Handbook for the Undergraduate Mathematics Courses
Supplement to the Handbook
Honour School of Mathematics
Syllabus and Synopses for Part B 2010–11
for examination in 2011

Contents

1 Foreword

The synopses for Part B will be available on the website at:

http://www.maths.ox.ac.uk/current-students/undergraduates/handbooks-synopses/

before the start of Michaelmas Term 2010.

See the current edition of the *Examination Regulations* for the full regulations governing these examinations.

Notice of misprints or errors of any kind, and suggestions for improvements in this booklet should be addressed to the Academic Assistant in the Mathematical Institute.

1.1 Honour School of Mathematics

1.1.1 Units and Half Units

In Part B each candidate shall offer a total of four units from the schedule of units and half units.

(a) A total of at least three units offered should be from the schedule of 'Mathematics Department units and half units'

(b) Candidates may offer at most one unit which is designated as an extended essay or a structured project[1]

Details of units for Part C will be published in 2011. Students staying on to take Part C will take a total of 3 units from the schedule of Part C units and half units.

Language Classes

Mathematics students may apply to take classes in a foreign language. In 2010-11 classes will be offered in French . Students' performances in these classes will not contribute to the degree classification awarded. However, successful completion of the course may be recorded on students' transcripts. See section 5 for more details.

1.2 Extract from Examination Conventions

1.2.1 The Examination Papers in Part B

Most courses in Mathematics are assessed by examination. Most subjects offered have a 'weight' of one unit, and will be examined in a 3-hour examination. In many of these subjects it will also be possible to take the first half, or either half, of the subject as a 'half unit'. Where this is the case, a half unit will usually be examined in an examination paper of $1\frac{1}{2}$ hours. Each half unit paper will contain **3** questions.

Rubric on 3-hour examination papers (Mathematics)

[1] Units which may be offered under this heading are indicated in the synopses.

The rubric on 3-hour examination papers will usually be: "you may submit answers to as many questions as you wish; the best two from each section will count towards the total mark".

Rubric on $1\frac{1}{2}$ hour examination papers (Mathematics)

The rubric on $1\frac{1}{2}$ hour examination papers will usually be: "you may submit answers to as many questions as you wish; the best two will count towards the total mark for this paper.".

Where not otherwise stated, an overview of the syllabus and form of the papers for each unit and half-unit is given in the lecture synopsis.

Part C (M–Level) courses available in the third year

All Part C courses (marked as 'M–Level') available to third year students will be examined using the same examination questions/projects as used for fourth-year students.

Marking of Papers

For Mathematics Department papers in Part B and Part C mark schemes for questions out of 25 will aim to ensure that the following qualitative criteria hold:

- 20-25 marks: a completely or almost completely correct answer, showing excellent understanding of the concepts and skill in carrying through the arguments and/or calculations; minor slips or omissions only.

- 13-19 marks: a good though not complete answer, showing understanding of the concepts and competence in handling the arguments and/or calculations. In this range, an answer might consist of an excellent answer to a substantial part of the question, or a good answer to the whole question which nevertheless shows some flaws in calculation or in understanding or in both.

This should be regarded only as a guide, conveying the intention of the examiners.

Marking of Extended Essays

All extended essays are independently marked by at least two assessors. The examiner responsible for extended essays will oversee the reconciliation of marks. If agreement is not possible an additional assessor will be appointed.

Analysis of marks

Part B

The Board of Examiners in Part B will assign USMs for whole unit and half unit papers taken in Part B and may recalibrate the raw marks to arrive at USMs reported to candidates. The whole unit papers are designed so that the raw marks sum to 100. Examiners will take into account relative difficulty of papers when assigning USMs. In order to achieve this, Examiners may use information on candidates' performances on the Part A examination when recalibrating the raw marks. They may also use other statistics to check that the USMs assigned fairly reflect the student performances on a paper.

The USMs awarded to a candidate for papers in Part B will be aggregated with the USMs from Part A to arrive at a classification.

1.3 Classification in the Honour School of Mathematics

Each candidate will receive a numerical mark on each paper in each Part of the examination in the University standardised range 0-100, such that

- a First Class performance (on that paper) is indicated by a mark of 70 to 100;
- an Upper Second Class performance (on that paper) is indicated by a mark of 60 to 69;
- a Lower Second Class performance (on that paper) is indicated by a mark of 50 to 59
- a Third Class performance (on that paper) is indicated by a mark of 40 to 49;
- a Pass performance (on that paper) is indicated by a mark of 30 to 39;
- a Fail performance (on that paper) is indicated by a mark of 0 to 29.

Aggregation of marks for the award of the classification on the successful completion of Parts A and B

All successful candidates will be awarded a classification after the Part B examination. This classification will be based on the following rules (agreed by the Mathematics Teaching Committee).

A *Strong Paper rule* is used for classification.

Strong Paper rule
A candidate will have satisfied the First Class, resp., Upper Second Class, resp., Lower Second Class strong paper rule if at least 3 papers from Parts A and B lie in that class (or better) and include at least one of them in Part B.

To give an example, a candidate will have satisfied the Upper Second Class strong paper rule if (the equivalent of) at least 3 of their whole unit paper USMs have at least Upper Second Class marks with (the equivalent of) at least one Upper Second Class whole unit at Part B level. Students may take half unit papers for Part B and for two half units (not making up a whole unit paper) to count as the equivalent of a whole unit of at least Upper Second Class, both half units must be of at least Upper Second Class.

The Strong Paper rule gives a *marks profile.*

The Part A USMs are given a weighting of 2, and the Part B USMs a weighting of 3 for a whole unit and 1.5 for a half unit.

In the following $Av\,USM = Average\,weighted\,USM\ for\ Parts\ A\ and\ B\ together$ (rounded up to whole number);

- First Class: $Av\ USM \geq 70$ and the First Class strong paper rule satisfied.
- Upper Second Class: $Av\,USM \geq 70$ and the First Class strong paper rule not satisfied **OR** $70 > Av\ USM \geq 60$ and the Upper Second class strong paper rule satisfied.
- Lower Second Class: $70 > Av\ USM \geq 60$ and the Upper Second Class strong paper rule not satisfied **OR** $60 > Av\ USM \geq 50$ and the Lower Second Class strong paper rule satisfied.
- Third Class: $50 > Av\ USM \geq 40$ **OR** $60 > Av\ USM \geq 50$ and the Lower Second Class strong paper rule not satisfied
- Pass: $40 > Av\ USM \geq 30$
- Fail: $Av\ USM < 30$

BA in Mathematics
Any candidate who satisfes the Examiners for Parts A and B (and who does not subsequently enter for and achieve Honours for Part C) may supplicate for the Honours degree of the Bachelor of Arts in Mathematics with the classification as described above, provided that they have fullfilled all the conditions for admission to a degree of the university.

MMath in Mathematics
In order to proceed to Part C, a candidate must achieve a lower second standard or better in Parts A and B together.

Candidates successfully completing Part C will receive a separate classification based on their University Standardised Marks in Part C papers.

Note that successful candidates may only supplicate for one degree - either a BA or an MMath. The MMath. has two classifications associated with it but a candidate will not be awarded a BA degree and an MMath. degree.

Class Descriptors

The average USM ranges used in the classifications reflect the following descriptions:

- First Class: the candidate shows excellent skills in reasoning, deductive logic and problem-solving. He/she demonstrates an excellent knowledge of the material, and is able to use that in unfamiliar contexts.
- Upper Second Class: the candidate shows good or very good skills in reasoning, deductive logic and problem-solving. He/she demonstrates a good or very good knowledge of much of the material.

- Lower Second Class: the candidate shows adequate basic skills in reasoning, deductive logic and problem-solving. He/she demonstrates a sound knowledge of much of the material.
- Third Class: the candidate shows reasonable understanding of at least part of the basic material and some skills in reasoning, deductive logic and problem-solving.
- Pass: the candidate shows some limited grasp of at least part of the basic material.

  [Note that the aggregation rules in some circumstances allow a stronger performance on some papers to compensate for a weaker performance on others.]
- Fail: little evidence of competence in the topics examined; the work is likely to show major misunderstanding and confusion, coupled with inaccurate calculations; the answers to questions attempted are likely to be fragmentary only.

Students who want to know details of the classification conventions for later parts of the degree should refer to the relevant synopsis which may be found at https://www.maths.ox.ac.uk/current-students/undergraduates/handbooks-synopses.

1.4 Registration for Part B courses 2010–11

CLASSES Students will have to register in advance for the classes they wish to take. Students will have to register by Friday of Week 9 of Trinity Term 2010 using the online registration system which can be accessed at https://www.maths.ox.ac.uk/course-registration. Further guidance on how to use the online system can be found at:
http://www.maths.ox.ac.uk/help/faqs/undergrads/course-registration.

Students who register for a course or courses for which there is a quota will have to select a "reserve choice" which they will take if they do not receive a place on the course with the quota. They may also have to give the reasons why they wish to take a course which has a quota, and provide the name of a tutor who can provide a supporting statement for them should the quota be exceeded. Where this is necessary students will be contacted by email after they have registered. In the event that the quota for a course is exceeded, the Mathematics Teaching Committee will decide who may have a place on the course on the basis of the supporting statements from the student and tutor. Students who are not allocated a place on the course with the quota will be registered for their "reserve" course. Students will be notified of this by email. In the case of the "Undergraduate Ambassadors' Scheme" students will have to attend a short interview in Week 0, Michaelmas Term. In the meantime they will also have to complete a separate application form, and indicate a "reserve" course.

Note on Intercollegiate Classes

Where undergraduate registrations for lecture courses fall below 5, classes will not run as part of the intercollegiate scheme but will be arranged informally by the lecturer.

LECTURES: Some combinations of subjects are not advised and lectures may clash. Details are given below. We will use the information on your registration forms to aim to keep clashes to a minimum. However, because of the large number of options available in Part B some clashes are inevitable, and we must aim to accommodate the maximum number of student preferences.

Lecture Timetabling in Part B 2010-11

The Teaching Committee has agreed that the following clashes be allowed.

Pure vs Applied

B1 Logic and Set Theory B2 Algebra B3 Geometry B9 Number Theory	may clash with	B6 Theoretical Mechanics B8 Topics in Applied Mathematics
B1 Logic and Set Theory B2 Algebra B3 Geometry B7 Mathematical Physics B9 Number Theory All M-level options	may clash with	B21 Numerical Solution of Differential Equations
B3 Geometry B7 Mathematical Physics All M-level options	may clash with	B22 Integer Programming

Mathematics vs Statistics

B1 Logic and Set Theory B2 Algebra B3 Geometry B6 Theoretical Mechanics B7 Mathematical Physics B9 Number Theory All M-level options	may clash with	OBS1 Applied Statistics OBS3 Stochastic Modelling
B3 Geometry B6 Theoretical Mechanics B7 Mathematical Physics All M-level options	may clash with	OBS4 Actuarial Science
B21 Numerical Solution of Differential Equations	may clash with	OBS1 Applied Statistics OBS2 Statistical Inference OBS3 Stochastic Modelling OBS4 Actuarial Science
All Part B Maths and M-level options		OBS2 Statistical Inference

Mathematics vs Comp

B3 Geometry B6 Theoretical Mechanics B7 Mathematical Physics B8 Topics in Applied Mathematics All M-level options	may clash with	OCS1 Functional Programming and Analysis of Algorithms

Mathematics vs Philosophy

B5 Differential Equations and Applications B6 Theoretical Mechanics B7 Mathematical Physics B8 Topics in Applied Mathematics B10 Martingales & Financial Mathematics B11 Communication Theory	may clash with	FHS Maths & Phil Core options

'Other' Mathematical and 'Non-Mathematical' subjects

OBS1 Applied Statistics OBS2 Statistical Inference OBS3 Stochastic Modelling OBS4 Actuarial Science	may clash with	OCS1 Functional Programming and Analysis of Algorithms	may clash with	All papers in Philosophy

1.5 Three-year/Four-year Course Registration

You should register your intention to take either the BA course or the MMath. course during your third year. You are advised to discuss the right course of action for you with your College Tutor, who will also advise you how to register. Any student whose performance in the Part A and B examinations together falls below **lower second standard** will not be permitted to proceed to Part C.

All students are registered on the MMath versions of each course. If you subsequently decide to change to the BA option you must inform your college office who will in turn inform central administration and the departments. Please be aware that any change to your course may impact the level of your maintenance funding and the time taken to receive your student loan (you are advised to contact Student Finance
http://www.direct.gov.uk/en/EducationAndLearning/UniversityAndHigherEducation/ StudentFinance for further enquiries). Please note also that if you intend to change option you are strongly advised to do so before you take the Part B examinations.

2 Mathematics Department units and half-units

2.1 B1:Foundations Logic and Set Theory

Level: H-level **Method of Assessment:** Written examination.

Weight: Whole-unit (OSS paper code 2640), or can be taken as either a half-unit in Logic or a half-unit in Set Theory.

2.1.1 B1a: Logic — Dr Koenigsmann — 16 MT

[Option **B1a** if taken as a half-unit. OSS paper code 2A40.]

Overview

To give a rigorous mathematical treatment of the fundamental ideas and results of logic that is suitable for the non-specialist mathematicians and will provide a sound basis for more advanced study. Cohesion is achieved by focussing on the Completeness Theorems and the relationship between probability and truth. Consideration of some implications of the Compactness Theorem gives a flavour of the further development of model theory. To give a concrete deductive system for predicate calculus and prove the Completeness Theorem, including easy applications in basic model theory.

