UNIVERSITY OF OXFORD
Mathematical Institute

HONOUR SCHOOL OF MATHEMATICS

SUPPLEMENT TO THE UNDERGRADUATE HANDBOOK – 2012 Matriculation

SYLLABUS AND SYNOPSES OF LECTURE COURSES

Part A
2013-14
for examination in 2014

These synopses can be found at:
http://www.maths.ox.ac.uk/current-students/undergraduates/handbooks-synopses/

Issued October 2013

Handbook for the Undergraduate Mathematics Courses
Supplement to the Handbook
Honour School of Mathematics
Syllabus and Synopses for Part A 2013–14
for examination in 2014

Contents

1 Foreword

Notice of misprints or errors of any kind, and suggestions for improvements in this booklet should be addressed to the Academic Administrator in the Mathematical Institute.

1.1 Honour School of Mathematics

[See the current edition of the *Examination Regulations* for the full regulations governing these examinations.]

For Part A, each candidate shall be required to offer 8 written papers. These papers must include:

A1 – Algebra 1 and Differential Equations 1 (3 hours)

A2 – Metric Spaces and Complex Analysis (3 hours)

ASO – Short Options (1.5 hours)

and five papers from the Long Options (each 1.5 hours long)

A3 Algebra 2

A4 Integration

A5 Topology

A6 Differential Equations 2

A7 Numerical Analysis

A8 Probability

A9 Statistics

A10 Waves and Fluids

A11 Quantum Theory

Paper ASO will examine the seven Short Options (Number Theory, Algebra 3, Projective Geometry, Multivariable Differentiation, Calculus of Variations, Graph Theory, Special Relativity). Students are recommended to take three of these Short Options.

Part A shall be taken on one occasion only (there will be no resits). At the end of the Part A examinations, a candidate will be awarded eight 'University Standardised Marks' (USMs). The USMs from Papers A1 and A2 will have twice the weight of the USMs awarded for the Long Options and Paper ASO. A weighted average of these USMs will be carried forward for the classification awarded at the end of the third year, with this average from the second year papers counting for 40%.

All students who complete Parts A and B will be classified. Those who have achieved honours and who wish to graduate at this point may supplicate for a BA.

Students wishing to take the four-year course should register to do so at the beginning of their third year, and will be permitted to do so on the basis of an upper second class performance, or better, in the third year classification. They will take Part C in their fourth year, be awarded a separate classification and, if successful, may supplicate for an MMath.

Examination Conventions can be found at:http://www.maths.ox.ac.uk/notices/undergrad

Syllabus and Synopses

The examination syllabus, as referred to in the *Examination Regulations*, and synopses have been approved by the Mathematics Teaching Committee for examination in 2014.

The **synopses** in this booklet give additional detail to the syllabus (for example, showing how the material is split by lectures) and are also accompanied by lists of recommended reading.

The Part A examination **syllabus** is the mathematical material of the synopses, as separately detailed by paper below, for examination in 2014.

Masters in Theoretical and Mathematical Physics

For the first time, in 2015, there will be a Mathematical Physics stream as an alternative to Part C. Hence the Part A cohort in 2013-14 are the first students to have this option. Students who move on to this stream and successfully complete the year will be awarded an MMathPhys.

Mathematics students interested in transferring to the MMathPhys will need to make an application during their third year. This option will not be available to students on the joint degrees.

Interested students should bring this up with their tutors. Full details relating to this masters will be in the MMathPhys handbook (which will be available by Week 0 of Michaelmas Term 2013), including details of those second and third year options which are suggested background or recommendations for the masters and a description of the application process and deadlines.

1.2 Honour School of Mathematics & Philosophy

See the current edition of the *Examination Regulations* for the full regulations governing these examinations. For the Schedule of Mathematics Papers for Part A, see the Supplement to the Undergraduate Handbook for the Honour School of Mathematics & Philosophy:

http://www.maths.ox.ac.uk/current-students/undergraduates/handbooks-synopses/mathsphil.

1.3 Honour School of Mathematics & Statistics

See the current edition of the *Examination Regulations* for the full regulations governing these examinations, and the details published by the Statistics Department:

http://www.stats.ox.ac.uk/current_students/bammath/course_handbooks.

1.4 Honour Schools of Computer Science and Mathematics & Computer Science

See the current edition of the *Examination Regulations* for the full regulations governing these examinations, and the details published in a handbook by the Department of Computer Science:

http://www.cs.ox.ac.uk/teaching/handbooks.html.

2 CORE MATERIAL

2.1 Syllabus

The examination syllabi of the two core papers A1 and A2 shall be the mathematical content of the synopses for the courses

- Algebra 1
- Differential Equations 1
- Metric Spaces and Complex Analysis

as detailed below.

2.2 Synopses of Lectures

This section contains the lecture synopses associated with the two core papers A1 and A2.

2.2.1 A1: Algebra 1: Linear Algebra — Prof Tillmann — 16 lectures MT

Overview

The core of linear algebra comprises the theory of linear equations in many variables, the theory of matrices and determinants, and the theory of vector spaces and linear maps. All these topics were introduced in the Prelims course. Here they are developed further to provide the tools for applications in geometry, modern mechanics and theoretical physics, probability and statistics, functional analysis and, of course, algebra and number theory. Our aim is to provide a thorough treatment of some classical theory that describes the behaviour of linear maps on a finite-dimensional vector space to itself, both in the purely algebraic setting and in the situation where the vector space carries a metric derived from an inner product.

Learning Outcomes

Students will deepen their understanding of Linear Algebra. They will be able to define and obtain the minimal and characteristic polynomials of a linear map on a finite-dimensional vector space, and will understand and be able to prove the relationship between them; they will be able to prove and apply the Primary Decomposition Theorem, and the criterion for diagonalisability. They will have a good knowledge of inner product spaces, and be able to apply the Bessel and Cauchy–Schwarz inequalities; will be able to define and use the adjoint of a linear map on a finite-dimensional inner product space, and be able to prove and exploit the diagonalisability of a self-adjoint map.

Synopsis

Definition of an abstract vector space over an arbitrary field. Examples. Linear maps. Division Algorithm in $F[x]$. Characteristic polynomials and minimal polynomials. Coincidence of roots. [2].

Quotient vector spaces. The first isomorphism theorem for vector spaces and rank-nullity. Induced linear maps. Applications: Triangular form for matrices over $\mathbb{C}$. Cayley-Hamilton Theorem. [2.5].

