UNIVERSITY OF OXFORD
Mathematical Institute

HONOUR SCHOOL OF MATHEMATICS

SUPPLEMENT TO THE UNDERGRADUATE HANDBOOK – 2013 Matriculation

SYNOPSES OF LECTURE COURSES

Part B 2015-16
for examination in 2016

These synopses can be found at:
http://www.maths.ox.ac.uk/members/students/undergraduate-courses/teaching-and-learning/handbooks-synopses

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Handbook for the Undergraduate Mathematics Courses
Supplement to the Handbook
Honour School of Mathematics
Syllabus and Synopses for Part B 2015–16
for examination in 2016

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1 Foreword

The synopses for Part B will be available on the website at:

https://www.maths.ox.ac.uk/members/students/undergraduate-courses/teaching-and-learning/handbooks-synopses

before the start of Michaelmas Term 2015.

See the current edition of the *Examination Regulations* for the full regulations governing these examinations.

Examination Conventions can be found at: https://www.maths.ox.ac.uk/members/students/undergraduate-courses/examinations-assessments/examination-conventions

1.1 Honour School of Mathematics

1.1.1 Units

In Part B each candidate shall offer a total of eight units from the schedule of units. Each unit is the equivalent of a sixteen hour lecture course.

(a) A total of at least four units offered should be from the schedule of 'Mathematics Department units'.

(b) Candidates may offer up to four units from the schedule of 'Other Units' but with no more than two from each category (Statistics options, Computer Science options, Other options).

(c) Candidates may offer at most one double unit which is designated as an extended essay or a structured project.[1]

All Mathematics Department lecture courses are independently available as units.

Details of Part C units, examinable in 2017, will be published before Michaelmas Term 2016.

1.2 Language Classes

Mathematics students may apply to take classes in a foreign language. In 2015-16 classes will be offered in French and Spanish. Students' performances in these classes will not contribute to the degree classification awarded. However, successful completion of the course may be recorded on students' transcripts. See section 5 for more details.

1.3 Registration for Part B courses 2015–16

CLASSES: Students will have to register in advance for the courses they wish to take. Students will have to register by Friday of Week 10 of Trinity Term 2015 using the online registration system which can be accessed at https://www.maths.ox.ac.uk/courses/registration/.

[1] Units which may be offered under this heading are indicated in the synopses.

Students will then be asked to sign up for classes at the start of Michaelmas Term 2015. Further information about this will be sent via email before the start of term.

Students who register for a course or courses for which there is a quota should consider registering for an additional course (by way of a "reserve choice") in case they do not receive a place on the course with the quota. They may also have to give the reasons why they wish to take a course which has a quota, and provide the name of a tutor who can provide a supporting statement for them should the quota be exceeded. Where this is necessary students will be contacted by email after they have registered. In the event that the quota for a course is exceeded, the Mathematics Teaching Committee will decide who may have a place on the course on the basis of the supporting statements from the student and tutor, and all relevant students will be notified of the decision by email. In the case of the "Undergraduate Ambassadors' Scheme" students will have to attend a short interview in Week 0, Michaelmas Term.

Where undergraduate registrations for lecture courses fall below 5, classes will not run as part of the intercollegiate scheme but will be arranged informally by the lecturer.

LECTURES: Every effort will be made when timetabling lectures to ensure that lectures do not clash. However, because of the large number of options this may sometimes be unavoidable. In the event of clashes being necessary, then students will be notified of the clashes by email and in any case options will only be allowed to clash when the take-up of both options is unlikely or inadvisable.

1.4 Three-year/Four-year Course Registration

You should register your intention to take either the BA course or the MMath. course during your third year. You are advised to discuss the right course of action for you with your College Tutor, who will also advise you how to register. Any student whose performance in the Part A and B examinations together falls below **upper second standard** will not be permitted to proceed to Part C.

All students are registered on the MMath versions of each course. If you subsequently decide to change to the BA option you must inform your college office who will in turn inform central administration and the departments. Please be aware that any change to your course may impact the level of your maintenance funding and the time taken to receive your student loan (you are advised to contact Student Finance www.direct.gov.uk/en/EducationAndLearning/UniversityAndHigherEducation/ StudentFinance for further enquiries). Please note also that if you intend to change option you are strongly advised to do so before you take the Part B examinations.

1.5 Course list by term

Table 1: Michaelmas Term Mathematics Department Units

Code	Title	Term
B1.1	Logic	MT
B2.1	Introduction to Representation Theory	MT
B3.1	Galois Theory	MT
B3.2	Geometry of Surfaces	MT
B3.5	Topology and Groups	MT
B4.1	Banach Spaces	MT
B5.2	Applied Partial Differential Equations	MT
B5.3	Viscous Flow	MT
B5.5	Mathematical Ecology and Biology	MT
B6.1	Numerical Solutions of Differential Equations I	MT
B6.3	Integer Programming	MT
B7.1	Classical Mechanics	MT
B7.2	Electromagnetism	MT
B8.1	Martingales Through Measure Theory	MT
B8.4	Communication Theory	MT
SB3a	Applied Probability	MT

Table 2: Hilary Term Mathematics Department Units

Code	Title	Term
B1.2	Set Theory	HT
B2.2	Commutative Algebra	HT
B3.3	Algebraic Curves	HT
B3.4	Algebraic Number Theory	HT
B4.2	Hilbert Spaces	HT
B5.1	Stochastic modelling of biological processes	HT
B5.4	Waves and Compressible Flow	HT
B5.6	Nonlinear Systems	HT
B6.2	Numerical Solution of Differential Equations II	HT
C7.3	Further Quantum Theory	HT
B8.2	Continuous Martingales and Stochastic Calculus	HT
B8.3	Mathematical Models of Financial Derivatives	HT
B8.5	Graph Theory	HT

2 Mathematics Department units

2.1 B1.1: Logic — Prof. Jochen Koenigsmann — 16 MT

Level: H-level **Method of Assessment:** Written examination.