Learning Outcomes

Students will be able to use the formal language of propositional and predicate calculus and be familiar with their deductive systems and related theorems. For example, they will know and be able to use the soundness, completeness and compactness theorems for deductive systems for predicate calculus.

Synopsis

The notation, meaning and use of propositional and predicate calculus. The formal language of propositional calculus: truth functions; conjunctive and disjunctive normal form; tautologies and logical consequence. The formal language of predicate calculus: satisfaction, truth, validity, logical consequence.

Deductive system for propositional calculus: proofs and theorems, proofs from hypotheses, the Deduction Theorem; Soundness Theorem. Maximal consistent sets of formulae; completeness; constructive proof of completeness.

Statement of Soundness and Completeness Theorems for a deductive system for predicate calculus; derivation of the Compactness Theorem; simple applications of the Compactness Theorem.

A deductive system for predicate calculus; proofs and theorems; prenex form. Proof of Completeness Theorem. Existence of countable models, the downward Löwenheim–Skolem Theorem.

Reading

1. R. Cori and D. Lascar, *Mathematical Logic: A Course with Exercises (Part I)* (Oxford University Press, 2001), sections 1, 3, 4.
2. A. G. Hamilton, *Logic for Mathematicians* (2nd edition, Cambridge University Press, 1988), pp.1–69, pp.73–76 (for statement of Completeness (Adequacy)Theorem), pp.99–103 (for the Compactness Theorem).
3. W. B. Enderton, *A Mathematical Introduction to Logic* (Academic Press, 1972), pp.101–144.
4. D. Goldrei, *Propositional and Predicate Calculus: A model of argument* (Springer, 2005).

Further Reading

1. R. Cori and D. Lascar, *Mathematical Logic: A Course with Exercises (Part II)* (Oxford University Press, 2001), section 8.

2.1.2 B1b: Set Theory — Dr Pila — 16 HT

[Option **B1b** if taken as a half-unit. OSS paper code 2B40.]

Overview

To introduce sets and their properties as a unified way of treating mathematical structures, including encoding of basic mathematical objects using set theoretic language. To emphasize the difference between intuitive collections and formal sets. To introduce and discuss the notion of the infinite, the ordinals and cardinality. The Axiom of Choice and its equivalents are presented as a tool.

Learning Outcomes

Students will have a sound knowledge of set theoretic language and be able to use it to codify mathematical objects. They will have an appreciation of the notion of infinity and arithmetic of the cardinals and ordinals. They will have developed a deep understanding of the Axiom of Choice, Zorn's Lemma and well-ordering principle, and have begun to appreciate the implications.

Synopsis

What is a set? Introduction to the basic axioms of set theory. Ordered pairs, cartesian products, relations and functions. Axiom of Infinity and the construction of the natural numbers; induction and the Recursion Theorem.

Cardinality; the notions of finite and countable and uncountable sets; Cantor's Theorem on power sets. The Tarski Fixed Point Theorem. The Schröder–Bernstein Theorem.

Isomorphism of ordered sets; well-orders. Transfinite induction; transfinite recursion [informal treatment only].

Comparability of well-orders.

The Axiom of Choice, Zorn's Lemma, the Well-ordering Principle; comparability of cardinals. Equivalence of WO, CC, AC and ZL. Ordinals. Arithmetic of cardinals and ordinals; in [ZFC],

Reading

1. D. Goldrei, *Classic Set Theory* (Chapman and Hall, 1996).
2. W. B. Enderton, *Elements of Set Theory* (Academic Press, 1978).

Further Reading

1. R. Cori and D. Lascar, *Mathematical Logic: A Course with Exercises (Part II)* (Oxford University Press, 2001), section 7.1–7.5.
2. R. Rucker, *Infinity and the Mind: The Science and Philosophy of the Infinite* (Birkhäuser, 1982). An accessible introduction to set theory.
3. J. W. Dauben, *Georg Cantor: His Mathematics and Philosophy of the Infinite* (Princton University Press, 1990). For some background, you may find JW Dauben's biography of Cantor interesting.
4. M. D. Potter, *Set Theory and its Philosophy: A Critical Introduction* (Oxford University Press, 2004). An interestingly different way of establishing Set Theory, together with some discussion of the history and philosophy of the subject.
5. G. Frege, *The Foundations of Arithmetic : A Logical-Mathematical Investigation into the Concept of Number* (Pearson Longman, 2007).
6. M. Schirn, *The Philosophy of Mathematics Today* (Clarendon, 1998). A recentish survey of the area at research level.
7. W. Sierpinski, *Cardinal and Ordinal Numbers* (Polish Scientific Publishers, 1965). More about the arithmetic of transfinite numbers.

2.2 B2: Algebra

Level: H-level **Method of Assessment:** Written examination.

Weight: Whole-unit (OSS paper code 2641), or available as a half-unit in B2a, or as a half-unit in B2b.

Recommended Prerequisites: All second year algebra.

2.2.1 B2a: Introduction to Representation Theory — Dr Henke — 16 MT

[Option **B2a** if taken as a half-unit. OSS paper code 2A41.]

Overview

This course gives an introduction to the representation theory of finite groups and finite dimensional algebras. Representation theory is a fundamental tool for studying symmetry by means of linear algebra: it is studied in a way in which a given group or algebra may act on vector spaces, giving rise to the notion of a representation.

We start in a more general setting, studying modules over rings, in particular over euclidean domains, and their applications. We eventually restrict ourselves to modules over algebras (rings that carry a vector space structure). A large part of the course will deal with the structure theory of semisimple algebras and their modules (representations). We will prove the Jordan-Hölder Theorem for modules. Moreover, we will prove that any finite-dimensional semisimple algebra is isomorphic to a product of matrix rings (Wedderburn's Theorem over $\mathbb{C}$).

In the later part of the course we apply the developed material to group algebras, and classify when group algebras are semisimple (Maschke's Theorem).

Learning Outcomes

Students will have a sound knowledge of the theory of non-commutative rings, ideals, associative algebras, modules over euclidean domains and applications. They will know in particular simple modules and semisimple algebras and they will be familiar with examples. They will appreciate important results in the course such as the Jordan-Hölder Theorem, Schur's Lemma, and the Wedderburn Theorem. They will be familiar with the classification of semisimple algebras over $\mathbb{C}$ and be able to apply this.

Synopsis

Noncommutative rings, one- and two-sided ideals. Associative algebras (over fields). Main examples: matrix algebras, polynomial rings and quotients of polynomial rings. Group algebras, representations of groups.

Modules over euclidean domains and applications such as finitely generated abelian groups, rational canonical forms. Modules and their relationship with representations. Simple and semisimple modules, composition series of a module, Jordan-Hölder Theorem. Semisimple algebras. Schur's Lemma, the Wedderburn Theorem, Maschke's Theorem.

Reading

1. K. Erdmann, *B2 Algebras*, Mathematical Institute Notes (2007).

2. G. D. James and M. Liebeck, *Representations and Characters of Finite Groups* (2nd edition, Cambridge University Press, 2001).

Further Reading

1. J. L. Alperin and R. B. Bell, *Groups and Representations,* Graduate Texts in Mathematics 162 (Springer-Verlag, 1995).

2. P. M. Cohn, *Classic Algebra* (Wiley & Sons, 2000). (Several books by this author available.)

3. C. W. Curtis, and I. Reiner, *Representation Theory of Finite Groups and Associative Algebras* (Wiley & Sons, 1962).

4. L. Dornhoff, *Group Representation Theory* (Marcel Dekker Inc., New York, 1972).

5. I. M. Isaacs, *Character Theory of Finite Groups* (AMS Chelsea Publishing, American Mathematical Society, Providence, Rhode Island, 2006).

6. J.-P. Serre, *Linear Representations of Finite Groups,* Graduate Texts in Mathematics 42 (Springer-Verlag, 1977).

2.2.2 B2b: Group Theory — Prof. Collins — 16 HT

[Option **B2b** if taken as a half-unit. OSS paper code 2B41.]

Recommended Prerequisites: Part A Group Theory is essential. Part B Introduction to Representation Theory is useful as "further algebraic thinking", and some of the results proved in that course will be stated (but not proved). In particular, character theory will be developed using Wedderburn's theorem (with Maschke's theorem assumed in the background). Students should also be well acquainted with linear algebra, especially inner products and conditions for the diagonalisability of matrices.

Overview

A finite group represents one of the simplest algebraic objects, having just one operation on a finite set, and historically groups arose from the study of permutations or, more generally, sets of bijective functions on a set closed under composition. Thus there is the scope for both a rich theory and a wide source of examples. Some of this has been seen in the Part A course Group Theory, and this course will build on that. In particular, the Jordan–Hölder theorem (covered there but not examined) shows that there are essentially two problems, to find the finite simple groups, and to learn how to put them together. The first of these dominated the second half of the 20th century and has been completed; this proved a massive task, encompassing in excess of 20,000 printed pages, and much remains to be done to distil the underlying ideas. In this course, our aim will be to introduce some of the very fundamental ideas that made this work possible. Much is classical, but it will be presented in a modern form, and one new inclusion in this course (compared with previous years), Alperin's fusion theorem, has links with the most recent work in the subject.

Learning Outcomes

By the end of this course, a student should feel comfortable with a number of techniques for studying finite groups, appreciate certain classes of finite simple groups that represent prototypes for almost all finite simple groups, and have seen the proofs of some of the "great" theorems.

MFoCS students may be expected to read beyoned the confines of the lectures from the "more sophisticated books" below to attempt the additional problems set for MFoCS students in preparation for working on the miniproject.

Synopsis

Review of isomorphism theorems (up to Jordan–Hölder), composition series, soluble groups; some examples of groups of (relatively) small order. Constructions of groups; semidirect products, notion of presentations. Cauchy's theorem, Sylow's theorems, the Frattini argument, nilpotent groups. Conjugation families and Alperin's fusion theorem. Alternating and projective special linear groups. Characters of complex representations. The class algebra and central idempotents, orthogonality relations, construction of character tables and properties derivable from a character table. The character ring as a subring of the algebra of complex-valued class functions. Burnside's $p^\alpha q^\beta$-theorem.

Reading

The bulk of the course is well covered by the web notes written in 2010 by Jan Grabowski, which will be reposted with minor corrections. (A few topics from those notes will not be covered as will be clear from this year's synopsis.) New web notes will be written to cover the one topic not covered in those notes, namely Alperin's Fusion Theorem.

Additional Reading

1. Geoff Smith and Olga Tabachnikova, *Topics in Group Theory,* Springer Undergraduate Mathematics Series (Springer-Verlag, 2000). ISBN 1-85233-235-2

2. G.D. James and M. Liebeck, *Representations and Characters of Groups* (Second edition, Cambridge University Press, 2001). ISBN 0-521-00392-X

The following more sophisticated books are useful for reference and, in approach, may better represent the spirit of this course:

3. J I Alperin and Rowen B Bell, *Groups and Representations*, Graduate Texts in Mathematics 162 (Springer-Verlag, 1995). ISBN 0-387-94526-1
4. M J Collins, *Representations and Characters of Finite Groups*, Cambridge Studies in Advanced Mathematics 22, Cambridge University Press, 1990; reprinted p/b 2008, ISBN 978-0-521-06764-5, esp pp 48-63.
5. D Gorenstein, *Finite Groups*, Harper and Row, 1968; reissued Chelsea, 1980.

All these, and many other books on finite group theory and introductory character theory, can be found in most college libraries.

2.3 B3: Geometry

Level: H-level **Method of Assessment:** Written examination.

Weight: Whole-unit (OSS paper code 2642), or can be taken as either a half-unit in Geometry of Surfaces or a half-unit in Algebraic Curves (but see "Prerequisites").

Recommended Prerequisites: 2nd year core algebra and analysis, 2nd year topology. Multivariable calculus and group theory would be useful but not essential. Also, B3a is helpful, but not essential, for B3b.

2.3.1 B3a: Geometry of Surfaces — Prof. Hitchin — 16 MT

[Option **B3a** if taken as a half-unit. OSS paper code 2A42.]

Overview

Different ways of thinking about surfaces (also called two-dimensional manifolds) are introduced in this course: first topological surfaces and then surfaces with extra structures which allow us to make sense of differentiable functions ('smooth surfaces'), holomorphic functions ('Riemann surfaces') and the measurement of lengths and areas ('Riemannian 2-manifolds').

These geometric structures interact in a fundamental way with the topology of the surfaces. A striking example of this is given by the Euler number, which is a manifestly topological quantity, but can be related to the total curvature, which at first glance depends on the geometry of the surface.

The course ends with an introduction to hyperbolic surfaces modelled on the hyperbolic plane, which gives us an example of a non-Euclidean geometry (that is, a geometry which meets all Euclid's axioms except the axioms of parallels).

Learning Outcomes

Students will be able to implement the classification of surfaces for simple constructions of topological surfaces such as planar models and connected sums; be able to relate the Euler characteristic to branching data for simple maps of Riemann surfaces; be able to describe the definition and use of Gaussian curvature; know the geodesics and isometries of the hyperbolic plane and their use in geometrical constructions.

Synopsis

The concept of a topological surface (or 2-manifold); examples, including polygons with pairs of sides identified. Orientation and the Euler characteristic. Classification theorem for compact surfaces (the proof will not be examined).

Riemann surfaces; examples, including the Riemann sphere, the quotient of the complex numbers by a lattice, and double coverings of the Riemann sphere. Holomorphic maps of Riemann surfaces and the Riemann–Hurwitz formula. Elliptic functions.

Smooth surfaces in Euclidean three-space and their first fundamental forms. The concept of a Riemannian 2-manifold; isometries; Gaussian curvature.