Bezout's Lemma in $F[x]$. Primary Decomposition Theorem. Diagonalizability and Triangularizability in terms of minimal polynomials. Proof of existence of Jordan canonical form over $\mathbb{C}$ (using primary decomposition and inductive proof of form for nilpotent linear maps). [3.5].

Dual spaces of finite-dimensional vector spaces. Dual bases. Dual of a linear map and description of matrix with respect to dual basis. Natural isomorphism between a finite-dimensional vector space and its second dual. Annihilators of subspaces, dimension formula. Isomorphism between U^* and V^*/U°. [3]

Recap on real inner product spaces. Definition of non-degenerate symmetric bilinear forms and description as isomorphism between V and V^*. Hermitian forms on complex vector spaces. Review of Gram-Schmidt. Orthogonal Complements. [2].

Adjoints for linear maps of inner product spaces. Uniqueness. Concrete construction via matrices [1].

Definition of orthogonal/unitary maps. Definition of the groups O_n, SO_n, U_n, SU_n. Diagonalizability of self-adjoint and unitary maps. [2].

Reading

Richard Kaye and Robert Wilson, *Linear Algebra* (OUP, 1998) ISBN 0-19-850237-0. Chapters 2–13. [Chapters 6, 7 are not entirely relevant to our syllabus, but are interesting.]

Further Reading

1. Paul R. Halmos, *Finite-dimensional Vector Spaces*, (Springer Verlag, Reprint 1993 of the 1956 second edition), ISBN 3-540-90093-4. §§1–15, 18, 32–51, 54–56, 59–67, 73, 74, 79. [Now over 50 years old, this idiosyncratic book is somewhat dated but it is a great classic, and well worth reading.]

2. Seymour Lipschutz and Marc Lipson, *Schaum's Outline of Linear Algebra* (3rd edition, McGraw Hill, 2000), ISBN 0-07-136200-2. [Many worked examples.]

3. C. W. Curtis, *Linear Algebra—an Introductory Approach* (4th edition, Springer, reprinted 1994).

4. D. T. Finkbeiner, *Elements of Linear Algebra* (Freeman, 1972). [Out of print, but available in many libraries.]

2.2.2 A1: Differential Equations 1 — Prof Grindrod — 16 lectures MT

Overview

The aim of this course is to introduce all students reading mathematics to the basic theory of ordinary and partial differential equations.

The course will be example-led and will concentrate on equations that arise in practice rather than those constructed to illustrate a mathematical theory. The emphasis will be on solving equations and understanding the possible behaviours of solutions, and the analysis will be developed as a means to this end.

The course will furnish undergraduates with the necessary skills to pursue any of the applied options in the third year and will also form the foundation for a deeper and more rigorous course in partial differential equations.

Learning Outcomes

On completion of the course, students will have acquired a sound knowledge of a range of techniques for solving linear ordinary and partial differential equations. They will have gained an appreciation of the importance of existence and uniqueness of solution and will be aware that explicit analytic solutions are the exception rather than the rule.

Synopsis

Picard's Existence Theorem: Picard's Theorem for first-order scalar ODEs with proof. Examples of blow-up and nonuniqueness, discussion of continuation and global existence; in particular global existence for equations with linear growth. Picard's Theorem for systems of ODEs, with proof using Contraction Mapping Principle *[CMP to be covered in metric space course]*. Application to scalar second order ODEs, with particular reference to linear equations. (5/6 lectures)

Classification of second order, semilinear PDEs; Normal form; Ideas of uniqueness and well-posedness. Illustration of suitable boundary conditions by example. Poisson's Equation and the Heat Equation: Maximum Principle *[done as in B5b at present]* leading to uniqueness and continuous dependence on the initial data. (5/6 lectures)

Theory of Fourier and Laplace transforms, inversion, convolution. Inversion of some standard Fourier and Laplace transforms via contour integration. Use of Fourier and Laplace transforms in solving ordinary and partial differential equations; in particular, use of Fourier transform in solving Laplace's equation and the Heat equation. (5 lectures)

Reading

The best single text is:

P. J. Collins, *Differential and Integral Equations* (O.U.P., 2006), Chapters 1-7, 14,15.

Alternatives

W. E. Boyce & R. C. DiPrima, *Elementary Differential Equations and Boundary Value Problems* (7th edition, Wiley, 2000).

Erwin Kreyszig, *Advanced Engineering Mathematics* (8th Edition, Wiley, 1999).

F. B. Hildebrand, *Methods of Applied Mathematics* (Dover, 1992).

W. A. Strauss, *Partial Differential Equations: an Introduction* (Wiley, 1992).

G. F. Carrier & C E Pearson, *Partial Differential Equations — Theory and Technique* (Academic, 1988).

H. A. Priestley, *Introduction to Complex Analysis* (Second edition, Oxford, 2003).

J. Ockendon, S. Howison, A. Lacey & A. Movchan, *Applied Partial Differential Equations* (Oxford, 1999). [More advanced.]

2.2.3 A2: Metric Spaces and Complex Analysis — Dr Earl — 32 lectures MT

Overview

The theory of functions of a complex variable is a rewarding branch of mathematics to study at the undergraduate level with a good balance between general theory and examples. It occupies a central position in mathematics with links to analysis, algebra, number theory, potential theory, geometry, topology, and generates a number of powerful techniques (for example, evaluation of integrals) with applications in many aspects of both pure and applied mathematics, and other disciplines, particularly the physical sciences.

In these lectures we begin by introducing students to the language of topology before using it in the exposition of the theory of (holomorphic) functions of a complex variable. The central aim of the lectures is to present Cauchy's Theorem and its consequences, particularly series expansions of holomorphic functions, the calculus of residues and its applications.

The course concludes with an account of the conformal properties of holomorphic functions and applications to mapping regions.

Learning Outcomes

Students will have been introduced to point-set topology and will know the central importance of complex variables in analysis. They will have grasped a deeper understanding of differentiation and integration in this setting and will know the tools and results of complex analysis including Cauchy's Theorem, Cauchy's integral formula, Liouville's Theorem, Laurent's expansion and the theory of residues.