Weight: Unit

Recommended Prerequisites: None

Overview

To give a rigorous mathematical treatment of the fundamental ideas and results of logic that is suitable for the non-specialist mathematicians and will provide a sound basis for more advanced study. Cohesion is achieved by focusing on the Completeness Theorems and the relationship between provability and truth. Consideration of some implications of the Compactness Theorem gives a flavour of the further development of model theory. To give a concrete deductive system for predicate calculus and prove the Completeness Theorem, including easy applications in basic model theory.

Learning Outcomes

Students will be able to use the formal language of propositional and predicate calculus and be familiar with their deductive systems and related theorems. For example, they will know and be able to use the soundness, completeness and compactness theorems for deductive systems for predicate calculus.

Synopsis

The notation, meaning and use of propositional and predicate calculus. The formal language of propositional calculus: truth functions; conjunctive and disjunctive normal form; tautologies and logical consequence. The formal language of predicate calculus: satisfaction, truth, validity, logical consequence.

Deductive system for propositional calculus: proofs and theorems, proofs from hypotheses, the Deduction Theorem; Soundness Theorem. Maximal consistent sets of formulae; completeness; constructive proof of completeness.

Statement of Soundness and Completeness Theorems for a deductive system for predicate calculus; derivation of the Compactness Theorem; simple applications of the Compactness Theorem.

A deductive system for predicate calculus; proofs and theorems; prenex form. Proof of Completeness Theorem. Existence of countable models, the downward Löwenheim–Skolem Theorem.

Reading

1. R. Cori and D. Lascar, *Mathematical Logic: A Course with Exercises (Part I)* (Oxford University Press, 2001), sections 1, 3, 4.
2. A. G. Hamilton, *Logic for Mathematicians* (2nd edition, Cambridge University Press, 1988), pp.1–69, pp.73–76 (for statement of Completeness (Adequacy)Theorem), pp.99–103 (for the Compactness Theorem).
3. W. B. Enderton, *A Mathematical Introduction to Logic* (Academic Press, 1972), pp.101–144.
4. D. Goldrei, *Propositional and Predicate Calculus: A model of argument* (Springer, 2005).
5. A. Prestel and C. N. Delzell, *Mathematical Logic and Model Theory* (Springer, 2010).

Further Reading

1. R. Cori and D. Lascar, *Mathematical Logic: A Course with Exercises (Part II)* (Oxford University Press, 2001), section 8.

2.2 B1.2: Set Theory — Prof. Jonathan Pila — 16 HT

Level: H-level **Method of Assessment:** Written examination.

Weight: Unit

Recommended Prerequisites: There are no formal prerequisites, but familiarity with some basic mathematical objects and notions such as: the rational and real number fields; the idea of surjective, injective and bijective functions, inverse functions, order relations; the notion of a continuous function of a real variable, sequences, series, and convergence, and the definitions of basic abstract structures such as fields, vector spaces, and groups (all covered in Mathematics I and II in Prelims) will be helpful at points.

Overview

To introduce sets and their properties as a unified way of treating mathematical structures, including encoding of basic mathematical objects using set theoretic language. To emphasize the difference between intuitive collections and formal sets. To introduce and discuss the notion of the infinite, the ordinals and cardinality. The Axiom of Choice and its equivalents are presented as a tool.

Learning Outcomes

Students will have a sound knowledge of set theoretic language and be able to use it to codify mathematical objects. They will have an appreciation of the notion of infinity and arithmetic of the cardinals and ordinals. They will have developed a deep understanding of the Axiom of Choice, Zorn's Lemma and well-ordering principle, and have begun to appreciate the implications.

Synopsis

What is a set? Introduction to the basic axioms of set theory. Ordered pairs, cartesian products, relations and functions. Axiom of Infinity and the construction of the natural numbers; induction and the Recursion Theorem.

Cardinality; the notions of finite and countable and uncountable sets; Cantor's Theorem on power sets. The Tarski Fixed Point Theorem. The Schröder–Bernstein Theorem.

Isomorphism of ordered sets; well-orders. Transfinite induction; transfinite recursion [informal treatment only].

Comparability of well-orders.

The Axiom of Choice, Zorn's Lemma, the Well-ordering Principle; comparability of cardinals. Equivalence of WO, CC, AC and ZL. Ordinals. Arithmetic of cardinals and ordinals; in [ZFC].

Reading

1. D. Goldrei, *Classic Set Theory* (Chapman and Hall, 1996).

2. H. B. Enderton, *Elements of Set Theory* (Academic Press, 1978).

Further Reading

1. R. Cori and D. Lascar, *Mathematical Logic: A Course with Exercises (Part II)* (Oxford University Press, 2001), section 7.1–7.5.

2. R. Rucker, *Infinity and the Mind: The Science and Philosophy of the Infinite* (Princeton University Press, 1995). An accessible introduction to set theory.