Geodesics. The Gauss–Bonnet Theorem (statement of local version and deduction of global version). Critical points of real-valued functions on compact surfaces.

The hyperbolic plane, its isometries and geodesics. Compact hyperbolic surfaces as Riemann surfaces and as surfaces of constant negative curvature.

Reading

1. A. Pressley, *Elementary Differential Geometry,* Springer Undergraduate Mathematics Series (Springer-Verlag, 2001). (Chapters 4–8 and 10–11.)
2. G. B. Segal, *Geometry of Surfaces*, Mathematical Institute Notes (1989).
3. R. Earl, *The Local Theory of Curves and Surfaces*, Mathematical Institute Notes (1999).
4. J. McCleary, *Geometry from a Differentiable Viewpoint* (Cambridge, 1997).

Further Reading

1. P. A. Firby and C. E. Gardiner, *Surface Topology* (Ellis Horwood, 1991) (Chapters 1–4 and 7).
2. F. Kirwan, *Complex Algebraic Curves,* Student Texts 23 (London Mathematical Society, Cambridge, 1992) (Chapter 5.2 only).
3. B. O'Neill, *Elementary Differential Geometry* (Academic Press, 1997).

2.3.2 B3b: Algebraic Curves — Prof. Joyce — 16 HT

[Option **B3b** if taken as a half-unit. OSS paper code 2B42.]

Overview

A real algebraic curve is a subset of the plane defined by a polynomial equation $p(x, y) = 0$. The intersection properties of a pair of curves are much better behaved if we extend this picture in two ways: the first is to use polynomials with complex coefficients, the second to extend the curve into the projective plane. In this course projective algebraic curves are studied, using ideas from algebra, from the geometry of surfaces and from complex analysis.

Learning Outcomes

Students will know the concepts of projective space and curves in the projective plane. They will appreciate the notion of nonsingularity and know some basic features of intersection theory. They will view nonsingular algebraic curves as examples of Riemann surfaces, and be familiar with divisors, meromorphic functions and differentials.

Synopsis

Projective spaces, homogeneous coordinates, projective transformations.

Algebraic curves in the complex projective plane. Euler's relation. Irreducibility, singular and nonsingular points, tangent lines.

Bezout's Theorem (the proof will not be examined). Points of inflection, and normal form of a nonsingular cubic.

Nonsingular algebraic curves as Riemann surfaces. Meromorphic functions, divisors, linear equivalence. Differentials and canonical divisors. The group law on a nonsingular cubic.

The Riemann–Roch Theorem (the proof will not be examined). The geometric genus. Applications.

Reading

1. F. Kirwan, *Complex Algebraic Curves,* Student Texts 23 (London Mathematical Society, Cambridge, 1992), Chapters 2–6.

2.4 B3.1a: Topology and Groups —Dr Papazoglou — 16 MT

Level: H-level. **Method of Assessment:** Written examination.

Weight: Half-unit, OSS paper code 2A63.

Prerequisites

2nd year Groups in Action, 2nd year Topology.

Overview

This course introduces the important link between topology and group theory. On the one hand, associated to each space, there is a group, known as its fundamental group. This can be used to solve topological problems using algebraic methods. On the other hand, many results about groups are best proved and understood using topology. For example, presentations of groups, where the group is defined using generators and relations, have a topological interpretation. The endpoint of the course is the Nielsen–Shreier Theorem, an important, purely algebraic result, which is proved using topological techniques.

Synopsis

Homotopic mappings, homotopy equivalence. Simplicial complexes. Simplicial approximation theorem.

The fundamental group of a space. The fundamental group of a circle. Application: the fundamental theorem of algebra. The fundamental groups of spheres.

Free groups. Existence and uniqueness of reduced representatives of group elements. The fundamental group of a graph.

Groups defined by generators and relations (with examples). Tietze transformations.

The free product of two groups. Amalgamated free products.

The Seifert–van Kampen Theorem.

Cell complexes. The fundamental group of a cell complex (with examples). The realization of any finitely presented group as the fundamental group of a finite cell complex.

Covering spaces. Liftings of paths and homotopies. A covering map induces an injection between fundamental groups. The use of covering spaces to determine fundamental groups: the circle again, and real projective n-space. The correspondence between covering spaces and subgroups of the fundamental group. Regular covering spaces and normal subgroups.

Cayley graphs of a group. The relationship between the universal cover of a cell complex, and the Cayley graph of its fundamental group. The Cayley 2-complex of a group.

The Nielsen–Schreier Theorem (every subgroup of a finitely generated free group is free) proved using covering spaces.

Reading

1. John Stillwell, *Classical Topology and Combinatorial Group Theory* (Springer-Verlag, 1993).

Additional Reading

1. D. Cohen, *Combinatorial Group Theory: A Topological Approach,* Student Texts 14 (London Mathematical Society, 1989), Chapters 1–7.
2. A. Hatcher, *Algebraic Topology* (CUP, 2001), Chapter. 1.
3. M. Hall, Jr, *The Theory of Groups* (Macmillan, 1959), Chapters. 1–7, 12, 17 .
4. D. L. Johnson, *Presentations of Groups,* Student Texts 15 (Second Edition, London Mathematical Society, Cambridge University Press, 1997). Chapters. 1–5, 10,13.
5. W. Magnus, A. Karrass, and D. Solitar, *Combinatorial Group Theory* (Dover Publications, 1976). Chapters. 1–4.

2.5 B4: Analysis

Level: H-level **Method of Assessment:** Written examination.

Weight: Whole-unit (OSS paper code 2643), or B4a may be taken as a half-unit.

Recommended Prerequisites: Part A Topology and Integration. [From Topology, only the material on metric spaces, including closures, will be used. From Integration, the only concepts which will be used are the convergence theorems and the theorems of Fubini and Tonelli, and the notions of measurable functions and null sets. No knowledge is needed of outer measure, or of any particular construction of the integral, or of any proofs.]

Overview

The two most important kinds of infinite–dimensional vector space are Banach spaces and Hilbert spaces; they provide the theoretical underpinnings for much of differential equations, and also for quantum theory in physics. This course provides an introduction to Banach spaces and Hilbert spaces. It combines familiar ideas from topology and linear algebra. It would be useful background for further work in analysis, differential equations, and so on.

2.5.1 B4a: Banach Spaces — Dr Kirchheim — 16 MT

[Option **B4a** if taken as a half-unit. OSS paper code 2A43.]

Learning Outcomes

Students will have a firm knowledge of real and complex normed vector spaces, with their geometric and topological properties. They will be familiar with the notions of completeness, separability and density, will know the properties of a Banach space and important examples, and will be able to prove results relating to the Hahn–Banach Theorem. They will have developed an understanding of the theory of bounded linear operators on a Banach space.

Synopses

Real and complex normed vector spaces, their geometry and topology. Completeness. Banach spaces, examples (ℓ^p, ℓ^∞, L^p, $C(K)$, spaces of differentiable functions).

Finite-dimensional normed spaces; equivalence of norms and completeness. Separable spaces; separability of subspaces.

Continuous linear functionals. Dual spaces. Hahn–Banach Theorem (proof for real separable spaces only) and applications, including density of subspaces.

Bounded linear operators, examples (including integral operators). Adjoint operators. Spectrum and resolvent. Spectral mapping theorem for polynomials.

Essential Reading

1. B.P. Rynne and M.A. Youngson, *Linear Functional Analysis* (Springer SUMS, 2nd edition, 2008), Chapters 2, 4, 5.
2. E. Kreyszig, *Introductory Functional Analysis with Applications* (Wiley, revised edition, 1989), Chapters 2, 4.24.3, 4.5, 7.17.4.

2.5.2 B4b: Hilbert Spaces — Prof. Batty — 16 HT

Learning Outcomes

Students will appreciate the role of completeness through the Baire category theorem and its consequences for operators on Banach spaces. They will have a demonstrable knowledge of the properties of a Hilbert space, including orthogonal complements, orthonormal sets, complete orthonormal sets together with related identities and inequalities. They will be familiar with the theory of linear operators on a Hilbert space, including adjoint operators, self-adjoint and unitary operators with their spectra. They will know the L^2-theory of Fourier series and be aware of the classical theory of Fourier series and other orthogonal expansions.

Synopses

Baire Category Theorem and its consequences for operators on Banach spaces (Uniform Boundedness, Open Mapping, Inverse Mapping and Closed Graph Theorems). Strong convergence of sequences of operators.

Hilbert spaces; examples including L^2-spaces. Orthogonality, orthogonal complement, closed subspaces, projection theorem. Riesz Representation Theorem.

Linear operators on Hilbert space, adjoint operators. Self-adjoint operators, orthogonal projections, unitary operators, and their spectra.

Orthonormal sets, Pythagoras, Bessels inequality. Complete orthonormal sets, Parseval.

L^2-theory of Fourier series, including completeness of the trigonometric system. Discussion of classical theory of Fourier series (including statement of pointwise convergence for piecewise differentiable functions, and exposition of failure for some continuous functions). Examples of other orthogonal expansions (Legendre, Laguerre, Hermite etc.).

Reading

Essential Reading

1. B.P. Rynne and M.A. Youngson, *Linear Functional Analysis* (Springer SUMS, 2nd edition, 2008), Chapters 3, 4.4, 6.
2. E. Kreyszig, *Introductory Functional Analysis with Applications* (Wiley, revised edition, 1989), Chapters 3, 4.7–4.9, 4.12–4.13, 9.1–9.2.
3. N. Young, *An Introduction to Hilbert Space* (Cambridge University Press, 1988), Chs 17.

Further Reading

1. E.M. Stein and R. Shakarchi, *Real Analysis: Measure Theory, Integration & Hilbert Spaces* (Princeton Lectures in Analysis III, 2005), Chapter 4.

B568a Introduction to Applied Mathematics — Dr Moroz — 6 MT

Overview

In week one of Michaelmas Term, an introductory course of six lectures will be provided. These lectures are a compulsory prerequisite for all students doing any of the courses in B5, B6 or B8. This courselet will cover basic material common to all these courses, and it will be assumed that students taking any of these courses have attended B568a. The courses B5a, B6a and B8a will consequently contain 14 lectures, and begin in second week. There will be one problem sheet associated with this course, and it is anticipated that written solutions will be provided. **No separate class will be scheduled.**

Learning Outcomes

Student will acquire the background knowledge to prepare them for applied mathematics options.

Synopsis

Modelling and conservation laws. [1 lecture]

Scaling and non-dimensionalisation. [1 lecture]

Asymptotic sequences. Regular and singular perturbation methods for algebraic equations. Simple boundary layer theory. [4 lectures]

Reading

1. S. D. Howison, *Practical Applied Mathematics: Modelling, Analysis, Approximation* (Cambridge University Press, Cambridge, 2005). Chapters. 1,2,3,13,16.

2.6 B5 Differential Equations and Applications

Level: H-level **Method of Assessment:** Written examination.

Weight: Whole-unit (OSS paper code 2644), or can be taken as either a half-unit in Techniques of Applied Mathematics or a half-unit in Applied Partial Differential Equations.

Recommended Prerequisites: Calculus of Variations and Fluid Mechanics from Part A are desirable but not essential. The introductory Michaelmas Term course B568a is a prerequisite for both parts of the course, and the material in that course will be assumed to be known.

2.6.1 B5a: Techniques of Applied Mathematics — Dr Münch — 14 MT

[Option **B5a** if taken as a half-unit. OSS paper code 2A44.]

Overview

This course develops mathematical techniques which are useful in solving 'real-world' problems involving differential equations, and is a development of ideas which arise in the second year differential equations course. The course aims to show in a practical way how equations 'work', what kinds of solution behaviours can occur, and some techniques which are useful in their solution.

Learning Outcomes

Students will know how differential equations can be used to model real-world phenomena and be able to describe the behaviour of the types of solutions that can occur. They will be familiar with the use of delta functions and the Fredholm Alternative and will be able to solve Sturm–Liouville systems. They will develop the theory of ODEs with regular singular points, including special functions.

Synopsis

Introduction to distributions; the delta function. Green's functions revisited. [3 lectures]

Fredholm alternative. [1 lecture]

Sturm–Liouville systems, adjoints, eigenfunction expansions. Integral equations and eigenfunctions. [6 lectures]

Singular points of differential equations; special functions. [4 lectures]

Reading

1. A. C. Fowler, *Techniques of Applied Mathematics*, Mathematical Institute Notes (2005).

2. J. P. Keener, *Principles of Applied Mathematics: Transformation and Approximation* (revised edition, Perseus Books, Cambridge, Mass., 2000).

3. E. J. Hinch, *Perturbation Methods* (Cambridge University Press, Cambridge, 1991).

4. J. R. Ockendon, S. D. Howison, A. A. Lacey and A. B. Movchan, *Applied Partial Differential Equations* (revised edition, Oxford University Press, Oxford, 2003).

5. R. Haberman, *Mathematical Models* (SIAM, Philadelphia, 1998).

6. S. D. Howison, *Practical Applied Mathematics: Modelling, Analysis, Approximation* (Cambridge University Press, Cambridge, 2005).

2.6.2 B5b: Applied Partial Differential Equations — Dr Howell — 16 HT

[Option **B5b** if taken as a half-unit. OSS paper code 2B44.]

Overview

This course continues the Part A Differential Equations course, and extends some of the techniques of B5a to partial differential equations. In particular, first-order conservation laws are solved and the idea of a shock is introduced; general nonlinear first-order partial differential equations are solved, the classification of second-order partial differential equations is extended to systems, with hyperbolic systems being solved by characteristic variables. Then Green's function, maximum principle and similarity variable methods are demonstrated for partial differential equations.

