part of (b). There were very few correct attempts for (b)(ii), which required a different idea to the rest of the question.

Q3: Few candidates did this, although there were some good solutions. Most of the question is a fairly direct application of methods from lectures. The last part of (b) was not technically difficult, but needed the idea of controlling the chromatic number by bounding the size of an independent set in G.

C8.4: Probabilistic Combinatorics

Question 1 was the most popular. With hindsight the first few parts were a bit too straightforward, and the final part too tricky, so the spread of marks was relatively narrow. (a) is very simple. (b) was mostly well done, though some candidates missed that the events that two disjoint edges are present as isolated edges are not independent. Others overcomplicated the variance calculation by using methods appropriate to other contexts, rather than the simple formulae from part (a). (c)(ii) is already slightly tricky, and there were very few good answers to (d)(ii). The idea is to find a random time T such that with high probability G_T has the two properties in (c). When this happens, no earlier or later graph in the random sequence can contain an isolated K_4.

Question 2 was least popular. The bookwork part (a) was mostly well done, though with some confusion about the role of dependence between events. Solutions to (b) were mixed; the idea is that it is a very much simpler version of a calculation in the final chapter of the course. Simply write the contribution from overlap s in the form $\Theta(n^{f(s)})$ and observe that $f(s)$ is quadratic (a happy parabola) and so maximized at an end. (c) is just calculation of the relevant exponents in this case, to check they are larger than 1. There were few good answers to (d). The idea is to fix a colouring, apply Janson, and take the union bound over colourings.

Question 3 was almost as popular as Question 1, but overall less well done. There was a good spread of marks here, though in some cases it seems more from candidates running out of time. (a) and (b) are bookwork. For (c)(i) the key is to apply the Chernoff bound but not the Local Lemma (a big hint is the form of the answer, a probability tending to 1). (c)(ii) is simple and was well done. (c)(iii) is actually a little tricky and there were few good answers. For part (d) the Local Lemma now comes in, but the key is the dependency digraph, which does not correspond to pairwise dependence.

Statistics Units

Reports on the following courses may be found in the Mathematics and Statistics examiners' report.

SC1 - Stochastic Models in Mathematical Genetics SC2 - Probability and Statistics for Network Analysis SC4 - Advanced Topics in Statistical Machine Learning SC5 - Advanced Simulation Methods SC7 - Bayes Methods SC9 - Interacting Particle Systems SC10- Algorithmic Foundations of Learning

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