Handbook for the Undergraduate Mathematics Courses
Supplement to the Handbook
Honour School of Mathematics
Syllabus and Synopses for Part B 2008–9
for examination in 2009

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1 Foreword

See the current edition of the *Examination Regulations* for the full regulations governing these examinations.

1.1 Honour School of Mathematics

In Part B each candidate shall offer a total of four units from the schedule of units and half-units.

(a) A total of at least three units offered should be from the schedule of 'Mathematics Department units and half-units'

(b) Candidates may offer at most one unit which is designated as an extended essay[1]

Details of units for Part C will be published in 2009. Students staying on to take the four-year course will take 3 units from Part C in their fourth year.

In the classification awarded at the end of the third year, unit paper marks in Part A will be given a 'weighting' of 2, and unit paper marks in Part B will be given a 'weighting' of 3.

For those students staying on to do the fourth year, a separate class will be awarded on the basis of the Part C marks.

Language Classes

Mathematics students are also invited to apply to take classes in a foreign language. In 2008-09 classes are offered in French. Students' performance in these classes will not contribute to the degree classification in Mathematics. However, successful completion of the course may be recorded on student transcripts. See section 5 for more details. If there are vacancies we will offer these to students in joint schools.

1.2 "Units" and "half-units" and methods of examination

Most courses in Mathematics are assessed by examination. Most subjects offered have a 'weight' of one unit, and will be examined in a 3-hour examination paper. In many of these subjects it will also be possible to take the first half, or either half, of the subject as a 'half-unit'. Where this is the case, a half-unit will usually be examined in an examination paper of $1\frac{1}{2}$ hours. From 2009, each half-unit paper will contain **3** questions.

Rubrics on 3-hour examination papers (Mathematics)

The rubric on 3-hour examination papers will usually be: "candidates may submit answers to as many questions as they wish; the best two from each section will count for the total mark".

Rubrics on $1\frac{1}{2}$ hour examination papers

The rubric on $1\frac{1}{2}$ hour examination papers will usually be: "candidates may submit answers to as many questions as they wish; the best two will count for the total mark.".

[1]Units which may be offered under this heading are indicated in the synopses.

All Computer Science options will be examined by a paper of $1\frac{1}{2}$ hours in length.

1.3 Honour School of Mathematics & Philosophy

In Part B each candidate shall offer the following

(i) A total of at least two units in Mathematics from the schedule of 'Mathematics Department Units' as specified in this handbook, to include B1 Foundations: Logic and Set Theory. In addition, candidates are permitted to offer O1 History of Mathematics as specified from the list of 'Other Mathematical Units' in this handbook, so long as they offer a total of three units in Mathematics. Candidates are eligible to apply for the course N1, the Undergraduate Ambassador's Scheme. See the Mathematics and Philosophy handbook for more details.

(ii) At least three subjects in *Philosophy* from subjects 101-18, 120-2 or 199 Thesis as prescribed in the Regulations of Philosophy in all Honour Schools including Philosophy, of which two must be subjects 102 and 122.

(iii) The total number of units and subjects together should be six.

Philosophy Examinations are generally of 3-hours duration. See the Maths & Philosophy handbook for further details.

1.4 Part C (M–Level) courses available in the third year

All Part C courses (marked as 'M–Level') available to third year students will be examined using the same examination questions/projects - as used for fourth-year students. The courses Topology and Groups and Algebraic Topology will not be available in your fourth year but there are likely to be other Geometry topics offered such as Lie Groups and Differentiable Manifolds.

1.5 Classification in the Honour School of Mathematics

Classification in the Honour School of Mathematics

Each candidate will receive a numerical mark on each paper in each Part of the examination in the University standardised range 0-100, such that

- a First Class performance (on that paper) is indicated by a mark of 70 to 100;
- an Upper Second Class performance (on that paper) is indicated by a mark of 60 to 69;
- a Lower Second Class performance (on that paper) is indicated by a mark of 50 to 59
- a Third Class performance (on that paper) is indicated by a mark of 40 to 49;
- a Pass performance (on that paper) is indicated by a mark of 30 to 39;
- a Fail performance (on that paper) is indicated by a mark of 0 to 29.

In order to arrive at such University standardised marks (or USMs) for each paper, the examiners will mark and assess papers in the ways described below.

Parts B and C

The Examination Papers

Where not otherwise stated, an overview of the syllabus and form of the papers for each unit and half-unit is given in the lecture synopsis.

Marking of Papers

For Mathematics Department papers in Part B and Part C mark schemes for questions out of 25 will aim to ensure that the following qualitative criteria hold:

- 20-25 marks: a completely or almost completely correct answer, showing excellent understanding of the concepts and skill in carrying through the arguments and/or calculations; minor slips or omissions only.

- 13-19 marks: a good though not complete answer, showing understanding of the concepts and competence in handling the arguments and/or calculations. In this range, an answer might consist of an excellent answer to a substantial part of the question, or a good answer to the whole question which nevertheless shows some flaws in calculation or in understanding or in both.

This should be regarded only as a guide, conveying the intention of the examiners.

In many cases candidates will be taking papers applicable to several Schools: one group of examiners will determine the USM algorithm for a given paper and the resulting USMs will then be used by the examiners responsible for the particular candidate.

Analysis of marks

Part A

At the end of the Part A examination, a candidate will be awarded a university standardised mark (USM) for each of the four papers. The Examiners will recalibrate the raw marks to arrive at the USMs reported to candidates. In arriving at this recalibration, the examiners will principally take into account the total sum over all four papers of the marks for each question, subject to the rubric on each paper.

The Examiners aim to ensure that all papers and all subjects within a paper are fairly and equally rewarded, but if in any case a paper, or a subject within a paper, appears to have been problematical, then the Examiners may take account of this in calculating USMs.

The USMs awarded to a candidate for papers in Part A will be carried forward into a classification as described below.

Part B

The Board of Examiners in Part B will assign USMs for whole-unit and half-unit papers taken in Part B and may recalibrate the raw marks to arrive at university standardised marks reported to candidates. The whole-unit papers are designed so that the raw marks sum to 100, however, Examiners will take into account the relative difficulty of papers when assigning USMs. In order to achieve this, Examiners may use information on candidates' performances on the Part A examination when recalibrating the raw marks. They may also use other statistics to check that the USMs assigned fairly reflect the students' performances on a paper.

The USMs awarded to a candidate for papers in Part B will be aggregated with the USMs from Part A to arrive at a classification.

Part C

The Board of Examiners in Part C will assign USMs for whole-unit and half-unit papers taken in Part C and may recalibrate the raw marks to arrive at university standardised marks reported to candidates. The whole-unit papers are designed so that the raw marks sum to 100, however, Examiners will take into account the relative difficulty of papers when assigning USMs. In order to achieve this, Examiners may use information on candidates' performances on the earlier Parts of the examination when recalibrating the raw marks. They may also use other statistics to check that the USMs assigned fairly reflect the students' performances on a paper.

The USMs awarded to a candidate for papers in Part C will be aggregated to arrive at a classification for Year 4.

Aggregation of marks for award of Part B from 2009 onwards

All successful candidates will be awarded a classification at the end of three years, after the Part B examination. This classification will be based on the following rules (agreed by the Mathematics Teaching Committee).

A *strong paper rule* is adopted for classification in 2009 and onwards.

By the *nth class strong paper rule* we mean that for a candidate to be classified at the nth class standard, at least 3 papers from Parts A and B must lie in the nth class or higher and at least one of these is at Part B. For example, for a First class award, a candidate would need at least 3 of their whole-unit paper USMs to be first class marks (with at least 1 first class whole-unit at Part B) together with a weighted average score of parts A and B over 70.

In effect we are looking at a *marks profile.*

Let $AvUSM - PartA\&B = Average\ weighted\ USM\ in\ Parts\ A\ and\ B\ together$ (rounded up to a whole number);

The Part A USMs are given a weighting of 2, and the Part B USMs a weighting of 3 for a whole-unit and 1.5 for a half-unit.

- First Class: $AvUSM - PartA\&B \geq 70$ and the first class strong paper rule satisfied.
- Upper Second Class: $AvUSM - PartA\&B \geq 70$ not satisfying the first class strong paper rule **OR** $70 > AvUSM - PartA\&B \geq 60$ and the upper second strong paper rule satisfied.
- Lower Second Class: $70 > AvUSM - PartA\&B \geq 60$ and not satisfying the upper second strong paper rule **OR** $60 > AvUSM - PartA\&B \geq 50$ and the lower second strong paper rule satisfied.
- Third Class: $50 > AvUSM - PartA\&B \geq 40$ **OR** $60 > AvUSM - PartA\&B \geq 50$ and not satisfying the lower second strong paper rule
- Pass: $40 > AvUSM - PartA\&B \geq 30$
- Fail: $AvUSM - PartA\&B < 30$

[Note: Half-unit papers count as half a paper when determining the average USM, or determining the number of strong papers.]

BA in Mathematics
All candidates who wish to leave at the end of their third year and who satisfy the Examiners may supplicate for a classified BA in Mathematics at the end of Part B based on the above classification.

MMath in Mathematics in 2009 and onwards
In order to proceed to Part C, a candidate must minimally achieve lower second standard in Part A and Part B together.

Candidates successfully studying for a fourth year will receive a separate classification based on their University standardised marks in Part C papers, according to the following rules (agreed by the Mathematics Teaching Committee).

$AvUSM - PartC = Average\ USM\ in\ Part\ C$ (rounded up to a whole number)

- First Class: $AvUSM - PartC \geq 70$
- Upper Second Class: $70 > AvUSM - PartC \geq 60$
- Lower Second Class: $60 > AvUSM - PartC \geq 50$

- Third Class: $50 > AvUSM - PartC \geq 40$

A 'Pass' will not be awarded for Year 4. Candidates achieving:

$$AvUSM - PartC < 40,$$

may supplicate for a BA with the classification obtained at the end of Part B.

[Note: Half-unit papers count as half a paper when determining the average.]

Candidates leaving after four years who satisfy the Examiners may supplicate for an MMath in Mathematics, with two associated classifications; for example:
MMath in Mathematics: Years 2 and 3 together - First class; Year 4 - First class

Note that successful candidates may supplicate for one degree only - either a BA or an MMath. The MMath is doubly classified but a candidate will not be awarded a BA degree and an MMath degree.

Class Descriptors

The average USM ranges used in the classifications reflect the following descriptions:

- First Class: the candidate shows excellent skills in reasoning, deductive logic and problem-solving. He/she demonstrates an excellent knowledge of the material, and is able to use that innovatively in unfamiliar contexts.
- Upper Second Class: the candidate shows good or very good skills in reasoning, deductive logic and problem-solving. He/she demonstrates a good or very good knowledge of much of the material.
- Lower Second Class: the candidate shows adequate basic skills in reasoning, deductive logic and problem-solving. He/she demonstrates a sound knowledge of much of the material.
- Third Class: the candidate shows reasonable understanding of at least part of the basic material and some skills in reasoning, deductive logic and problem-solving.
- Pass: the candidate shows some limited grasp of basic material demonstrated by the equivalent of an average of one meaningful attempt at a question on each unit of study. A stronger performance on some papers may compensate for a weaker performance on others.
- Fail: little evidence of competence in the topics examined; the work is likely to show major misunderstanding and confusion, coupled with inaccurate calculations; the answers to questions attempted are likely to be fragmentary only.

1.6 Registration for Part B courses 2008–9

CLASSES Students will have to register in advance for the classes they wish to take. Students will have to register by Friday of Week 9 of Trinity Term 2008 using the online registration system which can be accessed at https://www.maths.ox.ac.uk/course-registration. Students will need to login using OU Webauth. It will then be possible for students to select the units and half-units they wish to take from the drop-down menu. Further guidance on how to use the online system can be found at:
http://www.maths.ox.ac.uk/help/faqs/undergrads/course-registration.

Students who register for a course or courses for which there is a quota will have to select a "reserve choice" which they will take if they do not receive a place on the course with the quota. They may also have to give the reasons why they wish to take a course which has a quota, and provide the name of a tutor who can provide a supporting statement for them should the quota be exceeded. Where this is necessary students will be contacted by email after they have registered. In the event that the quota for a course is exceeded, the Mathematics Teaching Committee will decide who may have a place on the course on the basis of the supporting statements from the student and tutor. Students who are not allocated a place on the course with the quota will be registered for their "reserve" course. Students will be notified of this by email. In the case of the "Undergraduate Ambassadors' Scheme" students will have to attend a short interview in Week 0, Michaelmas Term. In the meantime they will also have to complete a separate application form, and indicate a "reserve" course.

LECTURES: Some combinations of subjects are not advised and lectures may clash. Details are given below. We will use the information on your registration forms to aim to keep clashes to a minimum. However, because of the large number of options available in Part B some clashes are inevitable, and we must aim to accommodate the maximum number of student preferences.

Lecture Timetabling in Part B 2008-9

The Teaching Committee has agreed that the following clashes be allowed.

Pure vs Applied

B1 Logic and Set Theory B2 Algebra B3 Geometry B9 Number Theory	may clash with	B6 Theoretical Mechanics B8 Topics in Applied Mathematics

Mathematics vs Statistics

B1 Logic and Set Theory B2 Algebra B3 Geometry B6 Theoretical Mechanics B7 Mathematical Physics B9 Number Theory All M-level options	may clash with	OBS1 Applied Statistics OBS3 Stochastic Modelling
B3 Geometry B6 Theoretical Mechanics B7 Mathematical Physics All M-level options	may clash with	OBS4 Actuarial Science
All Part B Maths and M-level options		OBS2 Statistical Inference

Mathematics vs Comp

B3 Geometry B6 Theoretical Mechanics B7 Mathematical Physics B8 Topics in Applied Mathematics All M-level options	may clash with	OCS1 Functional Programming and Analysis of Algorithms
B1 Logic and Set Theory B2 Algebra B3 Geometry B7 Mathematical Physics B9 Number Theory All M-level options	may clash with	B21 Numerical Solution of Differential Equations
B3 Geometry B7 Mathematical Physics All M-level options	may clash with	B22 Integer Programming

Mathematics vs Philosophy

B5 Differential Equations and Applications B6 Theoretical Mechanics B7 Mathematical Physics B8 Topics in Applied Mathematics B10 Martingales & Financial Mathematics B11 Communication Theory	may clash with	FHS Maths & Phil Core options

'Other' Mathematical and 'Non-Mathematical' subjects

OBS1 Applied Statistics OBS2 Statistical Inference OBS3 Stochastic Modelling OBS4 Actuarial Science	may clash with	OCS1 Functional Programming and Analysis of Algorithms B21 Numerical Solution of Differential Equations B22 Integer Programming	may clash with	All papers in Philosophy

1.7 Three-year/Four-year Course Registration

You should register your intention to take either the three-year course or the four-year course during your third year. You are advised to discuss the right course of action for you with your College Tutor, who will also advise you how to register. Any student whose performance in the second and third year examination together falls below **lower second standard** will not be permitted to proceed to the fourth year.

All students are registered on the MMath (4 year) versions of each course. If you subsequently decide to change to the BA (3 year) option you must inform your college office who will in turn inform central administration and the departments. Please be aware that any change to your course may impact the level of your maintenance funding and the time taken to receive your student loan (you are advised to contact Student Finance http://www.direct.gov.uk/en/EducationAndLearning/UniversityAndHigherEducation/ StudentFinance for further enquiries). Please note also that if you intend to change option you are strongly advised to do so before you take Year 3 examinations.

2 Mathematics Department units and half-units

2.1 B1: Logic and Set Theory

Level: H-level **Method of Assessment:** Written Examination

Weight: Whole-unit (OSS paper code 2640), or can be taken as either a half-unit in Logic or a half-unit in Set Theory.

2.1.1 B1a: Logic — Dr Koenigsmann — 16 MT

[Option **B1a** if taken as a half-unit. OSS paper code 2A40.]

Overview

To give a rigorous mathematical treatment of the fundamental ideas and results of logic that is suitable for the non-specialist mathematicians and will provide a sound basis for more advanced study. Cohesion is achieved by focussing on the Completeness Theorems and the relationship between probability and truth. Consideration of some implications of the Compactness Theorem gives a flavour of the further development of model theory. To give a concrete deductive system for predicate calculus and prove the Completeness Theorem, including easy applications in basic model theory.

Learning Outcomes

Students will be able to use the formal language of propositional and predicate calculus and be familiar with their deductive systems and related theorems. For example, they will know and be able to use the soundness, completeness and compactness theorems for deductive systems for predicate calculus.

Synopsis

The notation, meaning and use of propositional and predicate calculus. The formal language of propositional calculus: truth functions; conjunctive and disjunctive normal form; tautologies and logical consequence. The formal language of predicate calculus: satisfaction, truth, validity, logical consequence.