Learning Outcomes

Students will know a range of techniques to solve PDEs including non-linear first-order and second-order and their classification. They will be able to demonstrate various principles for solving PDEs including Green's function, maximum principle and similarity solutions.

Synopsis

First-order equations: conservation laws and shocks. Charpit's equations; eikonal equation. [4 lectures]

Systems of partial differential equations, characteristics. Shocks; viscosity solutions; weak solutions. [4 lectures]

Maximum principles, comparison methods, well-posed problems for the heat equation and for Laplace's equation. [3 lectures]

Similarity solutions. [2 lectures]

Fundamental solution for the heat equation and for Laplace's equation via delta functions and similarity solutions. [3 lectures]

Reading

1. Dr Norbury's web notes.
2. Institute lecture notes are now available (JN).
3. M. Renardy and R.C. Rogers, *An Introduction to Partial Differential Equations* (Springer-Verlag, New York, 2004).
4. J. P. Keener, *Principles of Applied Mathematics: Transformation and Approximation* (revised edition, Perseus Books, Cambridge, Mass., 2000).
5. J. R. Ockendon, S. D. Howison, A. A. Lacey and A. B. Movchan, *Applied Partial Differential Equations* (revised edition, Oxford University Press, Oxford, 2003).

2.7 B5.1a: Dynamics and Energy Minimization — Prof Ball — 16 MT

Level: H-level **Method of Assessment:** Written examination.

Weight: Half-unit (OSS Number to be confirmed)

Recommended Prerequisites: Elements from the following Part A courses: Differential Equations (Picard's theorem and phase plane arguments), Integration (convergence theorems and *Lp* spaces). Topology (metric spaces). However this material will be reviewed in the course.

Overview

The aim of this course is to discuss the mathematics needed to describe the approach to equilibrium in dissipative dynamical systems. It will describe the basic ideas of dynamical systems, Lyapunov functions and stability, and also provide an introduction to the theory of local minimizers in the one-dimensional calculus of variations. The ideas will be applied to the dynamical system generated by a semilinear PDE.

Synopsis

Part I, *Dynamical systems* (7 lectures). Introduction to the problem of the approach to equilibrium and its thermodynamic origins. ODEs in $\mathbb{R}^n$; local and global existence, continuous dependence on initial data, Lyapunov functions. Dynamical systems in $\mathbb{R}^n$ and metric spaces. $\omega-$ limit sets, invariance, La Salle invariance principle, Lyapunov stability.

Part II, *Local minimizers* in the 1D calculus of variations (5 lectures) Introduction to Sobolev spaces in 1D. Simplifed theory of global, weak and strong local minimizers in the calculus of variations.

Part III, *Applications to PDE.* (4 lectures) Discussion of one-dimensional semilinear parabolic PDE, the approach to equi- librium, and stability.

Reading

There is no single book that covers the course, which is a new compilation of material, and the lecturer aims to provide comprehensive notes. The follow- ing books contain useful material (but go well beyond the course in different directions):

For Part I

1. *Nonlinear differential equations and dynamical systems*, Ferdinand Verhulst, 2nd Edition, Springer, 1996.
2. *Nonlinear Ordinary Differential Equations,An Introduction to Dynamical Systems*, 4th Edition, D.W. Jordan and P. Smith (Oxford University Press, 2007).
3. *Dynamics and Bifurcations*, J.K. Hale and H. Kocak (Springer, 1991).

For Part II,

1. *One-dimensional Variational Problems*, G. Buttazzo, M. Giaquinta, S. Hilde- brandt, Oxford Lecture Series in Mathematics, Vol. 15 (Oxford University Press, 1998)

2. *Introduction to the Calculus of Variations*, U. Brechtken-Manderscheid (Chap- man and Hall, 1991).

3. *Introduction to the Calculus of Variations*, H. Sagan, (Dover, 1992).

For Part III,

1. *Infnite-Dimensional Dynamical Systems: An Introduction to Dissipative Parabolic PDEs and the Theory of Global Attractors*, James C. Robinson, (Cambridge Texts in Applied Mathematics, Cambridge University Press, 2001).

2.8 B6 Theoretical Mechanics

Level: H-level **Method of Assessment:** Written examination.

Weight: Whole-unit (OSS paper code 2645), or can be taken as either a half-unit in Viscous Flow, or a half-unit in Waves and Compressible Flow.

Recommended Prerequisites: The Part A (second-year) course 'Fluid Dynamics and Waves'. Though two half-units are intended to stand alone, they will complement each other. This course combines well with B5 Differential Equations and Applications. The introductory Michaelmas Term course B568a is a prerequisite for both parts of the course, and the material in that course will be assumed to be known.

2.8.1 B6a: Viscous Flow — Dr Oliver — 14 MT

[Option **B6a** if taken as a half-unit. OSS paper code 2A45.]

Overview

Viscous fluids are important in so many facets of everyday life that everyone has some intuition about the diverse flow phenomena that occur in practice. This course is distinctive in that it shows how quite advanced mathematical ideas such as asymptotics and partial differential equation theory can be used to analyse the underlying differential equations and hence give scientific understanding about flows of practical importance, such as air flow round wings and flow in oil reservoirs.

Learning Outcomes

Students will have developed an appreciation of diverse flow phenomena in various mediums including Poiseuille flow, Rayleigh flow, airflow around wings and flow in oil reservoirs. They will have a demonstrable knowledge of the mathematical theory necessary to analyse such phenomena.

Synopsis

Derivation of Navier–Stokes equations for an incompressible Newtonian fluid. Vorticity. Exact solutions for unidirectional flows; shear flow, Poiseuille flow, Rayleigh flow. Dimensional analysis, Reynolds number. Derivation of equations for high and low Reynolds number flows.

Derivation of Prandtl's boundary-layer equations. Similarity solutions for flow past a semi-infinite flat plate and for jets. Discussion of separation and application to the theory of flight. Jeffery–Hamel flow.

Slow flow past a circular cylinder and a sphere. Non-uniformity of the two dimensional approximation; Oseen's equation. Lubrication theory: bearings, thin films and Hele–Shaw cell. Flow in a porous medium and the Saftman–Taylor instability. Stability and the transition to turbulence.

Reading

1. D. J. Acheson, *Elementary Fluid Dynamics* (Oxford University Press, 1990), Chapters 2, 6, 7, 8.
2. H. Ockendon and J. R. Ockendon, *Viscous Flow* (Cambridge Texts in Applied Mathematics, 1995).
3. M. E. O'Neill and F. Chorlton, *Viscous and Compressible Fluid Dynamics* (Ellis Horwood, 1989), Chapters 2, 3, 4.1–4.3, 4.19–4.20, 4.22–4.24, 5.1–5.2, 5.6.

2.8.2 B6b: Waves and Compressible Flow —Dr Shipley — 16 HT

[Option **B6b** if taken as a half-unit. OSS paper code 2B45.]

Overview

Propagating disturbances, or waves, occur frequently in applied mathematics. This course will be centred on some prototypical examples from fluid dynamics, the two most familiar being surface gravity waves and waves in gases. The models for compressible flow will be derived and then analysed for small amplitude motion. This will shed light on the important phenomena of dispersion, group velocity and resonance, and the differences between supersonic and subsonic flow, as well as revealing the crucial dependence of the waves on the number of space dimensions.

Larger amplitude motion of liquids and gases will be described by incorporating non-linear effects, and the theory of characteristics for partial differential equations will be applied to understand the shock waves associated with supersonic flight.

Learning Outcomes

Students will have developed a sound knowledge of a range of mathematical models used to study waves (both linear and non-linear), will be able to describe examples of waves from fluid dynamics and will have analysed a model for compressible flow. They will have an awareness of shock waves and how the theory of characteristics for PDEs can be applied to study those associated with supersonic flight.

Synopsis

1–2 Equations of inviscid compressible flow including flow relative to rotating axes.

3–6 Models for linear wave propagation including Stokes waves, internal gravity waves, inertial waves in a rotating fluid, and simple solutions.

7–10 Theories for Linear Waves: Fourier Series, Fourier integrals, method of stationary phase, dispersion and group velocity. Flow past thin wings.

11–12 Nonlinear Waves: method of characteristics, simple wave flows applied to one-dimensional unsteady gas flow and shallow water theory.

13–16 Shock Waves: weak solutions, RankineHugoniot relations, oblique shocks, bores and hydraulic jumps.

Reading

1. H. Ockendon and J. R. Ockendon, *Waves and Compressible Flow* (Springer, 2004).

2. J. R. Ockendon, S. D. Howison, A. A. Lacey and A. B. Movchan, *Applied Partial Differential Equations* (revised edition, Oxford University Press, Oxford, 2003). Chapters 2.5, 4.5–7.

3. D. J. Acheson, *Elementary Fluid Dynamics* (Oxford University Press, 1990). Chapter 3

4. J. Billingham and A. C. King, *Wave Motion* (Cambridge University Press, 2000). Chapters 1–4, 7,8.

Background Reading

1. M. J. Lighthill, *Waves in Fluids* (Cambridge University Press, 1978).

2. G. B. Whitham, *Linear and Nonlinear Waves* (Wiley, 1973).

2.9 B7.1/C7.1: Quantum Mechanics; Quantum Theory and Quantum Computers

Level: H-level/M-level **Method of Assessment:** Written examination.

Weight: Whole-unit. B7.1a could be taken as a free-standing half-unit but C7.1b can not.

2.9.1 B7.1a: Quantum Mechanics and Electromagnetism — Prof. Tod — 16 MT

[Option **B7.1a** if taken as a half-unit. OSS paper code 2A86.]

Level: H-level

Recommended Prerequisites: Quantum Theory. Calculus of Variations. Classical Mechanics would be useful, but not essential.

Overview

Quantum theory was born out of the attempt to understand the interactions between matter and radiation. It transpired that light waves can behave like streams of particles, but other particles also have wave-like properties. Although there remain deep mathematical and physical questions at the frontiers of the subject, the resulting theory encompasses not just the mechanical but also the electrical and chemical properties of matter. Many of the key components of modern technology such as transistors and lasers were developed using quantum theory, and the theory has stimulated important 20th century advances in pure mathematics in, for example, functional analysis, algebra, and differential geometry. In spite of their revolutionary impact and central importance, the basic mathematical ideas are easily accessible and provide fresh and surprising applications of the mathematical techniques encountered in other branches of mathematics.

This introductory course explores some of the consequences of this, including a treatment of the hydrogen atom.

Learning Outcomes

Students will have gained a sound knowledge of the mathematical ideas related to the development of quantum theory. They will be able to apply mathematical techniques from earlier courses to a range of examples in quantum mechanics.

Synopsis

Maxwell's equations in vacuum and with sources. Lorentz force law. nPlane waves and polarization. Electrostatics and Magnetostatics. Energy density and the Poynting vector. Scalar and vector potentials. Gauge invariance. Maxwell's equations in the Lorentz gauage. The wave equation for potentials.

The mathematical structure of quantum mechanics and the postulates of quantum mechanics. Commutation relations. Poisson's brackets and Dirac's quantization scheme. Heisenberg's uncertainty principle. Creation and annihilation operators for the harmonic oscillator. Measurements and the interpretation of quantum mechanics. Schroedinger's cat. Spin-1/2 particles. Angular momentum in quantum mechanics. Particle in a central potential. The hydrogen atom.

Reading

1. K. C. Hannabuss, *Introduction to Quantum Theory* (Oxford University Press, 1997). Chapters 1–4, 6–8.

Further Reading

1. A popular non-technical account of the subject:

   A. Hey and P. Walters, *The New Quantum Universe* (Cambridge, 2003).

2. Also designed for a similar Oxford course:

   I. P. Grant, *Classical and Quantum Mechanics*, Mathematical Institute Notes (1991).

3. A classical account of the subject which goes well beyond this course:

   L. I. Schiff, *Quantum Mechanics* (3rd edition, Mc Graw Hill, 1968).

4. Some other books covering similar material:

   B. J. Brandsen and C. J. Joachain, *Introduction to Quantum Mechanics* (Longman, 1995).

   A. I. M. Rae, *Quantum Mechanics* (4th edition, Institute of Physics, 1993).

2.9.2 C7.1b: Quantum Theory and Quantum Computers — Dr Hannabuss — 16 HT — M-Level

Level: M-level **Method of Assessment:** Written examination.

Weight: Whole-unit (cannot be taken unless B7.1a is taken). OSS paper code 2686.

Prerequisites: B7.1a Quantum Mechanics.

Overview

This course builds directly on the first course in quantum mechanics and covers a series of important topics, particularly features of systems containing several particles. The behaviour of identical particles in quantum theory is more subtle than in classical mechanics, and an understanding of these features allows one to understand the periodic table of elements and the rigidity of matter. It also introduces a new property of entanglement linking particles which can be quite widely dispersed.

There are rarely neat solutions to problems involving several particles, so usually one needs some approximation methods. In very complicated systems, such as the molecules of gas in a container, quantum mechanical uncertainty is compounded by ignorance about other details of the system and requires tools of quantum statistical mechanics.

Two state quantum systems enable one to encode binary information in a new way which permits superpositions. This leads to a quantum theory of information processing, and by exploiting entanglement to other ideas such as quantum teleportation.

Learning Outcomes

Students will be able to demonstrate knowledge and understanding of quantum mechanics of many particle systems, statistics, entanglement, and applications to quantum computing.

Synopsis

Identical particles, symmetric and anti-symmetric states, Fermi–Dirac and Bose–Einstein statistics and atomic structure.