Synopsis

Metric Spaces (10 lectures)

Basic definitions: metric spaces, isometries, continuous functions ($\varepsilon - \delta$ definition), homeomorphisms, open sets, closed sets. Examples of metric spaces, including metrics derived from a norm on a real vector space, particularly l^1, l^2, l^∞ norms on $\mathbb{R}^n$, the sup norm on the bounded real-valued functions on a set, and on the bounded continuous real-valued functions on a metric space. The characterisation of continuity in terms of the pre-image of open sets or closed sets. The limit of a sequence of points in a metric space. A subset of a metric space inherits a metric. Discussion of open and closed sets in subspaces. The closure of a subset of a metric space. [3]

Completeness (but not completion). Completeness of the space of bounded real-valued functions on a set, equipped with the norm, and the completeness of the space of bounded continuous real-valued functions on a metric space, equipped with the metric. Lipschitz maps and contractions. Contraction Mapping Theorem. [2.5]

Connected metric spaces, path-connectedness. Closure of a connected space is connected, union of connected sets is connected if there is a non-empty intersection, continuous image of a connected space is connected. Path-connectedness implies connectedness. Connected open subset of a normed vector space is path-connected. [2]

Compactness. Heine-Borel theorem. The image of a compact set under a continuous map between metric spaces is compact. The equivalence of continuity and uniform continuity for functions on a compact metric space. Compact metric spaces are sequentially compact. Statement (but no proof) that sequentially compact metric spaces are compact. Compact metric spaces are complete. [2.5]

Complex Analysis (22 lectures)

Basic geometry and topology of the complex plane, including the equations of lines and circles. [1]

Complex differentiation. Holomorphic functions. Cauchy-Riemann equations (including $z, \bar{z}$ version). Real and imaginary parts of a holomorphic function are harmonic. [2]

Recap on power series and differentiation of power series. Exponential function and logarithm function. Fractional powers examples of multifunctions. The use of cuts as method of defining a branch of a multifunction. [3]

Extended complex plane, Riemann sphere, stereographic projection. Möbius transformations acting on the extended complex plane. Möbius transformations take circlines to circlines. [2]

Conformal mappings. Riemann mapping theorem (no proof): Möbius transformations, exponential functions, fractional powers; mapping regions (not Christoffel transformations or Joukowski's transformation). [3]

Path integration. Cauchy's Theorem. (Sketch of proof only students referred to various texts for proof.) Fundamental Theorem of Calculus in the path integral/holomorphic situation. [2]

Cauchy's Integral formulae. Taylor expansion. Liouville's Theorem. Identity Theorem. Morera's Theorem. [4]

Laurent's expansion. Classification of isolated singularities. Calculation of principal parts, particularly residues. [2]

Residue Theorem. Evaluation of integrals by the method of residues (straightforward examples only but to include the use of Jordan's Lemma and simple poles on contour of integration). [3]

Reading

1. W. A. Sutherland, *Introduction to Metric and Topological Spaces* (Second Edition, OUP, 2009).

2. H. A. Priestley, *Introduction to Complex Analysis* (second edition, OUP, 2003).

Further Reading

1. L. Ahlfors, *Complex Analysis* (McGraw-Hill, 1979).

2. Reinhold Remmert, *Theory of Complex Functions* (Springer, 1989) (Graduate Texts in Mathematics 122).

3 OPTIONS

3.1 Syllabus

The examination syllabi of the options paper, A3-A11, shall be the mathematical content of the synopses for the courses

- Algebra 2
- Integration
- Topology
- Differential Equations 2
- Numerical Analysis
- Probability
- Statistics
- Waves and Fluids
- Quantum Theory

as detailed below.

3.2 Synopses of Lectures

This section contains the lecture synopses associated with the options papers A3-A11.

3.2.1 A3: Algebra 2: Rings and Modules — Dr McGerty — 16 lectures HT

Overview

The first abstract algebraic object which are normally studied are groups, which arise naturally from the study of symmetries. The focus of this course is on rings, which generalise the kind of algebraic structure possessed by the integers: a ring has two operations, addition and multiplication, which interact in the usual way. The course begins by studying the fundamental concepts of rings: what are maps between them, when are two rings isomorphic etc. much as was done for groups. As an application, we get a general procedure for building fields, generalising the way one constructs the complex numbers from the reals. We then begin to study the question of factorization in rings, and find a class of rings, known as Principal Ideal Domains, where any element can be written uniquely as a product of prime elements generalising the case of the integers. Finally, we study modules, which roughly means we study linear algebra over certain rings rather than fields. This turns out to have useful applications to ordinary linear algebra and to abelian groups.

Learning Outcomes

Students should become familiar with rings and fields, and understand the structure theory of modules over a Euclidean domain along with its implications. The material underpins many later courses in algebra and number theory, and thus should give students a good background for studying these more advanced topics.

Synopsis

Definition of rings and examples: $\mathbb{Z}$, fields, polynomial rings (in more than one variable), matrix rings. The characteristic of a ring with an identity. Integral Domains. [1]

For the rest of the course we restrict our attention to rings which are commutative and have an identity.

Homomorphisms of rings. Quotient rings, ideals and the first isomorphism theorem and consequences, e.g. Chinese Remainder Theorem. Relation between ideals in R and R/I. [2]

Prime ideals and maximal ideals, relation to fields and integral domains. Examples of ideals. Fields of fractions: examples and uniqueness of extension of maps from an integral domain to the field of fractions. [2]

Application of quotients to constructing fields by adjunction of elements; examples to include $\mathbb{C} = \mathbb{R}[x]/(x^2 + 1)$ and some finite fields. Degree of a field extension, the tower law. [1]

Euclidean Domains. Examples. Principal Ideal Domains. EDs are PIDs. Unique factorisation for PIDs. Gauss's Lemma and Eisenstein's Criterion for irreducibility. [3]

Modules: Definition and examples: vector spaces, abelian groups, vector spaces with an endomorphism. Submodules and quotient modules and direct sums. The first isomorphism theorem. [2]

Row and column operations on matrices over a ring. Equivalence of matrices and canonical forms of matrices over a Euclidean Domain. [2]

Free modules and presentations of finitely generated modules. Structure of finitely generated modules of a Euclidean domain. [2]

Application to rational canonical form for matrices, and structure of finitely generated Abelian groups. [1]

Reading

1. Michael Artin, *Algebra* (2nd ed. Pearson, (2010). (Excellent text covering everything in this course and much more besides).

2. Neils Lauritzen, *Concrete Abstract Algebra*, CUP (2003) (Excellent on groups, rings and fields, and covers topics in the Number Theory course also. Does not cover material on modules).

3. P. B. Bhattacharya, S. K. Jain, S. R. Nagpaul, *Basic Abstract Algebra*, CUP (1994) (Covers all of the basic algebra material most undergraduate courses have).
4. B. Hartley, T. O. Hawkes, Chapman and Hall, *Rings, Modules and Linear Algebra*. (Possibly out of print, but many library should have it. Relatively concise and covers all the material in the course).