Deductive system for propositional calculus: proofs and theorems, proofs from hypotheses, the Deduction Theorem; Soundness Theorem. Maximal consistent sets of formulae; completeness; constructive proof of completeness.

Statement of Soundness and Completeness Theorems for a deductive system for predicate calculus; derivation of the Compactness Theorem; simple applications of the Compactness Theorem.

A deductive system for predicate calculus; proofs and theorems; prenex form. Proof of Completeness Theorem. Existence of countable models, the downward Löwenheim–Skolem Theorem.

Reading

1. R. Cori and D. Lascar, *Mathematical Logic: A Course with Exercises (Part I)* (Oxford University Press, 2001), sections 1, 3, 4.
2. A. G. Hamilton, *Logic for Mathematicians* (2nd edition, CUP, 1988), pp.1–69, pp.73–76 (for statement of Completeness (Adequacy)Theorem), pp.99–103 (for the Compactness Theorem).
3. W. B. Enderton, *A Mathematical Introduction to Logic* (Academic Press, 1972), pp.101–144.
4. D. Goldrei, *Propositional and Predicate Calculus: A model of argument* (Springer, 2005).

Further Reading

1. R. Cori and D. Lascar, *Mathematical Logic: A Course with Exercises (Part II)* (Oxford University Press, 2001), section 8.

2.1.2 B1b: Set Theory — Dr Knight — 16 HT

[Option **B1b** if taken as a half-unit. OSS paper code 2B40.]

Overview

To introduce sets and their properties as a unified way of treating mathematical structures, including encoding of basic mathematical objects using set theoretic language. To emphasize the difference between intuitive collections and formal sets. To introduce and discuss the notion of the infinite, the ordinals and cardinality. To consider the Axiom of Choice and its implications.

Learning Outcomes

Students will have a sound knowledge of set theoretic language and be able to use it to codify mathematical objects. They will have an appreciation of the notion of infinity and arithmetic of the cardinals and ordinals. They will have developed a deep understanding of the Axiom of Choice, Zorn's Lemma and well-ordering principle, and appreciate the implications.

Synopsis

What is a set? Introduction to the basic axioms of set theory. Ordered pairs, cartesian products, relations and functions. Axiom of Infinity and the construction of the natural numbers; induction and the Recursion Theorem.

Cardinality; the notions of finite and countable and uncountable sets; Cantor's Theorem on power sets. The Tarski Fixed Point Theorem. The Schröder–Bernstein Theorem.

Isomorphism of ordered sets; well-orders. Transfinite induction; transfinite recursion [informal treatment only].

Comparability of well-orders.

The Axiom of Choice, Zorn's Lemma, the Well-ordering Principle; comparability of cardinals. [Equivalence of WO, CC, AC and ZL]. Ordinals. Arithmetic of cardinals and ordinals [the proof that, in [ZFC], $\kappa.\kappa = \kappa$ for an infinite cardinal κ is excluded].

Reading

1. D. Goldrei, *Classic Set Theory* (Chapman and Hall, 1996).
2. W. B. Enderton, *Elements of Set Theory* (Academic Press, 1978).

Further Reading

1. R. Cori and D. Lascar, *Mathematical Logic: A Course with Exercises (Part II)* (Oxford University Press, 2001), section 7.1–7.5.

2.2 B2: Algebra

Level: H-level **Method of Assessment:** Written Examination.

Weight Whole-unit (OSS paper code 2641), or available as a half-unit in B2a, or as a half-unit in B2b.

Recommended Prerequisites: All second year algebra.

2.2.1 B2a: Introduction to Representation Theory — Dr Henke — 16MT

[Option **B2a** if taken as a half-unit. OSS paper code to follow.]

Overview

This course gives an introduction to the representation theory of finite groups and finite dimensional algebras. Representation theory is a fundamental tool for studying symmetry by means of linear algebra: it is studied in a way in which a given group or algebra may act on vector spaces, giving rise to the notion of a representation.

A large part of the course will deal with the structure theory of semisimple algebras and their modules (representations). We will prove the Jordan-Hölder Theorem for modules. Moreover, we will prove that any finite-dimensional semisimple algebra is isomorphic to a product of matrix rings (Wedderburn's Theorem over $\mathbb{C}$).

In the later part of the course we apply the developed material to group algebras, and classify when group algebras are semisimple (Maschke's Theorem). Moreover, we introduce the concept of a character of a group and study its properties, in particular how these are reflected in the structure of the group itself.

Learning Outcomes

Students will have a sound knowledge of the theory of non-commutative rings, ideals, associative algebras, modules, simple modules and semisimple algebras and will be familiar with examples. They will appreciate important results in the course such as the Jordan–Hölder Theorem, Schur's Lemma, and the Wedderburn Theorem. They will be familiar with the classification of semisimple algebras over $\mathbb{C}$ and be able to apply this. They will be familiar with the character theory of finite groups and they will have developed a toolkit to calculate with characters.

Synopsis

Noncommutative rings, one- and two-sided ideals. Associative algebras (over fields). Main examples: matrix algebras, polynomial rings and quotients of polynomial rings. Group algebras, representations of groups.

Modules and their relationship with representations. Simple and semisimple modules, composition series of a module, Jordan-Hölder Theorem. Semisimple algebras. Schur's Lemma, the Wedderburn Theorem, Maschke's Theorem.

Characters of finite groups. Orthogonality relations for irreducible characters. Character tables. Examples and applications.

Reading

K. Erdmann, *B2 Algebras*, Mathematical Institute Notes (2007).

G. D. James and M. Liebeck, *Representations and Characters of Finite Groups* (2nd edition, Cambridge University Press, 2001).

Further Reading

J. L. Alperin and R. B. Bell, *Groups and Representations,* Graduate Texts in Mathematics 162 (Springer-Verlag, 1995).

P. M. Cohn, *Classic Algebra* (Wiley & Sons, 2000). (Several books by this author available.)

C. W. Curtis, and I. Reiner, *Representation Theory of Finite Groups and Associative Algebras* (Wiley & Sons, 1962).

L. Dornhoff, *Group Representation Theory* (Marcel Dekker Inc., New York, 1972).

I. M. Isaacs, *Character Theory of Finite Groups* (AMS Chelsea Publishing, American Mathematical Society, Providence, Rhode Island, 2006).

J.-P. Serre, *Linear Representations of Finite Groups,* Graduate Texts in Mathematics 42 (Springer-Verlag, 1977).

2.2.2 B2b: Group Theory — Dr Grabowski — 16 HT

[Option B2b if taken as a half-unit. OSS paper code to follow.]

Overview

The second half turns to group theory, taking the concepts introduced in the first and second year and using them to prove results that allow us to probe the structure of groups. Up to now, you have mostly focussed on important basic examples, such as symmetric groups, but we will be able to deduce key properties of a group from as little information as just the prime decomposition of its order. The point we look towards is the classification of the finite simple groups. These groups are the building blocks from which all finite groups are made, in a strong sense that we will describe. Unfortunately, even the statement of the theorem is beyond this course but we can see how the programme to classify these groups begins. For example, by the end we will have shown that there are no non-Abelian simple groups of order strictly less than 60 but we will also have identified two infinite classes of non-Abelian simple groups, namely the alternating groups of degree five or more and the projective special linear groups over finite fields.

Learning Outcomes

Students will be able to state and prove the classification of finite Abelian groups and state the corresponding result for finitely generated Abelian groups. Additionally they will be able to state and prove some of the classic theorems of finite group theory including Cauchy's Theorem, Sylow's Theorems and the Jordan–Hölder Theorem.

By applying the techniques of the course they will be able to identify whether or not there can be a simple group of a given order. By the completion of this course we will be able to identify two infinite classes of finite simple groups and prove the non-existence of simple groups when the order of the group has certain forms (e.g. $|G|$ of the form $2p$, a product of two primes, a proper prime power etc.).

Synopsis

We will start with the "simplest" groups, the finite Abelian groups, and classify these. We will mostly concentrate on finite groups but much of what we do has relevance for infinite groups too. In particular, we will also classify the finitely generated Abelian groups.

Next we focus on the role of the primes in group theory, introducing Cauchy's Theorem on the existence of elements of prime order, which acts as a partial converse to Lagrange's Theorem. It is natural to study groups of prime power order: these groups are called p-groups and we will prove Sylow's theorems about p-subgroups of groups.

We then turn to some important tools that describe the "building blocks" of groups, namely composition series and the Jordan-Hölder Theorem. These naturally lead to the definition and study of soluble groups. As well as knowing about building blocks, one also needs to know how to put these together and so we study extensions and semi-direct products.

Finally, we examine groups of "small" order—and we really do mean small! We will identify all the possible groups (up to isomorphism) of order strictly less than 16 and, as mentioned above, rule out the possibility of non-Abelian simple groups with order taking certain forms. We will show that there are actually no non-Abelian simple groups of order strictly less than 60 but also prove that the alternating groups A_n are examples when the degree n is five or more. We will also study another family of simple groups, the projective special linear groups, which arise as matrix groups over finite fields.

Reading

1. Geoff Smith and Olga Tabachnikova. *Topics in Group Theory,* Springer Undergraduate Mathematics Series (Springer–Verlag, 2000). ISBN 1-85233-235-2.

Further Reading

There are many books on group theory available in Oxford libraries or to buy. Any of the books below will provide an alternative perspective to the recommended text and you will find others not on this list that are equally suitable.

2. John F. Humphreys. *A Course in Group Theory* (Oxford University Press, 1996). ISBN 0-19-853459-0.

3. Joseph J. Rotman. *An Introduction to the Theory of Groups,* Graduate Texts in Mathematics 148 (Fourth edition, Springer–Verlag, 1995). ISBN 3-540-94285-8.

4. W. Ledermann. *Introduction to Group Theory* (Longman (Oliver & Boyd), 1973). ISBN 0-582-44180-3. (with A.J. Wier, Second edition. Longman, 1996. ISBN 0-582-25954-1)

5. J. I. Alperin and Rowen B. Bell. *Groups and Representations,* Graduate Texts in Mathematics 162 (Springer–Verlag, 1995). ISBN 0-387-94526-1.

2.3 B3: Geometry

Level: H-level **Method of Assessment:** Written examination.

Weight: Whole-unit (OSS paper code 2642), or can be taken as either a half-unit in Geometry of Surfaces or a half-unit in Algebraic Curves (but see "Prerequisites").

Recommended Prerequisites: 2nd year core algebra and analysis, 2nd year topology. Multivariable calculus and groups in action would be useful but not essential. Also, B3a is helpful, but not essential, for B3b.

2.3.1 B3a: Geometry of Surfaces — Dr Dancer — 16 MT

[Option B3a if taken as a half-unit. OSS paper code 2A42.]

Overview

Different ways of thinking about surfaces (also called two-dimensional manifolds) are introduced in this course: first topological surfaces and then surfaces with extra structures which allow us to make sense of differentiable functions ('smooth surfaces'), holomorphic functions ('Riemann surfaces') and the measurement of lengths and areas ('Riemannian 2-manifolds').

These geometric structures interact in a fundamental way with the topology of the surfaces. A striking example of this is given by the Euler number, which is a manifestly topological quantity, but can be related to the total curvature, which at first glance depends on the geometry of the surface.

The course ends with an introduction to hyperbolic surfaces modelled on the hyperbolic plane, which gives us an example of a non-Euclidean geometry (that is, a geometry which meets all Euclid's axioms except the axioms of parallels).

Learning Outcomes

Students will be able to implement the classification of surfaces for simple constructions of topological surfaces such as planar models and connected sums; be able to relate the Euler characteristic to branching data for simple maps of Riemann surfaces; be able to describe the definition and use of Gaussian curvature; know the geodesics and isometries of the hyperbolic plane and their use in geometrical constructions.

Synopsis

The concept of a topological surface (or 2-manifold); examples, including polygons with pairs of sides identified. Orientation and the Euler characteristic. Classification theorem for compact surfaces (the proof will not be examined).

Riemann surfaces; examples, including the Riemann sphere, the quotient of the complex numbers by a lattice, and double coverings of the Riemann sphere. Holomorphic maps of Riemann surfaces and the Riemann–Hurwitz formula. Elliptic functions.

Smooth surfaces in Eucliean three-space and their first fundamental forms. The concept of a Riemannian 2-manifold; isometries; Gaussian curvature.

Geodesics. The Gauss–Bonnet Theorem (statement of local version and deduction of global version). Critical points of real-valued functions on compact surfaces.

The hyperbolic plane, its isometries and geodesics. Compact hyperbolic surfaces as Riemann surfaces and as surfaces of constant negative curvature.

Reading

1. A. Pressley, *Elementary Differential Geometry,* Springer Undergraduate Mathematics Series (Springer-Verlag, 2001). (Chapters 4–8 and 10–11.)
2. G. B. Segal, *Geometry of Surfaces*, Mathematical Institute Notes (1989).
3. R. Earl, *The Local Theory of Curves and Surfaces*, Mathematical Institute Notes (1999).
4. J. McCleary, *Geometry from a Differentiable Viewpoint* (Cambridge, 1997).

Further Reading

1. P. A. Firby and C. E. Gardiner, *Surface Topology* (Ellis Horwood, 1991) (Chapters 1–4 and 7).
2. F. Kirwan, *Complex Algebraic Curves,* Student Texts 23 (London Mathematical Society, Cambridge, 1992) (Chapter 5.2 only).
3. B. O'Neill, *Elementary Differential Geometry* (Academic Press, 1997).

2.3.2 B3b: Algebraic Curves — Prof. Hitchin — 16 HT

[Option **B3b** if taken as a half-unit. OSS paper code 2B42.]

Overview

A real algebraic curve is a subset of the plane defined by a polynomial equation $p(x, y) = 0$. The intersection properties of a pair of curves are much better behaved if we extend this picture in two ways: the first is to use polynomials with complex coefficients, the second to extend the curve into the projective plane. In this course projective algebraic curves are studied, using ideas from algebra, from the geometry of surfaces and from complex analysis.

Learning Outcomes

Students will know the concepts of projective space and curves in the projective plane. They will appreciate the notion of nonsingularity and know some basic features of intersection theory. They will view nonsingular algebraic curves as examples of Riemann surfaces, and be familiar with divisors, meromorphic functions and differentials.

Synopsis

Projective spaces, homogeneous coordinates, projective transformations.

Algebraic curves in the complex projective plane. Euler's relation. Irreducibility, singular and nonsingular points, tangent lines.

Bezout's Theorem (the proof will not be examined). Points of inflection, and normal form of a nonsingular cubic.

Nonsingular algebraic curves as Riemann surfaces. Meromorphic functions, divisors, linear equivalence. Differentials and canonical divisors. The group law on a nonsingular cubic.

The Riemann-Roch Theorem (the proof will not be examined). The geometric genus. Applications.

Reading

1. F. Kirwan, *Complex Algebraic Curves,* Student Texts 23 (London Mathematical Society, Cambridge, 1992), Chapters 2–6.

2.4 B4: Analysis

Level: H-level **Method of Assessment:** Written Examination.

Weight: Whole-unit (OSS paper code 2643), or B4a may be taken as a half-unit.

Recommended Prerequisites: For B4a, Part A Topology is desirable and Integration is useful.

For the whole-unit, Topology is desirable and Integration is highly desirable.

Overview

The two most important kinds of infinite-dimensional vector space are Banach spaces and Hilbert spaces; they provide the theoretical underpinnings for much of differential equations, and also for quantum theory in physics. This course provides an introduction to Banach spaces and Hilbert spaces. It combines familiar ideas from topology and linear algebra. It would be useful background for further work in analysis, differential equations, and so on.

2.4.1 B4a: Banach Spaces — Dr Edwards — 16 MT

[Option **B4a** if taken as a half-unit. OSS paper code 2A43.]

Learning Outcome

Students will have a firm knowledge of real and complex normed vector spaces, with their geometric and topological properties. They will be familiar with the notions of completeness, separability and density, will know the properties of a Banach space and its important examples, and will be able to prove results relating to the Hahn–Banach Theorem. They will have developed an understanding of the theory of bounded linear operators on a Banach space.

Synopses

Real and complex normed vector spaces, their geometry and topology. Completeness. Banach spaces, examples (ℓ^p, ℓ^∞, L^p, $C(K)$, spaces of differentiable functions).

Finite-dimensional normed spaces; equivalence of norms and completeness. Separable spaces; separability of subspaces.

Continuous linear functionals. Dual spaces. Hahn–Banach Theorem (proof for real separable spaces only) and applications, including density of subspaces.

Bounded linear operators, examples (including integral operators). Adjoint operators. Spectrum and resolvent. Spectral mapping theorem for polynomials.

Essential Reading

1. E. Kreyszig, *Introductory Functional Analysis with Applications* (Wiley, revised edition, 1989), Chs 2, 4.2–4.3, 4.5, 7.1–7.4.