Heisenberg representation, interaction representation, time dependent perturbation theory and Feynman–Dyson expansion. Approximation methods, Rayleigh–Schrödinger time-independent perturbation theory and variation principles. The virial theorem. Helium.

Mixed states, density operators. The example of spin systems. Purification. Gibbs states and the KMS condition.

Entanglement. The EPR paradox, Bell's inequalities, Aspect's experiment.

Quantum information processing, qubits and quantum computing. The no-cloning theorem, quantum teleportation. Quantum logic gates. Quantum operations. The quantum Fourier transform.

Reading

K. Hannabuss, *Introduction to Quantum Mechanics* (Oxford University Press, 1997). Chapters 10–12 and 14, 16, supplemented by lecture notes on quantum computers on the web

Further Reading

A popular non-technical account of the subject:

A. Hey and P. Walters, *The New Quantum Universe* (Cambridge, 2003).

Also designed for an Oxford course, though only covering some material:

I.P. Grant, *Classical and Quantum Mechanics*, Mathematical Institute Notes (1991).

A concise account of quantum information theory:

S. Stenholm and K.-A. Suominen, *Quantum Approach to Informatics* (Wiley, 2005).

An encyclopaedic account of quantum computing:

M. A. Nielsen and I. L. Chuang, *Quantum Computation* (Cambridge University Press, 2000).

Even more paradoxes can be found in:

Y. Aharanov and D. Rohrlich, *Quantum Paradoxes* (Wiley–VCH, 2005).

Those who read German can find further material on entanglement in:

J. Audretsch, *Verschränkte Systeme* (Wiley–VCH, 2005).

Other accounts of the first part of the course:

L. I. Schiff, *Quantum Mechanics* (3rd edition, Mc Graw Hill, 1968).

B. J. Bransden and C. J. Joachain, *Introduction to Quantum Mechanics* (Longman, 1995).

A. I. M. Rae, *Quantum Mechanics* (4th edition, Institute of Physics, 1993).

John Preskill's on-line lecture notes (http://www.theory.caltech.edu/ preskill/ph219/index.html).

2.10 B7.2: Relativity

Level: H-level **Method of Assessment:** Written examination.

Weight: half-unit.

Recommended Prerequisites: For B7.2b — B7.1a: Quantum Mechanics and Electromagnetism. Part A Quamtum Theory,

2.10.1 B7.2b: Special Relativity and Electromagnetism — Dr de la Ossa — 16 HT

[Note: in order to take B7.2b you must take B7.1a]

[OSS paper code 2A87.]

Level: H-level **Method of Assessment:** Written examination.

Recommended Prerequisites: Part A Quantum Theory.

Overview

Maxwell's electromagnetic theory revealed light to be an electromagnetic phenomenon whose speed of propagation proved to be observer-independent. This discovery led to the overthrow of classical Newtonian mechanics, in which space and time were absolute, and its replacement by Special Relativity and space-time. The aim of this course is to study

Einstein's special theory of relativity, to understand space-time, and to incorporate into it Maxwell's electrodynamics. These theories together with quantum theory are essential for an understanding of modern physics.

Synopsis

Constancy of the speed of light. Lorentz transformations and the invariance of the wave operator; time dilation, length contraction and the relativistic Doppler effect; the resolution of the simple 'paradoxes' of relativity. Four-vectors, four-velocity and four-momentum; equivalence of mass and energy; particle collisions and four-momentum conservation; four-acceleration and four-force, the example of the constant-acceleration world-line. Contravariant and covariant vectors and tensors; index notation.

Solving Poisson's equation and the wave equation with sources. Derivation of Maxwell's equations with sources from a variational principle.

Electromagnetism in four-dimensional form; the electromagnetic field tensor; the transformation las for the electric and magnetic fields; the Lorentz force law; the electromagnetic four potential and the energy-momentum tensor.

Reading

The preferred text is:
N. M. J. Woodhouse, *Special Relativity* (Springer, 2002).

N. M. J. Woodhouse, *General Relativity* (Springer, 2006).

An alternative is:
W. Rindler, *Introduction to Special Relativity* (2nd edition, Oxford University Press 1991).

Additional Reading

For the experimental background to special relativity, and in many libraries:
A. P. French, *Special Relativity* (MIT Introductory Physics Series, Nelson Thornes, 1971).

For advanced texts on electromagnetism, see:
W. J. Duffin, *Advanced Electricity and Magnetism* (McGraw–Hill, 1968).
J. D. Jackson, *Classical Electromagnetism* (Wiley, 1962).

2.11 B8 Topics in Applied Mathematics

Level: H-level **Method of Assessment:** Written examination.

Weight: Whole-unit (OSS paper code 2647), or can be taken as either a half-unit in Mathematical Ecology and Biology or a half-unit in Nonlinear Systems.

Recommended Prerequisites: Part A core material (especially differential equations). The introductory Michaelmas Term course B568a is a prerequisite for both parts of the course, and the material in that course will be assumed to be known.

2.11.1 B8a: Mathematical Ecology and Biology — Dr Baker — 14 MT

[Option **B8a** if taken as a half-unit. OSS paper code 2A47.]

Overview

Mathematical Ecology and Biology introduces the applied mathematician to practical applications in an area that is growing very rapidly. The course mainly focusses on situations where continuous models are appropriate and where these may be modelled by deterministic ordinary and partial differential equations. By using particular modelling examples in ecology, chemistry, biology, physiology and epidemiology, the course demonstrates how various applied mathematical techniques, such as those describing linear stability, phase planes, singular perturbation and travelling waves, can yield important information about the behaviour of complex models.

Learning Outcomes

Students will have developed a sound knowledge and appreciation of the ideas and concepts related to modelling biological and ecological systems using ordinary and partial differential equations.

Synopsis

Continuous and discrete population models for a single species, including Ludwig's 1978 insect outbreak models hysterisis and harvesting.
Modelling interacting populations, including predator-prey and the Principle of competitive exclusion.

Epidemic models.

Michaelis–Menten model for enzyme-substrate kinetics.

Excitable systems. Threshold phenomena (nerve pulses) and nerve signal propagation.

Travelling wave propagation with biological examples.

Biological pattern formation, including Turing's model for animal coat markings.

Reading

J.D. Murray, *Mathematical Biology, Volume I: An Introduction (2002); Volume II: Spatial Models and Biomedical Applications (2003)* (3rd edition, Springer–Verlag).

1. Volume I: 1.1, 1.2, 1.6, 2.1–2.4, 3.1, 3.3–3.6, 3.8, 6.1–6.3, 6.5, 6.6, 8.1, 8.2, 8.4, 8.5, 10.1, 10.2, 11.1–11.5, 13.1–13.5, Appendix A.

2. Volume II: 1.6, 2, 3.1, 3.2, 5.1, 5.2, 13.1–13.4.

Further Reading

1. J. Keener and J. Sneyd, *Mathematical Physiology* (First Edition Springer, Berlin, 1998) 1.1, 1.2, 9.1, 9.2.

2. N. F . Britton, Essential Mathematical Biology (Springer, London, 2003). 1.1, 1.2, 1.3, 1.5, 2.1, 2.3, 2.4, 2.5, 2.7, 3.1, 3.2, 3.3, 5.1, 5.2, 5.3, 5.6, 7.1, 7.2, 7.3, 7.4, 7.5.

2.11.2 B8b: Nonlinear Systems — Dr Porter — 16 HT

[Option **B8b** if taken as a half-unit. OSS paper code 2B47.]

Overview

This course aims to provide an introduction to the tools of dynamical systems theory which are essential for the realistic modelling and study of many disciplines, including mathematical ecology and biology, fluid dynamics, granular media, mechanics, and more.

The course will include the study of both nonlinear ordinary differential equations and maps. It will draw examples from appropriate model systems and various application areas. The problem sheets will require numerical computation (using programs such as Matlab).

Learning Outcomes

Students will have developed a sound knowledge and appreciation of some of the tools, concepts, and computations used in the study of nonlinear dynamical systems. They will also get some exposure to some modern research topics in the field.

Synopsis

1. Bifurcations and Nonlinear Oscillators [8 lectures]
   (a) Bifurcation theory: standard codimension one examples (saddle-node, Hopf, etc.), normal forms and codimension two examples (briefly).
   (b) Non-conservative oscillators: Van der Pol's equation, limit cycles.
   (c) Conservative oscillators (introduction to Hamiltonian systems): Duffing's equation, forced pendulum.
   (d) Synchronization: synchronization in non-conservative oscillators, phase-only oscillators (e.g., Kuramoto model).
2. Maps [2 lectures]
   (a) Stability and periodic orbits, bifurcations of one-dimensional maps.
   (b) Poincaré sections and first-return maps (leads to part 3 topics)
3. Chaos in Maps and Differential Equations [4 lectures]

(a) Maps: logistic map, Bernoulli shift map, symbolic dynamics, two-dimensional maps (examples could include Henon map, Chirikov–Taylor ["standard"] map, billiard systems)

(b) Differential equations: Lyapunov exponents, chaos in conservative systems (e.g., forced pendulum, Henon–Heiles), chaos in non-conservative systems (e.g., Lorenz equations)

4. Other topics [2 lectures or as time permits]

   Topics will vary from year to year and could include: dynamics on networks, solitary waves, spatio-temporal chaos, quantum chaos.

Reading

Students are by no means expected to read all these sources. These are suggestions intended to be helpful.

1. S. H. Strogatz, *Nonlinear Dynamics and Chaos with Applications to Physics, Biology, Chemistry and Engineering* (Westview Press, 2000).

2. E. Ott, *Chaos in Dynamical Systems* (Second edition, Cambridge University Press, Cambridge, 2002).

3. P. Cvitanovic, et al, *Chaos: Classical and Quantum* (Niels Bohr Institute, Copenhagen 2008). [Available for free online at http://www.chaosbook.org/]

4. R. H. Rand, *Lecture Notes on Nonlinear Vibrations*. [Available for free online at http://audiophile.tam.cornell.edu/randdocs/nlvibe52.pdf]

5. J. Guckenheimer and P. Holmes, *Nonlinear Oscillations, Dynamical Systems, and Bifurcation of Vector Fields* (Springer-Verlag, 1983).

6. G. L. Baker and J. P. Gollub, *Chaotic Dynamics: An Introduction* (Second edition, Cambridge University Press, Cambridge, 1996).

7. P. G. Drazin, *Nonlinear Systems* (Cambridge University Press, Cambridge, 1992).

8. S. Wiggins, *Introduction to Applied Nonlinear Dynamical Systems and Chaos* (Second edition, Springer, 2003).

9. S. H. Strogatz, 'From Kuramoto to Crawford: exploring the inset of synchronization in populations of coupled oscillators', *Physica D* 143 (2000) 1-20.

10. Various additional books and review articles (especially for some of the 'other topics').

2.12 B9: Number Theory

Level: H-level **Method of Assessment:** Written examination.

Weight: Whole-unit (OSS paper code 2648), or B9a can be taken as half-unit (but B9b cannot).

Recommended Prerequisites: All second-year algebra and arithmetic. Students who have not taken Part A Number Theory should read about quadratic residues in, for example, the appendix to Stewart and Tall. This will help with the examples.

2.12.1 B9a: Galois Theory — Prof. Kirwan — 16 MT

[Option **B9a** if taken as a half-unit. OSS paper code 2A48.]

Overview

The course starts with a review of second-year ring theory with a particular emphasis on polynomial rings, and a discussion of general integral domains and fields of fractions. This is followed by the classical theory of Galois field extensions, culminating in some of the classical theorems in the subject: the insolubility of the general quintic and impossibility of certain ruler and compass constructions considered by the Ancient Greeks.

Learning Outcomes

Understanding of the relation between symmetries of roots of a polynomial and its solubility in terms of simple algebraic formulae; working knowledge of interesting group actions in a nontrivial context; working knowledge, with applications, of a nontrivial notion of finite group theory (soluble groups); understanding of the relation between algebraic properties of field extensions and geometric problems such as doubling the cube and squaring the circle.

Synopsis

Review of polynomial rings, factorisation, integral domains. Reminder that any nonzero homomorphism of fields is injective. Fields of fractions.

Review of group actions on sets, Gauss' Lemma and Eisenstein's criterion for irreducibility of polynomials, field extensions, degrees, the tower law. Symmetric polynomials.

Separable extensions. Splitting fields and normal extensions. The theorem of the primitive element. The existence and uniqueness of algebraic closure. (Proofs not examinable)

Groups of automorphisms, fixed fields. The fundamental theorem of Galois theory.

Examples: Kummer extensions, cyclotomic extensions, finite fields and the Frobenius automorphism. Techniques for calculating Galois groups.

Soluble groups. Solubility by radicals, solubility of polynomials of degree at most 4, insolubility of the general quintic, impossibility of some ruler and compass constructions.

Reading

1. J. Rotman, *Galois Theory* (Springer-Verlag, NY Inc, 2001/1990).
2. I. Stewart, *Galois Theory* (Chapman and Hall, 2003/1989)
3. D.J.H. Garling, *A Course in Galois Theory* (Cambridge University Press I.N., 1987).
4. Herstein, *Topics in Algebra* (Wiley, 1975)

2.12.2 B9b: Algebraic Number Theory — Prof. Flynn — 16 HT

Overview

An introduction to algebraic number theory. The aim is to describe the properties of number fields, but particular emphasis in examples will be placed on quadratic fields, where it is easy to calculate explicitly the properties of some of the objects being considered. In such fields the familiar unique factorisation enjoyed by the integers may fail, and a key objective of the course is to introduce the class group which measures the failure of this property.

