



**UNIVERSITY OF OXFORD**  
**Mathematical Institute**

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**HONOUR SCHOOL OF MATHEMATICS**

**SUPPLEMENT TO THE UNDERGRADUATE  
HANDBOOK – 2012 Matriculation**

**SYNOPSIS OF LECTURE COURSES**

**Part B 2014-15  
for examination in 2015**

**These synopses can be found at:**  
**<http://www.maths.ox.ac.uk/members/students/undergraduate-courses/teaching-and-learning/handbooks-synopses>**

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Handbook for the Undergraduate Mathematics Courses  
 Supplement to the Handbook  
 Honour School of Mathematics  
 Syllabus and Synopses for Part B 2014–15  
 for examination in 2015

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## 1 Foreword

*The synopses for Part B will be available on the website at:*

<http://www.maths.ox.ac.uk/current-students/undergraduates/handbooks-synopses/>  
before the start of Michaelmas Term 2014.

See the current edition of the *Examination Regulations* for the full regulations governing these examinations.

Examination Conventions can be found at: <http://www.maths.ox.ac.uk/notices/undergrad>

### 1.1 Honour School of Mathematics

#### 1.1.1 Units

In Part B each candidate shall offer a total of eight units from the schedule of units. Each unit is the equivalent of a sixteen hour lecture course.

(a) A total of at least six units offered should be from the schedule of ‘Mathematics Department units’.

(b) Candidates may offer at most one double unit which is designated as an extended essay or a structured project.<sup>1</sup>

Most Mathematics Department lecture courses are independently available as units, the exceptions being:

1. B4.2 Hilbert Spaces, where B4.1 Banach Spaces is an essential prerequisite.
2. B3.4 Algebraic Number Theory, where B3.1 Galois Theory is an essential prerequisite.

Details of Part C units, examinable in 2016, will be published before Michaelmas Term 2015.

### 1.2 Language Classes

Mathematics students may apply to take classes in a foreign language. In 2014-15 classes will be offered in French and German. Students’ performances in these classes will not contribute to the degree classification awarded. However, successful completion of the course may be recorded on students’ transcripts. See section 5 for more details.

### 1.3 Registration for Part B courses 2014–15

**CLASSES:** Students will have to register in advance for the courses they wish to take. Students will have to register by Friday of Week 10 of Trinity Term 2014 using the online registration system which can be accessed at <https://www.maths.ox.ac.uk/courses/registration/>.

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<sup>1</sup>Units which may be offered under this heading are indicated in the synopses.

Students will then be asked to sign up for classes at the start of Michaelmas Term 2014. Further information about this will be sent via email before the start of term.

Students who register for a course or courses for which there is a quota should consider registering for an additional course (by way of a “reserve choice”) in case they do not receive a place on the course with the quota. They may also have to give the reasons why they wish to take a course which has a quota, and provide the name of a tutor who can provide a supporting statement for them should the quota be exceeded. Where this is necessary students will be contacted by email after they have registered. In the event that the quota for a course is exceeded, the Mathematics Teaching Committee will decide who may have a place on the course on the basis of the supporting statements from the student and tutor, and all relevant students will be notified of the decision by email. In the case of the “Undergraduate Ambassadors’ Scheme” students will have to attend a short interview in Week 0, Michaelmas Term.

Where undergraduate registrations for lecture courses fall below 5, classes will not run as part of the intercollegiate scheme but will be arranged informally by the lecturer.

**LECTURES:** Every effort will be made when timetabling lectures to ensure that lectures do not clash. However, because of the large number of options this may sometimes be unavoidable. In the event of clashes being necessary, then students will be notified of the clashes by email and in any case options will only be allowed to clash when the take-up of both options is unlikely or inadvisable.

#### 1.4 Three-year/Four-year Course Registration

You should register your intention to take either the BA course or the MMath. course during your third year. You are advised to discuss the right course of action for you with your College Tutor, who will also advise you how to register. Any student whose performance in the Part A and B examinations together falls below **upper second standard** will not be permitted to proceed to Part C.

All students are registered on the MMath versions of each course. If you subsequently decide to change to the BA option you must inform your college office who will in turn inform central administration and the departments. Please be aware that any change to your course may impact the level of your maintenance funding and the time taken to receive your student loan (you are advised to contact Student Finance [www.direct.gov.uk/en/EducationAndLearning/UniversityAndHigherEducation/StudentFinance](http://www.direct.gov.uk/en/EducationAndLearning/UniversityAndHigherEducation/StudentFinance) for further enquiries). Please note also that if you intend to change option you are strongly advised to do so before you take the Part B examinations.

## 1.5 Course list by term

Table 1: Michaelmas Term Mathematics Department Units

Code	Title	Term
B1.1	Logic	MT
B2.1	Introduction to Representation Theory	MT
B3.1	Galois Theory	MT
B3.2	Geometry of Surfaces	MT
B3.5	Topology and Groups	MT
B4.1	Banach Spaces	MT
B4.3	Dynamical Systems and Energy Minimization	MT
B5.1	Techniques of Applied Mathematics	MT
B5.3	Viscous Flow	MT
B5.5	Mathematical Ecology and Biology	MT
B6.1	Numerical Solutions of Differential Equations I	MT
B6.3	Integer Programming	MT
C7.2	Electromagnetism	MT
C7.3	Further Quantum Theory	MT
B8.1	Martingales Through Measure Theory	MT
B8.4	Communication Theory	MT
BS3	Applied Probability	MT

Table 2: Hilary Term Mathematics Department Units

Code	Title	Term
B1.2	Set Theory	HT
C2.6	Commutative Algebra	HT
B3.3	Algebraic Curves	HT
B3.4	Algebraic Number Theory	HT
B4.2	Hilbert Spaces	HT
B5.2	Applied Partial Differential Equations	HT
B5.4	Waves and Compressible Flow	HT
B5.6	Nonlinear Systems	HT
B6.2	Numerical Solution of Differential Equations II	HT
B7.1	Classical Mechanics	HT
C7.4	Introduction to Quantum Information	HT
B8.2	Continuous Martingales and Stochastic Calculus	HT
B8.3	Mathematical Models of Financial Derivatives	HT
B8.5	Graph Theory	HT

## 2 Mathematics Department units

### 2.1 B1.1: Logic — Prof. Koenigsmann — 16 MT

**Level:** H-level

**Method of Assessment:** Written examination.

**Weight:** Unit

**Recommended Prerequisites:** None

#### Overview

To give a rigorous mathematical treatment of the fundamental ideas and results of logic that is suitable for the non-specialist mathematicians and will provide a sound basis for more advanced study. Cohesion is achieved by focusing on the Completeness Theorems and the relationship between provability and truth. Consideration of some implications of the Compactness Theorem gives a flavour of the further development of model theory. To give a concrete deductive system for predicate calculus and prove the Completeness Theorem, including easy applications in basic model theory.

#### Learning Outcomes

Students will be able to use the formal language of propositional and predicate calculus and be familiar with their deductive systems and related theorems. For example, they will know and be able to use the soundness, completeness and compactness theorems for deductive systems for predicate calculus.

#### Synopsis

The notation, meaning and use of propositional and predicate calculus. The formal language of propositional calculus: truth functions; conjunctive and disjunctive normal form; tautologies and logical consequence. The formal language of predicate calculus: satisfaction, truth, validity, logical consequence.

Deductive system for propositional calculus: proofs and theorems, proofs from hypotheses, the Deduction Theorem; Soundness Theorem. Maximal consistent sets of formulae; completeness; constructive proof of completeness.

Statement of Soundness and Completeness Theorems for a deductive system for predicate calculus; derivation of the Compactness Theorem; simple applications of the Compactness Theorem.

A deductive system for predicate calculus; proofs and theorems; prenex form. Proof of Completeness Theorem. Existence of countable models, the downward Löwenheim–Skolem Theorem.



## Reading

1. R. Cori and D. Lascar, *Mathematical Logic: A Course with Exercises (Part I)* (Oxford University Press, 2001), sections 1, 3, 4.
2. A. G. Hamilton, *Logic for Mathematicians* (2nd edition, Cambridge University Press, 1988), pp.1–69, pp.73–76 (for statement of Completeness (Adequacy)Theorem), pp.99–103 (for the Compactness Theorem).
3. W. B. Enderton, *A Mathematical Introduction to Logic* (Academic Press, 1972), pp.101–144.
4. D. Goldrei, *Propositional and Predicate Calculus: A model of argument* (Springer, 2005).

## Further Reading

1. R. Cori and D. Lascar, *Mathematical Logic: A Course with Exercises (Part II)* (Oxford University Press, 2001), section 8.

## 2.2 B1.2: Set Theory — Prof. Pila — 16 HT

**Level:** H-level

**Method of Assessment:** Written examination.

**Weight:** Unit

**Recommended Prerequisites:** There are no formal prerequisites, but familiarity with some basic mathematical objects and notions such as: the rational and real number fields; the idea of surjective, injective and bijective functions, inverse functions, order relations; the notion of a continuous function of a real variable, sequences, series, and convergence, and the definitions of basic abstract structures such as fields, vector spaces, and groups (all covered in Mathematics I and II in Prelims) will be helpful at points.

### Overview

To introduce sets and their properties as a unified way of treating mathematical structures, including encoding of basic mathematical objects using set theoretic language. To emphasize the difference between intuitive collections and formal sets. To introduce and discuss the notion of the infinite, the ordinals and cardinality. The Axiom of Choice and its equivalents are presented as a tool.

### Learning Outcomes

Students will have a sound knowledge of set theoretic language and be able to use it to codify mathematical objects. They will have an appreciation of the notion of infinity and arithmetic of the cardinals and ordinals. They will have developed a deep understanding of the Axiom of Choice, Zorn's Lemma and well-ordering principle, and have begun to appreciate the implications.

## Synopsis

What is a set? Introduction to the basic axioms of set theory. Ordered pairs, cartesian products, relations and functions. Axiom of Infinity and the construction of the natural numbers; induction and the Recursion Theorem.

Cardinality; the notions of finite and countable and uncountable sets; Cantor's Theorem on power sets. The Tarski Fixed Point Theorem. The Schröder–Bernstein Theorem.

Isomorphism of ordered sets; well-orders. Transfinite induction; transfinite recursion [informal treatment only].

Comparability of well-orders.

The Axiom of Choice, Zorn's Lemma, the Well-ordering Principle; comparability of cardinals. Equivalence of WO, CC, AC and ZL. Ordinals. Arithmetic of cardinals and ordinals; in [ZFC],

## Reading

1. D. Goldrei, *Classic Set Theory* (Chapman and Hall, 1996).
2. H. B. Enderton, *Elements of Set Theory* (Academic Press, 1978).

## Further Reading

1. R. Cori and D. Lascar, *Mathematical Logic: A Course with Exercises (Part II)* (Oxford University Press, 2001), section 7.1–7.5.
2. R. Rucker, *Infinity and the Mind: The Science and Philosophy of the Infinite* (Princeton University Press, 1995). An accessible introduction to set theory.
3. J. W. Dauben, *Georg Cantor: His Mathematics and Philosophy of the Infinite* (Princeton University Press, 1990). For some background, you may find JW Dauben's biography of Cantor interesting.
4. M. D. Potter, *Set Theory and its Philosophy: A Critical Introduction* (Oxford University Press, 2004). An interestingly different way of establishing Set Theory, together with some discussion of the history and philosophy of the subject.
5. G. Frege, *The Foundations of Arithmetic : A Logical-Mathematical Investigation into the Concept of Number* (Pearson Longman, 2007).
6. M. Schirn, *The Philosophy of Mathematics Today* (Clarendon, 1998). A recentish survey of the area at research level.
7. W. Sierpinski, *Cardinal and Ordinal Numbers* (Polish Scientific Publishers, 1965). More about the arithmetic of transfinite numbers.

## 2.3 B2.1: Introduction to Representation Theory — Prof. Nikolov — 16 MT

**Level:** H-level

**Method of Assessment:** Written examination.

**Weight:** Unit

**Recommended Prerequisites:** Part A Algebra 2 is essential. Algebra 3 is recommended.

### Overview

This course gives an introduction to the representation theory of finite groups and finite dimensional algebras. Representation theory is a fundamental tool for studying symmetry by means of linear algebra: it is studied in a way in which a given group or algebra may act on vector spaces, giving rise to the notion of a representation.