3. J. W. Dauben, *Georg Cantor: His Mathematics and Philosophy of the Infinite* (Princton University Press, 1990). For some background, you may find JW Dauben's biography of Cantor interesting.

4. M. D. Potter, *Set Theory and its Philosophy: A Critical Introduction* (Oxford University Press, 2004). An interestingly different way of establishing Set Theory, together with some discussion of the history and philosophy of the subject.

5. W. Sierpinski, *Cardinal and Ordinal Numbers* (Polish Scientific Publishers, 1965). More about the arithmetic of transfinite numbers.

6. J. Stillwell, *Roads to Infinity* (CRC Press, 2010).

2.3 B2.1: Introduction to Representation Theory — Prof. Nikolay Nikolov — 16 MT

Level: H-level **Method of Assessment:** Written examination.

Weight: Unit

Recommended Prerequisites: Rings and Modules is essential. Group Theory is recommended.

Overview

This course gives an introduction to the representation theory of finite groups and finite dimensional algebras. Representation theory is a fundamental tool for studying symmetry by means of linear algebra: it is studied in a way in which a given group or algebra may act on vector spaces, giving rise to the notion of a representation.

A large part of the course will deal with the structure theory of semisimple algebras and their modules (representations). We will prove the Jordan-Hölder Theorem for modules. Moreover, we will prove that any finite-dimensional semisimple algebra is isomorphic to a product of matrix rings (Wedderburn's Theorem over $\mathbb{C}$).

In the later part of the course we apply the developed material to group algebras, and classify when group algebras are semisimple (Maschke's Theorem). All of this material will be applied to the study of characters and representations of finite groups.

Learning Outcomes

They will know in particular simple modules and semisimple algebras and they will be familiar with examples. They will appreciate important results in the course such as the Jordan-Hölder Theorem, Schur's Lemma, and the Wedderburn Theorem. They will be familiar with the classification of semisimple algebras over $\mathbb{C}$ and be able to apply this to representations and characters of finite groups.

Synopsis

Noncommutative rings, one- and two-sided ideals. Associative algebras (over fields). Main examples: matrix algebras, polynomial rings and quotients of polynomial rings. Group algebras, representations of groups.

Modules and their relationship with representations. Simple and semisimple modules, composition series of a module, Jordan-Hölder Theorem. Semisimple algebras. Schur's Lemma, the Wedderburn Theorem, Maschke's Theorem. Characters of complex representations. Orthogonality relations, finding character tables. Tensor product of modules. Induction and restriction of representations. Application: Burnside's $p^a q^b$ Theorem.

Reading

1. K. Erdmann, *B2 Algebras*, Mathematical Institute Notes (2007).

2. G. D. James and M. Liebeck, *Representations and Characters of Finite Groups* (2nd edition, Cambridge University Press, 2001).

Further Reading

1. J. L. Alperin and R. B. Bell, *Groups and Representations,* Graduate Texts in Mathematics 162 (Springer-Verlag, 1995).
2. P. M. Cohn, *Classic Algebra* (Wiley & Sons, 2000). (Several books by this author available.)
3. C. W. Curtis, and I. Reiner, *Representation Theory of Finite Groups and Associative Algebras* (Wiley & Sons, 1962).
4. L. Dornhoff, *Group Representation Theory* (Marcel Dekker Inc., New York, 1972).
5. I. M. Isaacs, *Character Theory of Finite Groups* (AMS Chelsea Publishing, American Mathematical Society, Providence, Rhode Island, 2006).
6. J.-P. Serre, *Linear Representations of Finite Groups,* Graduate Texts in Mathematics 42 (Springer-Verlag, 1977).

2.4 B2.2: Commutative Algebra — Prof. Nikolay Nikolov — 16HT

Level: M-level **Method of Assessment:** Written examination.

Weight: Unit

Recommended Prerequisites Rings and Modules is essential. Representation Theory and Galois Theory are recommended.

Overview

Amongst the most familiar objects in mathematics are the ring of integers and the polynomial rings over fields. These play a fundamental role in number theory and in algebraic geometry, respectively. The course explores the basic properties of such rings.

Synopsis

Modules, ideals, prime ideals, maximal ideals.
Noetherian rings; Hilbert basis theorem. Minimal primes.
Localization.
Polynomial rings and algebraic sets. Weak Nullstellensatz.
Nilradical and Jacobson radical; strong Nullstellensatz.
Artin-Rees Lemma; Krull intersection theorem.
Integral extensions. Prime ideals in integral extensions.
Noether Normalization Lemma.
Krull dimension; ‘Principal ideal theorem’; dimension of an affine algebra.

Reading

1. M. F. Atiyah and I. G. MacDonald: *Introduction to Commutative Algebra*, (Addison-Wesley, 1969).

2.5 B3.1: Galois Theory — Prof. Xenia de la Ossa — 16 MT

Level: H-level **Method of Assessment:** Written examination.

Weight: Unit

Recommended Prerequisites: Rings and Modules is essential and Group Theory is recommended. Students who have not taken Part A Number Theory should read about quadratic residues in, for example, the appendix to Stewart and Tall. This will help with the examples.

Overview

The course starts with a review of second-year ring theory with a particular emphasis on polynomial rings, and a discussion of general integral domains and fields of fractions. This is followed by the classical theory of Galois field extensions, culminating in some of the classical theorems in the subject: the insolubility of the general quintic and impossibility of certain ruler and compass constructions considered by the Ancient Greeks.