Learning Outcomes

Students will learn about the arithmetic of algebraic number fields. They will learn to prove theorems about integral bases, and about unique factorisation into ideals. They will learn to calculate class numbers, and to use the theory to solve simple Diophantine equations.

Synopsis

1. field extensions, minimum polynomial, algebraic numbers, conjugates, discriminants, Gaussian integers, algebraic integers, integral basis
2. examples: quadratic fields
3. norm of an algebraic number
4. existence of factorisation
5. factorisation in $\mathbb{Q}(\sqrt{d})$
6. ideals, $\mathbb{Z}$-basis, maximal ideals, prime ideals
7. unique factorisation theorem of ideals
8. relationship between factorisation of number and of ideals
9. norm of an ideal
10. ideal classes

11. statement of Minkowski convex body theorem
12. finiteness of class number
13. computations of class number to go on example sheets

Reading

1. I. Stewart and D. Tall, *Algebraic Number Theory* (Chapman and Hall Mathematics Series, May 1987).

Further Reading

1. D. Marcus, *Number Fields* (Springer-Verlag, New York–Heidelberg, 1977). ISBN 0-387-90279-1.

2.13 B10: Martingales and Financial Mathematics

Level: H-level **Method of Assessment:** Written examination.

Weight: Whole-unit (OSS paper code 2649), or can be taken as either a half-unit in B10a or a half-unit in B10b.

Recommended Prerequisites: For B10a, Part A Integration is a prerequisite, so that the corresponding material will be assumed to be known, and Topology is helpful but not essential. For B10b, Part A Probability is a prerequisite.

2.13.1 B10a: Martingales Through Measure Theory — Prof. Etheridge — 16 MT

[Option **B10a** if taken as a half-unit.]

Overview

Probability theory arises in the modelling of a variety of systems where the understanding of the “unknown” plays a key role, such as population genetics in biology, market evolution in financial mathematics, and learning features in game theory. It is also very useful in various areas of mathematics, including number theory and partial differential equations. The course introduces the basic mathematical framework underlying its rigorous analysis, and is therefore meant to provide some of the tools which will be used in more advanced courses in probability.

The first part of the course provides a review of measure theory from Integration Part A, and develops a deeper framework for its study. Then we proceed to develop notions of conditional expectation, martingales, and to show limit results for the behaviour of these martingales which apply in a variety of contexts.

Learning Outcomes

The students will learn about measure theory, random variables, independence, expectation and conditional expectation, product measures and discrete-parameter martingales.

Synopsis

A branching-process example. Review of σ-algebras, measure spaces. Uniqueness of extension of π-systems and Carathéodory's Extension Theorem [both without proof], monotone-convergence properties of measures, lim sup and lim inf of a sequence of events, Fatou's Lemma, reverse Fatou Lemma, first Borel–Cantelli Lemma.

Random variables and their distribution functions, σ-algebras generated by a collection of random variables. Independence of events, random variables and σ-algebras, π-systems criterion for independence, second Borel–Cantelli Lemma. The tail σ-algebra, Kolomogorov's 0–1 Law. Convergence in measure and convergence almost everywhere.

Integration and expectation, review of elementary properties of the integral and L^p spaces [from Part A Integration for the Lebesgue measure on $\mathbb{R}$]. Scheffé's Lemma, Jensen's inequality, orthogonal projection in L^2. The Kolmogorov Theorem and definition of conditional expectation, proof as least-squares-best predictor, elementary properties. The Radon–Nikodym Theorem [without proof, not examinable].

Filtrations, martingales, stopping times, discrete stochastic integrals, Doob's Optional-Stopping Theorem, Doob's Upcrossing Lemma and "Forward" Convergence Theorem, martingales bounded in L^2, Doob decomposition.

Uniform integrability and L^1 convergence, Levy's "Upward" and "Downward" Theorem, corollary to the Kolmogorov's Strong Law of Large Numbers, Doob's submartingale inequalities.

Examples and applications, including branching processes, and harmonic functions with boundary conditions on connected finite subsets of $\mathbb{Z}^d$.

Reading

1. D. Williams. *Probability with Martingales*, Cambridge University Press, 1995.
2. P. M. Tarres Lecture notes, *Appendix : Notes on Fubini's theorem on* $\mathbb{R}$*, Product measures, infinite products of probability triples*, Mathematical Institute, 2009.

Further Reading

1. Z. Brzeźniak and T. Zastawniak, Basic stochastic processes. A course through exercises. Springer Undergraduate Mathematics Series. (Springer-Verlag London, Ltd., 1999) [more elementary than D. Williams' book, but can provide with a complementary first reading].

2. M. Capinski and E. Kopp. *Measure, integral and probability*, Springer Undergraduate Mathematics Series. (Springer-Verlag London, Ltd., second edition, 2004).
3. R. Durrett. *Probability: Theory and Examples.* (Second Edition Duxbury Press, Wadsworth Publishing Company, 1996).
4. A. Etheridge. *A Course in Financial Calculus*, (Cambridge University Press, 2002).
5. J. Neveu. *Discrete-parameter Martingales.* (North-Holland, Amsterdam, 1975).
6. S. I. Resnick. *A Probability Path*, (Birkhäuser, 1999).

2.13.2 B10b: Mathematical Models of Financial Derivatives — Dr Jin — 16 HT

[Option **B10b** if taken as a half-unit. OSS paper code 2B49.]

Overview

The course aims to introduce students to mathematical modelling in financial markets. At the end of the course the student should be able to formulate a model for an asset price and then determine the prices of a range of derivatives based on the underlying asset using arbitrage free pricing ideas.

Learning Outcomes

Students will have a familiarity with the mathematics behind the models and analytical tools used in Mathematical Finance. This includes being able to formulate a model for an asset price and then determining the prices of a range of derivatives based on the underlying asset using arbitrage free pricing ideas.

Synopsis

Introduction to markets, assets, interest rates and present value; arbitrage and the law of one price: European call and put options, payoff diagrams. Introduction to Brownian motion, continuous-time martingales, informal treatment of Itô's formula and stochastic differential equations. Discussion of the connection with PDEs through the Feynman–Kac formula.

The Black–Scholes analysis via delta hedging and replication, leading to the Black–Scholes partial differential equation for a derivative price. General solution via Feynman–Kac and risk neutral pricing, explicit solution for call and put options.

Extensions to assets paying dividends, time-varying parameters. Forward and future contracts, options on them. American options, formulation as a free-boundary problem and a linear complementarity problem. Simple exotic options. Weakly path-dependent options including barriers, lookbacks and Asians.

Reading

1. T. Bjork, *Arbitrage Theory in Continuous Time* (Oxford University Press, 1998).
2. P. Wilmott, S. D. Howison and J. Dewynne, *Mathematics of Financial Derivatives* (Cambridge university Press, 1995).
3. A. Etheridge, *A Course in Financial Calculus* (Cambridge University Press, 2002).

Background

1. J. Hull, *Options Futures and Other Financial Derivative Products* (4th edition, Prentice Hall, 2001).
2. N. Taleb, *Dynamic Hedging* (Wiley, 1997).
3. P. Wilmott, *Derivatives* (Wiley, 1998).

2.14 B11a: Communication Theory — Dr Stirzaker — 16 MT

NB: B22a: Integer Programming is a very suitable complement to this course.

Level: H-level **Method of Assessment:** Written examination.

Weight: Half-unit OSS paper code 2650.

Recommended Prerequisites: Part A Probability would be helpful, but not essential.

Overview

The aim of the course is to investigate methods for the communication of information from a sender, along a channel of some kind, to a receiver. If errors are not a concern we are interested in codes that yield fast communication; if the channel is noisy we are interested in achieving both speed and reliability. A key concept is that of information as reduction in uncertainty. The highlight of the course is Shannon's Noisy Coding Theorem.

Learning Outcomes

(i) Know what the various forms of entropy are, and be able to manipulate them.

(ii) Know what data compression and source coding are, and be able to do it.

(iii) Know what channel coding and channel capacity are, and be able to use that.

Synopsis

Uncertainty (entropy); conditional uncertainty; information. Chain rules; relative entropy; Gibbs' inequality; asymptotic equipartition and typical sequences. Instantaneous and uniquely decipherable codes; the noiseless coding theorem for discrete memoryless sources; constructing compact codes.

The discrete memoryless channel; decoding rules; the capacity of a channel. The noisy coding theorem for discrete memoryless sources and binary symmetric channels.

Extensions to more general sources and channels.

Reading

1. D. J. A. Welsh, *Codes and Cryptography* (Oxford University Press, 1988), Chapters 1–3, 5.
2. G. Jones and J. M. Jones, *Information and Coding Theory* (Springer, 2000), Chapters 1–5.
3. T. Cover and J. Thomas, *Elements of Information Theory* (Wiley, 1991), Chapters 1–5, 8.

Further Reading

1. R. B. Ash, *Information Theory* (Dover, 1990).
2. D. MacKay, *Information Theory, Inference, and Learning Algorithms* (Cambridge, 2003). [Can be seen at: http://www.inference.phy.cam.ac.uk/mackay/itila. Do not infringe the copyright!]

2.15 B12a: Applied Probability — Dr Hammond — 16 MT

[This option was formerly OBS3a and is paper BS3 in the Honour School of Mathematics & Statistics. Teaching responsibility of the Department of Statistics.]

Level: H-Level

Method of Assessment: $1\frac{1}{2}$-hour written examination (3-hours if taken as a whole-unit with OBS3b)

Recommended Prerequisites: Part A Probability.

Weight: Half-unit (OSS paper code 2A72). Can be taken as a whole-unit with OBS3b.

The whole-unit (B12a and OBS3b) has been designed so that a student obtaining at least an upper second class mark on the whole unit can expect to gain exemption from the Institute of Actuaries' paper CT4, which is a compulsory paper in their cycle of professional actuarial examinations. The first half of the unit, clearly, and also the second half of the unit, apply much more widely than just to insurance models.

Overview

This course is intended to show the power and range of probability by considering real examples in which probabilistic modelling is inescapable and useful. Theory will be developed as required to deal with the examples.

Synopsis

Poisson processes and birth processes. Continuous-time Markov chains. Transition rates, jump chains and holding times. Forward and backward equations. Class structure, hitting times and absorption probabilities. Recurrence and transience. Invariant distributions and limiting behaviour. Time reversal. Renewal theory. Limit theorems: strong law of large numbers, strong law and central limit theorem of renewal theory, elementary renewal theorem, renewal theorem, key renewal theorem. Excess life, inspection paradox.

Applications in areas such as: queues and queueing networks - M/M/s queue, Erlang's formula, queues in tandem and networks of queues, M/G/1 and G/M/1 queues; insurance ruin models; applications in applied sciences.

Reading

1. J. R. Norris, *Markov Chains* (Cambridge University Press, 1997).
2. G. R. Grimmett and D. R. Stirzaker, *Probability and Random Processes* (3rd edition, Oxford University Press, 2001).
3. G. R. Grimmett and D. R. Stirzaker, *One Thousand Exercises in Probability* (Oxford University Press, 2001).
4. S. M. Ross, *Introduction to Probability Models* (4th edition, Academic Press, 1989).
5. D. R. Stirzaker: *Elementary Probability* (2nd edition, Cambridge University Press, 2003).

2.16 B21 Numerical Solution of Differential Equations

[From Part B2 in the Honour School of Computer Science.]

Level: H-level

Method of Assessment: $1\frac{1}{2}$-hour written examination for a half-unit, 3-hour written examination for a whole-unit.

Weight: Whole-unit, or can be taken either as a half-unit in NSDE I (OSS paper code 0286) or in NSDE II (OSS paper code 0287).

Recommended Prerequisites: None.

2.16.1 B21a Numerical Solution of Differential Equations I — Prof. Süli — 16 MT

Overview

To introduce and give an understanding of numerical methods for the solution of ordinary and partial differential equations, their derivation, analysis and applicability.

The MT lectures are devoted to numerical methods for initial value problems, while the HT lectures concentrate on the numerical solution of boundary value problems.

Learning Outcomes

At the end of the course the student will be able to:

1. construct one-step and linear multistep methods for the numerical solution of initial-value problems for ordinary differential equations and systems of such equations, and to analyse their stability and accuracy properties;

2. construct finite difference methods for the numerical solution of initial-boundary-value problems for second-order parabolic partial differential equations, and first-order hyperbolic equations, and to analyse their stability and accuracy properties.

Synopsis

The MT part of the course is devoted to the development and analysis of numerical methods for initial value problems. We begin by considering classical techniques for the numerical solution of ordinary differential equations. The problem of stiffness is then discussed in tandem with the associated questions of step-size control and adaptivity.

Initial value problems for ordinary differential equations: Euler, multistep and Runge–Kutta; stiffness; error control and adaptive algorithms. [Introduction (1 lecture) + 5 lectures]

The remaining lectures focus on the numerical solution of initial value problems for partial differential equations, including parabolic and hyperbolic problems.

Initial value problems for partial differential equations: parabolic equations, hyperbolic equations; explicit and implicit methods; accuracy, stability and convergence, Fourier analysis, CFL condition. [10 lectures]

Reading List

The course will be based on the following textbooks:

1. K. W. Morton and D. F. Mayers, *Numerical Solution of Partial Differential Equations* (Cambridge University Press, 1994). ISBN 0-521-42922-6 (Paperback edition) [Chapters 2, 3 (Secs. 3.1, 3.2), Chapter 4 (Secs. 4.1–4.6), Chapter 5].