3.2.2 A4: Integration — Dr Qian — 16 lectures HT

Overview

The course will exhibit Lebesgue's theory of integration in which integrals can be assigned to a huge range of functions on the real line, thereby greatly extending the notion of integration presented in Mods. The theory will be developed in such a way that it can be easily extended to a wider framework including summation of series and probability theory (although no knowledge of probability will be required), but measures other than Lebesgue's will only be lightly touched.

Operations such as passing limits, infinite sums, or derivatives, through integral signs, or reversing the order of double integrals, are often taken for granted in courses in applied mathematics. Actually, they can occasionally fail. Fortunately, there are powerful convergence and other theorems allowing such operations to be justified under conditions which are widely applicable. The course will display these theorems and a wide range of their applications.

This is a course in rigorous applications. Its principal aim is to develop understanding of the statements of the theorems and how to apply them carefully. Knowledge of technical proofs concerning the construction of Lebesgue measure and the integral will not be an essential part of the course, and such proofs will usually be omitted from the lectures.

Learning Outcomes

Synopsis

Measure spaces. Outer measure, null set, measurable set. The Cantor set. Lebesgue measure on the real line. Counting measure. Probability measures. Construction of a non-measurable set (non-examinable). Measurable function, simple function, integrable function. Reconciliation with the integral introduced in Prelims.

A simple comparison theorem. Integrability of polynomial and exponential functions over suitable intervals. Changes of variable. Fatou's Lemma (proof not examinable). Monotone Convergence Theorem (proof not examinable). Dominated Convergence Theorem. Corollaries and applications of the Convergence Theorems (including term-by-term integration of series).

Theorems of Fubini and Tonelli (proofs not examinable). Differentiation under the integral sign. Change of variables.

Brief introduction to L^p spaces. Hölder and Minkowski inequalities (proof not examinable).

Reading

1. A. Etheridge, *Integration,* Mathematical Institute Lecture Notes
2. M. Capinski & E. Kopp, *Measure, Integral and Probability* (Second Edition, Springer, 2004).
3. F. Jones, *Lebesgue Integration on Euclidean Space* (Second Edition, Jones & Bartlett, 2000).

Further Reading

1. R. G. Bartle, *The Elements of Integration* (Wiley, 1966) or *The Elements of Integration and Lebesgue Measure* (Wiley, 1995)
2. D. S. Kurtz & C. W. Swartz, *Theories of Integration* (Series in Real Analysis Vol.9, World Scientific, 2004).
3. H. A. Priestley, *Introduction to Integration* (OUP 1997).

   [Useful for worked examples, although adopts a different approach to construction of the integral].
4. H. L. Royden, *Real Analysis* (various editions; 4th edition has P. Fitzpatrick as co author).
5. E. M. Stein & R. Shakarchi, *Real Analysis: Measure Theory, Integration and Hilbert Spaces* (Princeton Lectures in Analysis III, Princeton University Press,2005).
6. T. Toa, *An Introduction to Measure Theory* (American Mathematical Society, 2011)

3.2.3 A5: Topology — Prof Lackenby — 16 lectures HT

Overview

Topology is the study of 'spatial' objects. Many key topological concepts were introduced in the Metric Spaces course, such as the open subsets of a metric space, and the continuity of a map between metric spaces. More advanced concepts such as connectedness and compactness were also defined and studied. Unlike in a metric space, there is no notion of distance between points in a topological space. Instead, one keeps track only of the open subsets, but this is enough to define continuity, connectedness and compactness. By dispensing with a metric, the fundamentals of proofs are often clarified and placed in a more general setting.

In the first part of the course, these topological concepts are introduced and studied. In the second part of the course, simplicial complexes are defined; these are spaces that are obtained by gluing together triangles and their higher-dimensional analogues in a suitable way. This is a very general construction: many spaces admit a homeomorphism to a simplicial complex, which is known as a triangulation of the space. At the end of the course, the proof of one of the earliest and most famous theorems in topology is sketched. This is the classification of compact triangulated surfaces.

Learning Outcomes

By the end of the course, a student should be able to understand and construct abstract arguments about topological spaces. Their topological intuition should also be sufficiently well-developed to be able to reason about concrete topological spaces such as surfaces.

Synopsis

Axiomatic definition of an abstract topological space in terms of open sets. Basic definitions: closed sets, continuity, homeomorphism, convergent sequences, connectedness and comparison with the corresponding definitions for metric spaces. Examples to include metric spaces (definition of topological equivalence of metric spaces), discrete and indiscrete topologies, cofinite topology. The Hausdorff condition. Subspace topology. [3 lectures]

Accumulation points of sets. Closure of a set. Interior of a set. Continuity if and only if $f\left(\overline{A}\right) \subseteq \overline{f(A)}$. [2 lectures]

Basis of a topology. Product topology on a product of two spaces and continuity of projections. [2 lectures]

Compact topological spaces, closed subset of a compact set is compact, compact subset of a Hausdorff space is closed. Product of two compact spaces is compact. A continuous bijection from a compact space to a Hausdorff space is a homeomorphism. Equivalence of sequential compactness and abstract compactness in metric spaces. [3 lectures]

Quotient topology. Quotient maps. Characterisation of when quotient spaces are Hausdorff in terms of saturated sets and in terms of the graph of the equivalence relation. Examples, including the torus, Klein bottle and real projective plane. [3 lectures]

Abstract simplicial complexes and their topological realisation. A triangulation of a space. Any compact triangulated surface is homeomorphic to the sphere with g handles ($g \geq 0$) or the sphere with h cross-caps ($h \geq 1$). (No proof that these surfaces are not homeomorphic, but a brief informal discussion of Euler characteristic.) [3 lectures]

Reading

W. A. Sutherland, *Introduction to Metric and Topological Spaces* (Oxford University Press, 1975). Chapters 2-6, 8, 9.1-9.4.
(New edition to appear shortly.)

J. R. Munkres, *Topology, A First Course* (Prentice Hall, 1974), chapters 2, 3, 7.

Further Reading

B. Mendelson, *Introduction to Topology* (Allyn and Bacon, 1975). (cheap paperback edition available).

G. Buskes, A. Van Rooij, *Topological Spaces* (Springer, 1997).

N. Bourbaki, *General Topology* (Springer, 1998).

J. Dugundji, *Topology* (Allyn and Bacon, 1966), chapters 3, 4, 5, 6, 7, 9, 11. [Although out of print, available in some libraries.]