Further Reading

1. G. F. Vincent-Smith, *B4: Analysis,* Mathematical Institute Notes (1991), Chs 1, 2, 5.2.

2.4.2 B4b: Hilbert Spaces — Prof. Joyce — 16 HT

Learning Outcome

Students will have a demonstrable knowledge of the properties of a Hilbert space, including orthogonal complements, orthonormal sets, complete orthonormal sets together with related identities and inequalities. They will know and be able to use important orthogonal expansions and will know the theory of Classical Fourier series and their convergence. They will be familiar with the theory of linear operators on a Hilbert space, including adjoint operators, self-adjoint and unitary operators with their spectra.

Synopses

Hilbert spaces; examples including L^2-spaces. Orthogonality, orthogonal complement, closed subspaces, projection theorem. Riesz Representation Theorem.

Orthonormal sets, Pythagoras, Bessel's inequality. Complete orthonormal sets, Parseval.

Orthogonal expansions, examples (Legendre, Laguerre, Hermite etc.) Classical Fourier series: Riemann–Lebesgue Lemma; Dirichlet kernel, pointwise convergence of Fourier series, Dini's test, Fejér's Theorem, Weierstrass' Approximation Theorem. Completeness of trigonometric system.

Linear operators on Hilbert space, adjoint operators. Self-adjoint operators, orthogonal projections, unitary operators, and their spectra.

Reading

Essential

1. E. Kreyszig, *Introductory Functional Analysis with Applications* (Wiley, revised edition, 1989), Ch 3.
2. N. Young, *An Introduction to Hilbert Space* (CUP, 1988), Chs 1–7.

Further

1. G. F. Vincent-Smith, *B4 Analysis*, Mathematical Institute Notes (1991), Chs 3, 4.
2. H. A. Priestley, *Introduction to Integration* (OUP, 1997), Chs 28–32.
3. A. Vretblad, *Fourier Analysis and its Applications* (Springer, 2003).

B568a Introduction to Applied Mathematics — Dr Porter — 6MT

Overview

In week one of Michaelmas Term, an introductory course of six lectures will be provided, which is a compulsory prerequisite for all students doing any of the courses in B5, B6 or B8. This courselet will cover basic material which is common to all these courses, and it will be assumed that students taking any of these courses have attended B568a. The courses B5a, B6a and B8a will consequently contain 14 lectures, and will begin in second week. There will be one problem sheet associated with this course, and it is anticipated that written solutions will be provided. **No separate class will be scheduled.**

Learning Outcomes

Acquired the background knowledge to prepare them for applied mathematics options.

Synopsis

Modelling and conservation laws.

Scaling and non-dimensionalisation.

Asymptotic sequences. Regular and singular perturbation methods for algebraic equations. Simple boundary layer theory.

Reading

1. S. D. Howison, *Practical Applied Mathematics: Modelling, Analysis, Approximation* (CUP, Cambridge, 2005). Chs. 1,2,3,13,16.

2.5 B5 Differential Equations and Applications

Level: H-level **Method of Assessment:** Written Examination.

Weight: Whole-unit (OSS paper code 2644), or can be taken as either a half-unit in Techniques of Applied Mathematics or a half-unit in Applied Partial Differential Equations.

Recommended Prerequisites: Calculus of Variations and Fluid Mechanics from Part A are desirable but not essential. The introductory Michaelmas Term course B568a is a prerequisite for both parts of the course, and the material in that course will be assumed to be known.

2.5.1 B5a: Techniques of Applied Mathematics — Prof. Howison — 14 MT

[Option **B5a** if taken as a half-unit. OSS paper code 2A44.]

Overview

This course develops mathematical techniques which are useful in solving 'real-world' problems involving differential equations, and is a development of ideas which arise in the second year differential equations course. The course aims to show in a practical way how equations 'work', what kinds of solution behaviours can occur, and some techniques which are useful in their solution.

Learning Outcomes

Students will know how differential equations can be used to model real-world phenomena and be able to describe the behaviour of the types of solutions that can occur. They will be familiar with the hysteresis and stability of ODEs and be able to solve Sturm-Liouville systems. They will develop the theory of PDEs, for example to model shocks and know similarity solutions.

Synopsis

Nonlinear oscillations. Multiple scale methods.

Ordinary differential equations: hysteresis and stability.

Sturm–Liouville systems, comparison methods. Integral equations and eigenfunctions.

Partial differential equations: shocks, similarity solutions.

Reading

1. A. C. Fowler, *Techniques of Applied Mathematics*, Mathematical Institute Notes (2005).
2. J. P. Keener, *Principles of Applied Mathematics: Transformation and Approximation* (revised edition, Perseus Books, Cambridge, Mass., 2000).
3. E. J. Hinch, *Perturbation Methods* (CUP, Cambridge, 1991).
4. J. R. Ockendon, S. D. Howison, A. A. Lacey and A. B. Movchan, *Applied Partial Differential Equations* (revised edition, OUP, Oxford, 2003).
5. R. Haberman, *Mathematical Models* (SIAM, Philadelphia, 1998).
6. S. D. Howison, *Practical Applied Mathematics: Modelling, Analysis, Approximation* (CUP, Cambridge, 2005).

2.5.2 B5b: Applied Partial Differential Equations — Dr Norbury — 16 HT

[Option **B5b** if taken as a half-unit. OSS paper code 2B44.]

Overview

This course continues the Part A Differential Equations course, and extends some of the techniques of B5a, to partial differential equations. In particular, general nonlinear first-order partial differential equations are solved, the classification of second-order partial differential equations is extended to systems, with hyperbolic systems being solved by characteristic variables. Then Green's function, maximum principle and similarity variable methods are demonstrated for partial differential equations, together with eigenfunction expansions.

Learning Outcomes

Students will know a range of techniques to solve PDEs including non-linear first-order and second-order and their classification. They will be able to demonstrate various principles for solving PDEs including Green's function, maximum principle and eigenfunctions.

Synopsis

Charpit's equations; eikonal equation.

Systems of partial differential equations, characteristics. Weak solutions. Riemann's function.

Maximum principles, comparison methods, well-posed problems, and Green's functions for the heat equation and for Laplace's equation.

Delta functions. Eigenfunction expansions.

Reading

1. Dr Norbury's web notes.
2. Institute lecture notes are now available (JN).
3. M. Renardy and R.C. Rogers, *An Introduction to Partial Differential Equations* (Springer–Verlag, New York, 2004).
4. J. P. Keener, *Principles of Applied Mathematics: Transformation and Approximation* (revised edition, Perseus Books, Cambridge, Mass., 2000).
5. J. R. Ockendon, S. D. Howison, A. A. Lacey and A. B. Movchan, *Applied Partial Differential Equations* (revised edition, OUP, Oxford, 2003).

2.6 B6 Theoretical Mechanics

Level: H-level **Method of Assessment:** Written Examination.

Weight: Whole-unit (OSS paper code 2645), or can be taken as either a half-unit in Viscous Flow, or a half-unit in Waves and Compressible Flow.

Recommended Prerequisites: The Part A (second-year) course 'Fluid Dynamics and Waves'. Though each half-unit is intended to stand alone, they will complement each other. This course combines well with B5 Differential Equations and Applications. The introductory Michaelmas Term course B568a is a prerequisite for both parts of the course, and the material in that course will be assumed to be known.

2.6.1 B6a: Viscous Flow — Dr Oliver — 14 MT

[Option **B6a** if taken as a half-unit. OSS paper code 2A45.]

Overview

Viscous fluids are important in so many facets of everyday life that everyone has some intuition about the diverse flow phenomena that occur in practice. This course is distinctive in that it shows how quite advanced mathematical ideas such as asymptotics and partial differential equation theory can be used to analyse the underlying differential equations and hence give scientific understanding about flows of practical importance, such as air flow round wings and flow in oil reservoirs.

Learning Outcomes

Students will have developed an appreciation of diverse flow phenomena in various mediums including Poiseuille flow, Rayleigh flow, airflow around wings and flow in oil reservoirs. They will have a demonstrable knowledge of the mathematical theory necessary to analyse such phenomena.

Synopsis

Derivation of Navier–Stokes equations for an incompressible Newtonian fluid. Vorticity. Energy equation and dissipation. Exact solutions for unidirectional flows; shear flow, Poiseuille flow, Rayleigh flow. Dimensional analysis, Reynolds number. Derivation of equations for high and low Reynolds number flows.

Derivation of Prandtl's boundary-layer equations. Similarity solutions for flow past a semi-infinite flat plate and for jets. Discussion of separation and application to the theory of flight. Jeffery–Hamel flow.

Slow flow past a circular cylinder and a sphere. Non-uniformity of the two dimensional approximation; Oseen's equation. Lubrication theory: bearings, thin films and Hele–Shaw cell. Flow in a porous medium. Stability and the transition to turbulence.

Reading

1. D. J. Acheson, *Elementary Fluid Dynamics* (OUP, 1990), Chs 2, 6, 7, 8.
2. H. Ockendon and J. R. Ockendon, *Viscous Flow* (Cambridge Texts in Applied Mathematics, 1995).
3. M. E. O'Neill and F. Chorlton, *Viscous and Compressible Fluid Dynamics* (Ellis Horwood, 1989), Chs 2, 3, 4.1–4.3, 4.19–4.20, 4.22–4.24, 5.1–5.2, 5.6.

2.6.2 B6b: Waves and Compressible Flow — t.b.c. — 16 HT

[Option **B6b** if taken as a half-unit. OSS paper code 2B45.]

Overview

Propagating disturbances, or waves, occur frequently in applied mathematics. This course will be centred on some prototypical examples from fluid dynamics, the two most familiar being surface gravity waves and waves in gases. The models for compressible flow will be derived and then analysed for small amplitude motion. This will shed light on the important phenomena of dispersion, group velocity and resonance, and the differences between supersonic and subsonic flow, as well as revealing the crucial dependence of the waves on the number of space dimensions.

Larger amplitude motion of liquids and gases will be described by incorporating non-linear effects, and the theory of characteristics for partial differential equations will be applied to understand the shock waves associated with supersonic flight.

Learning Outcomes

Students will have developed a sound knowledge of a range of mathematical models used to study waves (both linear and non-linear), will be able to describe examples of waves from fluid dynamics and will have analysed a model for compressible flow. They will have an awareness of shock waves and how the theory of characteristics for PDEs can be applied to study those associated with supersonic flight.

Synopsis

1–2 Equations of inviscid compressible flow including flow relative to rotating axes.

3–6 Models for linear wave propagation including Stokes waves, Inertial waves, Rossby waves and simple solutions.

7–10 Theories for Linear waves: Fourier Series, Fourier integrals, method of stationary phase, dispersion and group velocity. Flow past thin wings, Huyghens principle.

11–12 Nonlinear Waves: method of characteristics, simple wave flows applied to one-dimensional unsteady gas flow and shallow water theory.

13–16 Shock Waves: weak solutions, Rankine–Hugoniot relations, oblique shocks, bores and hydraulic jumps.

Reading

1. H. Ockendon and J. R. Ockendon, *Waves and Compressible Flow* (Springer, 2004).
2. J. R. Ockendon, S. D. Howison, A. A. Lacey and A. B. Movchan, *Applied Partial Differential Equations* (revised edition, OUP, Oxford, 2003). Chs 2.5, 4.5–7.
3. D. J. Acheson, *Elementary Fluid Dynamics* (OUP, 1990). Ch 3
4. J. Billingham and A. C. King, *Wave Motion* (CUP, 2000). Ch 1–4, 7,8.

Background Reading

1. M. J. Lighthill, *Waves in Fluids* (CUP, 1978).
2. G. B. Whitham, *Linear and Nonlinear Waves* (Wiley, 1973).

2.7 B7.1/C7.1: Quantum Mechanics; Quantum Theory and Quantum Computers

Level: H-level/M-level **Method of Assessment:** Written Examination. The rubric for the whole-unit is as for a Part B examination.

Weight: Whole-unit. B7.1a could be taken as a free-standing half-unit but C7.1b could not.

2.7.1 B7.1a: Quantum Mechanics — Dr Hannabuss — 16 MT

[Option **B7.1a** if taken as a half-unit. OSS paper code 2A86.]

Level: H-level

Recommended Prerequisites: Calculus of Variations. Classical Mechanics would be useful, but not essential.

Overview

Quantum theory was born out of the attempt to understand the interactions between matter and radiation. It transpired that light waves can behave like streams of particles, but other particles also have wave-like properties. Although there remain deep mathematical and physical questions at the frontiers of the subject, the resulting theory encompasses not just the mechanical but also the electrical and chemical properties of matter. Many of the key components of modern technology such as transistors and lasers were developed using quantum theory, and the theory has stimulated important 20th century advances in pure mathematics in, for example, functional analysis, algebra, and differential geometry. In spite of their revolutionary impact and central importance, the basic mathematical ideas are easily accessible and provide fresh and surprising applications of the mathematical techniques encountered in other branches of mathematics.

This introductory course explores some of the consequences of this, including a treatment of the hydrogen atom.

Learning Outcomes

Students will have gained a sound knowledge of the mathematical ideas related to the development of quantum theory. They will be able to apply mathematical techniques from earlier courses to a range of examples in quantum mechanics.

Synopsis

Generalised momenta, the Hamiltonian, Hamilton's equations of motion, Poisson brackets. De Broglie waves, the Schrödinger equation; stationary states, quantum states of a particle in a box; interpretation of the wave function, probability density and current. Boundary conditions; conservation of current, tunnelling, parity.

Expectation values of observables, eigenvalues and eigenfunctions.

The one-dimensional harmonic oscillator, higher-dimensional oscillators and normal modes.

The rotationally symmetric and general radial states of the hydrogen atom with fixed nucleus.

The mathematical structure of quantum mechanics. Commutation relations, Poisson brackets and Dirac's quantisation scheme.

Heisenberg's uncertainty principle. Creation and annihilation operators for the harmonic oscillator.

Measurements and the interpretation of quantum mechanics. Schrödinger's cat.

Angular momentum, commutation relations, spectrum and matrix representation. Orbital angular momentum, rotational symmetry and spin-$\frac{1}{2}$ particles. Application to a particle in a central potential and the hydrogen atom.

Reading

1. K. C. Hannabuss, *Introduction to Quantum Mechanics* (OUP, 1997). Chapters 1–4, 6–8.

Further reading

1. A popular non-technical account of the subject:

   A. Hey and P. Walters, *The New Quantum Universe* (Cambridge, 2003).

2. Also designed for a similar Oxford course:

   I. P. Grant, *Classical and Quantum Mechanics*, Mathematical Institute Notes (1991).

3. A classical account of the subject which goes well beyond this course:

   L. I. Schiff, *Quantum Mechanics* (3rd edition, Mc Graw Hill, 1968).

4. Some other books covering similar material:

   B. J. Brandsen and C. J. Joachain, *Introduction to Quantum Mechanics* (Longman, 1995).

   A. I. M. Rae, *Quantum Mechanics* (4th edition, Institute of Physics, 1993).

2.7.2 C7.1b: Quantum Theory and Quantum Computers — Dr Hannabuss — 16HT — M-Level

Level: M-level **Method of Assessment:** Written Examination

Weight: Whole-unit (cannot be taken unless B7.1a is taken). OSS paper code 2686.

Prerequisites: B7.1a Quantum Mechanics.

Overview

This course builds directly on the first course in quantum mechanics and covers a series of important topics, particularly features of systems containing several particles. The behaviour of identical particles in quantum theory is more subtle than in classical mechanics, and an understanding of these features allows one to understand the periodic table of elements and the rigidity of matter. It also introduces a new property of entanglement linking particles which can be quite widely dispersed.

There are rarely neat solutions to problems involving several particles, so usually one needs some approximation methods. In very complicated systems, such as the molecules of gas in a container, quantum mechanical uncertainty is compounded by ignorance about other details of the system and requires tools of quantum statistical mechanics.

Two state quantum systems enable one to encode binary information in a new way which permits superpositions. This leads to a quantum theory of information processing, and by exploiting entanglement to other ideas such as quantum teleportation.

Learning Outcomes

Students will be able to demonstrate knowledge and understanding of quantum mechanics of many particle systems, statistics, entanglement, and applications to quantum computing.

Synopsis

Identical particles, symmetric and anti-symmetric states, Fermi–Dirac and Bose–Einstein statistics and atomic structure.

Heisenberg representation, interaction representation, time dependent perturbation theory and Feynman–Dyson expansion. Approximation methods, Rayleigh–Schrödinger time-independent perturbation theory and variation principles. The Virial Theorem. The ground state of helium.

Entanglement. The EPR paradox, Bell's inequalities, Aspect's experiment. GHZ states.

Mixed states, density operators. The example of spin systems. Purification. Gibbs states and the KMS condition.