We start in a more general setting, studying modules over rings, in particular over euclidean domains, and their applications. We eventually restrict ourselves to modules over algebras (rings that carry a vector space structure). A large part of the course will deal with the structure theory of semisimple algebras and their modules (representations). We will prove the Jordan-Hölder Theorem for modules. Moreover, we will prove that any finite-dimensional semisimple algebra is isomorphic to a product of matrix rings (Wedderburn's Theorem over  $\mathbb{C}$ ).

In the later part of the course we apply the developed material to group algebras, and classify when group algebras are semisimple (Maschke's Theorem).

### Learning Outcomes

Students will have a sound knowledge of the theory of non-commutative rings, ideals, associative algebras, modules over euclidean domains and applications. They will know in particular simple modules and semisimple algebras and they will be familiar with examples. They will appreciate important results in the course such as the Jordan-Hölder Theorem, Schur's Lemma, and the Wedderburn Theorem. They will be familiar with the classification of semisimple algebras over  $\mathbb{C}$  and be able to apply this.

### Synopsis

Noncommutative rings, one- and two-sided ideals. Associative algebras (over fields). Main examples: matrix algebras, polynomial rings and quotients of polynomial rings. Group algebras, representations of groups.

Modules and their relationship with representations. Simple and semisimple modules, composition series of a module, Jordan-Hölder Theorem. Semisimple algebras. Schur's Lemma, the Wedderburn Theorem, Maschke's Theorem. Characters of complex representations. Orthogonality relations, finding character tables. Tensor product of modules. Induction and restriction of representations. Application: Burnside's  $p^a q^b$  Theorem.

## Reading

1. K. Erdmann, *B2 Algebras*, Mathematical Institute Notes (2007).
2. G. D. James and M. Liebeck, *Representations and Characters of Finite Groups* (2nd edition, Cambridge University Press, 2001).

## Further Reading

1. J. L. Alperin and R. B. Bell, *Groups and Representations*, Graduate Texts in Mathematics 162 (Springer-Verlag, 1995).
2. P. M. Cohn, *Classic Algebra* (Wiley & Sons, 2000). (Several books by this author available.)
3. C. W. Curtis, and I. Reiner, *Representation Theory of Finite Groups and Associative Algebras* (Wiley & Sons, 1962).
4. L. Dornhoff, *Group Representation Theory* (Marcel Dekker Inc., New York, 1972).
5. I. M. Isaacs, *Character Theory of Finite Groups* (AMS Chelsea Publishing, American Mathematical Society, Providence, Rhode Island, 2006).
6. J.-P. Serre, *Linear Representations of Finite Groups*, Graduate Texts in Mathematics 42 (Springer-Verlag, 1977).

## 2.4 C2.6: Commutative Algebra — Prof. Segal — 16HT

**Level:** M-level

**Method of Assessment:** Written examination.

**Weight:** Unit

**Recommended Prerequisites** Part A Algebra 2 is essential. Representation Theory and Galois Theory are recommended.

### Overview

Amongst the most familiar objects in mathematics are the ring of integers and the polynomial rings over fields. These play a fundamental role in number theory and in algebraic geometry, respectively. The course explores the basic properties of such rings.

### Synopsis

Modules, ideals, prime ideals, maximal ideals.

Noetherian rings; Hilbert basis theorem. Minimal primes.

Localization.

Polynomial rings and algebraic sets. Weak Nullstellensatz.

Nilradical and Jacobson radical; strong Nullstellensatz.

Artin-Rees Lemma; Krull intersection theorem.  
 Integral extensions. Prime ideals in integral extensions.  
 Noether Normalization Lemma.  
 Krull dimension; ‘Principal ideal theorem’; dimension of an affine algebra.

### Reading

1. M. F. Atiyah and I. G. MacDonald: *Introduction to Commutative Algebra*, (Addison-Wesley, 1969).

## 2.5 B3.1: Galois Theory — Prof de la Ossa — 16 MT

**Level:** H-level

**Method of Assessment:** Written examination.

**Weight:** Unit

**Recommended Prerequisites:** All second-year algebra and arithmetic. Part A Algebra 2 is essential and Algebra 3 is recommended. Students who have not taken Part A Number Theory should read about quadratic residues in, for example, the appendix to Stewart and Tall. This will help with the examples.

### Overview

The course starts with a review of second-year ring theory with a particular emphasis on polynomial rings, and a discussion of general integral domains and fields of fractions. This is followed by the classical theory of Galois field extensions, culminating in some of the classical theorems in the subject: the insolubility of the general quintic and impossibility of certain ruler and compass constructions considered by the Ancient Greeks.

### Learning Outcomes

Understanding of the relation between symmetries of roots of a polynomial and its solubility in terms of simple algebraic formulae; working knowledge of interesting group actions in a nontrivial context; working knowledge, with applications, of a nontrivial notion of finite group theory (soluble groups); understanding of the relation between algebraic properties of field extensions and geometric problems such as doubling the cube and squaring the circle.

### Synopsis

Review of polynomial rings, factorisation, integral domains. Reminder that any nonzero homomorphism of fields is injective. Fields of fractions.

Review of group actions on sets, Gauss’ Lemma and Eisenstein’s criterion for irreducibility of polynomials, field extensions, degrees, the tower law. Symmetric polynomials.

Separable extensions. Splitting fields and normal extensions. The theorem of the primitive element. The existence and uniqueness of algebraic closure (proofs not examinable).

Groups of automorphisms, fixed fields. The fundamental theorem of Galois theory.

Examples: Kummer extensions, cyclotomic extensions, finite fields and the Frobenius automorphism. Techniques for calculating Galois groups.

Soluble groups. Solubility by radicals, solubility of polynomials of degree at most 4, insolubility of the general quintic, impossibility of some ruler and compass constructions.

### Reading

1. J. Rotman, *Galois Theory* (Springer-Verlag, NY Inc, 2001/1990).
2. I. Stewart, *Galois Theory* (Chapman and Hall, 2003/1989)
3. D.J.H. Garling, *A Course in Galois Theory* (Cambridge University Press I.N., 1987).
4. Herstein, *Topics in Algebra* (Wiley, 1975)

## 2.6 B3.2: Geometry of Surfaces — Prof. Ritter — 16 MT

**Level:** H-level

**Method of Assessment:** Written examination.

**Weight:** Unit

**Recommended Prerequisites:** 2nd year core algebra and analysis, 2nd year topology. Multivariable calculus and group theory would be useful but not essential. Also, B3.2 is helpful, but not essential, for B3.3.

### Overview

Different ways of thinking about surfaces (also called two-dimensional manifolds) are introduced in this course: first topological surfaces and then surfaces with extra structures which allow us to make sense of differentiable functions ('smooth surfaces'), holomorphic functions ('Riemann surfaces') and the measurement of lengths and areas ('Riemannian 2-manifolds').

These geometric structures interact in a fundamental way with the topology of the surfaces. A striking example of this is given by the Euler number, which is a manifestly topological quantity, but can be related to the total curvature, which at first glance depends on the geometry of the surface.

The course ends with an introduction to hyperbolic surfaces modelled on the hyperbolic plane, which gives us an example of a non-Euclidean geometry (that is, a geometry which meets all Euclid's axioms except the axioms of parallels).

### Learning Outcomes

Students will be able to implement the classification of surfaces for simple constructions of topological surfaces such as planar models and connected sums; be able to relate the

Euler characteristic to branching data for simple maps of Riemann surfaces; be able to describe the definition and use of Gaussian curvature; know the geodesics and isometries of the hyperbolic plane and their use in geometrical constructions.

### Synopsis

The concept of a topological surface (or 2-manifold); examples, including polygons with pairs of sides identified. Orientation and the Euler characteristic. Classification theorem for compact surfaces (the proof will not be examined).

Riemann surfaces; examples, including the Riemann sphere, the quotient of the complex numbers by a lattice, and double coverings of the Riemann sphere. Holomorphic maps of Riemann surfaces and the Riemann–Hurwitz formula. Elliptic functions.

Smooth surfaces in Euclidean three-space and their first fundamental forms. The concept of a Riemannian 2-manifold; isometries; Gaussian curvature.

Geodesics. The Gauss–Bonnet Theorem (statement of local version and deduction of global version). Critical points of real-valued functions on compact surfaces.

The hyperbolic plane, its isometries and geodesics. Compact hyperbolic surfaces as Riemann surfaces and as surfaces of constant negative curvature.

### Reading

1. A. Pressley, *Elementary Differential Geometry*, Springer Undergraduate Mathematics Series (Springer-Verlag, 2001). (Chapters 4–8 and 10–11.)
2. G. B. Segal, *Geometry of Surfaces*, Mathematical Institute Notes (1989).
3. R. Earl, *The Local Theory of Curves and Surfaces*, Mathematical Institute Notes (1999).
4. J. McCleary, *Geometry from a Differentiable Viewpoint* (Cambridge, 1997).

### Further Reading

1. P. A. Firby and C. E. Gardiner, *Surface Topology* (Ellis Horwood, 1991) (Chapters 1–4 and 7).
2. F. Kirwan, *Complex Algebraic Curves*, Student Texts 23 (London Mathematical Society, Cambridge, 1992) (Chapter 5.2 only).
3. B. O’Neill, *Elementary Differential Geometry* (Academic Press, 1997).

## 2.7 B3.3: Algebraic Curves — Prof. Szendroi — 16 HT

**Level:** H-level

**Method of Assessment:** Written examination.

**Weight:** Unit

**Recommended Prerequisites:** 2nd year core algebra and analysis, 2nd year topology. Multivariable calculus and group theory would be useful but not essential. Projective Geometry is recommended. Also, B3.2 (Geometry of Surfaces) is helpful, but not essential.

### Overview

A real algebraic curve is a subset of the plane defined by a polynomial equation  $p(x, y) = 0$ . The intersection properties of a pair of curves are much better behaved if we extend this picture in two ways: the first is to use polynomials with complex coefficients, the second to extend the curve into the projective plane. In this course projective algebraic curves are studied, using ideas from algebra, from the geometry of surfaces and from complex analysis.

### Learning Outcomes

Students will know the concepts of projective space and curves in the projective plane. They will appreciate the notion of nonsingularity and know some basic features of intersection theory. They will view nonsingular algebraic curves as examples of Riemann surfaces, and be familiar with divisors, meromorphic functions and differentials.

### Synopsis

Projective spaces, homogeneous coordinates, projective transformations.

Algebraic curves in the complex projective plane. Irreducibility, singular and nonsingular points, tangent lines.

Bezout's Theorem (the proof will not be examined). Points of inflection, and normal form of a nonsingular cubic.

Nonsingular algebraic curves as Riemann surfaces. Meromorphic functions, divisors, linear equivalence. Differentials and canonical divisors. The group law on a nonsingular cubic.

The Riemann–Roch Theorem (the proof will not be examined). The geometric genus. Applications.

### Reading

1. F. Kirwan, *Complex Algebraic Curves*, Student Texts 23 (London Mathematical Society, Cambridge, 1992), Chapters 2–6.
2. W. Fulton, *Algebraic Curves*, 3rd ed., downloadable at [www.math.lsa.umich.edu/~wfulton](http://www.math.lsa.umich.edu/~wfulton)



## 2.8 B3.4: Algebraic Number Theory — Prof. Flynn — 16 HT

**Level:** H-level

**Method of Assessment:** Written examination.

**Weight:** Unit (cannot be taken unless B3.1 is taken)

**Prerequisites:** B9a Galois Theory, Algebra 2 and Number Theory.

**Recommended Prerequisites:** All second-year algebra and arithmetic. Students who have not taken Part A Number Theory should read about quadratic residues in, for example, the appendix to Stewart and Tall. This will help with the examples.

### Overview

An introduction to algebraic number theory. The aim is to describe the properties of number fields, but particular emphasis in examples will be placed on quadratic fields, where it is easy to calculate explicitly the properties of some of the objects being considered. In such fields the familiar unique factorisation enjoyed by the integers may fail, and a key objective of the course is to introduce the class group which measures the failure of this property.

### Learning Outcomes

Students will learn about the arithmetic of algebraic number fields. They will learn to prove theorems about integral bases, and about unique factorisation into ideals. They will learn to calculate class numbers, and to use the theory to solve simple Diophantine equations.