3.2.4 A6: Differential Equations 2 — Prof Please — 16 lectures HT

Overview

Learning Outcomes

Synopsis

Phase plane analysis: Phase planes, critical points, definition of stability, classification of critical points and linearisation, Bendixson-Dulac criterion. Examples including conservative nonlinear oscillators. (5 lectures)

Two-point boundary value problems for self-adjoint second-order ODEs; Green's function, eigenfunctions. (4 lectures)

Green's functions for Poisson's Equation and the Heat Equation.(3 lectures)

Approximation techniques: Asymptotic sequences. Regular and singular perturbation methods for algebraic equations.

Simple boundary layer theory. (4 lectures)

Reading

The best single text is:

P. J. Collins, *Differential and Integral Equations* (O.U.P., 2006), Chapters 1-7, 14,15.

Alternatives

W. E. Boyce & R. C. DiPrima, *Elementary Differential Equations and Boundary Value Problems* (7th edition, Wiley, 2000).

Erwin Kreyszig, *Advanced Engineering Mathematics* (8th Edition, Wiley, 1999).

F. B. Hildebrand, *Methods of Applied Mathematics* (Dover, 1992).

W. A. Strauss, *Partial Differential Equations: an Introduction* (Wiley, 1992).

G. F. Carrier & C E Pearson, *Partial Differential Equations — Theory and Technique* (Academic, 1988).

H. A. Priestley, *Introduction to Complex Analysis* (Second edition, Oxford, 2003).

J. Ockendon, S. Howison, A. Lacey & A. Movchan, *Applied Partial Differential Equations* (Oxford, 1999). [More advanced.]

3.2.5 A7: Numerical Analysis — Dr Macdonald — 16 lectures HT

Overview

Scientific computing pervades our lives: modern buildings and structures are designed using it, medical images are reconstructed for doctors using it, the cars and planes we travel on are designed with it, the pricing of "Instruments" in the financial market is done using it, tomorrow's weather is predicted with it. The derivation and study of the core, underpinning algorithms for this vast range of applications defines the subject of Numerical Analysis. This course gives an introduction to that subject.

Through studying the material of this course students should gain an understanding of numerical methods, their derivation, analysis and applicability. They should be able to solve certain mathematically posed problems using numerical algorithms. This course is designed to introduce numerical methods - i.e. techniques which lead to the (approximate) solution of mathematical problems which are usually implemented on computers. The course covers derivation of useful methods and analysis of their accuracy and applicability.

The course begins with a study of methods and errors associated with computation of functions which are described by data values (interpolation or data fitting). Following this we turn to numerical methods of linear algebra, which form the basis of a large part of computational mathematics, science, and engineering. Key ideas here include algorithms for linear equations, least squares, and eigenvalues built on LU and QR matrix factorizations. The course will also include the simple and computationally convenient approximation of curves: this includes the use of splines to provide a smooth representation of complicated curves, such as arise in computer aided design. Use of such representations leads to approximate methods of integration. Techniques for improving accuracy through extrapolation will also be described. The course requires elementary knowledge of functions and calculus and of linear algebra.

Although there are no assessed practicals for this course, the tutorial work involves a mix of written work and Matlab programming. Some minimal working knowledge of Matlab is recommended, but many examples will be provided. Some use of Matlab will be demonstrated in lectures.

Learning Outcomes

At the end of the course the student will know how to:

- Find the solution of linear systems of equations.
- Compute eigenvalues and eigenvectors of matrices.
- Approximate functions of one variable by polynomials and piecewise polynomials (splines).
- Compute good approximations to one-dimensional integrals.
- Increase the accuracy of numerical approximations by extrapolation.
- Use Matlab to achieve these goals.

Synopsis

Lagrange interpolation [1 lecture]

Newton-Cotes quadrature [2 lectures]

Gaussian elimination and LU factorization [2 lectures]

QR factorization [1 lecture]

Eigenvalues: Gershgorin's Theorem, symmetric QR algorithm [3 lectures]

Best approximation in inner product spaces, least squares, orthogonal polynomials [4 lectures]

Piecewise polynomials, splines [2 lectures]

Richardson Extrapolation. [1 lecture].

Reading

You can find the material for this course in many introductory books on Numerical Analysis such as

A. Quarteroni, R Sacco and F Saleri, *Numerical Mathematics* (Springer, 2000).

K. E. Atkinson, *An Introduction to Numerical Analysis* (2nd Edition, Wiley, 1989).

S. D. Conte and C. de Boor, *Elementary Numerical Analysis* (3rd Edition, Graw-Hill, 1980).

G. M. Phillips and P. J. Taylor, *Theory and Applications of Numerical Analysis* (2nd Edition, Academic Press, 1996).

W. Gautschi, *Numerical Analysis: An Introduction* (Birkhauser, 1977).

H. R. Schwarz, *Numerical Analysis: A Comprehensive Introduction* (Wiley, 1989).

But the main recommended book for this course is:

E. Süli and D. F. Mayers, *An Introduction to Numerical Analysis* (CUP, 2003). Of which the relevant chapters are: 6, 7, 2, 5, 9, 11.

3.2.6 A8: Probability — Dr Martin — 16 lectures MT

Overview

The first half of the course takes further the probability theory that was developed in the first year. The aim is to build up a range of techniques that will be useful in dealing with mathematical models involving uncertainty. The second half of the course is concerned with Markov chains in discrete time and Poisson processes in one dimension, both with developing the relevant theory and giving examples of applications.

Learning Outcomes

Synopsis

Continuous random variables. Jointly continuous random variables, independence, conditioning, functions of one or more random variables, change of variables. Examples including some with later applications in statistics. Moment generating functions and applications. Statements of the continuity and uniqueness theorems for moment generating functions. Characteristic functions (definition only). Convergence in distribution and convergence in probability. Markov and Chebyshev inequalities. Weak law of large numbers and central limit theorem for independent identically distributed random variables. Statement of the strong law of large numbers. Discrete-time Markov chains: definition, transition matrix, n-step transition probabilities, communicating classes, absorption, irreducibility, periodicity, calculation of hitting probabilities and mean hitting times. Recurrence and transience. Invariant distributions, mean return time, positive recurrence, convergence to equilibrium (proof not examinable), ergodic theorem (proof not examinable). Random walks (including symmetric and asymmetric random walks on Z, and symmetric random walks on Z^d). Poisson processes in one dimension: exponential spacings, Poisson counts, thinning and superposition.

Reading

G. R. Grimmett and D. R. Stirzaker, *Probability and Random Processes* (3rd edition, OUP, 2001). Chapters 4, 6.1-6.5, 6.8.