Learning Outcomes

Understanding of the relation between symmetries of roots of a polynomial and its solubility in terms of simple algebraic formulae; working knowledge of interesting group actions in a nontrivial context; working knowledge, with applications, of a nontrivial notion of finite group theory (soluble groups); understanding of the relation between algebraic properties of field extensions and geometric problems such as doubling the cube and squaring the circle.

Synopsis

Review of polynomial rings, factorisation, integral domains. Reminder that any nonzero homomorphism of fields is injective. Fields of fractions.

Review of group actions on sets, Gauss' Lemma and Eisenstein's criterion for irreducibility of polynomials, field extensions, degrees, the tower law. Symmetric polynomials.

Separable extensions. Splitting fields and normal extensions. The theorem of the primitive element. The existence and uniqueness of algebraic closure (proofs not examinable).

Groups of automorphisms, fixed fields. The fundamental theorem of Galois theory.

Examples: Kummer extensions, cyclotomic extensions, finite fields and the Frobenius automorphism. Techniques for calculating Galois groups.

Soluble groups. Solubility by radicals, solubility of polynomials of degree at most 4, insolubility of the general quintic, impossibility of some ruler and compass constructions.

Reading

1. J. Rotman, *Galois Theory* (Springer-Verlag, NY Inc, 2001/1990).
2. I. Stewart, *Galois Theory* (Chapman and Hall, 2003/1989)
3. D.J.H. Garling, *A Course in Galois Theory* (Cambridge University Press I.N., 1987).
4. Herstein, *Topics in Algebra* (Wiley, 1975)

2.6 B3.2: Geometry of Surfaces — Prof. Alexander Ritter — 16 MT

Level: H-level **Method of Assessment:** Written examination.

Weight: Unit

Recommended Prerequisites: Part A Topology. Introduction to Manifolds would be useful but not essential. Also, B3.2 is helpful, but not essential, for B3.3 (Algebraic Curves).

Overview

Different ways of thinking about surfaces (also called two-dimensional manifolds) are introduced in this course: first topological surfaces and then surfaces with extra structures which allow us to make sense of differentiable functions ('smooth surfaces'), holomorphic functions ('Riemann surfaces') and the measurement of lengths and areas ('Riemannian 2-manifolds').

These geometric structures interact in a fundamental way with the topology of the surfaces. A striking example of this is given by the Euler number, which is a manifestly topological quantity, but can be related to the total curvature, which at first glance depends on the geometry of the surface.

The course ends with an introduction to hyperbolic surfaces modelled on the hyperbolic plane, which gives us an example of a non-Euclidean geometry (that is, a geometry which meets all Euclid's axioms except the axioms of parallels).

Learning Outcomes

Students will be able to implement the classification of surfaces for simple constructions of topological surfaces such as planar models and connected sums; be able to relate the Euler characteristic to branching data for simple maps of Riemann surfaces; be able to describe the definition and use of Gaussian curvature; know the geodesics and isometries of the hyperbolic plane and their use in geometrical constructions.

Synopsis

The concept of a topological surface (or 2-manifold); examples, including polygons with pairs of sides identified. Orientation and the Euler characteristic. Classification theorem for compact surfaces (the proof will not be examined).

Riemann surfaces; examples, including the Riemann sphere, the quotient of the complex numbers by a lattice, and double coverings of the Riemann sphere. Holomorphic maps of Riemann surfaces and the Riemann–Hurwitz formula. Elliptic functions.

Smooth surfaces in Euclidean three-space and their first fundamental forms. The concept of a Riemannian 2-manifold; isometries; Gaussian curvature.

Geodesics. The Gauss–Bonnet Theorem (statement of local version and deduction of global version). Critical points of real-valued functions on compact surfaces.

The hyperbolic plane, its isometries and geodesics. Compact hyperbolic surfaces as Riemann surfaces and as surfaces of constant negative curvature.

Reading

1. A. Pressley, *Elementary Differential Geometry,* Springer Undergraduate Mathematics Series (Springer-Verlag, 2001). (Chapters 4–8 and 10–11.)

2. G. B. Segal, *Geometry of Surfaces*, Mathematical Institute Notes (1989).

3. R. Earl, *The Local Theory of Curves and Surfaces*, Mathematical Institute Notes (1999).

4. J. McCleary, *Geometry from a Differentiable Viewpoint* (Cambridge, 1997).

Further Reading

1. P. A. Firby and C. E. Gardiner, *Surface Topology* (Ellis Horwood, 1991) (Chapters 1–4 and 7).

2. F. Kirwan, *Complex Algebraic Curves,* Student Texts 23 (London Mathematical Society, Cambridge, 1992) (Chapter 5.2 only).

3. B. O'Neill, *Elementary Differential Geometry* (Academic Press, 1997).

2.7 B3.3: Algebraic Curves — Prof. Balasz Szendroi — 16 HT

Level: H-level **Method of Assessment:** Written examination.

Weight: Unit

Recommended Prerequisites: Part A Topology. Introduction to Manifolds would be useful but not essential. Projective Geometry is recommended. Also, B3.2 (Geometry of Surfaces) is helpful, but not essential.