### Synopsis

1. field extensions, minimum polynomial, algebraic numbers, conjugates, discriminants, Gaussian integers, algebraic integers, integral basis
2. examples: quadratic fields
3. norm of an algebraic number
4. existence of factorisation
5. factorisation in  $\mathbb{Q}(\sqrt{d})$
6. ideals,  $\mathbb{Z}$ -basis, maximal ideals, prime ideals
7. unique factorisation theorem of ideals
8. relationship between factorisation of number and of ideals
9. norm of an ideal
10. ideal classes
11. statement of Minkowski convex body theorem
12. finiteness of class number
13. computations of class number to go on example sheets

**Reading**

1. I. Stewart and D. Tall, *Algebraic Number Theory and Fermat's Last Theorem*. (Third Edition, Peters, 2002).

**Further Reading**

1. D. Marcus, *Number Fields* (Springer-Verlag, New York–Heidelberg, 1977). ISBN 0-387-90279-1.

**2.9 B3.5: Topology and Groups — Prof. Dancer — 16 MT****Level:** H-level.**Method of Assessment:** Written examination.**Weight:** Unit**Recommended Prerequisites** 2nd year Topology is essential. 2nd year Group Theory is recommended.**Overview**

This course introduces the important link between topology and group theory. On the one hand, associated to each space, there is a group, known as its fundamental group. This can be used to solve topological problems using algebraic methods. On the other hand, many results about groups are best proved and understood using topology. For example, presentations of groups, where the group is defined using generators and relations, have a topological interpretation. The endpoint of the course is the Nielsen–Shreier Theorem, an important, purely algebraic result, which is proved using topological techniques.

**Synopsis**

Homotopic mappings, homotopy equivalence. Simplicial complexes. Simplicial approximation theorem.

The fundamental group of a space. The fundamental group of a circle. Application: the fundamental theorem of algebra. The fundamental groups of spheres.

Free groups. Existence and uniqueness of reduced representatives of group elements. The fundamental group of a graph.

Groups defined by generators and relations (with examples). Tietze transformations.

The free product of two groups. Amalgamated free products.

The Seifert–van Kampen Theorem.

Cell complexes. The fundamental group of a cell complex (with examples). The realization of any finitely presented group as the fundamental group of a finite cell complex.

Covering spaces. Liftings of paths and homotopies. A covering map induces an injection between fundamental groups. The use of covering spaces to determine fundamental groups: the circle again, and real projective  $n$ -space. The correspondence between covering spaces and subgroups of the fundamental group. Regular covering spaces and normal subgroups.

Cayley graphs of a group. The relationship between the universal cover of a cell complex, and the Cayley graph of its fundamental group. The Cayley 2-complex of a group.

The Nielsen–Schreier Theorem (every subgroup of a finitely generated free group is free) proved using covering spaces.

### Reading

1. John Stillwell, *Classical Topology and Combinatorial Group Theory* (Springer-Verlag, 1993).

### Additional Reading

1. D. Cohen, *Combinatorial Group Theory: A Topological Approach*, Student Texts 14 (London Mathematical Society, 1989), Chapters 1–7.
2. A. Hatcher, *Algebraic Topology* (CUP, 2001), Chapter. 1.
3. M. Hall, Jr, *The Theory of Groups* (Macmillan, 1959), Chapters. 1–7, 12, 17 .
4. D. L. Johnson, *Presentations of Groups*, Student Texts 15 (Second Edition, London Mathematical Society, Cambridge University Press, 1997). Chapters. 1–5, 10,13.
5. W. Magnus, A. Karrass, and D. Solitar, *Combinatorial Group Theory* (Dover Publications, 1976). Chapters. 1–4.

## 2.10 B4.1: Banach Spaces — Prof. Belyaev — 16 MT

**Level:** H-level

**Method of Assessment:** Written examination.

**Weight:** Unit

**Recommended Prerequisites:** Part A Topology and Integration. [From Topology, only the material on metric spaces, including closures, will be used. From Integration, the only concepts which will be used are the convergence theorems and the theorems of Fubini and Tonelli, and the notions of measurable functions and null sets. No knowledge is needed of outer measure, or of any particular construction of the integral, or of any proofs.]

### Learning Outcomes

Students will have a firm knowledge of real and complex normed vector spaces, with their geometric and topological properties. They will be familiar with the notions of completeness, separability and density, will know the properties of a Banach space and important

examples, and will be able to prove results relating to the Hahn–Banach Theorem. They will have developed an understanding of the theory of bounded linear operators on a Banach space.

### Synopses

Real and complex normed vector spaces, their geometry and topology. Completeness. Banach spaces, examples ( $\ell^p$ ,  $\ell^\infty$ ,  $L^p$ ,  $C(K)$ , spaces of differentiable functions).

Finite-dimensional normed spaces; equivalence of norms and completeness. Separable spaces; separability of subspaces.

Continuous linear functionals. Dual spaces. Hahn–Banach Theorem (proof for real separable spaces only) and applications, including density of subspaces and separation of convex sets. Stone-Weierstrass Theorem.

Bounded linear operators, examples (including integral operators). Adjoint operators. Spectrum and resolvent. Spectral mapping theorem for polynomials.

### Reading

1. B.P. Rynne and M.A. Youngson, *Linear Functional Analysis* (Springer SUMS, 2nd edition, 2008), Chapters 2, 4, 5.
2. E. Kreyszig, *Introductory Functional Analysis with Applications* (Wiley, revised edition, 1989), Chapters 2, 4.2174.3, 4.5, 7.1177.4.

## 2.11 B4.2: Hilbert Spaces — Prof Priestley — 16 HT

**Level:** H-level

**Method of Assessment:** Written examination.

**Weight:** Unit (cannot be taken unless B4.1 is taken)

**Prerequisites:** B4.1 Banach Spaces

**Recommended Prerequisites:** A good working knowledge of Part A Core Analysis (both metric spaces and complex analysis) is expected. Part A Integration is desirable, but the only concepts which will be used are the convergence theorems and the theorems of Fubini and Tonelli, and the notions of measurable functions and null sets. No knowledge is needed of outer measure, or of any particular construction of the integral, or of any proofs.]

### Learning Outcomes

Students will appreciate the role of completeness through the Baire category theorem and its consequences for operators on Banach spaces. They will have a demonstrable knowledge of the properties of a Hilbert space, including orthogonal complements, orthonormal sets, complete orthonormal sets together with related identities and inequalities. They will be familiar with the theory of linear operators on a Hilbert space, including adjoint operators,

self-adjoint and unitary operators with their spectra. They will know the  $L^2$ -theory of Fourier series and be aware of the classical theory of Fourier series and other orthogonal expansions.

### Synopses

Hilbert spaces; examples including  $L^2$ -spaces. Orthogonality, orthogonal complement, closed subspaces, projection theorem. Riesz Representation Theorem.

Baire Category Theorem and its consequences for operators on Banach spaces (Uniform Boundedness, Open Mapping, Inverse Mapping and Closed Graph Theorems). Strong convergence of sequences of operators.

Linear operators on Hilbert space, adjoint operators. Self-adjoint operators, orthogonal projections, unitary operators, and their spectra.

Orthonormal sets, Pythagoras, Bessels inequality. Complete orthonormal sets, Parseval.

$L^2$ -theory of Fourier series, including completeness of the trigonometric system. Discussion of classical theory of Fourier series (including statement of pointwise convergence for piecewise differentiable functions, and exposition of failure for some continuous functions). Examples of other orthogonal expansions (Legendre, Laguerre, Hermite etc.).

### Reading

#### *Essential Reading*

1. B.P. Rynne and M.A. Youngson, *Linear Functional Analysis* (Springer SUMS, 2nd edition, 2008), Chapters 3, 4.4, 6.
2. E. Kreyszig, *Introductory Functional Analysis with Applications* (Wiley, revised edition, 1989), Chapters 3, 4.7–4.9, 4.12–4.13, 9.1–9.2.
3. N. Young, *An Introduction to Hilbert Space* (Cambridge University Press, 1988), Chs 1177.

#### *Further Reading*

1. E.M. Stein and R. Shakarchi, *Real Analysis: Measure Theory, Integration & Hilbert Spaces* (Princeton Lectures in Analysis III, 2005), Chapter 4.

## 2.12 B4.3: Dynamical Systems and Energy Minimization — Prof. Nguyen — 16 MT

**Level:** H-level

**Method of Assessment:** Written examination.

**Weight:** Unit

**Recommended Prerequisites:** Elements from the following Part A courses: Differential Equations (Picard's theorem and phase plane arguments), Integration (convergence theorems and  $L_p$  spaces). Topology (metric spaces). However this material will be reviewed in the course.

## Overview

The aim of this course is to discuss the mathematics needed to describe the approach to equilibrium in dissipative dynamical systems. It describes the basic ideas of dynamical systems, Lyapunov functions and stability, and also provides an introduction to the theory of local minimizers in the one-dimensional calculus of variations. The ideas are applied to the dynamical system generated by a semilinear PDE.

As well as addressing aspects of an important scientific question, the course is an introduction to some of the rigorous techniques that are central to the modern study of nonlinear PDE.

## Synopsis

Part I, *Dynamical systems* (7 lectures). Introduction to the problem of the approach to equilibrium and its thermodynamic origins. ODEs in  $\mathbb{R}^n$ ; local and global existence, continuous dependence on initial data, Lyapunov functions. Dynamical systems in  $\mathbb{R}^n$  and metric spaces.  $\omega$ -limit sets, invariance, La Salle invariance principle, Lyapunov stability.

Part II, *Local minimizers* in the 1D calculus of variations (5 lectures) Introduction to Sobolev spaces in 1D. Simplified theory of global, weak and strong local minimizers in the calculus of variations.

Part III, *Applications to PDE*. (4 lectures) Discussion of one-dimensional semilinear parabolic PDE, the approach to equilibrium, and stability.

## Reading

There is no single book that covers the course, which is a new compilation of material, and the lecturer aims to provide comprehensive notes. The following books contain useful material (but go well beyond the course in different directions):

For Part I

1. *Nonlinear differential equations and dynamical systems*, Ferdinand Verhulst, 2nd Edition, Springer, 1996.
2. *Nonlinear Ordinary Differential Equations, An Introduction to Dynamical Systems*, 4th Edition, D.W. Jordan and P. Smith (Oxford University Press, 2007).
3. *Dynamics and Bifurcations*, J.K. Hale and H. Kocak (Springer, 1991).

For Part II,

1. *One-dimensional Variational Problems*, G. Buttazzo, M. Giaquinta, S. Hildebrandt, Oxford Lecture Series in Mathematics, Vol. 15 (Oxford University Press, 1998)
2. *Introduction to the Calculus of Variations*, U. Brechtken-Manderscheid (Chapman and Hall, 1991).
3. *Introduction to the Calculus of Variations*, H. Sagan, (Dover, 1992).

For Part III,

1. *Infinite-Dimensional Dynamical Systems: An Introduction to Dissipative Parabolic PDEs and the Theory of Global Attractors*, James C. Robinson, (Cambridge Texts in Applied Mathematics, Cambridge University Press, 2001).

## 2.13 B5.1: Techniques of Applied Mathematics — Prof. Moulton — 16 MT

**Level:** H-level

**Method of Assessment:** Written examination.

**Weight:** Unit

**Recommended Prerequisites:** Calculus of Variations and Waves and Fluids from Part A are desirable but not essential. Part A Differential Equations 2 is recommended.

### Overview

This course develops mathematical techniques which are useful in solving ‘real-world’ problems involving differential equations, and is a development of ideas which arise in the second year differential equations course. The course aims to show in a practical way how equations ‘work’, what kinds of solution behaviours can occur, and some techniques which are useful in their solution.

### Learning Outcomes

Students will know how differential equations can be used to model real-world phenomena and be able to describe the behaviour of the types of solutions that can occur. They will be familiar with the use of delta functions and the Fredholm Alternative and will be able to solve Sturm–Liouville systems. They will develop the theory of ODEs with regular singular points, including special functions.

### Synopsis

The delta function, introduction to distributions, Green’s function revisited [4 lectures]

Sturm–Liouville systems, adjoints, eigenfunction expansions. [4 lectures]

Fredholm alternative. [1 lecture]

Integral equations. [2 lectures]

Singular points of differential equations; special functions, applications. [5 lectures]

### Reading

1. K. F. Riley, M. P. Hobson and S. J. Bence, *Mathematical Methods for Physics and Engineering*, 3rd Ed., Cambridge University Press, Cambridge, (2006).
2. A. C. Fowler, *Techniques of Applied Mathematics*, Mathematical Institute Notes (2005).
3. J. D. Logan, *Applied Mathematics*, 3rd Ed. Wiley Interscience, New York (2006).
4. I. Stakgold, *Green's Functions and Boundary Value Problems*, 2nd Ed., Wiley Interscience, New York (1998).
5. J. P. Keener, *Principles of Applied Mathematics* Perseus Books, Cambridge, Mass., 2000).
6. S. Howison, *Practical Applied Mathematics* (Cambridge University Press, Cambridge, 2005).