Quantum information processing, qubits and quantum computing. The no-cloning theorem, quantum teleportation. Quantum logic gates. Schmidt decomposition. Positive operator-valued measures. The quantum Fourier transform. Shor's factorisation algorithm.

Reading

1. K. C. Hannabuss, *Introduction to Quantum Mechanics* (OUP, 1997). Chapters 10–12 and 14, 16, supplemented by lecture notes on quantum computers on the web.

Further reading

1. A popular non-technical account of the subject:

   A. Hey and P. Walters, *The New Quantum Universe* (Cambridge, 2003).

2. Also designed for an Oxford course, though only covering some material:

   I. P. Grant, *Classical and Quantum Mechanics*, Mathematical Institute Notes (1991).

3. A concise account of quantum information theory:

   S. Stenholm and K.-A. Suominen, *Quantum Approach to Informatics* (Wiley, 2005).

4. An encyclopaedic account of quantum computing:

   M. A. Nielsen and I. L. Chuang, *Quantum Computation* (Cambridge University Press, 2000).

5. Even more paradoxes can be found in:

   Y. Aharanov and D. Rohrlich, *Quantum Paradoxes* (Wiley–VCH, 2005).

6. Those who read German can find further material on entanglement in:

   J. Audretsch, *Verschränkte Systeme* (Wiley–VCH, 2005).

7. Other accounts of the first part of the course:

   L. I. Schiff, *Quantum Mechanics* (3rd edition, Mc Graw Hill, 1968).

   B. J. Bransden and C. J. Joachain, *Introduction to Quantum Mechanics* (Longman, 1995).

   A. I. M. Rae, *Quantum Mechanics* (4th edition, Institute of Physics, 1993).

   John Preskill's on-line lecture notes (http://www.theory.caltech.edu/ preskill/ph219/index.html).

2.8 B7.2/C7.2: Relativity

Level: H-level/M-level **Method of Assessment:** Written Examination.
The rubric for the whole-unit is as for a Part B examination.

Weight: Whole-unit, or B7.2a can be taken as a half-unit (but C7.2b cannot).

Recommended Prerequisites: For B7.2a — Part A Electromagnetism, for C7.2b — B7.2a

2.8.1 B7.2a: Special Relativity and Electromagnetism — Dr de la Ossa — 16 MT

Level: H-level **Method of Assessment:** Written Examination.

[Options **B7.2a** if taken as a half-unit. OSS paper code 2A87.]

Recommended Prerequisites: Part A Electromagnetism.

Overview

Maxwell's electromagnetic theory revealed light to be an electromagnetic phenomenon whose speed of propagation proved to be observer-independent. This discovery led to the overthrow of classical Newtonian mechanics, in which space and time were absolute, and its replacement by Special Relativity and space-time. The aim of this course is to study Einstein's special theory of relativity, to understand space-time, and to incorporate into it

Maxwell's electrodynamics. These theories together with quantum theory are essential for an understanding of modern physics.

Synopsis

Constancy of the speed of light; Lorentz transformations and the invariance of the wave operator; time dilation, length contraction and the relativistic Doppler effect; the resolution of the simple 'paradoxes' of relativity. Four-vectors; four-velocity and four-momentum; equivalence of mass and energy; particle collisions and four-momentum conservation; four-acceleration and four-force; the example of the constant-acceleration world-line. Contravariant and covariant vectors and tensors; index notation.

Solving Poisson's equation and the wave-equation with sources. Derivation of Maxwell's equations with sources from a variational principle.

Electromagnetism in four-dimensional form; the electromagnetic field tensor; the transformation law for the electric and magnetic fields; the Lorentz four-force law; the electromagnetic four-potential and the energy-momentum tensor.

Reading

The preferred text is:
N. M. J. Woodhouse, *Special Relativity* (Springer, 2002).

N. M. J. Woodhouse, *General Relativity* (Springer, 2006).

An alternative is:
W. Rindler, *Introduction to Special Relativity* (2nd edition, OUP 1991).

Additional Reading

For the experimental background to special relativity, and in many libraries:
A. P. French, *Special Relativity* (MIT Introductory Physics Series, Nelson Thornes, 1971).

For advanced texts on electromagnetism, see:
W. J. Duffin, *Advanced Electricity and Magnetism* (McGraw–Hill, 1968).
J. D. Jackson, *Classical Electromagnetism* (Wiley, 1962).

2.8.2 C7.2b: General Relativity I — Prof. Chruściel — 16HT — M-Level

Level: M-Level **Method of Assessment:** Written examination.

Weight: Whole-unit, cannot be taken unless B7.2a is taken. OSS paper code 2687.

Prerequisites: B7.2a Relativity and Electromagnetism.

Overview

The course is intended as an elementary introduction to general relativity, the basic physical concepts of its observational implications, the new insights that it provides into the nature of space-time, and the structure of the universe. Familiarity with special relativity and electromagnetism as covered in B7.2a will be assumed. The lectures will review Newtonian gravitation, tensor calculus and continuum physics in special relativity, physics in curved space-time and the Einstein field equations. This will suffice for an account of simple applications to planetary motion, the bending of light and the existence of black holes.

This course starts by asking how the theory of gravitation can be made consistent with the special-relativistic framework. Physical considerations (the principle of equivalence, general covariance) are used to motivate and illustrate the mathematical machinery of tensor calculus. The technical development is kept as elementary as possible, emphasising the use of local inertial frames. A similar elementary motivation is given for Einstein's equations and the Schwarzschild solution. Orbits in the Schwarzschild solution are given a unified treatment which allows a simple account of the three classical tests of Einstein's theory. Finally, the analysis of extensions of the Schwarzschild solution show how the theory of black holes emerges and exposes the radical consequences of Einstein's theory for space-time structure. Cosmological solutions are not discussed.

Learning Outcomes

Students will have developed a knowledge and appreciation of the ideas and concepts described above.

Synopsis

Review of Newtonian gravitation theory and problems of constructing a relativistic generalisation. Review of Special Relativity. The equivalence principle. Tensor formulation of special relativity (including general particle motion, tensor form of Maxwell's equations and the energy momentum-tensor of dust). Curved space time. Local inertial coordinates. General coordinate transformations, elements of Riemannian geometry (including connections, curvature and geodesic deviation). Mathematical formulation of General Relativity, Einstein's equations (properties of the energy-momentum tensor will be needed in the case of dust only). The Schwarzschild solution; planetary motion, the bending of light, and black holes.

Reading

1. L. P. Hughston and K.P. Tod, *An Introduction to General Relativity* (LMS Student Text 5, CUP, 1990), Chs 1–18.

2. N. M. J. Woodhouse, *Notes on Special Relativity*, Mathematical Institute Notes. Revised edition; published in a revised form as *Special Relativity, Lecture notes in Physics m6* (Springer–Verlag, 1992), Chs 1–7.

Further Reading

1. B. Schutz, *A First Course in General Relativity* (CUP, 1990).
2. R. M. Wald, *General Relativity* (Chicago, 1984).
3. W. Rindler, *Essential Relativity* (Springer–Verlag, 2nd edition, 1990).

2.9 B8 Topics in Applied Mathematics

Level: H-level **Method of Assessment:** Written Examination

Weight: Whole-unit (OSS paper code 2647), or can be taken as either a half-unit in Nonlinear Systems or a half-unit in Mathematical Ecology and Biology.

Recommended Prerequisites: Part A core material (especially differential equations). The introductory Michaelmas Term course B568a is a prerequisite for both parts of the course, and the material in that course will be assumed to be known.

2.9.1 B8a: Mathematical Ecology and Biology — Dr Gaffney — 14 MT

[Option **B8a** if taken as a half-unit. OSS paper code 2A47.]

Overview

Mathematical Ecology and Biology introduces the applied mathematician to practical applications in an area that is growing very rapidly. The course mainly focusses on situations where continuous models are appropriate and where these may be modelled by deterministic ordinary and partial differential equations. By using particular modelling examples in ecology, chemistry, biology, physiology and epidemiology, the course demonstrates how various applied mathematical techniques, such as those describing linear stability, phase planes, singular perturbation and travelling waves, can yield important information about the behaviour of complex models.

Learning Outcomes

Students will have developed a sound knowledge and appreciation of the ideas and concepts related to modelling biological and ecological systems using ordinary and partial differential equations.

Synopsis

Continuous and discrete population models for a single species, including Ludwig's 1978 insect outbreak model for spruce budworm and hysterisis. Harvesting and strategies for sustainable fishing. Modelling interacting populations, including the Lotka-Volterra model for predator-prey (with application to hare-lynx interactions), and Okubo's 1989 model for red-grey squirrel competition. Principle of competitive exclusion.

Epidemic models.

Michaelis-Menten model for enzyme-substrate kinetics.

Excitable systems. Threshold phenomena (nerve pulses).

Travelling wave propagation with biological examples.

Biological pattern formation. Turing's model for animal coat markings.

Nerve signal propagation.

Reading

J.D. Murray, *Mathematical Biology, Volume I: An Introduction (2002); Volume II: Spatial Models and Biomedical Applications (2003)* (3rd edition, Springer–Verlag).

1. Volume I: 1.1, 1.2, 1.6, 2.1–2.4, 3.1, 3.3–3.6, 3.8, 6.1–6.3, 6.5, 6.6, 8.1, 8.2, 8.4, 8.5, 10.1, 10.2, 11.1–11.5, 13.1–13.5, Appendix A.
2. Volume II: 1.6, 2, 3.1, 3.2, 5.1, 5.2, 13.1–13.4.

Further Reading

1. J. Keener and J. Sneyd, *Mathematical Physiology* (Springer, Berlin, 1998) 1.1, 1.2, 9.1, 9.2.
2. H. Meinhardt, *The Algorithmic Beauty of Sea Shells* (2nd enlarged edition, Springer, Berlin, 2000).

2.9.2 B8b: Nonlinear Systems — Dr Moroz — 16 HT

[Option **B8b** if taken as a half-unit. OSS paper code 2B47.]

Overview

This course aims to provide an introduction to the tools of dynamical systems theory, which are essential in the realistic modelling and study of many disciplines, including Mathematical Ecology and Biology, Fluid mechanics, Economics, Mechanics and Celestial Mechanics. The course will include the study of both deterministic ordinary differential equations, as well as non-linear difference equations, drawing examples from the various areas of application, whenever possible and appropriate. The course will include the use of numerical software involving Matlab in the homework exercises.

Learning Outcomes

Students will have developed a sound knowledge and appreciation of some of the tools and concepts used in the study of dynamical systems.

Synopsis

Bifurcations for Ordinary Differential Equations

Bifurcations for simple ordinary differential equations: saddle-node, transcritical, pitchfork, Hopf. Centre stable and unstable manifolds. Normal forms. The Hopf Bifurcation Theorem. Lyapunov functions. [6 lectures]

Bifurcations For Maps

Poincaré section and first-return maps. Brief review of multipliers, stability and periodic cycles. Elementary bifurcations of one-dimensional maps: saddle-node, transcritical, pitchfork, period-doubling. Two-dimensional maps. Hénon and Standard map. [6 lectures]

Chaos

Logistic map. Bernoulli shift map and symbolic dynamics. Smale Horseshoes. Lorenz equations. [4 lectures]

Reading

1. G. L. Baker and J. P. Gollub, *Chaotic Dynamics: An Introduction* (2nd ed., C.U.P., Cambridge, 1996).

2. P. G. Drazin, *Nonlinear Systems* (C.U.P., Cambridge, 1992).

2.10 B9: Number Theory

Level: H-level **Method of Assessment:** Written Examination

Weight: Whole-unit (OSS paper code 2648), or B9a can be taken as half-unit (but B9b cannot).

Recommended Prerequisites: All second-year algebra and arithmetic. Students who have not taken Part A Number Theory should read about quadratic residues in, for example, the appendix to Stewart and Tall. This will help with the examples.

2.10.1 B9a: Galois Theory —Dr Szendroi — 16 MT

[Option **B9a** if taken as a half-unit. OSS paper code 2A48.]

Overview

The course starts with a review of second-year ring theory with a particular emphasis on polynomial rings. We also discuss general integral domains and fields of fractions. This is followed by the classical theory of Galois field extensions, culminating in a discussion of some of the classical theorems in the subject: the insolubility of the general quintic and impossibility of certain ruler and compass constructions considered by the Ancient Greeks.

Learning Outcomes

Understanding of the relation between symmetries of roots of a polynomial and its solubility in terms of simple algebraic formulae; working knowledge of interesting group actions in a nontrivial context; working knowledge, with applications, of a nontrivial notion of finite group theory (soluble groups); understanding of the relation between algebraic properties of field extensions and geometric problems such as doubling the cube and squaring the circle.

Synopsis

Review of polynomial rings, factorisation, integral domains. Any nonzero homomorphism of fields is injective. Fields of fractions.

Review of group actions on sets, Gauss' Lemma and Eisenstein's criterion for irreducibility of polynomials, field extensions, degrees, the tower law. Symmetric polynomials.

Separable extensions. Splitting fields. The theorem of the primitive element. The existence and uniqueness of algebraic closure.

Groups of automorphisms, fixed fields. The fundamental theorem of Galois theory.

Examples: Kummer extensions, cyclotomic extensions, finite fields and the Frobenius automorphism. Techniques for calculating Galois groups.

Soluble groups. Solubility by radicals, solubility of polynomials of degree at most 4, insolubility of the general quintic, impossibility of some ruler and compass constructions.

Reading

1. J. Rotman, *Galois Theory* (Springer–Verlag, NY Inc, 2001/1990).
2. I. Stewart, *Galois Theory* (Chapman and Hall, 2003/1989)
3. D.J.H. Garling, *A Course in Galois Theory* (Cambridge University Press I.N., 1987).
4. Herstein, *Topics in Algebra* (Wiley, 1975)

2.10.2 B9b: Algebraic Number Theory — Prof. Heath-Brown — 16 HT

Overview

An introduction to algebraic number theory. The aim is to describe the properties of number fields, but particular emphasis in examples will be placed on quadratic fields, where it is easy to calculate explicitly the properties of some of the objects being considered. In such fields the familiar unique factorisation enjoyed by the integers may fail, and a key objective of the course is to introduce the class group which measures the failure of this property.

Learning Outcomes

Students will learn about the arithmetic of algebraic number fields. They will learn to prove theorems about integral bases, and about unique factorisation into ideals. They will learn to calculate class numbers, and to use the theory to solve simple Diophantine equations.

Synopsis

1. field extensions, minimum polynomial, algebraic numbers, conjugates, discriminants, Gaussian integers, algebraic integers, integral basis
2. examples: quadratic fields
3. norm of an algebraic number
4. existence of factorisation
5. factorisation in $\mathbb{Q}(\sqrt{d})$
6. ideals, $\mathbb{Z}$-basis, maximal ideals, prime ideals
7. unique factorisation theorem of ideals
8. relationship between factorisation of number and of ideals
9. norm of an ideal
10. ideal classes
11. statement of Minkowski convex body theorem
12. finiteness of class number
13. computations of class number to go on example sheets

Reading

1. I. Stewart and D. Tall, *Algebraic Number Theory* (Chapman and Hall Mathematics Series, May 1987).

Further Reading

1. D. Marcus, *Number Fields* (Springer–Verlag, New York–Heidelberg, 1977). ISBN 0-387-90279-1.

2.11 B10: Martingales and Financial Mathematics

Level: H-level **Method of Assessment:** Written Examination

Weight: Whole-unit (OSS paper code 2649), or can be taken as either a half-unit in B10a or a half-unit in B10b.

Recommended Prerequisites: For B10a, Part A Integration is a prerequisite, so that the corresponding material will be assumed to be known, and Topology is helpful but not essential. For B10b, Part A Probability is a prerequisite.

2.11.1 B10a: Martingales Through Measure Theory — Dr Tarres— 16 MT

[Option **B10a** if taken as a half-unit. OSS paper code 2A49.]

Overview

Probability theory arises in the modelling of a variety of systems where the understanding of the "unknown" plays a key role, such as population genetics in biology, market evolution in financial mathematics, and learning features in game theory. It is also very useful in various areas of mathematics, including number theory and partial differential equations. The course introduces the basic mathematical framework underlying its rigorous analysis, and is therefore meant to provide some of the tools which will be used in more advanced courses in probability.

The first part of the course provides a review of measure theory from Integration Part A, and develops a deeper framework for its study. Then we proceed to develop notions of conditional expectation, martingales, and to show limit results for the behaviour of these martingales which apply in a variety of contexts.

Learning Outcomes

The students will learn about measure theory, random variables, independence, expectation and conditional expectation, product measures and discrete-parameter martingales.