Overview

A real algebraic curve is a subset of the plane defined by a polynomial equation $p(x, y) = 0$. The intersection properties of a pair of curves are much better behaved if we extend this picture in two ways: the first is to use polynomials with complex coefficients, the second to extend the curve into the projective plane. In this course projective algebraic curves are studied, using ideas from algebra, from the geometry of surfaces and from complex analysis.

Learning Outcomes

Students will know the concepts of projective space and curves in the projective plane. They will appreciate the notion of nonsingularity and know some basic features of intersection theory. They will view nonsingular algebraic curves as examples of Riemann surfaces, and be familiar with divisors, meromorphic functions and differentials.

Synopsis

Projective spaces, homogeneous coordinates, projective transformations.

Algebraic curves in the complex projective plane. Irreducibility, singular and nonsingular points, tangent lines.

Bezout's Theorem (the proof will not be examined). Points of inflection, and normal form of a nonsingular cubic.

Nonsingular algebraic curves as Riemann surfaces. Meromorphic functions, divisors, linear equivalence. Differentials and canonical divisors. The group law on a nonsingular cubic.

The Riemann–Roch Theorem (the proof will not be examined). The geometric genus. Applications.

Reading

1. F. Kirwan, *Complex Algebraic Curves,* Student Texts 23 (London Mathematical Society, Cambridge, 1992), Chapters 2–6.

2. W. Fulton, *Algebraic Curves,* 3rd ed., downloadable at www.math.lsa.umich.edu/∼wfulton

2.8 B3.4: Algebraic Number Theory — Prof. Minhyong Kim — 16 HT

Level: H-level **Method of Assessment:** Written examination

Weight: Unit

Prerequisites: Rings and Modules and Number Theory. B3.1 Galois Theory is an essential pre-requisite.

Recommended Prerequisites: All second-year algebra and arithmetic. Students who have not taken Part A Number Theory should read about quadratic residues in, for example, the appendix to Stewart and Tall. This will help with the examples.

Overview

An introduction to algebraic number theory. The aim is to describe the properties of number fields, but particular emphasis in examples will be placed on quadratic fields, where it is easy to calculate explicitly the properties of some of the objects being considered. In such fields the familiar unique factorisation enjoyed by the integers may fail, and a key objective of the course is to introduce the class group which measures the failure of this property.

Learning Outcomes

Students will learn about the arithmetic of algebraic number fields. They will learn to prove theorems about integral bases, and about unique factorisation into ideals. They will learn to calculate class numbers, and to use the theory to solve simple Diophantine equations.

Synopsis

1. field extensions, minimum polynomial, algebraic numbers, conjugates, discriminants, Gaussian integers, algebraic integers, integral basis
2. examples: quadratic fields
3. norm of an algebraic number
4. existence of factorisation
5. factorisation in $\mathbb{Q}(\sqrt{d})$
6. ideals, $\mathbb{Z}$-basis, maximal ideals, prime ideals
7. unique factorisation theorem of ideals
8. relationship between factorisation of number and of ideals
9. norm of an ideal
10. ideal classes
11. statement of Minkowski convex body theorem
12. finiteness of class number
13. computations of class number to go on example sheets

Reading

1. I. Stewart and D. Tall, *Algebraic Number Theory and Fermat's Last Theorem.* (Third Edition, Peters, 2002).

Further Reading

1. D. Marcus, *Number Fields* (Springer-Verlag, New York–Heidelberg, 1977). ISBN 0-387-90279-1.

2.9 B3.5: Topology and Groups — Prof. Marc Lackenby — 16 MT

Level: H-level. **Method of Assessment:** Written examination.

Weight: Unit

Recommended Prerequisites Part A Topology is essential and Group Theory is recommended.

Overview

This course introduces the important link between topology and group theory. On the one hand, associated to each space, there is a group, known as its fundamental group. This can be used to solve topological problems using algebraic methods. On the other hand, many results about groups are best proved and understood using topology. For example, presentations of groups, where the group is defined using generators and relations, have a topological interpretation. The endpoint of the course is the Nielsen–Shreier Theorem, an important, purely algebraic result, which is proved using topological techniques.

Learning Outcomes

Students will develop a sound understanding of simplicial complexes, cell complexes and their fundamental groups. They will be able to use algebraic methods to analyse topological spaces. They will also be able to address questions about groups using topological techniques.

Synopsis

Homotopic mappings, homotopy equivalence. Simplicial complexes. Simplicial approximation theorem.

The fundamental group of a space. The fundamental group of a circle. Application: the fundamental theorem of algebra. The fundamental groups of spheres.

Free groups. Existence and uniqueness of reduced representatives of group elements. The fundamental group of a graph.

Groups defined by generators and relations (with examples). Tietze transformations.

The free product of two groups. Amalgamated free products.

The Seifert–van Kampen Theorem.

Cell complexes. The fundamental group of a cell complex (with examples). The realization of any finitely presented group as the fundamental group of a finite cell complex.

Covering spaces. Liftings of paths and homotopies. A covering map induces an injection between fundamental groups. The use of covering spaces to determine fundamental groups: the circle again, and real projective n-space. The correspondence between covering spaces and subgroups of the fundamental group. Regular covering spaces and normal subgroups.

Cayley graphs of a group. The relationship between the universal cover of a cell complex, and the Cayley graph of its fundamental group. The Cayley 2-complex of a group.

The Nielsen–Schreier Theorem (every subgroup of a finitely generated free group is free) proved using covering spaces.