R. Grimmett and D. R. Stirzaker, *One Thousand Exercises in Probability* (OUP, 2001).

G. R. Grimmett and D J A Welsh, *Probability: An Introduction* (OUP, 1986). Chapters 6, 7.4, 8, 11.1-11.3.

J. R. Norris, *Markov Chains* (CUP, 1997). Chapter 1.

D. R. Stirzaker, *Elementary Probability* (Second edition, CUP, 2003). Chapters 7-9 excluding 9.9.

3.2.7 A9: Statistics — Dr Laws — 16 lectures HT

Overview

Building on the first year course, this course develops statistics for mathematicians, emphasising both its underlying mathematical structure and its application to the logical interpretation of scientific data. Advances in theoretical statistics are generally driven by the need to analyse new and interesting data which come from all walks of life.

Part A Probability is recommended for this course, but is not essential. If you are not doing Part A Probability then you should make sure that you are familiar with Prelims work on Probability, and you may also need to familiarise yourself with a couple of lectures' worth of material from Part A Probability. Should you be interested in taking courses involving statistics in Parts B or C, then it would be strongly advisable to take Part A Probability.

Learning Outcomes

At the end of the course students should have an understanding of: the use of probability plots to investigate plausible probability models for a set of data; maximum likelihood estimation and large sample properties of maximum likelihood estimators; hypothesis tests and confidence intervals (and the relationship between them). They should have a corresponding understanding of similar concepts in Bayesian inference.

Synopsis

Order statistics, probability plots.

Estimation: observed and expected information, statement of large sample properties of maximum likelihood estimators in the regular case, methods for calculating maximum likelihood estimates, large sample distribution of sample estimators using the delta method.

Hypothesis testing: simple and composite hypotheses, size, power and p-values, Neyman-Pearson lemma, distribution theory for testing means and variances in the normal model, generalized likelihood ratio, statement of its large sample distribution under the null hypothesis, analysis of count data.

Confidence intervals: exact intervals, approximate intervals using large sample theory, relationship to hypothesis testing.

Probability and Bayesian Inference. Posterior and prior probability densities. Constructing priors including conjugate priors, subjective priors, Jeffreys priors. Bayes estimators and credible intervals. Statement of asymptotic normality of the posterior. Model choice via posterior probabilities and Bayes factors.

Examples: statistical techniques will be illustrated with relevant datasets in the lectures.

Reading

F. Daly, D.J. Hand, M.C. Jones, A.D. Lunn and K.J. McConway, *Elements of Statistics* (Addison Wesley, 1995) Chapters 7-10 (and Chapters 1-6 for background).

J.A. Rice, *Mathematical Statistics and Data Analysis* (2nd edition, Wadsworth, 1995) Sections 8.5, 8.6, 9.1-9.7, 9.9, 10.3-10.6, 11.2, 11.3, 13.3, 13.4.

T Leonard and J.S.J. Hsu, *Bayesian Methods* (CUP, 1999), Chapters 2 and 3.

Further Reading

G. Casella and R. L. Berger, *Statistical Inference* (2nd edition, Wadsworth, 2001).

A.C. Davison, *Statistical Models* (Cambridge University Press, 2003), Chapter 11.

3.2.8 A10: Waves and Fluids — Dr Dellar — 16 lectures HT

Overview

This course introduces students to the mathematical theory of inviscid fluids. The theory provides insight into physical phenomena such as flight, vortex motion, and water waves. The course also explains important concepts such as conservation laws and dispersive waves and, thus, serves as an introduction to the mathematical modelling of continuous media.

Learning Outcomes

Synopsis

Incompressible flow. Convective derivative, streamlines and particle paths. Euler's equations of motion for an inviscid fluid. Bernoulli's Theorem. Vorticity, circulation and Kelvin's Theorem. The vorticity equation and vortex motion.

Irrotational incompressible flow; velocity potential. Two-dimensional flow, stream function and complex potential. Line sources and vertices. Method of images, circle theorem and Blasius's Theorem.

Uniform flow past a circular cylinder. Circulation, lift. Use of conformal mapping to determine flow past a flat wing. Water waves, including effects of finite depth and surface tension. Dispersion, simple introduction to group velocity.

Reading

D. J. Acheson, *Elementary Fluid Dynamics* (OUP, 1997). Chapters 1, 3.1-3.5, 4.1-4.8, 4.10-4.12, 5.1, 5.2, 5.6, 5.7.

R.P. Feynman, R.B. Leighton, M. Sands, *The Feynman Lectures on Physics*, volume II, (Addison Wesley 1964) Chapter 40 http://www.feynmanlectures.info/docroot/II_40.html

3.2.9 A11: Quantum Theory — Prof Tod — 16 lectures MT

Overview

Quantum theory was born out of the attempt to understand the interactions between radiation, described by Maxwell's theory of electromagnetism, and matter, described by Newton's mechanics.

Although there remain deep mathematical and physical questions at the frontiers of the subject, the resulting theory encompasses not just the mechanical but also the electrical and chemical properties of matter. Many of the key components of modern technology such as transistors and lasers were developed using quantum theory.

In quantum theory particles also have some wave-like properties. This introductory course explores some of the consequences of this culminating in a treatment of the hydrogen atom.

Learning Outcomes

By the end of this course, students will be able to solve the Schroedinger equation in a range of simple situations and understand its significance. In particular they will be able to calculate the energy levels of hydrogen-like atoms. They will also learn the abstract, algebraic formulation of quantum mechanics which complements and sometimes replaces the solution of the Schroedinger equation.

Synopsis

Wave-particle duality; Schrödinger's equation; stationary states; quantum states of a particle in a box (infinite squarewell potential).

Interpretation of the wave function; boundary conditions; probability density and conservation of current; scattering theory; transmission and reflection coefficients; parity.

The one-dimensional harmonic oscillator; higher-dimensional oscillators and normal modes; degeneracy. The rotationally symmetric states of the hydrogen atom with fixed nucleus.

The mathematical structure of quantum mechanics and the postulates of quantum mechanics.

Commutation relations. Heisenberg's uncertainty principle.

Creation and annihilation operators for the harmonic oscillator. Measurements and the collapse of the wave function.

Schrödinger's cat. Angular momentum in quantum mechanics. The particular case of spin-1/2. Particle in a central potential. General states of the hydrogen atom.

Reading

B. H. Bransden and C.J Joachain *Quantum Mechanics* (Second edition, Pearson Education Limited, 2000). Chapters 1-4.

P.C. W. Davies and D.S. Betts, *Quantum Mechanics (Physics and its Applications)* (2nd edition, Taylor & Francis Ltd, 1994). Chapters 1,2,4.