2. E. Süli and D. Mayers, *An Introduction to Numerical Analysis* (Cambridge University Press, 2003). ISBN 0-521-00794-1 (Paperback edition) [Chapter 12].

3. A. Iserles, *A First Course in the Numerical Analysis of Differential Equations* (Cambridge University Press, 1996). ISBN 0-521-55655-4 (Paperback edition) [Chapters 1–5, 13, 14].

2.16.2 B21b Numerical Solution of Differential Equations II — Dr Wathen — 16 HT

Overview

To introduce and give an understanding of numerical methods for the solution of ordinary and partial differential equations, their derivation, analysis and applicability. The MT lectures are devoted to numerical methods for initial value problems, while the HT lectures concentrate on the numerical solution of boundary value problems.

Learning Outcomes

Students will understand and have experience of the theory for: Construction of shooting methods for boundary value problems in one independent variable

Elementary numerical analysis of elliptic partial differential equations analysis of iterative methods for solution of large linear systems of equations

Synopsis

The HT part of the course is concerned with numerical methods for boundary value problems. We begin by developing numerical techniques for the approximation of boundary value problems for second-order ordinary differential equations. Boundary value problems for ordinary differential equations: shooting and finite difference methods. [Introduction (1 lecture) + 2 lectures]

Then we consider finite difference schemes for elliptic boundary value problems. This is followed by an introduction to the theory of direct and iterative algorithms for the solution of large systems of linear algebraic equations which arise from the discretisation of elliptic boundary value problems.

Boundary value problems for PDEs: finite difference discretisation; Poisson equation. Associated methods of sparse numerical algebra: sparse Gaussian elimination, iterative methods. [13 lectures]

Reading List

This course does not follow any particular textbook, but the following essentially cover the material:

1. A. Iserles, *A First Course in the Numerical Analysis of Differential Equations* (Cambridge University Press, 1996), Chapters 7,10,11.

2. K. W. Morton and D. F. Mayers, *Numerical Solution of Partial Differential Equations* (Cambridge University Press, 1994.
Or the more recent 2nd edition, 2005), Chapters 6 and 7.

3. G. D. Smith, *Numerical Solution of Partial Differential Equations: Finite Difference Methods* (Clarendon Press, Oxford, 1985 (and any later editions)), has some of the material in chapter 5.

2.17 B22a Integer Programming — Dr Hauser — 16 MT

[From Part B2 in the Honour School of Computer Science. Teaching responsibility of the Computing Laboratory but this may transfer to Mathematics during 2009/10.]

Level: H-level **Method of Assessment:** $1\frac{1}{2}$ -hour written examination.

Weight: Half-unit. OSS paper code 0282.

Recommended Prerequisites: None.

Overview

In many areas of practical importance linear optimisation problems occur with integrality constraints imposed on some of the variables. In optimal crew scheduling for example, a pilot cannot be fractionally assigned to two different flights at the same time. Likewise, in combinatorial optimisation an element of a given set either belongs to a chosen subset or it does not. Integer programming is the mathematical theory of such problems and of algorithms for their solution. The aim of this course is to provide an introduction to some of the general ideas on which attacks to integer programming problems are based: generating bounds through relaxations by problems that are easier to solve, and branch-and-bound.

Learning Outcomes

Students will understand some of the theoretical underpinnings that render certain classes of integer programming problems tractable ("easy" to solve), and they will learn how to solve them algorithmically. Furthermore, they will understand some general mechanisms by which intractable problems can be broken down into tractable subproblems, and how these mechanisms are used to design good heuristics for solving the intractable problems. Understanding these general principles will render the students able to guide the modelling phase of a real-world problem towards a mathematical formulation that has a reasonable chance of being solved in practice.

Synopsis

1. Course outline. What is integer programming (IP)? Some classical examples.

2. Further examples, hard and easy problems.
3. Alternative formulations of IPs, linear programming (LP) and the simplex method.
4. LP duality, sensitivity analysis.
5. Optimality conditions for IP, relaxation and duality.
6. Total unimodularity, network flow problems.
7. Optimal trees, submodularity, matroids and the greedy algorithm.
8. Augmenting paths and bipartite matching.
9. The assignment problem.
10. Dynamic programming.
11. Integer knapsack problems.
12. Branch-and-bound.
13. More on branch-and-bound.
14. Lagrangian relaxation and the symmetric travelling salesman problem.
15. Solving the Lagrangian dual.
16. Branch-and-cut.

Course Materials

1. L. A. Wolsey, *Integer Programming* (John Wiley & Sons, 1998), parts of chapters 1–5 and 7.
2. Lecture notes and problem sheets posted on the course web page.

Time Requirements

The course consists of 16 lectures and 6 problem classes. There are no practicals. It is estimated that 8–10 hours of private study are needed per week for studying the lecture notes and relevant chapters in the textbook, and for solving the problem sheets, so that the total time requirement is circa 12 hours per week.

2.18 BE "Mathematical" Extended Essay

Level: H-level **Method of Assessment:** Written extended essay.

Weight: Whole unit (7,500 words). OSS code 9921.

An essay on a mathematical topic may be offered as a whole unit. It is equivalent to a 32-hour lecture course. Generally, students will have approximately 8 hours of supervision distributed over the two terms.

Students offering an essay should read the *Guidance Notes on Extended Essays and Dissertations in Mathematics* by the Projects Committee available at:

http://www.maths.ox.ac.uk/current-students/undergraduates/projects/.

Application You must apply to the Mathematics Project Committee in advance for approval. Proposals should be addressed to The Chairman of the Projects Committee, c/o Mrs Helen Lowe, Room DH61, Dartingtion House, and must be received before 12 noon on Friday of Week 0 of Michaelmas Full Term. Note that a BE essay must have a substantial mathematical content. The application form is available at http://www.maths.ox.ac.uk/current-students/undergraduates/projects/.

Once your title has been approved, it may only be changed by approval of the Chairman of the Projects Committee.

Assessment Each project is blind double marked. The marks are reconciled through discussion between the two assessors which is overseen by the examiners. Please see the *Guidance Notes on Extended Essays and Dissertations in Mathematics* for detailed marking criteria and class descriptors.

Submission THREE copies of your essay, identified by your candidate number only, should be sent to the Chairman of Examiners, FHS of Mathematics Part B, Examination Schools, Oxford, to arrive no later than **12noon on Friday of week 9, Hilary Term 2011**. An electronic copy of your dissertation should also be submitted via the Mathematical Institute website. Further details may be found in the *Guidance Notes on Extended Essays and Dissertations in Mathematics.*

2.19 BSP:Structured Project(Mathematical Modelling and Numerical Computation), MT and HT

Level: H-level

Assessment: Written work, oral presentation, and peer review.

Weight: Whole unit.

Recommended prerequisites: None.

Quota: The maximum number of students that can be accepted for 2010–11 will be 20.

Learning outcomes

This option is designed to help students understand applications of mathematics to live research problems and to learn some of the necessary techniques. For those who plan to stay on for the MMath or beyond, the course will provide invaluable preliminary training. For those who plan to leave after the BA, it will offer insights into what mathematical research can involve, and training in key skills that will be of long term benefit in any career.

Students will gain experience of:

- Applications of numerical computation to current research problems.
- Reading and understanding research papers.
- Working with new people in new environments.
- Meeting the expectations of different disciplines.
- Presenting a well structured written report, using LaTeX.
- Making an oral presentation to a non-specialist audience.
- Reading and assessing the work of other students.
- Independent study and time management.

Students will be expected to:

a. Learn about a current research problem by reading one or more relevant research papers together with appropriate material from textbooks.

b. Carry out the required calculations using Maple or Matlab. Students are not expected to engage in original research but there will be scope for able students to envisage new directions.

c. Write up the problem and their findings in a report that is properly supported with detail, discussion, and good referencing.

d. Give an oral presentation to a non-specialist audience.

e. Undertake peer review.

Projects for 2010–2011, from which students will select one, will include applications to (i) biology (ii) finance (iii) earth sciences.

Teaching

At the beginning of the course students will be given clear written instructions for their project and the work that is expected throughout the year.

Michaelmas Term

There will be group meetings with the two supervisors (Cath Wilkins and Jackie Stedall) at the beginning and end of MT to deal with queries and to set out expectations. Between those meetings students will read around their chosen topic and take preparatory courses in LaTeX and Matlab, both of which are available from the department and are well documented online. Regular individual contact with supervisors by email, or if necessary in person, will be encouraged.

Hilary Term

Week 1
Lectures on key skills and expectations
(a) computing (Cath Wilkins)
(b) writing up and presentation (Jackie Stedall)

Weeks 3, 5
Each student to meet with both supervisors.

Week 7
Oral presentations.

Week 9
Submission of written paper.

Easter vacation
Peer review

Assessment

Students (and tutors) have sometimes expressed doubts about the predictability or reliability of project assessment. We are therefore concerned:

i. to make the assessment scheme as transparent as possible both to students and to assessors;

ii. that students who produce good project work should be able to achieve equivalent grades to students who write good exam papers.

The mark breakdown will be as follows:

a. Written work 70%, comprising:

general explanation and discussion of the problem 35%

mathematical calculations and commentary 35%

b. Oral presentation 20%

c. Peer review 10%

Note on (b):
The ability to present technical information to non-specialist audience is a crucial long term skill. Students will be expected to approach this task professionally, and will be offered advice and constructive feedback.

Note on (c):
This is a new kind assessment in Oxford mathematics, though other universities have used it with great success. As with journal peer review, the anonymity of both writer and reviewer will be strictly maintained. Each student will be expected to read one other student's project write-up and to make a careful and well explained judgement on it. Credit for this will go to the reviewer, not to the writer, whose work will already have been assessed by examiners in the usual way.

3 Other Mathematical units and half-units

3.1 O1: History of Mathematics — Dr Stedall — 16 lectures in MT and reading course of 8 seminars in HT

Level: H-level

Assessment: 2-hour written examination paper for the MT lectures and 3000-word essay for the reading course.

Weight: Whole unit.

Recommended prerequisites: None.

Quota: The maximum number of students that can be accepted for 2010–11 will be 20.

Learning outcomes

This course is designed to provide the historical background to some of the mathematics familiar to students from A-level and the first four terms of undergraduate study, and looks at a period from approximately the mid-sixteenth century to the end of the nineteenth century. The course will be delivered through 16 lectures in Michaelmas Term, and a reading course consisting of 8 seminars (equivalent to a further 16 lectures) in Hilary Term. Guidance will be given throughout on reading, note-taking, and essay-writing.

Students will gain:

- an understanding of university mathematics in its historical context;
- an enriched understanding of the mathematical content of the topics covered by the course

  together with skills in:

- reading and analysing historical mathematical sources;
- reading and analysing secondary sources;
- efficient note-taking;
- essay-writing (from 1000 to 3000 words);
- construction of references and bibliographies;
- oral discussion and presentation.

Lectures

The Michaelmas Term lectures will cover the following material:

- Introduction.
- Seventeenth century: analytic geometry; the development of calculus; Newton's *Principia.*
- Eighteenth century: from calculus to analysis; functions, limits, continuity; equations and solvability.
- Nineteenth century: group theory and abstract algebra; the beginnings of modern analysis; sequences and series; integration; complex analysis; linear algebra.

Classes to accompany the lectures will be held in Weeks 3, 5, 6, and 7. For each class students will be expected to prepare one piece of written work (1000 words) and one discussion topic.

Reading course

The Hilary Term part of the course is run as a reading course during which we will study two or three primary texts in some detail, using original sources and secondary literature. Details of the books to be read in HT 2011 will be decided and discussed towards the end of MT 2010. Students will be expected to write two essays (2000 words each) during the first six weeks of term. The course will then be examined by an essay of 3000 words to be completed during Weeks 7 to 9.

Recommended reading

Jacqueline Stedall, *Mathematics emerging: a sourcebook 1540–1900*, (Oxford University Press, 2008).

Victor Katz, *A history of mathematics* (brief edition), (Pearson Addison Wesley, 2004), or:

Victor Katz, *A history of mathematics: an introduction* (third edition), (Pearson Addison Wesley, 2009).

Benjamin Wardhaugh, *How to read historical mathematics*, (Princeton, 2010).

Supplementary reading

John Fauvel and Jeremy Gray (eds), *The history of mathematics: a reader*, (Macmillan, 1987).

Assessment

The Michaelmas Term material will be examined in a two-hour written paper at the end of Trinity Term. Candidates will be expected to answer two half-hour questions (commenting on extracts) and one one-hour question (essay). The paper will account for 50% of the marks for the course. The Reading Course will be examined by a 3000-word essay at the end of Hilary Term. The title will be set at the beginning of Week 7 and two copies of the project must be submitted to the Examination Schools by midday on Friday of Week 9. The miniproject will account for 50% of the marks for the course.

3.2 MS: Statistics Units and Half-units

The other half units that students in Part B may take are drawn from Part B of the Honour School of Mathematics and Statistics. For full details of these half units see the syllabus and synopses for Part B of the Honour School Mathematics and Statistics, which are available on the Web at http://www.stats.ox.ac.uk/current_students/bammath/course_handbooks/

The Statistics half units available are as follows:

- OBS1 Applied Statistics
- OBS2a Foundations of Statistical Inference
- OBS3b Statistical Lifetime Models (can only be taken as a whole unit with B12a)
- OBS4 Actuarial Science

3.3 Computer Science: Half Units

The other half units that students in Part B may take are drawn from Part B of the Honour School of Mathematics and Computing. For full details of these half units see the syllabus and synopses for Part B of the Honour School Mathematics and Computing, which are available on the Web at http://www.comlab.ox.ac.uk/teaching/mcs/PartB/

The Computer Science half units available are as follows:

- OCS1a Functional Programming
- OCS1b Design and Analysis of Algorithms
- OCS3a Reasoning about Information Update
- OCS4a Logic of Multi-agent Information Flow

3.4 OE "Other Mathematical" Extended Essay

Level: H-level **Method of Assessment:** Written essay.