Reading

1. John Stillwell, *Classical Topology and Combinatorial Group Theory* (Springer-Verlag, 1993).

Additional Reading

1. D. Cohen, *Combinatorial Group Theory: A Topological Approach,* Student Texts 14 (London Mathematical Society, 1989), Chapters 1–7.

2. A. Hatcher, *Algebraic Topology* (CUP, 2001), Chapter. 1.

3. M. Hall, Jr, *The Theory of Groups* (Macmillan, 1959), Chapters. 1–7, 12, 17 .

4. D. L. Johnson, *Presentations of Groups,* Student Texts 15 (Second Edition, London Mathematical Society, Cambridge University Press, 1997). Chapters. 1–5, 10,13.

5. W. Magnus, A. Karrass, and D. Solitar, *Combinatorial Group Theory* (Dover Publications, 1976). Chapters. 1–4.

2.10 B4.1: Banach Spaces — Prof. Dmitry Belyaev — 16 MT

Level: H-level **Method of Assessment:** Written examination.

Weight: Unit

Recommended Prerequisites: Part A Integration is recommended; the only concepts which will be used are the convergence theorems and the theorems of Fubini and Tonelli, and the notions of measurable functions and null sets. No knowledge is needed of outer measure, or of any particular construction of the integral, or of any proofs.

Learning Outcomes

Students will have a firm knowledge of real and complex normed vector spaces, with their geometric and topological properties. They will be familiar with the notions of completeness, separability and density, will know the properties of a Banach space and important examples, and will be able to prove results relating to the Hahn–Banach Theorem. They will have developed an understanding of the theory of bounded linear operators on a Banach space.

Synopses

Real and complex normed vector spaces, their geometry and topology. Completeness. Banach spaces, examples (ℓ^p, ℓ^∞, L^p, $C(K)$, spaces of differentiable functions).

Finite-dimensional normed spaces; equivalence of norms and completeness. Separable spaces; separability of subspaces.

Continuous linear functionals. Dual and reflexive spaces. Hahn–Banach Theorem (proof for real separable spaces only) and applications, including density of subspaces. Stone-Weierstrass Theorem.

Bounded linear operators, examples (including integral operators). Adjoint operators. Spectrum and resolvent. Spectral mapping theorem for polynomials.

Reading

1. B.P. Rynne and M.A. Youngson, *Linear Functional Analysis* (Springer SUMS, 2nd edition, 2008), Chapters 2, 4, 5.

2. E. Kreyszig, *Introductory Functional Analysis with Applications* (Wiley, revised edition, 1989), Chapters 2, 4.2174.3, 4.5, 7.1177.4.

2.11 B4.2: Hilbert Spaces — Prof. Hilary Priestley — 16 HT

Level: H-level **Method of Assessment:** Written examination

Weight: Unit

Prerequisites: B4.1 Banach Spaces is an essential pre-requisite.

Recommended Prerequisites: A good working knowledge of Part A Core Analysis (both metric spaces and complex analysis) is expected. Part A Integration is desirable, but the only concepts which will be used are the convergence theorems and the theorems of Fubini and Tonelli, and the notions of measurable functions and null sets. No knowledge is needed of outer measure, or of any particular construction of the integral, or of any proofs.]

Learning Outcomes

Students will appreciate the role of completeness through the Baire category theorem and its consequences for operators on Banach spaces. They will have a demonstrable knowledge of the properties of a Hilbert space, including orthogonal complements, orthonormal sets, complete orthonormal sets together with related identities and inequalities. They will be familiar with the theory of linear operators on a Hilbert space, including adjoint operators, self-adjoint and unitary operators with their spectra. They will know the L^2-theory of Fourier series and be aware of the classical theory of Fourier series and other orthogonal expansions.

Synopses

Hilbert spaces; examples including L^2-spaces. Orthogonality, orthogonal complement, closed subspaces, projection theorem. Riesz Representation Theorem.

Linear operators on Hilbert space, adjoint operators. Self-adjoint operators, orthogonal projections, unitary operators.

Baire Category Theorem and its consequences for operators on Banach spaces (Uniform Boundedness, Open Mapping, Inverse Mapping and Closed Graph Theorems). Strong convergence of sequences of operators.

Spectral theory in Hilbert spaces, in particular spectra of self-adjoint and unitary operators.

Orthonormal sets, Pythagoras, Bessels inequality. Complete orthonormal sets, Parseval.

L^2-theory of Fourier series, including completeness of the trigonometric system. Discussion of classical theory of Fourier series (including statement of pointwise convergence for piecewise differentiable functions, and exposition of failure for some continuous functions). Examples of other orthogonal expansions (Legendre, Laguerre, Hermite etc.).

Reading

1. B.P. Rynne and M.A. Youngson, *Linear Functional Analysis* (Springer SUMS, 2nd edition, 2008), Chapters 3, 4.4, 6.

2. E. Kreyszig, *Introductory Functional Analysis with Applications* (Wiley, revised edition, 1989), Chapters 3, 4.7–4.9, 4.12–4.13, 9.1–9.2.

3. N. Young, *An Introduction to Hilbert Space* (Cambridge University Press, 1988), Chs 1177.

Further Reading

1. E.M. Stein and R. Shakarchi, *Real Analysis: Measure Theory, Integration & Hilbert Spaces* (Princeton Lectures in Analysis III, 2005), Chapter 4.