R.P Feynman, R.B Leighton, M. Sands *The Feynman Lectures on Physics, Volume 3* (Addison-Wesley, 1998). Chapters 1,2 (for physical background).

K.C Hannabuss, *An Introduction to Quantum Theory* (Oxford University Press 1997). Chapters 1-4.

A.I.M. Rae, *Quantum Mechanics* (4th Edition, Taylor & Francis Ltd, 2002). Chapters 1-3.

4 SHORT OPTIONS

4.1 Syllabus

The examination syllabi of the short options paper ASO shall be the mathematical content of the synopses for the courses

- Number Theory
- Algebra 3
- Projective Geometry
- Multivariate Differentiation
- Calculus of Variations
- Graph Theory
- Special Relativity

as detailed below.

4.2 Synopses of Lectures

This section contains the lecture synopses associated with the short options paper ASO.

4.2.1 Number Theory — Dr Lauder — 8 lectures TT

Overview

Number theory is one of the oldest parts of mathematics. For well over two thousand years it has attracted professional and amateur mathematicians alike. Although notoriously 'pure' it has turned out to have more and more applications as new subjects and new technologies have developed. Our aim in this course is to introduce students to some classical and important basic ideas of the subject.

Learning Outcomes

Students will learn some of the foundational results in the theory of numbers due to mathematicians such as Fermat, Euler and Gauss. They will also study a modern application of this ancient part of mathematics.

Synopsis

The ring of integers; congruences; ring of integers modulo ; the Chinese Remainder Theorem. [2 lectures]

Wilson's Theorem; Fermat's Little Theorem for prime modulus; Euler's phi-function. Euler's generalisation of Fermat's Little Theorem to arbitrary modulus; primitive roots. [2 lectures]

Quadratic residues modulo primes. Quadratic reciprocity. [2 lectures]

Factorisation of large integers; basic version of the RSA encryption method. [2 lectures]

Reading

Alan Baker, *A Concise Introduction to the Theory of Numbers* (Cambridge University Press, 1984) ISBN: 0521286549 Chapters 1,3,4.

David Burton, *Elementary Number Theory* (McGraw-Hill, 2001).

Dominic Welsh, *Codes and Cryptography*, (Oxford University Press, 1988), ISBN 0-19853-287-3. Chapter 11.

4.2.2 Algebra 3: Group Theory — Prof Henke — 8 lectures TT

Overview

This group theory course develops the theory begun in prelims, and this course will build on that. After recalling basic concepts, the focus will be on two circles of problems.

1. The concept of free group and its universal property allow to define and describe groups in terms of generators and relations.

2. The notion of composition series and the Jordan-Hölder Theorem explain how to see, for instance, finite groups as being put together from finitely many simple groups. This leads to the problem of finding and classifying finite simple groups. Conversely, it will be explained how to put together two given groups to get new ones.

Moreover, the concept of symmetry will be formulated in terms of group actions and applied to prove some group theoretic statements.

Learning Outcomes

Students will learn to construct and describe groups. They will learn basic properties of groups and get familiar with important classes of groups. They will understand the crucial concept of simple groups. They will get a better understanding of the notion of symmetry by using group actions.

Synopsis

Groups, subgroups, normal subgroups. Review of the First Isomorphism Theorem and proof of Second and Third Isomorphism Theorems. [1]

Free groups. Uniqueness of reduced words and universal mapping property. Cayley graphs. Examples. [1]

Normal subgroups of free groups and generators and relations for groups. Examples. [1]

Simple groups, statement that A_n is simple (proof for $n = 5$). Definition and proof of existence of composition series for finite groups. Statement of the Jordan-Hölder Theorem. Examples. The derived subgroup and solvable groups. [2]

Discussion of semi-direct products and extensions of groups. Examples. [1]

Recollection of the definition of a group action. Application of group actions to prove the Sylow theorems. Applications to classification of groups of small order. [2]

Reading

1. Armstrong, M. A. *Groups and symmetry*, Undergraduate Texts in Mathematics. Springer-Verlag, New York, 1988. xii+186 pp.

2. Alperin, J. L.; Bell, Rowen B. *Groups and representations*, Graduate Texts in Mathematics, 162. Springer-Verlag, New York, 1995. x+194 pp.

3. Neumann, Peter M.; Stoy, Gabrielle A.; Thompson, Edward C. *Groups and geometry*, Oxford Science Publications. The Clarendon Press, Oxford University Press, Oxford, 1994. x+254 pp.

Further reading

1. Kurzweil, Hans; Stellmacher, Bernd *The theory of finite groups. An introduction*, Translated from the 1998 German original. Universitext. Springer-Verlag, New York, 2004. xii+387 pp.

2. Rotman, Joseph J. *An introduction to the theory of groups*, Fourth edition. Graduate Texts in Mathematics, 148. Springer-Verlag, New York, 1995. xvi+513 pp.

3. Robinson, Derek J. S. *A course in the theory of groups*, Graduate Texts in Mathematics, 80. Springer-Verlag, New York, 1993. xviii+481 pp.

4. Joyner, David, *Adventures in group theory. Rubik's cube, Merlin's machine, and other mathematical toys*, Second edition. Johns Hopkins University Press, Baltimore, MD, 2008. xviii+310 pp.

5. Aschbacher, M. *Finite group theory*, Second edition. Cambridge Studies in Advanced Mathematics, 10. Cambridge University Press, Cambridge, 2000. xii+304 pp.

4.2.3 Projective Geometry — Prof Dancer — 8 lectures TT

Overview

Projective spaces provide a means of extending vector spaces by adding points at infinity. The resulting geometry is in some respects better-behaved than that of vector spaces, especially as regards intersection properties. Projective geometry is a good application of many concepts from linear algebra, such as bilinear forms and duality. It also provides an introduction to algebraic geometry proper, that is, the study of spaces defined by algebraic equations, as many such spaces are best viewed as living inside projective spaces.

Learning Outcomes

Students will be familiar with the idea of projective space and the linear geometry associated to it, including examples of duality and applications to Diophantine equations.

Synopsis

1-2: Projective Spaces (as $P(V)$ of a vector space V). Homogeneous Co-ordinates. Linear Subspaces.

3-4: Projective Transformations. General Position. Desargues Theorem. Cross-ratio.

5: Dual Spaces. Duality.

6-7: Symmetric Bilinear Forms. Conics. Singular conics, singular points. Projective equivalence of non-singular conics.