## 2.14 B5.2: Applied Partial Differential Equations — Prof. Byrne — 16 HT

**Level:** H-level

**Method of Assessment:** Written examination.

**Weight:** Unit

**Recommended Prerequisites:** Differential Equations 1 and Differential Equations 2 from Part A are prerequisites, and the material in these courses will be assumed to be known. Calculus of Variations and Waves and Fluids from Part A and Techniques of Applied Mathematics from Part B are desirable but not essential.

### Overview

This course continues the Part A Differential Equations courses, and extends some of the techniques of B5.1 to partial differential equations. In particular, first-order conservation laws are solved and the idea of a shock is introduced; general nonlinear and quasi-linear first-order partial differential equations are solved, the classification of second-order partial differential equations is extended to systems, with hyperbolic systems being solved by characteristic variables. Then Riemann's function, maximum principle and similarity variable methods are demonstrated for partial differential equations.



## Learning Outcomes

Students will know a range of techniques to solve PDEs including non-linear first-order and second-order and their classification. They will be able to demonstrate various principles for solving PDEs including the method of characteristics, the maximum principle, similarity solutions and the Riemann function.

## Synopsis

First-order equations: conservation laws and shocks. Charpit's equations; eikonal equation. [5 lectures]

Systems of partial differential equations, characteristics. Shocks; viscosity solutions; weak solutions. [4 lectures]

Maximum principles, well-posed problems for the heat equation and for Laplace's equation. [3 lectures]

Similarity solutions. [2 lectures]

Riemann functions for hyperbolic partial differential equations. [2 lectures]

## Reading

1. Dr Norbury's web notes.
2. Institute lecture notes are now available (JN).
3. M. Renardy and R.C. Rogers, *An Introduction to Partial Differential Equations* (Springer-Verlag, New York, 2004).
4. J. P. Keener, *Principles of Applied Mathematics: Transformation and Approximation* (revised edition, Perseus Books, Cambridge, Mass., 2000).
5. J. R. Ockendon, S. D. Howison, A. A. Lacey and A. B. Movchan, *Applied Partial Differential Equations* (revised edition, Oxford University Press, Oxford, 2003).

### 2.15 B5.3: Viscous Flow — Prof. Waters — 16 MT

**Level:** H-level

**Method of Assessment:** Written examination.

**Weight:** Unit

**Recommended Prerequisites:** The Part A (second-year) course 'Waves and Fluids'. 'DEs2' would be desirable. This course combines well with B5.2 Applied Partial Differential Equations. Though the two units are intended to stand alone, they will complement each other.

## Overview

Viscous fluids are important in so many facets of everyday life that everyone has some intuition about the diverse flow phenomena that occur in practise. This course is distinctive in that it shows how quite advanced mathematical ideas such as asymptotics and partial differential equation theory can be used to analyse the underlying differential equations and hence give scientific understanding about flows of practical importance, such as air flow round wings, oil flow in a journal bearing and the flow of a large raindrop on a windscreen.

## Learning Outcomes

Students will have developed an appreciation of diverse viscous flow phenomena and they will have a demonstrable knowledge of the mathematical theory necessary to analyse such phenomena.

## Synopsis

Euler's identity and Reynolds' transport theorem. The continuity equation and incompressibility condition. Cauchy's stress theorem and properties of the stress tensor. Cauchy's momentum equation. The incompressible Navier-Stokes equations. Vorticity. Energy. Exact solutions for unidirectional flows; Couette flow, Poiseuille flow, Rayleigh layer, Stokes layer. Dimensional analysis, Reynolds number. Derivation of equations for high and low Reynolds number flows.

Thermal boundary layer on a semi-infinite flat plate. Derivation of Prandtl's boundary-layer equations and similarity solutions for flow past a semi-infinite flat plate. Discussion of separation and application to the theory of flight.

Slow flow past a circular cylinder and a sphere. Non-uniformity of the two dimensional approximation; Oseen's equation. Lubrication theory: bearings, squeeze films, thin films; Hele-Shaw cell and the Saffman-Taylor instability.

## Reading

1. D. J. Acheson, *Elementary Fluid Dynamics* (Oxford University Press, 1990), Chapters 2, 6, 7, 8.
2. H. Ockendon and J. R. Ockendon, *Viscous Flow* (Cambridge Texts in Applied Mathematics, 1995).

## Further reading

3. G.K. Batchelor, *An Introduction to Fluid Dynamics* (CUP, 2000). ISBN 0521663962.
4. C.C. Lin & L.A. Segel, *Mathematics Applied to Deterministic Problems in Natural Sciences*(Society of Industrial and Applied Mathematics, 1998). ISBN 0898712297.
5. L.A Segel, *Mathematics Applied to Continuum Mechancis*(Society for Industrial and Applied Mathematics, 2007). ISBN 0898716209.

## 2.16 B5.4: Waves and Compressible Flow — Prof. Hewitt — 16 HT

**Level:** H-level

**Method of Assessment:** Written examination.

**Weight:** Unit

**Recommended Prerequisites:** Part A ‘Waves and Fluids’ (and Integral Transforms from 2015/16). This course combines well with B5.2 Applied Partial Differential Equations and B5.3 Viscous Flow.

### Overview

Propagating disturbances, or waves, occur frequently in applied mathematics. This course will be centred on some prototypical examples from fluid dynamics, the two most familiar being surface gravity waves and waves in gases. The models for compressible flow will be derived and then analysed for small amplitude motion. This will shed light on the important phenomena of dispersion, group velocity and resonance, and the differences between supersonic and subsonic flow, as well as revealing the crucial dependence of the waves on the number of space dimensions. Larger amplitude motion of liquids and gases will be described by incorporating non-linear effects, and the theory of characteristics for partial differential equations will be applied to understand the shock waves associated with supersonic flight.

### Learning Outcomes

Students will have developed a sound knowledge of a range of mathematical models used to study waves (both linear and non-linear), will be able to describe examples of waves from fluid dynamics and will have analysed a model for compressible flow. They will have an awareness of shock waves and how the theory of characteristics for PDEs can be applied to study those associated with supersonic flight.

### Synopsis

Equations of inviscid compressible flow including flow relative to rotating axes.

Models for linear wave propagation including Stokes waves, internal gravity waves, inertial waves in a rotating fluid, and simple solutions.

Theories for Linear Waves: Fourier Series, Fourier integrals, method of stationary phase, dispersion and group velocity. Flow past thin wings.

Nonlinear Waves: method of characteristics, simple wave flows applied to one-dimensional unsteady gas flow and shallow water theory.

Shock Waves: weak solutions, RankineHugoniot relations, oblique shocks, bores and hydraulic jumps.

### Reading

1. H. Ockendon and J. R. Ockendon, *Waves and Compressible Flow* (Springer, 2004).

2. D. J. Acheson, *Elementary Fluid Dynamics* (Oxford University Press, 1990). Chapter 3
3. J. Billingham and A. C. King, *Wave Motion* (Cambridge University Press, 2000). Chapters 1–4, 7,8.
4. M. J. Lighthill, *Waves in Fluids* (Cambridge University Press, 1978).
5. G. B. Whitham, *Linear and Nonlinear Waves* (Wiley, 1973).

## 2.17 B5.5: Mathematical Ecology and Biology — Prof. Maini — 16 MT

**Level:** H-level

**Method of Assessment:** Written examination.

**Weight:** Unit

**Recommended Prerequisites:** Part A core material (especially differential equations).

### Overview

Mathematical Ecology and Biology introduces the applied mathematician to practical applications in an area that is growing very rapidly. The course mainly focusses on situations where continuous models are appropriate and where these may be modelled by deterministic ordinary and partial differential equations. By using particular modelling examples in ecology, chemistry, biology, physiology and epidemiology, the course demonstrates how various applied mathematical techniques, such as those describing linear stability, phase planes, singular perturbation and travelling waves, can yield important information about the behaviour of complex models.

### Learning Outcomes

Students will have developed a sound knowledge and appreciation of the ideas and concepts related to modelling biological and ecological systems using ordinary and partial differential equations.

### Synopsis

Continuous and discrete population models for a single species, including Ludwig's 1978 insect outbreak models, hysteresis and harvesting. Introduction to delay differential equation models.

Modelling interacting populations, including predator-prey and the principle of competitive exclusion. Discrete models for several species.

Epidemic models.

Michaelis–Menten model for enzyme-substrate kinetics.

Travelling wave propagation with biological examples.

Biological pattern formation, including Turing's model for animal coat markings, and chemotaxis models.

Excitable systems. Threshold phenomena (nerve pulses) and nerve signal propagation.

### Reading

J.D. Murray, *Mathematical Biology, Volume I: An Introduction (2002); Volume II: Spatial Models and Biomedical Applications (2003)* (3rd edition, Springer-Verlag).

1. Volume I: 1.1, 1.2, 1.3, 1.6, 2.1–2.4, 3.1, 3.3–3.6, 3.8, 6.1–6.3, 6.5, 6.6, 8.1, 8.2, 8.4, 8.5, 10.1, 10.2, 11.1–11.5, 13.1–13.5, Appendix A.
2. Volume II: 1.6, 2, 3.1, 3.2, 5.1, 5.2, 13.1–13.4.

Dr R.E. Baker, online lecture notes.

### Further Reading

1. J. Keener and J. Sneyd, *Mathematical Physiology* (First Edition Springer, Berlin, 1998) 1.1, 1.2, 9.1, 9.2.
2. N. F. Britton, *Essential Mathematical Biology* (Springer, London, 2003). 1.1, 1.2, 1.3, 1.5, 2.1, 2.3, 2.4, 2.5, 2.7, 3.1, 3.2, 3.3, 5.1, 5.2, 5.3, 5.6, 7.1, 7.2, 7.3, 7.4, 7.5.

## 2.18 B5.6: Nonlinear Systems — Prof. Moroz — 16 HT

**Level:** H-level

**Method of Assessment:** Written examination.

**Weight:** Unit

**Recommended Prerequisites:** Part A core material (especially differential equations) and Part A Differential Equations 2.

### Overview

This course aims to provide an introduction to the tools of dynamical systems theory which are essential for the realistic modelling and study of many disciplines, including mathematical ecology and biology, fluid dynamics, granular media, mechanics, and more.

The course will include the study of both nonlinear ordinary differential equations and maps. It will draw examples from appropriate model systems and various application areas. The problem sheets will require numerical computation (using programs such as Matlab).

### Learning Outcomes

Students will have developed a sound knowledge and appreciation of some of the tools, concepts, and computations used in the study of nonlinear dynamical systems. They will also get some exposure to some modern research topics in the field.

## Synopsis

### 1. Bifurcations

Bifurcation theory: standard codimension one examples (saddle-node, pitchfork, transcritical, Hopf); normal forms; Conservative and non-conservative systems.

### 2. Nonlinear Oscillations

Van der Pol and Duffing's equations. Poincaré-Lindstedt method; Method of multiple scales; Krylov-Bogoliubov method of Averaging; Relaxation and forced oscillations; Synchronization of coupled oscillators.

### 3. Maps

Poincaré sections and first-return maps. Stability and periodic orbits; bifurcations of one-dimensional maps. Two-dimensional maps: Hénon map, Chirikov (standard) map.

### 4. Chaos in Maps and Differential Equations

Maps: Logistic map, Bernoulli shift map, symbolic dynamics, Skinny Baker's map, Smale's Horseshoe Map.

Differential equations: Lorenz equations, Rossler equations.

## Reading

**Students are by no means expected to read all these sources. These are suggestions intended to be helpful.**

1. S. H. Strogatz, *Nonlinear Dynamics and Chaos with Applications to Physics, Biology, Chemistry and Engineering* (Westview Press, 2000).
2. E. Ott, *Chaos in Dynamical Systems* (Second edition, Cambridge University Press, Cambridge, 2002).
3. R. H. Rand, *Lecture Notes on Nonlinear Vibrations*. [Available for free online at <http://audiophile.tam.cornell.edu/randdocs/nlvibe52.pdf>]
4. P. G. Drazin, *Nonlinear Systems* (Cambridge University Press, Cambridge, 1992).
5. S. Wiggins, *Introduction to Applied Nonlinear Dynamical Systems and Chaos* (Second edition, Springer, 2003).
6. S. H. Strogatz, 'From Kuramoto to Crawford: exploring the inset of synchronization in populations of coupled oscillators', *Physica D* 143 (2000) 1-20.