Synopsis

A branching-process example. Review of σ-algebras, measure spaces. Uniqueness of extension of π-systems and Carathéodory's Extension Theorem [both without proof], monotone-convergence properties of measures, $\limsup$ and $\liminf$ of a sequence of events, Fatou's Lemma, reverse Fatou Lemma, first Borel–Cantelli Lemma.

Random variables and their distribution functions, representation of a random variables with prescribed distribution function, σ-algebras generated by a collection of random variables. Independence of events, random variables and σ-algebras, π-systems criterion for independence, second Borel-Cantelli Lemma. The tail σ-algebra, Kolomogorov's 0–1 Law.

Integration and expectation, review of elementary properties of the integral and L^p spaces [done in Part A Integration for the Lebesgue measure on $\mathbb{R}$]. Scheffé's Lemma, Jensen's inequality, orthogonal projection in L^2. Product measures, [Fubini's Theorem, infinite products of probability triples]. The Kolmogorov Theorem and definition of conditional expectation, proof as least-squares-best predictor, elementary properties. [The Radon-Nikodym Theorem.]

Filtrations, martingales, stopping times, discrete stochastic integrals, Doob's Optional-Stopping Theorem, Doob's Upcrossing Lemma and "Forward" Convergence Theorem, martingales bounded in L^2, Doob decomposition.

Uniform integrability and L^1 convergence, Levy's "Upward" and "Downward" Theorem, corollary to the Kolmogorov's Strong Law of Large Numbers, Doob's submartingale and L^p inequalities. Examples and applications, including branching processes, harmonic functions with boundary conditions on connected finite subsets of $\mathbb{Z}^d$ [not examinable].

[ ]=covered informally, without proofs.

Reading

1. D. Williams, *Probability with Martingales* (Cambridge University Press, 1995).

Further Reading

1. Z. Brzeźniak and T. Zastawniak, *Basic Stochastic Processes. A course through exercises*, Springer Undergraduate Mathematics Series (Springer-Verlag London, Ltd., 1999) [more elementary than D. Williams' book, but can provide with a complementary first reading].

2. M. Capinski and E. Kopp, *Measure, Integral and Probability*, Springer Undergraduate Mathematics Series (Springer-Verlag London, Ltd., second edition, 2004).

3. R. Durrett, *Probability: Theory and Examples,* (Second Edition, Duxbury Press, Wadsworth Publishing Company, 1996).

4. A. Etheridge, *A Course in Financial Calculus* (Cambridge University Press, 2002).

5. J. Neveu, *Discrete-parameter Martingales* (North–Holland, Amsterdam, 1975).

6. S. I. Resnick, *A Probability Path*, (Birkhaäuser, 1999).

2.11.2 B10b: Mathematical Models of Financial Derivatives — Dr Jin — 16 HT

[Option **B10b** if taken as a half-unit. OSS paper code 2B49.]

Overview

The course aims to introduce students to mathematical modelling in financial markets. At the end of the course the student should be able to formulate a model for an asset price and then determine the prices of a range of derivatives based on the underlying asset using arbitrage free pricing ideas.

Learning Outcomes

Students will have a familiarity with the mathematics behind the models and analytical tools used in Mathematical Finance. This includes being able to formulate a model for an asset price and then determining the prices of a range of derivatives based on the underlying asset using arbitrage free pricing ideas.

Synopsis

Introduction to markets, assets, interest rates and present value; arbitrage and the law of one price: European call and put options, payoff diagrams. Introduction to Brownian motion, continuous-time martingales, informal treatment of Itô's formula and stochastic differential equations. Discussion of the connection with PDEs through the Feynman-Kac formula.

The Black-Scholes analysis via delta hedging and replication, leading to the Black-Scholes partial differential equation for a derivative price. General solution via Feynman-Kac and risk neutral pricing, explicit solution for call and put options.

Extensions to assets paying dividends, time-varying parameters. Forward and future contracts, options on them. American options, formulation as a free-boundary problem and a linear complementarity problem. Simple exotic options. Weakly path-dependent options including barriers, lookbacks and Asians.

Reading

1. T. Bjork, *Arbitrage Theory in Continuous Time* (OUP, 1998).
2. P. Wilmott, S. D. Howison and J. Dewynne, *Mathematics of Financial Derivatives* (CUP, 1995).
3. A. Etheridge, *A Course in Financial Calculus* (CUP, 2002).

Background

1. J. Hull, *Options Futures and Other Financial Derivative Products* (4th edition, Prentice Hall, 2001).
2. N. Taleb, *Dynamic Hedging* (Wiley, 1997).
3. P. Wilmott, *Derivatives* (Wiley, 1998).

2.12 B11a: Communication Theory — Dr Stirzaker — 16 MT

NB: B22a: Integer Programming is a very suitable complement to this course.

Level: H-level **Method of Assessment:** Written Examination

Weight: Half-unit OSS paper code 2650.

Recommended Prerequisites: Part A Probability would be helpful, but not essential.

Overview

The aim of the course is to investigate methods for the communication of information from a sender, along a channel of some kind, to a receiver. If errors are not a concern we are interested in codes that yield fast communication, whilst if the channel is noisy we are interested in achieving both speed and reliability. A key concept is that of information as reduction in uncertainty. The highlight of the course is Shannon's Noisy Coding Theorem.

Learning Outcomes

(i) Know what the various forms of entropy are, and be able to manipulate them.

(ii) Know what data compression and source coding are, and be able to do it.

(iii) Know what channel coding and channel capacity are, and be able to use that.

Synopsis

Uncertainty (entropy); conditional uncertainty; information. Chain rules; relative entropy; Gibbs' inequality; asymptotic equipartition. Instantaneous and uniquely decipherable codes; the noiseless coding theorem for discrete memoryless sources; constructing compact codes.

The discrete memoryless channel; decoding rules; the capacity of a channel. The noisy coding theorem for discrete memoryless sources and binary symmetric channels.

Extensions to more general sources and channels.

Error-detection and error-correction. Constraints upon the choice of good codes; the Hamming (sphere-packing) and the Gilbert–Varshamov bounds.

Reading

1. D. J. A. Welsh, *Codes and Cryptography* (OUP, 1988), Chs 1–3, 5.

2. G. Jones and J. M. Jones, *Information and Coding Theory* (Springer, 2000), Ch 1–5.

3. T. Cover and J. Thomas, *Elements of Information Theory* (Wiley, 1991), Ch 1–5, 8.

Further Reading

1. R. B. Ash, *Information Theory* (Dover, 1990).

2. D. MacKay, *Information Theory, Inference, and Learning Algorithms* (Cambridge, 2003). [Can be seen at: http://www.inference.phy.cam.ac.uk/mackay/itila. Do not infringe the copyright!]

2.13 B12a: Applied Probability — Dr Goldschmidt — 16 MT

[This option was formerly OBS3a and is paper BS3 in the Honour School of Mathematics & Statistics.]

Level: H-Level **Method of Assessment:** $1\frac{1}{2}$-hour examination (3-hours if taken as a whole-unit with OBS3b)

Weight: Half-unit (OSS paper code 2A72). Can be taken as a whole-unit with OBS3b.

The whole-unit (B12a and OBS3b) has been designed so that a student obtaining at least an upper second class mark on the whole-unit can expect to gain exemption from the Institute of Actuaries' paper CT4, which is a compulsory paper in their cycle of professional actuarial examinations. The first half of the unit, clearly, and also the second half of the unit, apply much more widely than just to insurance models.

Overview

This course is intended to show the power and range of probability by considering real examples in which probabilistic modelling is inescapable and useful. Theory will be developed as required to deal with the examples.

Synopsis

Poisson processes and birth processes. Continuous-time Markov chains. Transition rates, jump chains and holding times. Forward and backward equations. Class structure, hitting times and absorption probabilities. Recurrence and transience. Invariant distributions and limiting behaviour. Time reversal.

Applications of Markov chains in areas such as queues and queueing networks — M/M/s queue, Erlang's formula, queues in tandem and networks of queues, M/G/1 and G/M/1 queues; insurance ruin models; epidemic models; applications in applied sciences.

Renewal theory. Limit theorems: strong law of large numbers, strong law and central limit theorem of renewal theory, elementary renewal theorem, renewal theorem, key renewal theorem. Excess life, inspection paradox. Applications.

Reading

1. J. R. Norris, *Markov Chains* (Cambridge University Press, 1997).

2. G. R. Grimmett and D. R. Stirzaker, *Probability and Random Processes* (3rd edition, Oxford University Press, 2001).

3. G. R. Grimmett and D. R. Stirzaker, *One Thousand Exercises in Probability* (Oxford University Press, 2001).

4. S. M. Ross, *Introduction to Probability Models* (4th edition, Academic Press, 1989).

5. D. R. Stirzaker: *Elementary Probability* (2nd edition, Cambridge University Press, 2003).

2.14 B21 Numerical Solution of Differential Equations

[From Part B2 in the Honour School of Computer Science. Teaching responsibility of the Computing Laboratory.]

Level: H-level

Method of Assessment: $1\frac{1}{2}$-hour exam for a half-unit, 3-hour examination for a whole-unit.

Weight: Whole-unit, or can be taken either as a half-unit in NSDE I (OSS paper code 286) or in NSDE II (OSS paper code 287).

Recommended Prerequisites: none.

2.14.1 B21a Numerical Solution of Differential Equations I — Prof. Süli — 16 MT

Overview

To introduce and give an understanding of numerical methods for the solution of ordinary and partial differential equations, their derivation, analysis and applicability.

The MT lectures are devoted to numerical methods for initial value problems, while the HT lectures concentrate on the numerical solution of boundary value problems.

Learning Outcomes

At the end of the course the student will be able to:

1. Construct one-step and linear multistep methods for the numerical solution of initial-value problems for ordinary differential equations and systems of such equations, and to analyse their stability and accuracy properties;

2. Construct finite difference methods for the numerical solution of initial-boundary-value problems for second-order parabolic partial differential equations, and first-order hyperbolic equations, and to analyse their stability and accuracy properties.

Synopsis

The MT part of the course is devoted to the development and analysis of numerical methods for initial value problems. We begin by considering classical techniques for the numerical solution of ordinary differential equations. The problem of stiffness is then discussed in tandem with the associated questions of step-size control and adaptivity.

Initial value problems for ordinary differential equations: Euler, multistep and Runge–Kutta; stiffness; error control and adaptive algorithms. [Introduction (1 lecture) + 5 lectures]

The remaining lectures focus on the numerical solution of initial value problems for partial differential equations, including parabolic and hyperbolic problems.

Initial value problems for partial differential equations: parabolic equations, hyperbolic equations; explicit and implicit methods; accuracy, stability and convergence, Fourier analysis, CFL condition. [10 lectures]

Reading List

The course will be based on the following textbooks:

1. K. W. Morton and D. F. Mayers, *Numerical Solution of Partial Differential Equations* (Cambridge University Press, 1994). ISBN 0-521-42922-6 (Paperback edition) [Chapters 2, 3 (Secs. 3.1, 3.2), Chapter 4 (Secs. 4.1–4.6), Chapter 5].

2. E. Süli and D. Mayers, *An Introduction to Numerical Analysis* (Cambridge University Press, 2003). ISBN 0-521-00794-1 (Paperback edition) [Chapter 12].

3. A. Iserles, *A First Course in the Numerical Analysis of Differential Equations* (Cambridge University Press, 1996). ISBN 0-521-55655-4 (Paperback edition) [Chapters 1–5, 13, 14].

2.14.2 B21b Numerical Solution of Differential Equations II — Dr Sobey — 16 HT

Overview

To introduce and give an understanding of numerical methods for the solution of ordinary and partial differential equations, their derivation, analysis and applicability.

The MT lectures are devoted to numerical methods for initial value problems, while the HT lectures concentrate on the numerical solution of boundary value problems.

Learning Outcomes

Students will understand and have experience of the theory for:

(i) Construction of shooting methods for boundary value problems in one independent variable

(ii) Elementary numerical analysis of elliptic partial differential equations

(iii) Analysis of iterative methods for solution of large linear systems of equations

Synopsis

The HT part of the course is concerned with numerical methods for boundary value problems. We begin by developing numerical techniques for the approximation of boundary value problems for second-order ordinary differential equations.

Boundary value problems for ordinary differential equations: shooting and finite difference methods. [Introduction (1 lecture) + 2 lectures]

Then we consider finite difference schemes for elliptic boundary value problems. This is followed by an introduction into the theory of direct and iterative algorithms for the solution of large systems of linear algebraic equations which arise from the discretisation of elliptic boundary value problems.

Boundary value problems for PDEs: finite difference discretisation; Poisson equation. Associated methods of sparse numerical algebra: sparse Gaussian elimination, classical iterations, multigrid iterations. [13 lectures]

Reading List

This course does not follow any particular textbook, but the following essentially cover the material:

1. A. Iserles, *A First Course in the Numerical Analysis of Differential Equations* (Cambridge University Press, 1996), Chapters 7,10,11.
2. K. W. Morton and D. F. Mayers, *Numerical Solution of Partial Differential Equations* (Cambridge University Press, 1994).
   Or the more recent 2nd edition (2005), Chapters 6 and 7.
3. G. D. Smith, *Numerical Solution of Partial Differential Equations: Finite Difference Methods* (Clarendon Press, Oxford, 1985 (and any later editions)), has some of the material in chapter 5.

2.15 B22a Integer Programming — Dr Hauser — 16 HT

[From Part B2 in the Honour School of Computer Science. Teaching responsibility of the Computing Laboratory.]

Level: H-level **Method of Assessment:** $1\frac{1}{2}$ -hour exam.

Weight: Half-unit. OSS paper code 282.

Recommended Prerequisites: None.

Overview

In many areas of practical importance linear optimisation problems occur with integrality constraints imposed on some of the variables. In optimal crew scheduling for example, a pilot cannot be fractionally assigned to two different flights at the same time. Likewise, in combinatorial optimisation an element of a given set either belongs to a chosen subset or it does not. Integer programming is the mathematical theory of such problems and of algorithms for their solution. The aim of this course is to provide an introduction to some of the general ideas on which attacks to integer programming problems are based: generating bounds through relaxations by problems that are easier to solve, and branch-and-bound.

Learning Outcomes

Students will understand some of the theoretical underpinnings that render certain classes of integer programming problems tractable ("easy" to solve), and they will learn how to solve them algorithmically. Furthermore, they will understand some general mechanisms by which intractable problems can be broken down into tractable subproblems, and how these mechanisms are used to design good heuristics for solving the intractable problems. Understanding these general principles will render the students able to guide the modelling phase of a real-world problem towards a mathematical formulation that has a reasonable chance of being solved in practice.

Syllabus

Simplex algorithm for linear programming in dictionary form, linear programming duality and sensitivity analysis, alternative formulations of integer programmes, ideal formulations of integer programmes, optimality conditions for integer programming, integer programming duality, linear programming relaxation, combinatorial relaxation and Lagrangian relaxation of integer programming problems, total unimodularity, network flow models, submodularity, matroids and the greedy algorithm, maximum weight subtree problems, augmenting paths, bipartite matching, the assignment problem, integer knapsack problems, dynamic programming, branch-and-bound, the symmetric travelling salesman problem, the subgradient algorithm, elementary branch-and-cut approaches.

Synopsis

1. Course outline. What is integer programming (IP)? Some classical examples.
2. Further examples, hard and easy problems.
3. Alternative formulations of IPs, linear programming (LP) and the simplex method.
4. LP duality, sensitivity analysis.
5. Optimality conditions for IP, relaxation and duality.

6. Total unimodularity, network flow problems.
7. Optimal trees, submodularity, matroids and the greedy algorithm.
8. Augmenting paths and bipartite matching.
9. The assignment problem.
10. Dynamic programming.
11. Integer knapsack problems.
12. Branch-and-bound.
13. More on branch-and-bound.
14. Lagrangian relaxation and the symmetric travelling salesman problem.
15. Solving the Lagrangian dual.
16. Branch-and-cut.

Course Materials

1. L. A. Wolsey, *Integer Programming* (John Wiley & Sons, 1998), parts of chapters 1–5 and 7.
2. Lecture notes and problem sheets posted on the course web page.

Time Requirements

The course consists of 16 lectures and 6 problem classes. There are no practicals. It is estimated that 8–10 hours of private study are needed per week for studying the lecture notes and relevant chapters in the textbook, and for solving the problem sheets, so that the total time requirement is circa 12 hours per week.

Part C (M-level) courses available in the third year

2.16 C3.1: Topology & Groups and Algebraic Topology — M-Level

Level: M-level. **Method of examination:** Written Examination.

Weight: Whole-unit , or can be taken either as a half-unit in Topology and Groups or a half-unit in Algebraic Topology. OSS code to follow.

2.16.1 C3.1a: Topology and Groups — Prof. Lackenby — 16MT

[Option **C3.1a** if taken as a half-unit. OSS code to follow.]