Weight: Whole unit (7,500 words). OSS paper code 9923.

An essay on an other mathematical topic may be offered as a whole unit. It is equivalent to a 32-hour lecture course. Generally, students will have approximately 8 hours of supervision distributed over the two terms.

Students offering an essay should read the *Guidance Notes on Extended Essays and Dissertations in Mathematics* by the Projects Committee available at:

http://www.maths.ox.ac.uk/current-students/undergraduates/projects/.

Application You must apply to the Mathematics Project Committee in advance for approval. Proposals should be addressed to The Chairman of the Projects Committee, c/o Mrs Helen Lowe, Room DH61, Dartingtion House, and must be received before 12 noon on Friday of Week 0 of Michaelmas Full Term. The application form is available at http://www.maths.ox.ac.uk/current-students/undergraduates/projects/.

Once your title has been approved, it may only be changed by approval of the Chairman of the Projects Committee.

Assessment Each project is blind double marked. The marks are reconciled through discussion between the two assessors which is overseen by the examiners. Please see the *Guidance Notes on Extended Essays and Dissertations in Mathematics* for detailed marking criteria and class descriptors.

Submission THREE copies of your essay, identified by your candidate number only, should be sent to the Chairman of Examiners, FHS of Mathematics Part B, Examination Schools, Oxford, to arrive no later than **12noon on Friday of week 9, Hilary Term 2011**. An electronic copy of your dissertation should also be submitted via the Mathematical Institute website. Further details may be found in the *Guidance Notes on Extended Essays and Dissertations in Mathematics.*

4 Non-Mathematical units and half-units

4.1 N1: Mathematics Education and Undergraduate Ambassadors Scheme

Level: H-level **Method of Assessment:** See individual synopses for each half unit

Weight: Whole-unit (OSS paper code: tbc), or N1a may be taken as a half-unit.

4.1.1 N1a Mathematics Education —Prof Watson & Dr Stylianides —

Level: H-level

Method of Assessment: Two examined written assignments and a short presentation.
(1500 word written assignment (not examined) due: end of week 2
2500 word written assignment (examined) due: end of week 5
Presentation (examined) week 8
3000 word written assignment (examined) due: week 1 Hilary Term)

Weight: Half Unit. OSS paper code tbc.

Quota: There will be a quota of approximately 20 students for this course.

Recommended Prerequisites: None

Overview

The Mathematics Education option will be a half-unit, run in Michaelmas Term. The course is appropriate for all students of the appropriate degree courses, whether or not they are interested in teaching subsequently. Final credit will be based on two examined written assignments (35 % each), one of which will be submitted at the start of Hilary Term, and a presentation (30%). Teaching will be 22 hours of contact time which will include lecture, seminar, class and tutorial formats as follows:

- A two-hour lecture/class per week. These will be interactive and involve discussion and other tasks as well as input from the lecturer.
- Two two-hour workshops in preparation for written assignments
- Two tutorials of one hour per student pair, the first for feedback about a trial writing task, the second to prepare for the presentation (The latter two types of constitute 6 tutorial hours).

Learning Outcomes

1. Understanding:
    - the psychology of learning mathematics;

- the nature of mathematics and the curriculum;
- relations between teaching and learning at primary, secondary and tertiary level;
- the role of mathematics education in society;
- issues associated with communicating mathematics.

2. Understanding connections between mathematics, education issues and the mathematical experience of learners.

3. The ability to express ideas about the study and learning of mathematics in writing, verbally, and in other forms of communication.

Synopsis

1. Introduction to mathematics education as a field of study.
   - Issues and problems; relation to learning mathematics. Introduction to course, the education library, and the expected forms of study including pair work. Setting assignment.
2. Relations between teaching and learning
   - Theories about teaching and learning, e.g. variation theory; deep/surface approaches; observable characteristics in comparative studies; task design.
3. Mathematics education in society.
   - Achievement according to class, gender, ethnicity in UK and elsewhere
4. Communicating mathematics.
   - Verbal; symbolic; diagram; gestural; dynamic representation.

Reading

Main Texts:

1. Tall, D. (1991) *Advanced Mathematical Thinking.* (Mathematics Education Library, 11). Dordrecht: Kluwer

2. Gates, P. (ed.) (2001) *Issues in Mathematics Teaching.* London: RoutledgeFalmer

3. Mason, J., Burton, L. & Stacey K. (2010) *Thinking Mathematically.* Any edition by any publisher will do.

4. Polya, G. (1957) *How to Solve It.* Any edition by any publisher will do.

5. Davis, P. And Hersh, R. (1981) *The Mathematical Experience.* Any edition by any publisher will do.

6. Mason, J. & Johnston-Wilder, S. (2004) *Fundamental Constructs in Mathematics Education*, London: RoutledgeFalmer.

7. Carpenter, T., Dossey, J. & Koehler, J. (2004) *Classics in Mathematics Education Research.* Reston, VA: National Council of Teachers of Mathematics.

Important websites:

1. ncetm.org.uk

4.1.2 N1b Undergraduate Ambassadors' Scheme — Dr Earl — mainly HT

[This course is no longer available as a half-unit it must be taken alongside N1a mathematics Education]

Method of Assessment: Journal of activities, Oral presentation, Course report and project, Teacher report.

Quota: There will be a quota of approximately 10 students for this course.

Co-ordinator: Dr Earl

Option available to Mathematics, Mathematics & Statistics, Mathematics & Philosophy students.

Learning Outcomes

The Undergraduate Ambassadors' Scheme (UAS) was begun by Simon Singh in 2002 to give university undergraduates a chance to experience assisting and, to some extent, teaching in schools, and to be credited for this. The option focuses on improving students' communication, presentation, cooperation and organizational skills and sensitivity to others' learning difficulties.

Course Description and Timing:

The Oxford UAS option, N1, is a half-unit, mainly run in Hilary Term. A quota will be in place, of approximately 10 students, and so applicants for the UAS option will be asked to name a second alternative half-unit. The course is appropriate for all students, whether or not they are interested in teaching subsequently.

A student on the course will be assigned to a mathematics teacher in a local secondary school (in the Oxford, Kidlington, Wheatley area) for half a day per week during Hilary Term. Students will be expected to keep a journal of their activities, which will begin by assisting in the class, but may widen to include teaching the whole class for a part of a period, or working separately with a smaller group of the class. Students will be required at one point to give a presentation to one of their school classes relating to a topic from university mathematics, and will also run a small project based on some aspect of mathematics education with advice from the course co-ordinator and teacher/s. Final credit will be based on the journal (20%), the presentation (30%), an end of course report (approximately 3000 words) including details of the project (35%), together with a report from the teacher (15%).

Short interviews will take place on Thursday or Friday of 0th week in Michaelmas term to select students for this course. The interview (of roughly 15 minutes) will include a presentation by the student on an aspect of mathematics of their choosing. Students will be chosen on the basis of their ability to communicate mathematics, and two references will be sought from college tutors on these qualities. Applicants will be quickly notified of the decision.

During Michaelmas term there will be a Training Day, in conjunction with the Oxford Department of Education, as preparation for working with pupils and teachers, and to provide more detail on the organisation of teaching in schools. Those on the course will also need to fill in a CRB form, or to have done so already. By the end of term students will have been assigned to a teacher and have made a first, introductory, visit to their school. The course will begin properly in Hilary term with students helping in schools for half a day each week. Funds are available to cover travel expenses. Support classes will be provided throughout Hilary for feedback and to discuss issues such as the planning of the project. The deadline for the journal and report will be noon on Friday of 0th week of Trinity term.

Any further questions on the UAS option should be passed on to the option's co-ordinator, Richard Earl (earl@maths.ox.ac.uk).

Reading List

Clare Tickly, Anne Watson, Candia Morgan, *Teaching School Subjects: Mathematics* (Routledge Falmer, 2004).

4.2 Philosophy: Units and Half-units

The other units that students in Part B may take are drawn from courses provided by the Faculty of Philosophy. For full details of these units see the Philosophy Lecture Prospectus, which is available on the Web at http://www.philosophy.ox.ac.uk/ prior to the start of each term (usually in 0th Week).

The Philosophy units available are as follows:

- N102 Knowledge and Reality
- N122 Philosophy of Mathematics
- N101 History of Philosophy from Descartes to Kant

[Paper 102, 122 and 101 in all Honour Schools including Philosophy. Teaching responsibility of the Philosophy Faculty.]

5 Language Classes: French

Language courses in French may be offered by the University Language Centre.

Students in the FHS Mathematics may apply to take language classes. In 2010-2011, French language classes will be run in MT and HT. We have a limited number of places but if we have spare places we will offer these to joint school students, Mathematics and Computer Science, Mathematics and Philosophy and Mathematics and Statistics.

Two levels of French courses are offered, a lower level for those with a good pass at GCSE, and a higher level course for those with A/S or A level. Acceptance on either course will depend on satisfactory performance in the Preliminary Qualifying Test held in Week 1 of Michaelmas Term (Monday, 17.00-19.00 at the Language Centre). Classes at both levels will take place on Mondays, 17.00-19.00.

Performance on the course will not contribute to the class of degree awarded. However, upon successful completion of the course, students will be issued with a certificate of achievement which will be sent to their college.

Places at these classes are limited, so students are advised that they must indicate their interest at this stage. If you are interested but are unable to attend this presentation for some reason please contact the Academic Administrator in the Mathematical Institute (academic.administrator@maths.ox.ac.uk; (6)15203) as soon as possible.

Aims and rationale

The general aim of the language courses is to develop the student's ability to communicate (in both speech and writing) in French to the point where he or she can function in an academic or working environment in a French-speaking country.

The course has been designed with general linguistic competence in the four skills of reading, writing, speaking and listening in mind. There will be opportunities for participants to develop their own particular interests through presentations and assignments.

Form and Content

The course will consist of a thirty-two contact hour course, together with associated work. Classes will be held in the Language Centre at two hours per week in Michaelmas and Hilary Terms.

The content of the course is based on coursebooks together with a substantial amount of supplementary material prepared by the language tutors. Participants should expect to spend an additional two hours per week on preparation and follow-up work.

The course aims to cover similar ground in terms of grammar, spoken and written language and topics. Areas covered will include:

Grammar:

- all major tenses will be presented and/or revised, including the subjunctive
- passive voice
- pronouns
- formation of adjectives, adverbs, comparatives

- use of prepositions
- time expressions

Speaking

- guided spoken expression for academic, work and leisure contact (e.g. giving presentations, informal interviews, applying for jobs)
- expressing opinions, tastes and preferences
- expressing cause, consequence and purpose

Writing

- Guided letter writing for academic and work contact
- Summaries and short essays

Listening

- Listening practice of recorded materials (e.g. news broadcasts, telephone messages, interviews)
- developing listening comprehension strategies

Topics (with related readings and vocabulary, from among the following)

- life, work and culture
- the media in each country
- social and political systems
- film, theatre and music
- research and innovation
- sports and related topics
- student-selected topics

Teaching staff

The course is taught by Language Centre tutors or Modern Languages Faculty instructors.

Teaching and learning approaches

The course uses a communicative methodology which demands active participation from students. In addition, there will be some formal grammar instruction. Students will be expected to prepare work for classes and homework will be set. Students will also be given training in strategies for independent language study, including computer-based language learning, which will enable them to maintain and develop their language skills after the course.

Entry

Two classes in French at (probably at Basic and Threshold levels) will be formed according to level of French at entry. The minimum entry standard is a good GCSE Grade or equivalent. Registration for each option will take place at the start of Michaelmas Term. A preliminary qualifying test is then taken which candidates must pass in order to be allowed to join the course.

Learning Outcomes

The learning outcomes will be based on internationally agreed criteria for specific levels (known as the ALTE levels), which are as follows:

Basic Level (corresponds to ALTE Level 2 "Can-do" statements)

- Can express opinions on professional, cultural or abstract matters in a limited way, offer advice and understand instructions.
- Can give a short presentation on a limited range of topics.
- Can understand routine information and articles, including factual articles in newspapers, routine letters from hotels and letters expressing personal opinions.
- Can write letters or make notes on familiar or predictable matters.

Threshold Level (corresponds to ALTE Level 3 "Can-do" statements)

- Can follow or give a talk on a familiar topic or keep up a conversation on a fairly wide range of topics, such as personal and professional experiences, events currently in the news.
- Can give a clear presentation on a familiar topic, and answer predictable or factual questions.
- Can read texts for relevant information, and understand detailed instructions or advice.
- Can make notes that will be of reasonable use for essay or revision purposes.
- Can make notes while someone is talking or write a letter including non- standard requests.

Assessment

There will a preliminary qualifying test in Michaelmas Term. There are three parts to this test: Reading Comprehension, Listening Comprehension, and Grammar. Candidates who have not studied or had contact with French for some time are advised to revise thoroughly, making use of the Language Centre's French resources.

Students' achievement will be assessed through a variety of means, including continuous assessment of oral performance, a written final examination, and a project or assignment chosen by individual students in consultation with their language tutor.

Reading comprehension, writing, listening comprehension and speaking are all examined. The oral component may consist of an interview, a presentation or a candidate's performance in a formal debate or discussion.