## 2.19 B6.1 Numerical Solution of Differential Equations I — Prof. Sobey — 16 MT

**Level:** H-level

**Method of Assessment:** Written examination.

**Weight:** Unit

**Recommended Prerequisites:** None.

### Overview

To introduce and give an understanding of numerical methods for the solution of ordinary and partial differential equations, their derivation, analysis and applicability.

The MT lectures are devoted to numerical methods for initial value problems, while the subsequent HT course concentrates on the numerical solution of boundary value problems.

### Learning Outcomes

At the end of the course the student will be able to:

1. construct one-step and linear multistep methods for the numerical solution of initial-value problems for ordinary differential equations and systems of such equations, and to analyse their stability and accuracy properties;
2. construct finite difference methods for the numerical solution of initial-boundary-value problems for second-order parabolic partial differential equations, and first-order hyperbolic equations, and to analyse their stability and accuracy properties.

### Synopsis

The MT course is devoted to the development and analysis of numerical methods for initial value problems. We begin by considering classical techniques for the numerical solution of ordinary differential equations. The problem of stiffness is discussed in tandem with the associated questions of step-size control and adaptivity:

Initial value problems for ordinary differential equations: Euler, multistep and Runge–Kutta; stability; stiffness; error control: symplectic and adaptive algorithms.

The remaining lectures focus on the numerical solution of initial value problems for partial differential equations, including parabolic and hyperbolic problems:

Initial value problems for partial differential equations: parabolic equations, hyperbolic equations; explicit and implicit methods; accuracy, stability and convergence, Fourier analysis, CFL condition.

### Reading List

The course will be based on the following textbooks:

1. K. W. Morton and D. F. Mayers, *Numerical Solution of Partial Differential Equations* (Cambridge University Press, 1994). ISBN 0-521-42922-6 (Paperback edition) [Chapters 2, 3 (Secs. 3.1, 3.2), Chapter 4 (Secs. 4.1–4.6), Chapter 5].
2. E. Süli and D. Mayers, *An Introduction to Numerical Analysis* (Cambridge University Press, 2006). ISBN 0-521-00794-1 (Paperback edition) [Chapter 12].
3. A. Iserles, *A First Course in the Numerical Analysis of Differential Equations* (Cambridge University Press, 1996). ISBN 0-521-55655-4 (Paperback edition) [Chapters 1–5, 13, 14].

## 2.20 B6.2 Numerical Solution of Differential Equations II — Prof. Tanner — 16 HT

**Level:** H-level

**Method of Assessment:** Written examination.

**Weight:** Unit

**Recommended Prerequisites:** None.

### Overview

To introduce and give an understanding of numerical methods for the solution of ordinary and partial differential equations, their derivation, analysis and applicability. The MT lectures are devoted to numerical methods for initial value problems, while the HT lectures concentrate on the numerical solution of boundary value problems.

### Learning Outcomes

Students will understand and have experience of the theory for: Construction of shooting methods for boundary value problems in one independent variable

Elementary numerical analysis of elliptic partial differential equations analysis of iterative methods for solution of large linear systems of equations

### Synopsis

The HT part of the course is concerned with numerical methods for boundary value problems. We begin by developing numerical techniques for the approximation of boundary value problems for second-order ordinary differential equations. Boundary value problems for ordinary differential equations: shooting and finite difference methods. [Introduction (1 lecture) + 2 lectures]

Then we consider finite difference schemes for elliptic boundary value problems. This is followed by an introduction to the theory of direct and iterative algorithms for the solution of large systems of linear algebraic equations which arise from the discretisation of elliptic boundary value problems.



Boundary value problems for PDEs: finite difference discretisation; Poisson equation. Associated methods of sparse numerical algebra: sparse Gaussian elimination, iterative methods. [13 lectures]

### Reading List

This course does not follow any particular textbook, but the following essentially cover the material:

1. A. Iserles, *A First Course in the Numerical Analysis of Differential Equations* (Cambridge University Press, 1996), Chapters 7,10,11.
2. K. W. Morton and D. F. Mayers, *Numerical Solution of Partial Differential Equations* (Cambridge University Press, 1994. Or the more recent 2nd edition, 2005), Chapters 6 and 7.
3. G. D. Smith, *Numerical Solution of Partial Differential Equations: Finite Difference Methods* (Clarendon Press, Oxford, 1985 (and any later editions)), has some of the material in chapter 5.

## 2.21 B6.3 Integer Programming — Prof. Hauser — 16 MT

**Level:** H-level

**Method of Assessment:** Written examination.

**Weight:** Unit

**Recommended Prerequisites:** None.

### Overview

In many areas of practical importance linear optimisation problems occur with integrality constraints imposed on some of the variables. In optimal crew scheduling for example, a pilot cannot be fractionally assigned to two different flights at the same time. Likewise, in combinatorial optimisation an element of a given set either belongs to a chosen subset or it does not. Integer programming is the mathematical theory of such problems and of algorithms for their solution. The aim of this course is to provide an introduction to some of the general ideas on which attacks to integer programming problems are based: generating bounds through relaxations by problems that are easier to solve, and branch-and-bound.

### Learning Outcomes

Students will understand some of the theoretical underpinnings that render certain classes of integer programming problems tractable (“easy” to solve), and they will learn how to solve them algorithmically. Furthermore, they will understand some general mechanisms by which intractable problems can be broken down into tractable subproblems, and how these mechanisms are used to design good heuristics for solving the intractable problems.

Understanding these general principles will render the students able to guide the modelling phase of a real-world problem towards a mathematical formulation that has a reasonable chance of being solved in practice.

### Synopsis

1. Course outline. What is integer programming (IP)? Some classical examples.
2. Further examples, hard and easy problems.
3. Alternative formulations of IPs, linear programming (LP) and the simplex method.
4. LP duality, sensitivity analysis.
5. Optimality conditions for IP, relaxation and duality.
6. Total unimodularity, network flow problems.
7. Optimal trees, submodularity, matroids and the greedy algorithm.
8. Augmenting paths and bipartite matching.
9. The assignment problem.
10. Dynamic programming.
11. Integer knapsack problems.
12. Branch-and-bound.
13. More on branch-and-bound.
14. Lagrangian relaxation and the symmetric travelling salesman problem.
15. Solving the Lagrangian dual.
16. Branch-and-cut.

### Reading

1. L. A. Wolsey, *Integer Programming* (John Wiley & Sons, 1998), parts of chapters 1–5 and 7.

### Time Requirements

The course consists of 16 lectures and 6 problem classes. There are no practicals. It is estimated that 8–10 hours of private study are needed per week for studying the lecture notes and relevant chapters in the textbook, and for solving the problem sheets, so that the total time requirement is circa 12 hours per week.

## 2.22 B7.1: Classical Mechanics — Prof. Sparks — 16HT

**Level:** H-level

**Method of Assessment:** Written examination.

**Weight:** Unit

**Recommended Prerequisites:** Calculus of Variations.

### Overview

This course builds on the Prelims Dynamics course and the Part A option Calculus of Variations. The objective is to describe how physical theories may be described by the Lagrangian and Hamiltonian formalisms. As well as being elegant and computationally useful, these formalisms give important insights into symmetries and conservation laws, and are the language used to describe all modern theories of physics. Applications in the course will include a fuller treatment of rigid body motion, and (as time permits) some elementary features of integrable systems and (non-relativistic) field theories.

### Learning Outcomes

Students will be able to demonstrate knowledge and understanding of the Lagrangian and Hamiltonian formalisms. They will understand how symmetries and conserved quantities are described in this language, and be able to apply the ideas developed to small oscillations around equilibria, rigid body motion and some other elementary systems.

### Synopsis

The principle of least action. Generalized coordinates and constraints. Symmetries and Noether's Theorem. Examples with simple systems.

Equilibria. Small oscillations about a stable equilibrium and normal modes, with examples.

Rigid bodies. Revision of angular velocity, angular momentum and the inertia tensor. Euler's equations and tops. Euler angles and the description of  $SO(3)$ .

Legendre transformations and the Hamiltonian. Liouville's theorem. Poisson brackets. Canonical transformations. Phase space and its geometry.

As time permits: Elementary features of integrable systems. Lagrangian densities for (non-relativistic) field theories.

### Reading

H. Goldstein, C. Poole, J. Safko, *Classical Mechanics (Third Edition)*, Addison-Wesley (2002)

L. D. Landau, E. M. Lifshitz, *Mechanics (Course of Theoretical Physics, Vol. 1) (Third Edition)*, Butterworth-Heinemann (1976)

N. M. J. Woodhouse, *Introduction to Analytical Mechanics*, OUP (1987)

### 2.23 C7.2: Electromagnetism — Prof. Alday — 16MT

**Level:** M-level

**Method of Assessment:** Written examination.

**Weight:** Unit

**Recommended Prerequisites:** None

#### Overview

The idea is to have a classical course on Electromagnetism, similar to the one in a theoretical Physics degree. We will follow closely the book by Jackson, first 8 chapters.

#### Learning Outcomes

Students will have a clear understanding of what electromagnetism is, they will dominate many techniques and will be able to solve most classic problems of electromagnetism. This course should also enable them to continue learning by themselves, or take more advanced courses.

#### Synopsis

Basics of electrostatics;  
 Boundary value problems in electrostatics;  
 Multipoles, electrostatics of macroscopic media, dielectrics;  
 Magnetostatics;  
 Time-varying fields, Maxwell equations, conservation laws;  
 Plane electromagnetic waves;  
 Wave guides and resonant cavities [if time allows]

#### Reading

The lectures will follow:

J.D. Jackson, *Classical Electrodynamics*, John Wiley. (1962), chapters 1 to 8.

#### Further Reading

R. Feynman, *Lectures in Physics*, Vol.2. Electromagnetism, Addison Wesley.

L.D. Landau and E.M. Lifshitz, *A classical theory of fields* Volume 2.

### 2.24 C7.3: Further Quantum Theory — Prof. Mason — 16MT

**Level:** M-level

**Method of Assessment:** Written examination.

**Weight:** Unit

**Recommended Prerequisites:** Part A Quantum Theory.

## Overview

This course builds directly on the first course in quantum mechanics and covers a series of important topics, particularly features of systems containing several particles. The behaviour of identical particles in quantum theory is more subtle than in classical mechanics, and an understanding of these features allows one to understand the periodic table of elements and the rigidity of matter.

There are rarely neat solutions to problems involving several particles, so usually one needs some approximation methods. These are developed so as to study both energy levels of interacting Hamiltonians, and scattering.

## Learning Outcomes

Students will be able to demonstrate knowledge and understanding of quantum mechanics of many particle systems, and atomic structure.

## Synopsis

Addition of angular momentum and Hamiltonians with a spin interaction.

Identical particles, symmetric and anti-symmetric states, Fermi-Dirac and Bose-Einstein statistics and atomic structure.

Approximation methods, Rayleigh-Schrödinger time-independent perturbation theory and variation principles. Born-Oppenheimer approximation. The virial theorem. Helium. WKB approximation.

Heisenberg representation, interaction representation, time dependent perturbation theory and Feynman-Dyson expansion.

Scattering theory (the S matrix, scattering states, Coulomb scattering).

## Reading

The lectures will follow:

S. Weinberg, *Lectures on quantum mechanics*, CUP, (2013). Sections 4.3-5, 5.1-7, 6.1-3, 7.1-4.

See also:

K. C. Hannabuss, *Introduction to quantum mechanics*, OUP (1997). Chapter 16.1-4, 11.1-5, 12.1-4, 14.1-4 covers roughly half the course.

J. Binney and D. Skinner, *The physics of quantum mechanics*, PUP, 2011.

A popular non-technical account of the subject: A. Hey and P. Walters, *The New Quantum Universe* (Cambridge, 2003).

Also designed for an Oxford course, though only covering some material: I. .P Grant, *Classical and Quantum Mechanics*, Mathematical Institute Notes (1991).

### Further Reading

See also: L. I. Schiff, *Quantum Mechanics* (3rd edition, Mc Graw Hill, 1968).

B. J. Bransden and C. J. Joachain, *Introduction to Quantum Mechanics* (Longman, 1995).

A. I. M. Rae, *Quantum Mechanics* (4th edition, Institute of Physics, 1993).

## 2.25 C7.4: Introduction to Quantum Information — Prof. Ekert — 16HT

**Level:** M-level

**Method of Assessment:** Written examination.

**Weight:** Unit

### Recommended Prerequisites:

Quantum Theory.