Prerequisites

2nd year Groups in action 2nd year Topology

Learning outcomes

This course introduces the important link between topology and group theory. On the one hand, associated to each space, there is a group, known as its fundamental group. This can be used to solve topological problems using algebraic methods. On the other hand, many results about groups are best proved and understood using topology. For example, presentations of groups, where the group is defined using generators and relations, have a topological interpretation. The endpoint of the course is the Nielsen-Shreier Theorem, an important, purely algebraic result, which is proved using topological techniques.

Synopsis

Homotopic mappings, homotopy equivalence. Simplicial complexes. Simplicial approximation theorem.

The fundamental group of a space. The fundamental group of a circle. Application: the fundamental theorem of algebra. The fundamental groups of spheres.

Free groups. Existence and uniqueness of reduced representatives of group elements. The fundamental group of a graph.

Groups defined by generators and relations (with examples). Tietze transformations.

The free product of two groups. Amalgamated free products.

The Seifert van Kampen Theorem.

Cell complexes. The fundamental group of a cell complex (with examples). The realization of any finitely presented group as the fundamental group of a finite cell complex.

Covering spaces. Liftings of paths and homotopies. A covering map induces an injection between fundamental groups. The use of covering spaces to determine fundamental groups: the circle again, and real projective n-space. The correspondence between covering spaces and subgroups of the fundamental group. Regular covering spaces and normal subgroups.

Cayley graphs of a group. The relationship between the universal cover of a cell complex, and the Cayley graph of its fundamental group. The Cayley 2-complex of a group.

The Nielsen-Schreier Theorem (every subgroup of a finitely generated free group is free) proved using covering spaces.

Reading

1. John Stillwell, *Classical Topology and Combinatorial Group Theory* (Springer-Verlag, 1993).

Additional Reading

1. D. Cohen, *Combinatorial Group Theory: A Topological Approach,* Student Texts 14 (London Mathematical Society, 1989), Chs 1-7.
2. A. Hatcher, *Algebraic Topology* (CUP, 2001), Ch. 1.
3. M. Hall, Jr, *The Theory of Groups* (Macmillan, 1959), Chs. 1-7, 12, 17 .
4. D. L. Johnson, *Presentations of Groups,* Student Texts 15 (Second Edition, London Mathematical Society, Cambridge University Press, 1997). Chs. 1-5, 10,13.
5. W. Magnus, A. Karrass, and D. Solitar, *Combinatorial Group Theory* (Dover Publications, 1976). Chs. 1-4.

2.16.2 C3.1b Algebraic Topology — Prof. Tillmann — 16 HT

[Option **C3.1b** if taken as a half-unit. OSS code to follow.]

Prerequisites

Ideally students will also take the Topology and Groups course in Michaelmas, though this is not absolutely necessary.

Overview

Homology theory is a subject that pervades much of modern mathematics. Its basic ideas are used in nearly every branch, pure and applied. In this course, the homology groups of topological spaces are studied. These powerful invariants have many attractive applications. For example we will prove that the dimension of a vector space is a topological invariant and the fact that a 'hairy ball can not be combed'.

Learning outcomes

At the end of the course, students are expected to understand the basic algebraic and geometric ideas that underpin homology and cohomology theory. These include the cup product and Poincaré Duality for manifolds. They should be able to choose between the different homology theories and to use calculational tools such as the Mayer-Vietoris sequence to effectively compute the homology and cohomology of easy examples, including projective spaces, surfaces, certain simplicial spaces and cell complexes. At the end of the course, students should also have developed a sense of how the ideas of homology and cohomology may be applied to problems from other branches of mathematics.

Synopsis

Chain complexes of Abelian groups and their homology. Short exact sequences. Delta (and simplicial) complexes and their homology. Euler characteristic.

Singular homology of topological spaces. Relation of the first homology group to the fundamental group. Relative homology and the Five Lemma. Homotopy invariance and excision (details of proofs not examinable). Mayer-Vietoris Sequence. Equivalence of simplicial and singular homology. Axioms of homology.

Degree of a self-map of a sphere. Cell complexes and cellular homology. Application: the hairy ball theorem.

Cohomology of spaces and the Universal Coefficient Theorem (proof not examinable). Cup products. Künneth Theorem (without proof). Topological manifolds and orientability. The fundamental class of an orientable, closed manifold and the degree of a map between manifolds of the same dimension. Poincaré Duality (without proof).

Reading

1. A. Hatcher, *Algebraic Topology* (CUP, 2001). Chapter 3 and 4.
2. G. Bredon, *Topology and Geometry* (Springer, 1997). Chapter 4 and 5.
3. J. Vick, *Homology Theory,* Graduate Texts in Mathematics 145 (Springer, 1973).

2.17 C5.1a: Methods of Functional Analysis for Partial Differential Equations — Dr Dyson — 16MT — M-Level

Level: M-level **Method of Assessment:** Written Examination

Weight: Half-unit. OSS paper code 2A65.

Recommended Prerequisites: Part A Lebesgue integration. However, what is required is an ability to use integration techniques giving proper justification. There will be a 'Users' Guide to Integration' on the subject website so that anyone who has not done Part A Integration can read it up over the summer.

Overview

The course will introduce some of the modern techniques in partial differential equations that are central to the theoretical and numerical treatments of linear and nonlinear partial differential equations arising in science, geometry and other fields.

It provides valuable background for the Part C courses on Calculus of Variations, Fixed Point Methods for Nonlinear PDEs, and Finite Element Methods.

Learning outcomes

Students will learn techniques and results, such as Sobolev spaces, weak convergence, weak solutions, the direct method of calculus of variations, embedding theorems, the Lax-Milgram

Theorem, the Fredholm Alternative and the Hilbert-Schmidt Theorem and how to apply these to obtain existence and uniqueness results for linear and nonlinear elliptic partial differential equations.

Synopsis

Part I Function Spaces:

Why are function spaces important for partial differential equations?

Definition of Banach spaces, separability and dual spaces. Definition of Hilbert space. The spaces $L^p(\Omega)$, $1 \leq p \leq \infty$, where $\Omega \subset \mathbb{R}^n$ is open. Minkowski and Hölder inequalities. Statement that $L^p(\Omega)$ is a Banach space, and that the dual of L^p is $L^{p'}$, for $1 \leq p < \infty$. Statement that L^2 is a Hilbert space.

Weak and weak* convergence in L^p spaces. Examples. A bounded sequence in the dual of a separable Banach space has a weak* convergent subsequence.

Mollifiers and the density of smooth functions in L^p for $1 \leq p < \infty$.

Definition of weak derivatives and their uniqueness. Definition of Sobolev space $W^{m,p}(\Omega)$, $1 \leq p \leq \infty$. $H^m(\Omega) = W^{m,2}(\Omega)$. Definition of $W_0^{1,p}(\Omega)$, $1 \leq p < \infty$. Brief introduction to distributions.

Part II Elliptic Problems:

The direct method of calculus of variations: The Poincaré inequality. Proof of the existence and uniqueness of a weak solution to Poisson's equation $-\Delta u = f$, with zero Dirichlet boundary conditions and $f \in L^2(\Omega)$, with Ω bounded. Discussion of regularity of solutions.

The Lax-Milgram Theorem and Gårding's inequality. Existence and uniqueness of weak solutions to general linear uniformly elliptic equations.

Embedding theorems (proofs omitted except $W^{1,1}(a,b) \hookrightarrow C[a,b]$).

Compact operators and self adjoint operators. Fredholm Alternative and Hilbert-Schmidt Theorem. Examples including $-\Delta$ with zero Dirichlet boundary conditions.

A nonlinear elliptic problem treated by the direct method.

Reading

Lawrence C. Evans, *Partial Differential Equations,* Graduate Studies in Mathematics (American Mathematical Society, 2004).

M. Renardy and R. C. Rogers *An Introduction to Partial Differential Equations* (Springer-Verlag, New York, 2004).

Additional Reading

E. Kreyszig, *Introductory Functional Analysis with Applications* (Wiley, revised edition, 1989)

J. Rauch, *Partial Differential Equations* (Springer-Verlag, New York, 1992).

2.18 BE "Mathematical" Extended Essay

Level: H-level **Method of Assessment:** Written extended essay

Weight: One unit. OSS code 9921.

See the "Projects Guidance Notes" on the website at http://www.maths.ox.ac.uk/current-students/undergraduates/projects/ for more information on this option and an application form.

Extended essays will be assigned USMs according to the same principles as Mathematics papers. In arriving at these marks, the relative weights (for BE Essays) given to content, mathematics, and presentation will be 25%, 50% and 25%, respectively. Here is a brief explanation of these terms:

Mathematics: proofs and assertions should be correct, and the mathematics should be appropriate for the level of study. In applied topics, the derivation of the model should be properly justified.

Content: the examiners are looking for some of your own thoughts and contributions: you must do more than rehash text books and lecture notes; you should use original sources; you must not plagiarise.

Presentation: the mathematics must be clear and well laid out; the English should be clear and grammatically correct; sources should be properly acknowledged, references should be properly cited. Give some thought to notation, choice of typeface, and numbering of equations and sections. Do not fail to number the pages. Be sure to supply complete and accurate references for all the sources used in completing the project, and be sure to cite them properly in the text.

Excellent brief advice on mathematical writing is to be found on the London Mathematical Society website http://www.lms.ac.uk/publications/documents/writing.pdf

3 Other Mathematical units and half-units

3.1 O1: History of Mathematics — Dr Stedall — 16 lectures in MT and reading course with 8 seminars in HT

Level: H-level

Method of Assessment: 2 hour written examination paper for the MT lectures and 3000–3500 word mini-project for the reading course.

Weight: Whole-unit. OSS paper code 2654 (exam), X017 (mini-project).

Recommended Prerequisites: None.

Quota: The maximum number of students that can be accepted for 2008–09 will be 30.

Learning outcomes

This course is designed to provide the historical background to some of the mathematics familiar to students from A-level and four terms of undergraduate study, and looks at a period from approximately the mid sixteenth century to the end of the nineteenth century. The course will be delivered through 16 lectures in Michaelmas Term, and a Reading Course consisting of 8 seminars (equivalent to a further 16 lectures) in Hilary Term. Guidance will be given throughout on reading, note-taking, and essay-writing skills. Students will gain:

1. an understanding of university mathematics in its historical context;
2. an enriched understanding of mathematical content in the topics covered by the course together with skills in:
3. historical analysis of primary source material;
4. selective reading from a variety of secondary sources;
5. efficient note-taking;
6. writing well-argued essays (ranging in length from 800 to 3500 words);
7. accurate referencing and construction of bibliographies;
8. verbal presentation and discussion.

Lectures: The following is intended as an approximate guide to the content of the Michaelmas Term lecture course.

1. Week 1
   Introduction
2. Week 2
   (a) Analytic geometry
   (b) Development of calculus
3. Week 3
   (a) Newton's *Principia*
   (b) Eighteenth-century calculus and analysis
4. Week 4
   (a) Probability
   (b) Functions, limits, continuity
5. Week 5
   (a) Polynomial equations and solvability
   (b) Groups and fields
6. Week 6
   (a) Integrations
   (b) Complex analysis

7. Week 7
   (a) Sequences and series
   (b) Foundations

8. Week 8
   (a) Linear algebra
   (b) Geometry

The lecturers will set six sets of exercises (including suggested reading, extracts from primary sources on which the student will be expected to comment, and short essay titles). Written work will be discussed in six weekly intercollegiate classes in the usual way.

Reading Courses:

In Hilary Term each student will select a Reading Course from a choice of options. In this part of the course students will pursue a detailed study of a particular topic, using original sources and secondary literature. Courses will be run according to demand, requiring ideally a minimum of five and a maximum of eight students on each. Students will be expected to write three essays during the first six weeks of term. The course will be examined by a mini project of 3000–3500 words to be completed during Weeks 7 to 9. The options for Hilary Term 2009 will be discussed with students in Week 6 of Michaelmas Term.

Reading

1. Victor Katz, *A History of Mathematics: An Introduction* (2nd edition, Addison Wesley Longman, 1998).

2. John Fauvel and Jeremy Gray (eds), *The History of Mathematics: A Reader* (Macmillan, 1987).

3. Jacqueline Stedall, *Mathematics Emerging: A Sourcebook 1540–1900* (OUP, 2008).

Introductory and background reading:

1. Dirk Struik, *A Concise History of Mathematics* (4th revised edition, Dover, 1987).

2. Luke Hodgkin, *A History of Mathematics: From Mesopotamia to Modernity* (OUP, 2005).

3. William E Dunham, *The Calculus Gallery. Masterpieces from Newton to Lebesgue* (Princeton University Press, 2005). ISBN: 0-691-09565-5.

Assessment

The Michaelmas Term lectures will be examined in a two-hour written paper at the end of Trinity Term. Candidates will be expected to answer two half-hour questions (commenting on extracts) and one one-hour question (essay). The paper will account for 50% of the marks for the course. The Reading Course will be examined by a 3000–3500 word mini-project at the end of Hilary Term. The title will be set at the beginning of Week 7 and two copies of the project must be submitted to the Examination Schools by midday on Friday of Week 9. The mini-project will account for 50% of the marks for the course.

3.2 OBS1: Applied Statistics

[Paper BS1 in the Honour School of Mathematics & Statistics. Teaching responsibility of the Department of Statistics. See the website of Statistics Department for definitive details.]

Level: H-level

Method of Assessment: 2-hour examination plus assessed practical assignments. The practical assignments contribute $\frac{1}{3}$ of the marks for OBS1. As practicals are a formal part of the examination for OBS1 it is compulsory to submit all practicals.

Weight: Whole-unit. OSS paper code 2670.

Recommended Prerequisites: Part A Probability and Statistics.

3.2.1 Applied Statistics I — Dr Nicholls — 16 MT
Applied Statistics II — Dr Meinshausen — 10 HT

Overview

The course aims to develop the theory of statistical methods, and also to introduce students to the analysis of data using a statistical package. The main topics are: Simulation based inference, Practical aspects of linear models, Logistic regression and generalized linear models, and Robust and computer-intensive methods.

Synopsis

Michaelmas Term (16 lectures)

Practical aspects of linear models and analysis of variance: review of multiple regression, model selection, fit criteria, use of residuals, outliers, leverage, Box-Cox transformations, model interpretation. Logistic regression. Linear exponential families and generalized linear models, scale parameter, link functions, canonical link. Maximum likelihood fitting and iterated weighted least squares. Asymptotic theory: statement and application to inference, analysis of deviance, model checking, residual. Inference using simulation methods.

Hilary Term (10 lectures)

Nonparametric inference. Permutation tests. Rank statistics. Robust estimation. Influence curve. Breakdown point. Robust and resistant regression. Smoothing methods (kernels, splines, local polynomials). Bootstrapping.

Reading (Michaelmas Term)

A. C. Davison, *Statistical Models* (CUP, 2003).

A. J. Dobson, *An Introduction to Generalized Linear Models* (Chapman and Hall, 1990).

D. Lunn, *Notes* (2003).

Reading (Hilary Term)

P. J. Rousseeuw and A. M. Leroy, *Robust Regression and Outlier Detection* (Wiley, 1987), pp 1–194.

J. D. Gibbons, *Nonparametric Statistical Inference* (Marcel Dekker, 1985), pp 1–193, 273–290.

R. H. Randles and D. A. Wolfe, *Introduction to the Theory of Nonparametric Statistics* (Wiley, 1979), pp 1–322.

Further Reading

F. L. Ramsey and D. W. Schafer, *The Statistical Sleuth: A Course in Methods of Data Analysis* (2nd edition, Duxbury, 2002)

W. N. Venables and B. D. Ripley, *Modern Applied Statistics with S* (Springer, 2002)

L. Wasserman, *All of Nonparametric Statistics* (Springer, 2004)

Practicals

In addition to the lectures there will be five supervised practicals. Four of these contain problems whose written solutions will be assessed as part of the unit examination. Similar practical applications will be used as illustrations in lectures.

3.3 OBS2: Statistical Inference

[Teaching responsibility of the Department of Statistics. See the website of Statistics Department for definitive details.]

Level: H-level **Method of Assessment:** 3-hour or $1\frac{1}{2}$-hour examination

Weight: One unit (OSS paper code 2671), or the first 16 lectures can be taken as a half-unit OBS2a (OSS paper code 2A71).

Prerequisites: The Part A (second-year) course 'Statistics'. OBS2b cannot be taken alone. OBS2a is a prerequisite for OBS2b.

Quota: None.

3.3.1 OBS2a: Foundations of Statistical Inference — Dr Clifford — 16 MT

[Option **OBS2a** if taken as a half-unit. OSS paper code 2A71.]

Level: H-level

Prerequisite: The Part A (second-year) course 'Statistics'.