2.12 B5.1: Stochastic Modelling of Biological Processes — Prof. Ruth Baker — 16 HT

Level: H-level **Method of Assessment:** Written examination.

Weight: Unit

Recommended Prerequisites: Part A Differential Equations II and Probability, and Part B Applied Probability, would also be an advantage, but are not necessary.

Overview

Stochastic Modelling of Biological Processes provides an introduction to stochastic methods for modelling biological systems. The course starts with stochastic modelling of chemical reactions, introducing stochastic simulation algorithms and mathematical methods which can be used for analysis of stochastic models. Different models of molecular diffusion (on-lattice and off-lattice models, velocity-jump processes) and their properties are studied, before moving to stochastic reaction-diffusion models. Compartment-based and molecular-based approaches to stochastic reaction-diffusion modelling (Brownian dynamics) are discussed together with stochastic spatially-distributed models (pattern formation).

Learning Outcomes

The student will learn: (i) mathematical techniques for the analysis of stochastic models; (ii) how stochastic models can be efficiently simulated using a computer; (iii) connections and differences between different stochastic methods, and between stochastic and deterministic modelling.

Synopsis

Stochastic simulation of chemical reactions in well-stirred systems: Gillespie algorithm, chemical master equation, analysis of simple systems, deterministic vs. stochastic modelling, systems with multiple favourable states, stochastic resonance, stochastic focusing.

Stochastic differential equations: numerical methods, Fokker-Planck equation, first exit time, backward Kolmogorov equation, chemical Fokker-Planck equation.

Compartment-based stochastic reaction-diffusion models: reaction-diffusion master equation, pattern formation, morphogen gradients, Turing patterns.

Molecular-based approaches to reaction-diffusion modelling, Brownian dynamics, reaction radius.

Metropolis-Hastings algorithm: Markov chain Monte Carlo methods.

Bacterial chemotaxis: reaction-diffusion-advection processes, velocity-jump processes.

Reading

1. R. Erban, S. J. Chapman and P. K. Maini, *A practical guide to stochastic simulation of reaction-diffusion processes* (2007) Available at http://arxiv.org/abs/0704.1908

Further Reading

Students are by no means expected to read all these sources. There are suggestions intended to be helpful to students interested exploring the subjects covered in further detail.

1. H. Berg, *Random Walks in Biology* (Princeton University Press, 1993).
2. D. T. Gillespie, *Markov Processes, an Introduction for Physical Scientists* (Gulf Professional Publishing, 1992).
3. D. T. Gillespie and E. Seitaridou, *Simple Brownian Diffusion: An Introduction to the Standard Theoretical Models* (Oxford University Press, 2012).
4. D. J. Wilkinson, *Stochastic Modelling for Systems Biology* (2nd Edition, CRC Press, 2011).
5. L. S. Allen, *An Introduction to Stochastic Processes with Applications to Biology* (2nd Edition, CRC Press, 2010).

2.13 B5.2: Applied Partial Differential Equations — Prof. Derek Moulton — 16 MT

Level: H-level **Method of Assessment:** Written examination.

Weight: Unit

Recommended Prerequisites: Part A Differential Equations 2 is essential. Calculus of Variations and Waves and Fluids from Part A are desirable but not essential. Integral Transforms from Part A is strongly desirable.

Overview

This course continues the Part A Differential Equations courses. In particular, first-order conservation laws are solved and the idea of a shock is introduced; general nonlinear and quasi-linear first-order partial differential equations are solved, the classification of second-order partial differential equations is extended to systems, with hyperbolic systems being solved by characteristic variables. Then Riemann's function, Green's function, maximum principle and similarity variable methods are demonstrated for partial differential equations.

Learning Outcomes

Students will know a range of techniques to solve PDEs including non-linear first-order and second-order and their classification. They will be able to demonstrate various principles for solving PDEs including the method of characteristics, the maximum principle, similarity solutions and the Riemann function.

Synopsis

First-order equations: conservation laws and shocks. Charpit's equations; eikonal equation. [4 lectures]

Systems of partial differential equations, characteristics. Shocks; viscosity solutions; weak solutions. [4 lectures]

Green's function, maximum principles, well-posed problems for the heat equation and for Laplace's equation. [4 lectures]

Similarity solutions. [2 lectures]

Riemann functions for hyperbolic partial differential equations. [2 lectures]

Reading

1. Dr Norbury's web notes.

2. Institute lecture notes are now available (JN).

3. M. Renardy and R.C. Rogers, *An Introduction to Partial Differential Equations* (Springer-Verlag, New York, 2004).

4. J. P. Keener, *Principles of Applied Mathematics: Transformation and Approximation* (revised edition, Perseus Books, Cambridge, Mass., 2000).

5. J. R. Ockendon, S. D. Howison, A. A. Lacey and A. B. Movchan, *Applied Partial Differential Equations* (revised edition, Oxford University Press, Oxford, 2003).

2.14 B5.3: Viscous Flow — Prof. Sarah Waters — 16 MT

Level: H-level **Method of Assessment:** Written examination.

Weight: Unit

Recommended Prerequisites: The Part A (second-year) courses 'Waves and Fluids' and 'DEs2' would be desirable. This course combines well with B5.2 Applied Partial Differential Equations. Though the two units are intended to stand alone, they will complement each other.