The course material should be of interest to physicists, mathematicians, computer scientists, and engineers. The following will be assumed as prerequisites for this course:

- elementary probability, complex numbers, vectors and matrices;
- Dirac bra-ket notation;
- a basic knowledge of quantum mechanics especially in the simple context of finite dimensional state spaces (state vectors, composite systems, unitary matrices, Born rule for quantum measurements);
- basic ideas of classical theoretical computer science (complexity theory) would be helpful but are not essential.

Prerequisite notes will be provided giving an account of the necessary material. It would be desirable for you to look through these notes slightly before the start of the course.

### Overview

The classical theory of computation usually does not refer to physics. Pioneers such as Turing, Church, Post and Goedel managed to capture the correct classical theory by intuition alone and, as a result, it is often falsely assumed that its foundations are self-evident and purely abstract. They are not! Computers are physical objects and computation is a physical process. Hence when we improve our knowledge about physical reality, we may also gain new means of improving our knowledge of computation. From this perspective it should not be very surprising that the discovery of quantum mechanics has changed our understanding of the nature of computation. In this series of lectures you will learn how

inherently quantum phenomena, such as quantum interference and quantum entanglement, can make information processing more efficient and more secure, even in the presence of noise.

### Synopsis

1. Bits, gates, networks, Boolean functions, reversible and probabilistic computation
2. "Impossible" logic gates, amplitudes, quantum interference
3. One, two and many qubits
4. Entanglement and entangling gates
5. From interference to quantum algorithms
6. Algorithms, computational complexity and Quantum Fourier Transform
7. Phase estimation and quantum factoring
8. Non-local correlations and cryptography
9. Bell's inequalities
10. Density matrices and CP maps
11. Decoherence and quantum error correction

### Reading

Beyond the Quantum Horizon by D. Deutsch and A. Ekert, *Scientific American*, Sep 2012.

Less reality more security by A. Ekert, *Physics World*, Sep 2009.

The Limits of Quantum Computers, by S. Aaronson, *Scientific American*, Mar 2008.

A Do-It-Yourself Quantum Eraser by R. Hillmer and P. Kwiat, *Scientific American*, May 2007.

Quantum Seeing in the Dark by P. Kwiat et al, *Scientific American*, Nov 1996

Physical Limits of Computation by C.H. Bennett and R. Landauer, *Scientific American*, Jul 1985.

## 2.26 B8.1: Martingales Through Measure Theory — Prof. Riordan — 16 MT

**Level:** H-level

**Method of Assessment:** Written examination.

**Weight:** Unit

**Recommended Prerequisites:** Part A Integration is a prerequisite, so that the corresponding material will be assumed to be known. Part A Probability is a prerequisite.

## Overview

Probability theory arises in the modelling of a variety of systems where the understanding of the “unknown” plays a key role, such as population genetics in biology, market evolution in financial mathematics, and learning features in game theory. It is also very useful in various areas of mathematics, including number theory and partial differential equations. The course introduces the basic mathematical framework underlying its rigorous analysis, and is therefore meant to provide some of the tools which will be used in more advanced courses in probability.

The first part of the course provides a review of measure theory from Integration Part A, and develops a deeper framework for its study. Then we proceed to develop notions of conditional expectation, martingales, and to show limit results for the behaviour of these martingales which apply in a variety of contexts.

## Learning Outcomes

The students will learn about measure theory, random variables, independence, expectation and conditional expectation, product measures and discrete-parameter martingales.

## Synopsis

A branching-process example. Review of  $\sigma$ -algebras, measure spaces. Uniqueness of extension of  $\pi$ -systems and Carathéodory’s Extension Theorem [both without proof], monotone-convergence properties of measures, limsup and liminf of a sequence of events, Fatou’s Lemma, reverse Fatou Lemma, first Borel–Cantelli Lemma.

Random variables and their distribution functions,  $\sigma$ -algebras generated by a collection of random variables. Product spaces. Independence of events, random variables and  $\sigma$ -algebras,  $\pi$ -systems criterion for independence, second Borel–Cantelli Lemma. The tail  $\sigma$ -algebra, Kolmogorov’s 0–1 Law. Convergence in measure and convergence almost everywhere.

Integration and expectation, review of elementary properties of the integral and  $L^p$  spaces [from Part A Integration for the Lebesgue measure on  $\mathbb{R}$ ]. Scheffé’s Lemma, Jensen’s inequality. The Radon–Nikodym Theorem [without proof]. Existence and uniqueness of conditional expectation, elementary properties. Relationship to orthogonal projection in  $L^2$ .

Filtrations, martingales, stopping times, discrete stochastic integrals, Doob’s Optional-Stopping Theorem, Doob’s Upcrossing Lemma and “Forward” Convergence Theorem, martingales bounded in  $L^2$ , Doob decomposition, Doob’s submartingale inequalities.

Uniform integrability and  $L^1$  convergence, backwards martingales and Kolmogorov’s Strong Law of Large Numbers.

Examples and applications, including branching processes.



## Reading

1. D. Williams, *Probability with Martingales*, Cambridge University Press, 1995.
2. P. M. Tarres, Lecture notes, *Appendix : Notes on Fubini's theorem on  $\mathbb{R}$ , Product measures, infinite products of probability triples*, Mathematical Institute, 2009.

## Further Reading

1. Z. Brzeźniak and T. Zastawniak, Basic stochastic processes. A course through exercises. Springer Undergraduate Mathematics Series. (Springer-Verlag London, Ltd., 1999) [more elementary than D. Williams' book, but can provide with a complementary first reading].
2. M. Capinski and E. Kopp, *Measure, integral and probability*, Springer Undergraduate Mathematics Series. (Springer-Verlag London, Ltd., second edition, 2004).
3. R. Durrett, *Probability: Theory and Examples*. (Second Edition Duxbury Press, Wadsworth Publishing Company, 1996).
4. A. Etheridge, *A Course in Financial Calculus*, (Cambridge University Press, 2002).
5. J. Neveu, *Discrete-parameter Martingales*. (North-Holland, Amsterdam, 1975).
6. S. I. Resnick, *A Probability Path*, (Birkhäuser, 1999).

## 2.27 B8.2: Continuous Martingales and Stochastic Calculus — Prof. Obłój — 16 HT

**Level:** H-level

**Method of Assessment:** Written examination.

**Weight:** Unit

**Recommended Prerequisites:** B8.1 Martingales through Measure Theory is a prerequisite. Consequently, Part A Integration and Part A Probability are also prerequisites.

### Overview

Stochastic processes - random phenomena evolving in time - are encountered in many disciplines from biology, through geology to finance. This course focuses on mathematics needed to describe stochastic processes evolving continuously in time and introduces the basic tools of stochastic calculus which are at the cornerstone of modern probability theory. The motivating example of a stochastic process is Brownian motion, also called the Wiener process - a mathematical object initially proposed by Bachelier and Einstein, which originally modelled displacement of a pollen particle in a fluid. The paths of Brownian motion, or of any continuous martingale, are of infinite variation (they are in fact nowhere differentiable and have non-zero quadratic variation) and one of the aims of the course is to define a theory of integration along such paths equipped with a suitable integration by parts formula (Itô formula).

## Learning Outcomes

The students will develop an understanding of Brownian motion and continuous martingales in continuous time. They will become familiar with stochastic calculus and in particular be able to use Itô's formula.

## Synopsis

Stochastic process: collection of random variables vs a random variable on the path space. Brownian motion - definition, construction and basic properties, regularity of paths. Filtrations and stopping times, first hitting times. Brownian motion - martingale and strong Markov properties, reflection principle. Martingales - definitions, regularisation and convergence theorems, optional sampling theorem, maximal and Doob's  $L^p$  inequalities. Quadratic variation, local martingales, semimartingales. Recall of Stieltjes integral. Stochastic integration and Itô's formula with applications.

## Reading

There are a large number of textbooks which cover the course material with a varying degree of detail/rigour. Precise references for reading from two excellent reference books will be given. These are:

- D. Revuz and M. Yor, "Continuous martingales and Brownian motion", Springer (Revised 3<sup>rd</sup> ed.), 2001. Selected pages from Chapters 0–4: *exact pages covering each lecture will be indicated in the course materials.*
- I. Karatzas and S. Shreve, "Brownian motion and stochastic calculus", Springer (2<sup>nd</sup> ed.), 1991. Selected pages from Chapters 1–3: *exact pages covering each lecture will be indicated in the course materials.*

## Further Reading

Further helpful references include:

- R. Durrett, "Stochastic Calculus: A practical introduction", CRC Press, 1996. Sections 1.1 – 2.10.
- F. Klebaner, "Introduction to Stochastic Calculus with Applications", 3rd edition, Imperial College Press, 2012. Chapters 1, 2, 3.1–3.11, 4.1–4.5, 7.1–7.8, 8.1–8.7.
- J. M. Steele, "Stochastic Calculus and Financial Applications", Springer, 2010. Chapters 3 – 8.
- B. Oksendal, "Stochastic Differential Equations: An introduction with applications", 6th edition, Springer (Universitext), 2007. Chapters 1 – 3.
- S. Shreve, "Stochastic calculus for finance", Vol 2: Continuous-time models, Springer Finance, Springer-Verlag, New York, 2004. Chapters 3 – 4.

## 2.28 B8.3: Mathematical Models of Financial Derivatives — Prof. Dewynne — 16 HT

**Level:** H-level

**Method of Assessment:** Written examination.

**Weight:** Unit

**Recommended Prerequisites:** B8.1 would be good background. Part A Probability is a prerequisite. Part A Integration is also good background, though not a prerequisite.

### Overview

The course aims to introduce students to derivative security valuation in financial markets. At the end of the course the student should be able to formulate a model for an asset price and then determine the prices of a range of derivatives based on the underlying asset using arbitrage free pricing ideas.

### Learning Outcomes

Students will have a familiarity with the mathematics behind the models and analytical tools used in Mathematical Finance. This includes being able to formulate a model for an asset price and then determining the prices of a range of derivatives based on the underlying asset using arbitrage free pricing ideas.

### Synopsis

Introduction to markets, assets, interest rates and present value; arbitrage and the law of one price: European call and put options, payoff diagrams. Probability spaces, random variables, conditional expectation, discrete-time martingales. The binomial model; European and American claim pricing.

Introduction to Brownian motion and its quadratic variation, continuous-time martingales, informal treatment of Itô's formula and stochastic differential equations. Discussion of the connection with PDEs through the Feynman–Kac formula.

The Black–Scholes analysis via delta hedging and replication, leading to the Black–Scholes partial differential equation for a derivative price. General solution via Feynman–Kac and risk neutral pricing, explicit solution for call and put options.

American options, formulation as a free-boundary problem. Simple exotic options. Weakly path-dependent options including barriers, lookbacks and Asians. Implied volatility. Introduction to stochastic volatility. Robustness of Black-Scholes formula.

### Reading

1. S.E Shreve, *Stochastic Calculus for Finance*, vols I and II, (Springer 2004).
2. T. Bjork, *Arbitrage Theory in Continuous Time* (Oxford University Press, 1998).

3. P. Wilmott, S. D. Howison and J. Dewynne, *Mathematics of Financial Derivatives* (Cambridge university Press, 1995).
4. A. Etheridge, *A Course in Financial Calculus* (Cambridge University Press, 2002).

### Background on Financial Derivatives

1. J. Hull, *Options Futures and Other Financial Derivative Products* (4th edition, Prentice Hall, 2001).

## 2.29 B8.4: Communication Theory — Dr Griffiths — 16 MT

**Level:** H-level

**Method of Assessment:** Written examination.

**Weight:** Unit

**Recommended Prerequisites:** Part A Probability would be helpful, but not essential.

### Overview

The aim of the course is to investigate methods for the communication of information from a sender, along a channel of some kind, to a receiver. If errors are not a concern we are interested in codes that yield fast communication; if the channel is noisy we are interested in achieving both speed and reliability. A key concept is that of information as reduction in uncertainty. The highlight of the course is Shannon's Noisy Coding Theorem.

### Learning Outcomes

- (i) Know what the various forms of entropy are, and be able to manipulate them.
- (ii) Know what data compression and source coding are, and be able to do it.
- (iii) Know what channel coding and channel capacity are, and be able to use that.

### Synopsis

Uncertainty (entropy); conditional uncertainty; information. Chain rules; relative entropy; Gibbs' inequality; asymptotic equipartition and typical sequences. Instantaneous and uniquely decipherable codes; the noiseless coding theorem for discrete memoryless sources; constructing compact codes.