Learning Outcomes

Understanding how data can be interpreted in the context of a statistical model. Working knowledge and understanding of key-elements of model-based statistical inference, including awareness of similarities, relationships and differences between Bayesian and frequentist approaches.

Synopsis

Exponential families: Curved and linear exponential families; canonical parametrization; likelihood equations. Sufficiency: Factorization theorem; sufficiency in exponential families.

Frequentist estimation: unbiasedness; method of moments; the Cramér–Rao information inequality; statement of the large sample distribution of the MLE; proof for curved exponential families assuming consistency.

The Bayesian paradigm: subjective probability; prior to posterior analysis; conjugacy; examples from exponential families. Choice of prior distribution: proper and improper priors; Jeffreys' and maximum entropy priors. Hierarchical Bayes models, graphical representation.

Computational techniques: Markov chain Monte Carlo methods; sampling importance resampling; data examples.

Decision theory: risk function; randomized decision rules; admissibility. Rao–Blackwell theorem: Rao–Blackwellization; illustration with squared error loss. Minimax rules, Bayes rules and admissibility. Hypothesis testing as decision problem.

Empirical Bayes methods. James Stein estimator. Shrinkage.

Reading

1. G. A. Young and R. L. Smith, *Essentials of Statistical Inference* (Cambridge University Press, 2005).
2. T. Leonard and J. S. J. Hsu, *Bayesian Methods* (Cambridge University Press, 2005).

Further reading

1. D. R. Cox, *Principles of Statistical Inference* (Cambridge University Press, 2006).
2. D. Sorensen and D. Gianola, *Likelihood, Bayesian and MCMC Methods in Quantitative Genetics* (Springer, NY, 2002).

3. Y. Pawitan, *In All Likelihood: Statistical Modelling and Inference using Likelihood* (OUP, 2001).

3.3.2 OBS2b: Further Statistical Inference — Prof. Lauritzen — 16 HT

Level: H-level

Prerequisite: OBS2a

Learning Outcomes

Awareness of the increasing complexity of data sets in the modern world and the need to develop appropriate statistical methodology. Understanding of fundamental methods associated with statistical inference for multi-parameter models and high-dimensional data. Understanding of Bayesian and frequentist methods for the statistical analysis of data that arrive sequentially.

Synopsis

Ancillarity; conditional inference; dealing with nuisance parameters. Comparison of Bayesian and frequentist approaches.

Generalized linear models and exponential families: Newton–Raphson iteration and the method of scoring for multi-parameter problems.

Approximating integrals that arise in statistical applications: saddle-point expansions; Laplace's approximation.

Model comparison and model selection: Bayes factors; asymptotic approximation of Bayes factors; AIC vs. BIC.

The multivariate normal distribution; Wishart and inverse Wishart distributions. Frequentist inference for multivariate normal models: Wilks' test and Hotelling's T^2. Bayesian inference for multivariate normal models.

Sequential frequentist methods: sequential probability ratio tests. Sequential Bayesian methods: Kalman filter.

Reading

1. G. A. Young and R. L. Smith, *Essentials of Statistical Inference* (Cambridge University Press, 2005).
2. T. Leonard and J. S. J. Hsu, *Bayesian Methods* (Cambridge University Press, 2005).

Further reading

1. D. R. Cox, *Principles of Statistical Inference* (Cambridge University Press, 2006).

2. D. Sorensen and D. Gianola, *Likelihood, Bayesian and MCMC Methods in Quantitative Genetics* (Springer, New York, 2002).

3. Y. Pawitan, *In All Likelihood: Statistical Modelling and Inference using Likelihood* (OUP, 2001).

3.4 OBS3: Stochastic Modelling

[Paper BS3 in the Honour School of Mathematics & Statistics. Teaching responsibility of the Department of Statistics. See the website of Statistics Department for definitive details.]

Level: H-level **Method of Assessment:** 3-hour

Weight: Whole-unit (OSS paper code 2672), cannot be taken unless B12a is taken.

Recommended Prerequisites: Part A Probability and Statistics.

Overview This unit has been designed so that a student obtaining at least an upper second class mark on the whole-unit can expect to gain exemption from the Institute of Actuaries' paper CT4, which is a compulsory paper in their cycle of professional actuarial examinations. The first half of the unit, clearly, and also the second half of the unit, apply much more widely than just to insurance models.

3.4.1 OBS3b: Statistical Lifetime-Models — Dr Steinsaltz — 16 HT

Overview

The second half of the unit follows on from the first half on Applied Probability. Models introduced there are examined more specifically in the context of measuring "lifetimes", in the broad sense. In a life insurance context Markov transitions may model the passage from 'alive' to 'dead', possibly with intermediate stages like 'loss of a limb' or 'critically ill'. The same models are used to model fertility transitions, the progression of a disease, and the reliability of a mechanical device. The aim is to develop statistical methods to estimate transition rates, and to use these transition rates to construct life tables that form the basis in the calculation of life insurance premiums and pension projections. We will also cover the basics of survival analysis, to model the influence of covariates (e.g., weight, smoking, use of a medication) on lifespans.

Synopsis

Life tables: Basic notation, life expectancy and remaining life expectancy, curtate lifetimes. Census approximations, Lexis diagrams.

Survival models: general lifetime distributions, force of mortality (hazard rate), survival function, specific mortality laws, the single decrement model, multiple-decrement model and mortality in mixed populations.

Estimation procedures for lifetime distributions: empirical lifetime distributions, censoring and truncation, Kaplan–Meier estimate, Nelson–Aalen estimate. Parametric models, accelerated life models including Weibull, log-normal, log-logistic. Plot-based methods for model selection. Proportional hazards, partial likelihood, semiparametric estimation of survival functions, use and overuse of proportional hazards in insurance calculations and epidemiology.

Two-state and multiple-state Markov models, with simplifying assumptions. Estimation of Markovian transition rates: Maximum likelihood estimators, time-varying transition rates, census approximation. Application to reliability, medical statistics, ecology.

Graduation, including fitting Gompertz–Makeham model, comparison with standard life table: tests including chi-square test and grouping of signs test, serial correlations test; smoothness.

Demographic projection, Stable population age structures. The Lee-Carter model. Application to pension plans.

Reading

1. *Subject 104[CT4] Survival models[Modelling] Core Reading 2004[2005].* Faculty & Institute of Actuaries (2003 – 2004).
2. D. R. Cox and D. Oakes, *Analysis of Survival Data* (Chapman & Hall, 1984).

Further Reading

1. J. P. Klein and M. L. Moeschberger, *Survival Analysis* (Springer, 1997).
2. C. T. Le, *Applied Survival Analysis* (Wiley, 1997).
3. H. U. Gerber, *Life Insurance Mathematics* (3rd edition, Springer, 1997).
4. N. L. Bowers et al., *Actuarial mathematics* (2nd edition, Society of Actuaries 1997).

3.5 OBS4: Actuarial Science

[Paper BS4 in the Honour School of Mathematics & Statistics. Teaching responsibility of the Department of Statistics. See the website of Statistics Department for definitive details.]

Level: H-level **Method of Assessment:** 3-hour examination.

Weight: OBS4 can only be taken as one whole-unit. OSS paper code 2677.

Recommended Prerequisites: Part A Probability is useful, but not essential. If you have not done Part A Probability, make sure that you are familiar with Mods work on Probability.

3.5.1 OBS4a: Actuarial Science I — Dr Winkel — 16 MT

Overview

This unit is supported by the Institute of Actuaries. It has been designed to give the undergraduate mathematician an introduction to the financial and insurance worlds in which the practising actuary works. Students will cover the basic concepts of risk management models for investment and mortality, and for discounted cash flows. In the examination, a student obtaining at least an upper second class mark on this unit can expect to gain exemption from the Institute of Actuaries' paper CT1, which is a compulsory paper in their cycle of professional actuarial examinations.

Synopsis

Fundamental nature of actuarial work. Use of generalised cash flow model to describe financial transactions. Time value of money using the concepts of compound interest and discounting. Interest rate models. Present values and accumulated values of a stream of equal or unequal payments using specified rates of interest. Interest rates in terms of different time periods. Equation of value, rate of return of a cash flow, existence criteria.

Loan repayment schemes. Investment project appraisal, funds and weighted rates of return. Inflation modelling, inflation indices, real rates of return, inflation-adjustments. Valuation of fixed-interest securities, taxation and index-linked bonds.

Uncertain payments, corporate bonds, fair prices and risk. Single decrement model, present values and accumulated values of a stream of payments taking into account the probability of the payments being made according to a single decrement model. Annuity functions and assurance functions for a single decrement model. Risk and premium calculation.

Reading

All of the following are available from the Publications Unit, Institute of Actuaries, 4 Worcester Street, Oxford OX1 2AW

1. *Subject 102[CT1] Financial Mathematics Core Reading 2004[2005]* . Faculty & Institute of Actuaries (2003[2004]).

2. J. J. McCutcheon and W. F. Scott, *An Introduction to the Mathematics of Finance* (Heinemann, 1986).

3. P. Zima and R. P. Brown, *Mathematics of Finance* (McGraw–Hill Ryerson, 1993).

4. H. U. Gerber, *Life Insurance Mathematics* (3rd edition, Springer, 1997).

5. N. L. Bowers et al, *Actuarial mathematics* (2nd edition, Society of Actuaries, 1997).

3.5.2 OBS4b: Actuarial Science II — Mr Clarke — 16 HT

Synopsis

Liabilities under a simple assurance contract or annuity contract. Premium reserves, Thiele's differential equation. Expenses and office premiums.

The no-arbitrage assumption, arbitrage-free pricing. Price and value of forward contracts, effect of fixed income or fixed dividend yield from the asset. Futures, options and other financial products.

Investment and risk characteristics of investments. Term structure of interest rates, spot rates and forward rates, yield curves. Stability of investment portfolios, analysis of small changes in interest rates, Redington immunisation.

Simple stochastic interest rate models, mean-variance models, log-normal models. Mean, variance and distribution of accumulated values of simple sequences of payments.

Reading

All of the following are available from the Publications Unit, Institute of Actuaries, 4 Worcester Street, Oxford OX1 2AW

1. *Subject 102[CT1] Financial Mathematics Core Reading 2004[2005]*, Faculty & Institute of Actuaries (2003[2004]).
2. J. J. McCutcheon and W. F. Scott, *An Introduction to the Mathematics of Finance* (Heinemann, 1986).
3. H. U. Gerber, *Life Insurance Mathematics* (3rd edition, Springer, 1997).
4. N. L. Bowers et al, *Actuarial mathematics* (2nd edition, Society of Actuaries, 1997).

3.6 OCS1: Functional Programming, Design and Analysis of Algorithms

3.6.1 OCS1a: Functional Programming — Prof. Jeavons — 16 MT

Level: H-level. **Method of Assessment:** 3-hour exam.

Weight: Can only be taken with Design and Analysis of Algorithms as one whole-unit. OSS paper code 4208.

Recommended Prerequisites: none.

Overview

This is a first course in programming. It makes use of a programming language called Haskell, in which programs can be viewed as mathematical functions. This makes the language very powerful, so that we can easily construct programs that would be difficult or very large in other languages.

An important theme of the course is how to apply mathematical reasoning to programs, so as to prove that a program performs its task correctly, or to derive it by algebraic manipulation from a simpler but less efficient program for the same problem.

The course provides hands-on experience of programming through two lab exercises: the first one aims to make you acquainted with the mechanics of writing Haskell programs, and the second one tackles a more challenging programming task.

Learning Outcomes

At the end of the course the student will be able to:

1. Write programs in a functional style;
2. Reason formally about functional programs;
3. Use polymorphism and higher-order functions;
4. Reason about the time and space complexity of programs.

Syllabus

Principles of functional programming: expressions, evaluations, functions, and types. Type definitions and built-in types: numbers, characters, strings and lists. Basic operations on lists, including map, fold and filter, together with their algebraic properties. Recursive definitions and structural induction. Simple program calculation. Infinite lists and their uses. Further data structures: binary trees, general trees. Use of trees for representing sets and symbolic data. Normal order reduction and lazy evaluation. Simple cost models for functional programs; time and space complexity.

Synopsis

Programming with a functional notation: sessions and scripts, expressions and values. Evaluation of expressions. Case study: Approximating square roots. Reduction strategies: innermost vs outermost. [1]

Types and strong-typing. Basic types: Booleans and truth values. Simple programs involving pattern matching. Polymorphism and type classes. Functional application and currying. Functional composition. More types: characters, strings, tuples. Type synonyms. [2]

Lists and their operations; list comprehensions. The functions map, foldl, foldr, concat and filter. Many small examples illustrating the use of these functions in a compositional style of programming. [3]

Time complexity. Asymptotic notation. Advice on writing efficient programs; use of accumulating parameters. [2]

Recursion and induction. The algebraic properties of list functions and their proof by equational reasoning. Simple manipulation of programs to achieve efficiency. [2]

Infinite lists and their applications: Pascal's triangle, digits of a number, sieve of Eratosthenes. Infinite lists as limits. Proving properties of infinite lists: induction, take lemma. Cyclic structures. [2] Non-linear data structures. Binary trees and the relationship between size and depth. Binary search trees for representing sets. Insertion and searching in a binary search tree. Representing and evaluating arithmetic expressions. [3]

More advice on writing efficient programs: halve instead of decrease; tupling and accumulation. Space complexity and the use of strict. [1]

More substantial examples if time allows.

Reading List

Course text:

1. Richard Bird, *Introduction to Functional Programming using Haskell* (second edition, Prentice–Hall International, 1998).

Alternatives:

1. Richard Bird and Philip Wadler, *Introduction to Functional Programming* (Prentice–Hall International, 1988).
2. Simon Thompson, Haskell, *The Craft of Functional Programming* (Addison–Wesley, 1996).
3. Paul Hudak, *The Haskell School of Expression* (Cambridge University Press, 2000).

3.6.2 OCS1b: Design and Analysis of Algorithms — Prof. Ong — 16 HT

Level: H-level. **Method of Assessment:** 3-hour exam.

Weight: Can only be taken with Functional Programming as one whole-unit. OSS paper code 4208.

Recommended Prerequisites: none.

Overview

This core course covers good principles of algorithm design, elementary analysis of algorithms, and fundamental data structures. The emphasis is on choosing appropriate data structures and designing correct and efficient algorithms to operate on these data structures.

Learning Outcomes

This is a first course in data structures and algorithm design. Students will:

1. learn good principles of algorithm design;
2. learn how to analyse algorithms and estimate their worst-case and average-case behaviour (in easy cases);
3. become familiar with fundamental data structures and with the manner in which these data structures can best be implemented;
4. become accustomed to the description of algorithms in both functional and procedural styles;
5. learn how to apply their theoretical knowledge in practice (via the practical component of the course).

Syllabus

Basic strategies of algorithm design: top-down design, divide and conquer, average and worst-case criteria, asymptotic costs. Simple recurrence relations for asymptotic costs. Choice of appropriate data structures: arrays, lists, stacks, queues, trees, heaps, priority queues, graphs, hash tables. Applications to sorting and searching, matrix algorithms, shortest-path and spanning tree problems. Introduction to discrete optimisation algorithms: dynamic programming, greedy algorithms. Graph algorithms: depth first and breadth first search.

Synopsis

Program costs: time and space. Worst case and average case analysis. Asymptotics and "big O" notation. Polynomial and exponential growth. Asymptotic estimates of costs for simple algorithms. Use of induction and generating functions. [2]

Data structures and their representations: arrays, lists, stacks, queues, trees, heaps, priority queues, graphs. [3]

Algorithm design strategies: top down design, divide and conquer. Application to sorting and searching and to matrix algorithms. Solution of relevant recurrence relations. [4]

Graph algorithms: examples of depth-first and breadth-first search algorithms. Topological sorting, connected components. [3]

Introduction to discrete optimisation algorithms: dynamic programming, greedy algorithms, shortest path problems. [2]

Linear sorting and comparator networks (if time).

Reading List

Recommended texts

1. T. H. Cormen, C. E. Leiserson, R. L. Rivest and C. Stein, *Introduction to Algorithms* (2nd edition, MIT Press, 2001 (or the 1st edition, published in 1990)).
2. M. T. Goodrich and R. Tommassia, *Algorithm Design* (Wiley, 2002).
3. S. Dasgupta, C. Papadimitriou, and U. Vazirani, *Algorithms* (McGraw–Hill Higher Education, 2006).

3.7 OCS3a Lambda Calculus & Types — Dr Ker — 16 HT

Level: H-level. **Method of Assessment:** $1\frac{1}{2}$-hour examination.

Weight: Half-unit. OSS paper code 285.

Recommended Prerequisites: none.