7-8: Correspondence between P^1 and a non-singular conic. Simple applications to Diophantine Equations.

Reading

1. N.J. Hitchin, *Maths Institute notes on Projective Geometry*, found at http://people.maths.ox.ac.uk/hitchin/hitchinnotes/hitchinnotes.html

2. M. Reid and B. Szendrői, *Geometry and topology*, Cambridge University Press, 2005 (Chapter 5).

4.2.4 Multivariate Differentiation — Prof Dancer — 8 lectures TT

Overview

In this course, the notion of the total derivative for a function $f\colon \mathbb{R}^m \to \mathbb{R}^n$ is introduced. Roughly speaking, this is an approximation of the function near each point in $\mathbb{R}^n$ by a linear transformation. This is a key concept which pervades much of mathematics, both pure and applied. It allows us to transfer results from linear theory locally to nonlinear functions.

For example, the Inverse Function Theorem tells us that if the derivative is an invertible linear mapping at a point then the function is invertible in a neighbourhood of this point. Another example is the tangent space at a point of a surface in $\mathbb{R}^3$, which is the plane that locally approximates the surface best.

Learning Outcomes

Students will understand the concept of derivative in n dimensions and the implict and inverse function theorems which give a bridge between suitably nondegenerate infinitesimal information about mappings and local information. They will understand the concept of manifold and see some examples such as matrix groups.

Synopsis

Definition of a derivative of a function from R^m to R^n; examples; elementary properties; partial derivatives; the chain rule; the gradient of a function from R^n to R; Jacobian. Continuous partial derivatives imply differentiability, Mean Value Theorems. [3 lectures]

The Inverse Function Theorem and the Implicit Function Theorem (proofs non-examinable). [2 lectures]

The definition of a submanifold of R^m. Its tangent and normal space at a point, examples, including two-dimensional surfaces in R^3. [2 lectures]

Lagrange multipliers. [1 lecture]

Reading

Theodore Shifrin, *Multivariable Mathematics* (Wiley, 2005). Chapters 3-6.

T. M. Apostol, *Mathematical Analysis: Modern Approach to Advanced Calculus (World Students)* (Addison Wesley, 1975). Chapters 12 and 13.

S. Dineen, *Multivariate Calculus and Geometry* (Springer, 2001). Chapters 1-4.

J. J. Duistermaat and J A C Kolk, *Multidimensional Real Analysis I, Differentiation* (Cambridge University Press, 2004).

M. Spivak, *Calculus on Manifolds: A modern approach to classical theorems of advanced calculus*, W. A. Benjamin, Inc., New York-Amsterdam, 1965.

Further Reading

William R. Wade, *An Introduction to Analysis* (Second Edition, Prentice Hall, 2000). Chapter 11.

M. P. Do Carmo, *Differential Geometry of Curves and Surfaces* (Prentice Hall, 1976).

Stephen G. Krantz and Harold R. Parks, *The Implicit Function Theorem: History, Theory and Applications* (Birkhaeuser, 2002).

4.2.5 Calculus of Variations — Dr Hodges — 8 lectures TT

Overview

The calculus of variations concerns problems in which one wishes to find the minima or extrema of some quantity over a system that has functional degrees of freedom. Many important problems arise in this way across pure and applied mathematics and physics. They range from the problem in geometry of finding the shape of a soap bubble, a surface that minimizes its surface area, to finding the configuration of a piece of elastic that minimises its energy. Perhaps most importantly, the principle of least action is now the standard way to formulate the laws of mechanics and basic physics.

In this course it is shown that such variational problems give rise to a system of differential equations, the Euler-Lagrange equations. Furthermore, the minimizing principle that underlies these equations leads to direct methods for analysing the solutions to these equations. These methods have far reaching applications and will help develop students technique.

Learning Outcomes

Students will be able to formulate variational problems and analyse them to deduce key properties of system behaviour.

Synopsis

The basic variational problem and Euler's equation. Examples, including axi-symmetric soap films.

Extension to several dependent variables. Hamilton's principle for free particles and particles subject to holonomic constraints. Equivalence with Newton's second law. Geodesics on surfaces. Extension to several independent variables.

Examples including Laplace's equation. Lagrange multipliers and variations subject to constraint. Eigenvalue problems for Sturm-Liouville equations. Legendre Polynomials.

Reading

Arfken Weber, *Mathematical Methods for Physicists* (5th edition, Academic Press, 2005). Chapter 17.

Further Reading

N. M. J. Woodhouse, *Introduction to Analytical Dynamics* (1987). Chapter 2 (in particular 2.6). (This is out of print, but still available in most College libraries.)

M. Lunn, *A First Course in Mechanics* (OUP, 1991). Chapters 8.1, 8.2.

P. J. Collins, *Differential and Integral Equations* (O.U.P., 2006). Chapters 11, 12.

4.2.6 Graph Theory — Prof McDiarmid — 8 lectures TT

Overview

This course introduces some central topics in graph theory.

Learning Outcomes

Students should have an appreciation of the flavour of methods and results in graph theory.

Synopsis

Introduction. Trees and their characterisation, Cayley's theorem. Euler circuits, Dirac's theorem on Hamilton circuits. Bipartite matchings and Hall's theorem. Ramsey Theory.

Reading

R. J. Wilson, *Introduction to Graph Theory*, 5th edition, Prentice Hall, 2010.

D.B. West, *Introduction to Graph Theory*, 2nd edition, Prentice Hall, 2001.

4.2.7 Special Relativity — Dr Hodges — 8 lectures TT

Overview

The unification of space and time into a four-dimensional space-time is essential to the modern understanding of physics. This course will build on first-year algebra, geometry, and applied mathematics to show how this unification is achieved. The results will be illustrated throughout by reference to the observed physical properties of light and elementary particles.

Learning Outcomes

Students will be able to describe the fundamental phenomena of relativistic physics within the algebraic formalism of four-vectors. They will be able to solve simple problems involving Lorentz transformations. They will acquire a basic understanding of how the four-dimensional picture completes and supersedes the physical theories studied in first-year work.

Synopsis

Constancy of the speed of light. Lorentz transformations and the invariance of the wave operator; time dilation, length contraction, the relativistic Doppler effect and aberration.

Index notation, four-vectors, four-velocity and four-momentum; equivalence of mass and energy; particle collisions and four-momentum conservation; equivalence of mass and energy:

$E = mc^2$; four-acceleration and four-force, the example of the constant-acceleration world-line.

Reading

N. M. J. Woodhouse, *Special Relativity*, (Springer, 2002).