The discrete memoryless channel; decoding rules; the capacity of a channel. The noisy coding theorem for discrete memoryless sources and binary symmetric channels.

Extensions to more general sources and channels.

**Reading**

1. D. J. A. Welsh, *Codes and Cryptography* (Oxford University Press, 1988), Chapters 1–3, 5.
2. T. Cover and J. Thomas, *Elements of Information Theory* (Wiley, 1991), Chapters 1–5, 8.

**Further Reading**

1. R. B. Ash, *Information Theory* (Dover, 1990).
2. D. MacKay, *Information Theory, Inference, and Learning Algorithms* (Cambridge, 2003). [Can be seen at: <http://www.inference.phy.cam.ac.uk/mackay/itila>. Do not infringe the copyright!]
3. G. Jones and J. M. Jones, *Information and Coding Theory* (Springer, 2000), Chapters 1–5.
4. Y. Suhov & M. Kelbert, *Information Theory and Coding by Example* (Cambridge University Press, not yet published - available at the end of 2013), Relevant examples.

**2.30 B8.5: Graph Theory — Prof. Riordan — 16HT****Level:** H-level**Method of Assessment:** Written examination.**Weight:** Unit**Recommended Prerequisites:** Part A Graph Theory is recommended.**Overview**

Graphs (abstract networks) are among the simplest mathematical structures, but nevertheless have a very rich and well-developed structural theory. Since graphs arise naturally in many contexts within and outside mathematics, Graph Theory is an important area of mathematics, and also has many applications in other fields such as computer science.

The main aim of the course is to introduce the fundamental ideas of Graph Theory, and some of the basic techniques of combinatorics.

**Learning Outcomes**

The student will have developed a basic understanding of the properties of graphs, and an appreciation of the combinatorial methods used to analyze discrete structures.

## Synopsis

Introduction: basic definitions and examples. Trees and their characterization. Euler circuits; long paths and cycles. Vertex colourings: Brooks' theorem, chromatic polynomial. Edge colourings: Vizing's theorem. Planar graphs, including Euler's formula, dual graphs. Maximum flow - minimum cut theorem: applications including Menger's theorem and Hall's theorem. Tutte's theorem on matchings. Extremal Problems: Turan's theorem, Zarankiewicz problem, Erdős-Stone theorem.

## Reading

B. Bollobas, *Modern Graph Theory*, Graduate Texts in Mathematics 184 (Springer-Verlag, 1998)

## Further Reading

J. A. Bondy and U. S. R. Murty, *Graph Theory: An Advanced Course*, Graduate Texts in Mathematics 244 (Springer-Verlag, 2007).

R. Diestel, *Graph Theory*, Graduate Texts in Mathematics 173 (third edition, Springer-Verlag, 2005).

D. West, *Introduction to Graph Theory* (second edition, Prentice-Hall, 2001).

## 2.31 SB3a: Applied Probability — Dr Winkel — 16 MT

[Teaching responsibility of the Department of Statistics. Please note, this course is offered from the schedule of Mathematics Department Units ]

**Level:** H-Level

**Method of Assessment:** Written examination.

**Weight:** Unit

The double-unit (SB3a and SB3b) has been designed so that a student obtaining at least an upper second class mark on the double unit can expect to gain exemption from the Institute of Actuaries' paper CT4, which is a compulsory paper in their cycle of professional actuarial examinations. The first unit, clearly, and also the second unit, apply much more widely than just to insurance models.

**Recommended Prerequisites:** Part A Probability.

## Overview

This course is intended to show the power and range of probability by considering real examples in which probabilistic modelling is inescapable and useful. Theory will be developed as required to deal with the examples.

## Synopsis

Poisson processes and birth processes. Continuous-time Markov chains. Transition rates, jump chains and holding times. Forward and backward equations. Class structure, hitting times and absorption probabilities. Recurrence and transience. Invariant distributions and limiting behaviour. Time reversal. Renewal theory. Limit theorems: strong law of large numbers, strong law and central limit theorem of renewal theory, elementary renewal theorem, renewal theorem, key renewal theorem. Excess life, inspection paradox.

Applications in areas such as: queues and queueing networks - M/M/s queue, Erlang's formula, queues in tandem and networks of queues, M/G/1 and G/M/1 queues; insurance ruin models; applications in applied sciences.

## Reading

1. J. R. Norris, *Markov Chains* (Cambridge University Press, 1997).
2. G. R. Grimmett and D. R. Stirzaker, *Probability and Random Processes* (3rd edition, Oxford University Press, 2001).
3. G. R. Grimmett and D. R. Stirzaker, *One Thousand Exercises in Probability* (Oxford University Press, 2001).
4. S. M. Ross, *Introduction to Probability Models* (4th edition, Academic Press, 1989).
5. D. R. Stirzaker: *Elementary Probability* (2nd edition, Cambridge University Press, 2003).

## 2.32 BEE “Mathematical” Extended Essay

**Level:** H-level

**Method of Assessment:** Written extended essay.

**Weight:** Double unit (7,500 words).

An essay on a mathematical topic may be offered for examination at Part B as a double unit. It is equivalent to a 32-hour lecture course. Generally, students will have 8 hours of supervision distributed over Michaelmas and Hilary terms. In addition there are lectures on writing mathematics and using LaTeX in Michaelmas and Hilary terms. See the lecture list for details.

Students considering offering an essay should read the *Guidance Notes on Extended Essays and Dissertations in Mathematics* available at:

<https://www.maths.ox.ac.uk/members/students/undergraduate-courses/teaching-and-learning/projects>

### Application

Students must apply to the Mathematics Projects Committee for approval of their proposed topic in advance of beginning work on their essay. Proposals should be addressed to the Chairman of the Projects Committee, c/o Mrs Helen Lowe, Room S0.16, Mathematical Institute and are accepted from the end of Trinity Term. All proposals must be received before 12noon on Friday of Week 0 of Michaelmas Full Term. Note that a BE essay must have a substantial mathematical content. The application form is available at:

<https://www.maths.ox.ac.uk/members/students/undergraduate-courses/teaching-and-learning/projects>

Once a title has been approved, it may only be changed by approval of the Chairman of the Projects Committee.

### Assessment

Each project is blind double marked. The marks are reconciled through discussion between the two assessors, overseen by the examiners. Please see the *Guidance Notes on Extended Essays and Dissertations in Mathematics* for detailed marking criteria and class descriptors.

### Submission

THREE copies of your essay, identified by your candidate number only, should be sent to the Chairman of Examiners, FHS of Mathematics Part B, Examination Schools, Oxford, to arrive no later than **12noon on Monday of week 10, Hilary Term 2015**. An electronic copy of your dissertation should also be submitted via the Mathematical Institute website. Further details may be found in the *Guidance Notes on Extended Essays and Dissertations in Mathematics*.



### 2.33 BSP, Structured projects, MT and HT

**Level:** H-level                    **Assessment:** Written work, oral presentation, and peer review.

**Weight:** Double unit.

**Recommended prerequisites:** None.

**Quota:** Students will be able to choose a project from a menu of three or four possibilities. There is likely to be a cap of between four to eight on the numbers allowed to take any one project.

#### Learning outcomes

This option is designed to help students understand applications of mathematics to live research problems and to learn some of the necessary techniques. For those who plan to stay on for the MMath or beyond, the course will provide invaluable preliminary training. For those who plan to leave after the BA, it will offer insights into what mathematical research can involve, and training in key skills that will be of long term benefit in any career.

Students will gain experience of:

- Applications of numerical computation to current research problems.
- Reading and understanding research papers.
- Working with new people in new environments.
- Meeting the expectations of different disciplines.
- Presenting a well structured written report, using LaTeX.
- Making an oral presentation to a non-specialist audience.
- Reading and assessing the work of other students.
- Independent study and time management.

Students will be expected to:

- a. Learn about a current research problem by reading one or more relevant research papers together with appropriate material from textbooks.
- b. Carry out the required calculations using Maple, MuPAD or Matlab. Students are not expected to engage in original research but there will be scope for able students to envisage new directions.

- c. Write up the problem and their findings in a report that is properly supported with detail, discussion, and good referencing.
- d. Give an oral presentation to a non-specialist audience.
- e. Undertake peer review.

In past years projects have included applications to biology, finance, and earth sciences. It is expected that a similar menu of topics, from which students will select one, will be available for 2014-2015.

## Teaching

At the beginning of the course students will be given written instructions for their chosen project.

### *Michaelmas Term*

There will be a group meeting with the two organisers (Cath Wilkins and Jackie Stedall) at the beginning of MT to set out expectations and deal with queries. The organisers will meet again with students individually at the end of MT. Between those meetings students will read around their chosen topic and take preparatory courses in LaTeX and Matlab, both of which are available from the department and are well documented online. Individual contact with the organisers by email, or if necessary in person, will be encouraged.

### *Hilary Term*

#### Week 1

Lecture on expectations for the term, and advice on writing up.

#### Weeks 2 to 8

Students will meet regularly with their specialist supervisors. In addition, each student will meet at least once with one of the organisers, who will together maintain an overview of the student's progress.

#### Week 10

Submission of written paper.

### *Easter vacation*

Peer review

### *Trinity Term*

#### Week 1

Oral presentation

## Assessment

Students (and tutors) have sometimes expressed doubts about the predictability or reliability of project assessment. We are therefore concerned:

- i. to make the assessment scheme as transparent as possible both to students and to assessors;
- ii. that students who produce good project work should be able to achieve equivalent grades to students who write good exam papers.

The mark breakdown will be as follows:

- a. Written work 75%, of which:
  - 50% of available marks will be for general explanation and discussion of the problem
  - 50% of available marks will be for mathematical calculations and commentary
- b. Oral presentation 15%
- c. Peer review 10%

Note on (c):

This is a new kind assessment in Oxford mathematics, though other universities have used it with great success. As with journal peer review, the anonymity of both writer and reviewer will be strictly maintained. Each student will be expected to read one other student's project write-up and to make a careful and well explained judgement on it. Credit for this will go to the reviewer, not to the writer, whose work will already have been assessed by examiners in the usual way.

### 3 Other Mathematical units

#### 3.1 BO1.1: History of Mathematics — Dr Chris Hollings — 16 lectures in MT and reading course of 8 seminars in HT

**Level:** H-level

**Assessment:** 2-hour written examination paper for the MT lectures and 3000-word essay for the reading course.

**Weight:** Double unit.

**Recommended prerequisites:** None.

**Quota:** The maximum number of students that can be accepted will be 20.

#### Learning outcomes

This course is designed to provide the historical background to some of the mathematics familiar to students from A-level and the first four terms of undergraduate study, and looks at a period from approximately the mid-sixteenth century to the end of the nineteenth century. The course will be delivered through 16 lectures in Michaelmas Term, and a reading course consisting of 8 seminars (equivalent to a further 16 lectures) in Hilary Term. Guidance will be given throughout on reading, note-taking, and essay-writing.

Students will gain:

- an understanding of university mathematics in its historical context;
- an enriched understanding of the mathematical content of the topics covered by the course

together with skills in:

- reading and analysing historical mathematical sources;
- reading and analysing secondary sources;
- efficient note-taking;
- essay-writing (from 1000 to 3000 words);
- construction of references and bibliographies;
- oral discussion and presentation.

## Lectures

The Michaelmas Term lectures will cover the following material:

- Introduction.
- Seventeenth century: analytic geometry; the development of calculus; Newton's *Principia*.
- Eighteenth century: from calculus to analysis; functions, limits, continuity; equations and solvability.
- Nineteenth century: group theory and abstract algebra; the beginnings of modern analysis; sequences and series; integration; complex analysis; linear algebra.

Classes to accompany the lectures will be held in Weeks 3, 5, 6, 7. For each class students will be expected to prepare one piece of written work (1000 words) and one discussion topic.

## Reading course

The Hilary Term part of the course is run as a reading course during which we will study two or three primary texts in some detail, using original sources and secondary literature. Details of the books to be read in HT 2015 will be decided and discussed towards the end of MT 2014. Students will be expected to write two essays (2000 words each) during the first six weeks of term. The course will then be examined by an essay of 3000 words to be completed during Weeks 7 to 9.

## Recommended reading

Jacqueline Stedall, *Mathematics emerging: a sourcebook 1540–1900*, (Oxford University Press, 2008).

Victor Katz, *A history of mathematics* (brief edition), (Pearson Addison Wesley, 2004), or:

Victor Katz, *A history of mathematics: an introduction* (third edition), (Pearson Addison Wesley, 2009).