Overview

As a language for describing functions, any literate computer scientist would expect to understand the vocabulary of the lambda calculus. It is folklore that various forms of the lambda calculus are the prototypical functional programming languages, but the pure theory of the lambda calculus is also extremely attractive in its own right. This course introduces the terminology and philosophy of the lambda calculus, and then covers a range of self-contained topics studying the language and some related structures. Topics covered include the equational theory, term rewriting and reduction strategies, combinatory logic, Turing completeness and type systems. As such, the course will also function as a brief introduction to many facets of theoretical computer science, illustrating each (and showing the connections with practical computer science) by its relation to the lambda calculus.

There are no prerequisites, but the course will assume familiarity with constructing mathematical proofs. Some basic knowledge of computability would be useful for one of the topics (the Models of Computation course is much more than enough), but is certainly not necessary.

Learning Outcomes

The course is an introductory overview of the foundations of computer science with particular reference to the lambda-calculus. Students will

1. understand the syntax and equational theory of the untyped lambda-calculus, and gain familiarity with manipulation of terms;
2. be exposed to a variety of inductive proofs over recursive structures;

3. learn techniques for analysing term rewriting systems, with particular reference to -reduction;
4. see the connections between lambda-calculus and computabilty, and an example of how an undecidability proof can be constructed;
5. see the connections and distinctions between lambda-calculus and combinatory logic;
6. learn about simple type systems for the lambda-calculus, and how to prove a strong normalization result;
7. understand how to deduce types for terms, and prove correctness of a principal type algorithm.

Synopsis

Chapter 0 (1 lecture)

Introductory lecture. Preparation for use of inductive definitions and proofs.

Chapters 1–3 (5 lectures)

Terms, free and bound variables, alpha-conversion, substitution, variable convention, contexts, the formal theory lambda beta, the eta rule, fixed point combinators, lambda-theories.

Reduction. Compatible closure, reflexive transitive closure, diamond and Church–Rosser properties for general notions of reduction. beta reduction, proof of the Church–Rosser property (via parallel reduction), connection between beta-reduction and lambda beta, consistency of lambda beta. Inconsistency of equating all terms without beta-normal form.

Reduction strategies, head and leftmost reduction. Standard reductions. Proof that leftmost reduction is normalising. Statement, without proof, of Genericity Lemma, and simple applications.

Chapter 4 (2 lectures)

Church numerals, definability of total recursive functions. Second Recursion Theorem, Scott–Curry Theorem, undecideability of equality in lambda beta. Briefly, extension to partial functions.

Chapter 5 (2 lectures)

Untyped combinatory algebras. Abstraction algorithm, combinatory completeness, translations to and from untyped lambda-calculus, mismatches between combinary logic and lambda-calculus, basis. Term algebras.

Chapters 6–8 (6 lectures)

Simple type assignment a la Curry using Hindley's TA lambda system. Contexts and deductions. Subject Construction Lemma, Subject Reduction Theorem and failure of Subject Expansion. Briefly, a system with type invariance under equality.

Tait's proof of strong normalisation. Consequences: no fixed point combinators, poor definability power. Pointer to literature on PCF as the obvious extension of simple types to cover all computable functions.

Type substitutions and unification, Robinson's algorithm. Principal Type algorithm and correctness.

Reading List

Essential

1. Andrew Ker, lecture notes. Available from reception at the start of term. Comprehensive notes on the entire course, including practice questions and class exercises.

Useful Background

1. H. P. Barendregt, *The Lambda Calculus* (North–Holland, revised edition, 1984).

2. J. R. Hindley, *Basic Simple Type Theory,* Cambridge Tracts in Theoretical Computer Science 42 (CUP, 1997).

3. C. Hankin, Lambda Calculi, *A Guide for Computer Scientists,* Graduate Texts in Computer Science (OUP, 1994).

4. J. R. Hindley & J. P. Seldin, *Introduction to Combinators and Lambda-Calculus* (Cambridge University Press, 1986).

5. J.-Y. Girard, Y. Lafont, & P. Taylor, *Proofs and Types,* Cambridge Tracts in Theoretical Computer Science 7 (CUP, 1989).

3.8 OE "Other Mathematical" Extended Essay

Level: H-level **Method of Assessment:** Written essay

Weight: One unit. OSS paper code 9923.

See the "Projects Guidance Notes" on the website at http://www.maths.ox.ac.uk/current-students/undergraduates/projects/ for more information on this option and an application form.

Extended essays will be assigned USMs according to the same principles as Mathematics papers. For OE Essays on historical, statistical, computer-related or similar topics the examiners will assign 75% for content and 25% for style and presentation. However, the content should have substantial connections with mathematics. Here is a brief explanation of these terms:

Content: the examiners are looking for some of your own thoughts and contributions: you must do more than rehash text books and lecture notes; you should use original sources; you must not plagiarise.

Presentation: the mathematics must be clear and well laid out; the English should be clear and grammatically correct; sources should be properly acknowledged, references should be properly cited. Give some thought to notation, choice of typeface, and numbering of equations and sections. Do not fail to number the pages. Be sure to supply complete and accurate references for all the sources used in completing the project, and be sure to cite them properly in the text.

Excellent brief advice on mathematical writing is to be found on the London Mathematical Society website http://www.lms.ac.uk/publications/documents/writing.pdf

4 Non-Mathematical units and half-units

4.1 N1 Undergraduate Ambassadors' Scheme — Dr Earl — mainly HT

Level: H-level

Method of Assessment: Journal of activities, Oral presentation, Course report and project, Teacher report.

Weight: Half-unit. OSS paper code 9919.

Recommended Prerequisites: None

Quota: There will be a quota of approximately 16 students for this course.

Co-ordinator: Dr Earl

Option available to Mathematics, Mathematics & Statistics, Mathematics & Philosophy students.

Learning Outcomes

The Undergraduate Ambassadors' Scheme (UAS) was begun by Simon Singh in 2002 to give university undergraduates a chance to experience assisting and, to some extent, teaching in schools, and to be credited for this. The option focuses on improving students' communication, presentation, cooperation and organizational skills and sensitivity to others' learning difficulties.

Course Description and Timing:

The Oxford UAS option, N1, is a half-unit, mainly run in Hilary Term. A quota will be in place, of approximately 16 students, and so applicants for the UAS option will be asked to name a second alternative half-unit. The course is appropriate for all students, whether or not they are interested in teaching subsequently.

A student on the course will be assigned to a mathematics teacher in a local secondary school (in the Oxford, Kidlington, Wheatley area) for half a day per week during Hilary Term. Students will be expected to keep a journal of their activities, which will begin by assisting in the class, but may widen to include teaching the whole class for a part of a period, or working separately with a smaller group of the class. Students will be required at one point to give a presentation to one of their school classes relating to a topic from university mathematics, and will also run a small project based on some aspect of mathematics education with advice from the course co-ordinator and teacher/s. Final credit will be based on the journal (20%), the presentation (30%), an end of course report (approximately 3000 words) including details of the project (35%), together with a report from the teacher (15%).

Short interviews will take place on Thursday or Friday of 0th week in Michaelmas term to select students for this course. The interview (of roughly 15 minutes) will include a presentation by the student on an aspect of mathematics of their choosing. Students will be chosen on the basis of their ability to communicate mathematics, and two references will be sought from college tutors on these qualities. Applicants will be quickly notified of the decision.

During Michaelmas term there will be a Training Day, in conjunction with the Oxford Department of Education, as preparation for working with pupils and teachers, and to provide more detail on the organisation of teaching in schools. Those on the course will also need to fill in a CRB form, or to have done so already. By the end of term students will have been assigned to a teacher and have made a first, introductory, visit to their school. The course will begin properly in Hilary term with students helping in schools for half a day each week. Funds are available to cover travel expenses. Support classes will be provided throughout Hilary for feedback and to discuss issues such as the planning of the project. The deadline for the journal and report will be noon on Friday of 0th week of Trinity term.

Any further questions on the UAS option should be passed on to the option's co-ordinator, Richard Earl (earl@maths.ox.ac.uk).

Reading List

Clare Tickly, Anne Watson, Candia Morgan, *Teaching School Subjects: Mathematics* (RoutledgeFalmer, 2004).

4.2 N101 History of Philosophy from Descartes to Kant — Dr Lodge — MT

[Paper 101 in all Honour Schools including Philosophy. Teaching responsibility of the Philosophy Faculty.]

For further details on the Philosophy courses (including details of method of assessment, etc.) please refer to the Philosophy Lectures Prospectus which is published at http://www.philosophy.ox.ac.uk/ prior to the start of each term (usually in 0th Week).

Level H-level **Method of Assessment:** 3-hour examination

Weight: One unit. OSS paper code 101.

Recommended Prerequisites: none.

Learning Outcomes

For those taking Finals in 2007 and thereafter, paper 101 will have a new format: see below.

Candidates will be expected to show critical appreciation of the main philosophical ideas of the period. The subject will be studied in connection with the following texts: Descartes, Meditations, Objections and Replies; Spinoza, Ethics; Leibniz, Monadology, Discourse on Metaphysics; Locke, Essay Concerning Human Understanding; Berkeley, Principles of Human Knowledge, Three Dialogues Between Hylas and Philonous; Hume, Treatise of Human Nature; Kant, Critique of Pure Reason. The paper will consist of three sections; Section A will include questions about Descartes, Spinoza, and Leibniz; Section B will include questions about Locke, Berkeley and Hume; Section C will include questions about Kant. Candidates will be required to answer three questions, with at least one question from Section A and at least one question from Section B. [Examination Regulations 2006, p. 461.]

4.3 N102 Knowledge and Reality — Dr Wedgwood and Prof. Hawthorne — MT

[Paper 102 in all Honour Schools including Philosophy. Teaching responsibility of the Philosophy Faculty.]

For further details on the Philosophy courses (including details of method of assessment, etc.) please refer to the Philosophy Lectures Prospectus which is published at http://www.philosophy.ox.ac.uk/ prior to the start of each term (usually in 0th Week).

Level H-level **Method of Assessment:** 3-hour examination

Weight: One unit. OSS paper code 102.

Recommended Prerequisites: none.

Learning Outcomes

Candidates will be expected to show knowledge in some of the following areas: knowledge and justification; perception; memory; induction; other minds; *a priori* knowledge; necessity and possibility; reference; truth; facts and propositions; definition; existence; identity, including personal identity; substances, change, events; properties; causation; space; time; essence; natural kinds; realism and idealism; primary and secondary qualities. There will also be a section on Philosophy of Science. Candidates' answers must not be confined to questions from the section on Philosophy of Science. [Examination Regulations 2006, p. 461.]

4.4 N122 Philosophy of Mathematics — Dr Uzquiano — MT

[Paper 122 in all Honour Schools including Philosophy. Teaching responsibility of the Philosophy Faculty.]

For further details on the Philosophy courses (including details of method of assessment, etc.) please refer to the Philosophy Lectures Prospectus which is published at http://www.philosophy.ox.ac.uk/ prior to the start of each term (usually in 0th Week).

Level H-level **Method of Assessment:** 3-hour examination

Weight: One unit. OSS paper code 122.

Recommended Prerequisites 101 History of Philosophy from Descartes to Kant, or 102 Knowledge and Reality, or 108 The Philosophy of Logic and Language, or 117 Frege, Russell, and Wittgenstein, or 119 Formal Logic, or 120 Intermediate Philosophy of Physics.

Learning Outcomes

Questions may be set which relate to the following issues: Incommensurables in the development of Greek geometry. Comparisons between geometry and other branches of mathematics. The significance of non-Euclidean geometry. The problem of mathematical rigour in the development of the calculus. The place of intuition in mathematics (Kant, Poincaré). The idea that mathematics needs foundations. The role of logic and set theory (Dedekind, Cantor, Frege, Russell). The claim that mathematics must be constructive (Brouwer). The finitary study of formal systems as a means of justifying infinitary mathematics (Hilbert). Limits to the formalization of mathematics (Gödel). Anti-foundational views of mathematics. Mathematical objects and structures. The nature of infinity. The applicability of mathematics. [Examination Regulations 2006, p. 465.]

5 Language Classes

Language courses in French offered by the University Language Centre.

Students in the FHS Mathematics may apply to take language classes. In 2008–2009, French language classes will be run in MT and HT. We have a limited number of places but if we have spare places we will offer these to joint school students, Mathematics and Computer Science, Mathematics and Philosophy and Mathematics and Statistics

Students wishing to take language classes should attend the qualifying test on Monday of Week 1 Michaelmas Term from 5-7pm in the Language Centre, Woodstock Road.

Two levels of courses are offered, a lower level for those with a good pass at GCSE, and a higher level course for those with A/S or A level. Acceptance on either course will depend on satisfactory performance in the Preliminary Qualifying Test held in Week 1 of Michaelmas Term (Monday, 5-7pm at the Language Centre). Classes at both levels will take place on Mondays, 5-7pm.

Performance on the course will not contribute to the class of degree awarded. However, upon successful completion of the course, students will be issued with a certificate of achievement which will be sent to their college. This should also be noted on the transcript.

Places at these classes are limited, so students are advised that they must indicate their interest at this stage. If you are interested but are unable to attend the qualifying test for some reason please contact the Academic Administrator in the Mathematical Institute (academic.administrator@maths.ox.ac.uk; (6)15203) as soon as possible.

Aims and rationale

The general aim of the language courses is to develop the student's ability to communicate (in both speech and writing) in French to the point where he or she can function in an academic or working environment in a French-speaking country.

The courses have been designed with general linguistic competence in the four skills of reading, writing, speaking and listening in mind. There will be opportunities for participants to develop their own particular interests through presentations and assignments.

Form and Content

Each course will consist of a thirty-two contact hour course, together with associated work. Classes will be held in the Language Centre at two hours per week in Michaelmas and Hilary Terms.

The content of the courses is based on coursebooks together with a substantial amount of supplementary material prepared by the language tutors. Participants should expect to spend an additional two hours per week on preparation and follow-up work.

Each course aims to cover similar ground in terms of grammar, spoken and written language and topics. Areas covered will include:

Grammar:

- all major tenses will be presented and/or revised, including the subjunctive
- passive voice
- pronouns
- formation of adjectives, adverbs, comparatives
- use of prepositions

- time expressions

Speaking

- guided spoken expression for academic, work and leisure contact (e.g. giving presentations, informal interviews, applying for jobs)
- expressing opinions, tastes and preferences
- expressing cause, consequence and purpose

Writing

- Guided letter writing for academic and work contact
- Summaries and short essays

Listening

- Listening practice of recorded materials (e.g. news broadcasts, telephone messages, interviews)
- developing listening comprehension strategies

Topics (with related readings and vocabulary, from among the following)

- life, work and culture
- the media in each country
- social and political systems
- film, theatre and music
- research and innovation
- sports and related topics
- student-selected topics

Teaching staff

The courses are taught by Language Centre tutors or Modern Languages Faculty instructors.

Teaching and learning approaches

Each course uses a communicative methodology which demands active participation from students. In addition, there will be some formal grammar instruction. Students will be expected to prepare work for classes and homework will be set. Students will also be given training in strategies for independent language study, including computer-based language learning, which will enable them to maintain and develop their language skills after the course.

Entry

Two classes at (probably at Basic and Threshold levels) will be formed according to level of French at entry. The minimum entry standard is a good GCSE Grade or equivalent. Registration for each option will take place at the start of Michaelmas Term. A preliminary qualifying test is then taken which candidates must pass in order to be allowed to join the course.

Learning Outcomes

The learning outcomes will be based on internationally agreed criteria for specific levels (known as the ALTE levels), which are as follows:

Basic Level (corresponds to ALTE Level 2 "Can-do" statements)

- Can express opinions on professional, cultural or abstract matters in a limited way, offer advice and understand instructions.
- Can give a short presentation on a limited range of topics.
- Can understand routine information and articles, including factual articles in newspapers, routine letters from hotels and letters expressing personal opinions.
- Can write letters or make notes on familiar or predictable matters.

Threshold Level (corresponds to ALTE Level 3 "Can-do" statements)

- Can follow or give a talk on a familiar topic or keep up a conversation on a fairly wide range of topics, such as personal and professional experiences, events currently in the news.
- Can give a clear presentation on a familiar topic, and answer predictable or factual questions.
- Can read texts for relevant information, and understand detailed instructions or advice.
- Can make notes that will be of reasonable use for essay or revision purposes.
- Can make notes while someone is talking or write a letter including non- standard requests.

Assessment

There will a preliminary qualifying test in Michaelmas Term. There are three parts to this test: Reading Comprehension, Listening Comprehension, and Grammar. Candidates who have not studied or had contact with French for some time are advised to revise thoroughly, making use of the Language Centre's French resources.

Students' achievement will be assessed through a variety of means, including continuous assessment of oral performance, a written final examination, and a project or assignment chosen by individual students in consultation with their language tutor.

Reading comprehension, writing, listening comprehension and speaking are all examined. The oral component may consist of an interview, a presentation or a candidate's performance in a formal debate or discussion.