Benjamin Wardhaugh, *How to read historical mathematics*, (Princeton, 2010).

## Supplementary reading

John Fauvel and Jeremy Gray (eds), *The history of mathematics: a reader*, (Macmillan, 1987).

## Assessment

The Michaelmas Term material will be examined in a two-hour written paper at the end of Trinity Term. Candidates will be expected to answer two half-hour questions (commenting on extracts) and one one-hour question (essay). The paper will account for 50% of the marks for the course. The Reading Course will be examined by a 3000-word essay at the end of Hilary Term. The title will be set at the beginning of Week 7 and two copies of the project must be submitted to the Examination Schools by midday on Monday of Week 10. This essay will account for 50% of the marks for the course.

### 3.2 MS: Statistics Units and Double Units

Students in Part B may take units drawn from Part B of the Honour School of Mathematics and Statistics. For full details of these units see the syllabus and synopses for Part B of the Honour School Mathematics and Statistics, which are available on the web at [http://www.stats.ox.ac.uk/current\\_students/bammath/course\\_handbooks/](http://www.stats.ox.ac.uk/current_students/bammath/course_handbooks/)

The Statistics units available are as follows:

- SB1 Applied Statistics (double unit)
- SB2a Foundations of Statistical Inference (unit)
- SB3b Statistical Lifetime Models (unit; can only be taken as a double unit with SB3a)
- SB4a Actuarial Science I (unit; please note that if taken as a unit this course will not qualify candidates for an exemption from actuarial science exams)
- SB4b Actuarial Science II (unit; can only be taken as a double unit with SBS4a)

### 3.3 Computer Science: Units

Students in Part B may take units drawn from Part B of the Honour School of Mathematics and Computing. For full details of these units see the Department of Computer Science's website (<http://www.cs.ox.ac.uk/teaching/courses/>)

The Computer Science units available are as follows:

- OCS1 Lambda Calculus and Types (unit)
- OCS2 Computational Complexity (unit)

### 3.4 BOE “Other Mathematical” Extended Essay

**Level:** H-level

**Method of Assessment:** Written essay.

**Weight:** Double unit (7,500 words). OSS paper code 9923.

An essay on a topic related to mathematics may be offered for examination at Part B as a double unit. It is equivalent to a 32-hour lecture course. Generally, students will have 8 hours of supervision distributed over Michaelmas and Hilary terms. In addition there are lectures on writing mathematics and using LaTeX in Michaelmas and Hilary terms. See the lecture list for details.

Students considering offering an essay should read the *Guidance Notes on Extended Essays and Dissertations in Mathematics* available at:

<https://www.maths.ox.ac.uk/members/students/undergraduate-courses/teaching-and-learning/projects>.

#### Application

Students must apply to the Mathematics Projects Committee for approval of their proposed topic in advance of beginning work on their essay. Proposals should be addressed to the Chairman of the Projects Committee, c/o Mrs Helen Lowe, Room S0.16, Mathematical Institute and are accepted from the end of Trinity Term. All proposals must be received before 12noon on Friday of Week 0 of Michaelmas Full Term. The application form is available at <https://www.maths.ox.ac.uk/members/students/undergraduate-courses/teaching-and-learning/projects>.

Once a title has been approved, it may only be changed by approval of the Chairman of the Projects Committee.

#### Assessment

Each project is blind double marked. The marks are reconciled through discussion between the two assessors, overseen by the examiners. Please see the *Guidance Notes on Extended Essays and Dissertations in Mathematics* for detailed marking criteria and class descriptors.

#### Submission

THREE copies of your essay, identified by your candidate number only, should be sent to the Chairman of Examiners, FHS of Mathematics Part B, Examination Schools, Oxford, to arrive no later than **12noon on Monday of week 9, Hilary Term 2015**. An electronic copy of your dissertation should also be submitted via the Mathematical Institute website. Further details may be found in the *Guidance Notes on Extended Essays and Dissertations in Mathematics*.

## 4 Non-Mathematical units

### 4.0.1 BN1.2 Undergraduate Ambassadors' Scheme — Mr Andrews. — mainly HT

**Weight:** Unit

**Method of Assessment:** Journal of activities, Oral presentation, Course report and project, Teacher report.

**Quota:** There will be a quota of approximately 10 students for this course.

**Co-ordinator:** Nick Andrews.

Option available to Mathematics, Mathematics & Statistics, Mathematics & Philosophy students.

#### Learning Outcomes

The Undergraduate Ambassadors' Scheme (UAS) was begun by Simon Singh in 2002 to give university undergraduates a chance to experience assisting and, to some extent, teaching in schools, and to be credited for this. The option focuses on improving students' communication, presentation, cooperation and organizational skills and sensitivity to others' learning difficulties.

#### Course Description and Timing:

The Oxford UAS option, BN1.2, is a unit, mainly run in Hilary Term. A quota will be in place, of approximately 10 students, and so applicants for the UAS option will be asked to name a second alternative unit. The course is appropriate for all students, whether or not they are interested in teaching subsequently.

A student on the course will be assigned to a mathematics teacher in a local secondary school (in the Oxford, Kidlington, Wheatley area) for half a day per week during Hilary Term. Students will be expected to keep a journal of their activities, which will begin by assisting in the class, but may widen to include teaching the whole class for a part of a period, or working separately with a smaller group of the class. Students will be required at one point to give a presentation to one of their school classes relating to a topic from university mathematics, and will also run a small project based on some aspect of mathematics education with advice from the course co-ordinator and teacher/s. Final credit will be based on the journal (20%), the presentation (30%), an end of course report (approximately 3000 words) including details of the project (35%), together with a report from the teacher (15%).

Short interviews will take place on Thursday or Friday of 0th week in Michaelmas term to select students for this course. The interview (of roughly 15 minutes) will include a presentation by the student on an aspect of mathematics of their choosing. Students will be chosen on the basis of their ability to communicate mathematics, and two references will be sought from college tutors on these qualities. Applicants will be quickly notified of the decision.



Those on the course will also need to fill in a CRB form, or to have done so already. By the end of Michaelmas term students will have been assigned to a teacher and have made a first, introductory, visit to their school. The course will begin properly in Hilary term with students helping in schools for half a day each week. Funds are available to cover travel expenses. Support classes will be provided throughout Hilary for feedback and to discuss issues such as the planning of the project. The deadline for the journal and report will be noon on Monday of 1st week of Trinity term.

Any further questions on the UAS option should be passed on to the option's co-ordinator, via (director-ugrad-studies@maths.ox.ac.uk).

### Reading List

Clare Tickly, Anne Watson, Candia Morgan, *Teaching School Subjects: Mathematics* (Routledge Falmer, 2004).

### 4.1 Philosophy: Double Units

Students in Part B may take units drawn from courses provided by the Faculty of Philosophy. For full details of these units see the Philosophy Faculty website <http://www.philosophy.ox.ac.uk/>.

The Philosophy units available are as follows:

- N101 Early Modern Philosophy (double unit).
- N102 Knowledge and Reality (double unit).
- N122 Philosophy of Mathematics (double unit).
- N127 Philosophical Logic (double unit).

[Paper 101, 102, 122 and 127 in all Honour Schools including Philosophy. Teaching responsibility of the Philosophy Faculty.]

## 5 Language Classes: French and German

Language courses in French and German or Spanish (in alternate years) are offered by the University Language Centre.

Students in the FHS Mathematics may apply to take language classes. In 2014-2015, French and German language classes will be run in MT and HT. We have a limited number of places but if we have spare places we will offer these to joint school students, Mathematics and Computer Science, Mathematics and Philosophy and Mathematics and Statistics.

Two levels of French courses are offered, a lower level for those with a good pass at GCSE, and a higher level course for those with A/S or A level. Acceptance on either course will depend on satisfactory performance in the Preliminary Qualifying Test held in Week 1 of Michaelmas Term (Monday, 17.00-19.00 at the Language Centre). Classes at both levels will take place on Mondays, 17.00-19.00. A single class in German or Spanish at a lower or higher level will be offered on the basis of the performances in the Preliminary Qualifying Test, held at the same time as the French test. Classes will also be held on Mondays, 17-00-19.00.

Performance on the course will not contribute to the class of degree awarded. However, upon successful completion of the course, students will be issued with a certificate of achievement which will be sent to their college.

Places at these classes are limited, so students are advised that they must indicate their interest at this stage. If you are interested please contact Nia Roderick (roderick@maths.ox.ac.uk or tel. 01865 615205), Academic Assistant in the Mathematical Institute, as soon as possible.

### **Aims and rationale**

The general aim of the language courses is to develop the student's ability to communicate (in both speech and writing) in French, German and Spanish to the point where he or she can function in an academic or working environment in a French-speaking, German-speaking or Spanish-speaking country.

The courses have been designed with general linguistic competence in the four skills of reading, writing, speaking and listening in mind. There will be opportunities for participants to develop their own particular interests through presentations and assignments.

### **Form and Content**

Each course will consist of a thirty-two contact hour course, together with associated work. Classes will be held in the Language Centre at two hours per week in Michaelmas and Hilary Terms.

The content of the courses is based on coursebooks together with a substantial amount of supplementary material prepared by the language tutors. Participants should expect to spend an additional two hours per week on preparation and follow-up work.

Each course aims to cover similar ground in terms of grammar, spoken and written language and topics. Areas covered will include:

### Grammar:

- all major tenses will be presented and/or revised, including the subjunctive
- passive voice
- pronouns
- formation of adjectives, adverbs, comparatives
- use of prepositions
- time expressions

### Speaking

- guided spoken expression for academic, work and leisure contact (e.g. giving presentations, informal interviews, applying for jobs)
- expressing opinions, tastes and preferences
- expressing cause, consequence and purpose

### Writing

- Guided letter writing for academic and work contact
- Summaries and short essays

### Listening

- Listening practice of recorded materials (e.g. news broadcasts, telephone messages, interviews)
- developing listening comprehension strategies

### Topics (with related readings and vocabulary, from among the following)

- life, work and culture
- the media in each country
- social and political systems
- film, theatre and music
- research and innovation
- sports and related topics
- student-selected topics

## Teaching staff

The courses are taught by Language Centre tutors or Modern Languages Faculty instructors.

## Teaching and learning approaches

Each course uses a communicative methodology which demands active participation from students. In addition, there will be some formal grammar instruction. Students will be expected to prepare work for classes and homework will be set. Students will also be given training in strategies for independent language study, including computer-based language learning, which will enable them to maintain and develop their language skills after the course.

## Entry

Two classes in French and one in German or Spanish (probably at Basic and Threshold levels) will be formed according to level of French/German/Spanish at entry. The minimum entry standard is a good GCSE Grade or equivalent. Registration for each option will take place at the start of Michaelmas Term. A preliminary qualifying test is then taken which candidates must pass in order to be allowed to join the course.

## Learning Outcomes

The learning outcomes will be based on internationally agreed criteria for specific levels (known as the ALTE levels), which are as follows:

### Basic Level (corresponds to ALTE Level 2 “Can-do” statements)

- Can express opinions on professional, cultural or abstract matters in a limited way, offer advice and understand instructions.
- Can give a short presentation on a limited range of topics.
- Can understand routine information and articles, including factual articles in newspapers, routine letters from hotels and letters expressing personal opinions.
- Can write letters or make notes on familiar or predictable matters.

### Threshold Level (corresponds to ALTE Level 3 “Can-do” statements)

- Can follow or give a talk on a familiar topic or keep up a conversation on a fairly wide range of topics, such as personal and professional experiences, events currently in the news.
- Can give a clear presentation on a familiar topic, and answer predictable or factual questions.
- Can read texts for relevant information, and understand detailed instructions or advice.
- Can make notes that will be of reasonable use for essay or revision purposes.
- Can make notes while someone is talking or write a letter including non- standard requests.

**Assessment**

There will be a preliminary qualifying test in Michaelmas Term. There are three parts to this test: Reading Comprehension, Listening Comprehension, and Grammar. Candidates who have not studied or had contact with French, German or Spanish for some time are advised to revise thoroughly, making use of the Language Centre's French, German or Spanish resources.

Students' achievement will be assessed through a variety of means, including continuous assessment of oral performance, a written final examination, and a project or assignment chosen by individual students in consultation with their language tutor.

Reading comprehension, writing, listening comprehension and speaking are all examined. The oral component may consist of an interview, a presentation or a candidate's performance in a formal debate or discussion.