Overview

Viscous fluids are important in so many facets of everyday life that everyone has some intuition about the diverse flow phenomena that occur in practise. This course is distinctive in that is shows how quite advanced mathematical ideas such as asymptotics and partial differential equation theory can be used to analyse the underlying differential equations and hence give scientific understanding about flows of practical importance, such as air flow round wings, oil flow in a journal bearing and the flow of a large raindrop on a windscreen.

Learning Outcomes

Students will have developed an appreciation of diverse viscous flow phenomena and they will have a demonstrable knowledge of the mathematical theory necessary to analyse such phenomena.

Synopsis

Euler's identity and Reynolds' transport theorem. The continuity equation and incompressibility condition. Cauchy's stress theorem and properties of the stress tensor. Cauchy's momentum equation. The incompressible Navier-Stokes equations. Vorticity. Energy. Exact solutions for unidirectional flows; Couette flow, Poiseuille flow, Rayleigh layer, Stokes layer. Dimensional analysis, Reynolds number. Derivation of equations for high and low Reynolds number flows.

Thermal boundary layer on a semi-infinite flat plate. Derivation of Prandtl's boundary-layer equations and similarity solutions for flow past a semi-infinite flat plate. Discussion of separation and application to the theory of flight.

Slow flow past a circular cylinder and a sphere. Non-uniformity of the two dimensional approximation; Oseen's equation. Lubrication theory: bearings, squeeze films, thin films; Hele–Shaw cell and the Saffman-Taylor instability.

Reading

1. D. J. Acheson, *Elementary Fluid Dynamics* (Oxford University Press, 1990), Chapters 2, 6, 7, 8.

2. H. Ockendon and J. R. Ockendon, *Viscous Flow* (Cambridge Texts in Applied Mathematics, 1995).

Further reading

1. M. Van Dyke, *An Album of Fluid Motion* (Parabolic Pr, 1982). ISBN 0915760029.
2. G.K. Batchelor, *An Introduction to Fluid Dynamics* (CUP, 2000). ISBN 0521663962.
3. C.C. Lin & L.A. Segel, *Mathematics Applied to Deterministic Problems in Natural Sciences*(Society of Industrial and Applied Mathematics, 1998). ISBN 0898712297.
4. L.A Segel, *Mathematics Applied to Continuum Mechancis*(Society for Industrial and Applied Mathematics, 2007). ISBN 0898716209.

2.15 B5.4: Waves and Compressible Flow — Prof. Ian Hewitt — 16 HT

Level: H-level **Method of Assessment:** Written examination.

Weight: Unit

Recommended Prerequisites: Part A 'Waves and Fluids' and Integral Transforms. This course combines well with B5.2 Applied Partial Differential Equations and B5.3 Viscous Flow.

Overview

Propagating disturbances, or waves, occur frequently in applied mathematics. This course will be centred on some prototypical examples from fluid dynamics, the two most familiar being surface gravity waves and waves in gases. The models for compressible flow will be derived and then analysed for small amplitude motion. This will shed light on the important phenomena of dispersion, group velocity and resonance, and the differences between supersonic and subsonic flow, as well as revealing the crucial dependence of the waves on the number of space dimensions. Larger amplitude motion of liquids and gases will be described by incorporating non-linear effects, and the theory of characteristics for partial differential equations will be applied to understand the shock waves associated with supersonic flight.

Learning Outcomes

Students will have developed a sound knowledge of a range of mathematical models used to study waves (both linear and non-linear), will be able to describe examples of waves from fluid dynamics and will have analysed a model for compressible flow. They will have an awareness of shock waves and how the theory of characteristics for PDEs can be applied to study those associated with supersonic flight.

Synopsis

Equations of inviscid compressible flow including flow relative to rotating axes.

Models for linear wave propagation including Stokes waves, internal gravity waves, inertial waves in a rotating fluid, and simple solutions.

Theories for Linear Waves: Fourier Series, Fourier integrals, method of stationary phase, dispersion and group velocity. Flow past thin wings.

Nonlinear Waves: method of characteristics, simple wave flows applied to one-dimensional unsteady gas flow and shallow water theory.

Shock Waves: weak solutions, RankineHugoniot relations, oblique shocks, bores and hydraulic jumps.

Reading

1. H. Ockendon and J. R. Ockendon, *Waves and Compressible Flow* (Springer, 2004).
2. D. J. Acheson, *Elementary Fluid Dynamics* (Oxford University Press, 1990). Chapter 3
3. J. Billingham and A. C. King, *Wave Motion* (Cambridge University Press, 2000). Chapters 1–4, 7,8.
4. M. J. Lighthill, *Waves in Fluids* (Cambridge University Press, 1978).
5. G. B. Whitham, *Linear and Nonlinear Waves* (Wiley, 1973).

2.16 B5.5: Mathematical Ecology and Biology — Prof. Philip Maini — 16 MT

Level: H-level **Method of Assessment:** Written examination.

Weight: Unit

Recommended Prerequisites: Part A core material (especially differential equations).

Overview

Mathematical Ecology and Biology introduces the applied mathematician to practical applications in an area that is growing very rapidly. The course mainly focusses on situations where continuous models are appropriate and where these may be modelled by deterministic ordinary and partial differential equations. By using particular modelling examples in ecology, chemistry, biology, physiology and epidemiology, the course demonstrates how various applied mathematical techniques, such as those describing linear stability, phase planes, singular perturbation and travelling waves, can yield important information about the behaviour of complex models.

Learning Outcomes

Students will have developed a sound knowledge and appreciation of the ideas and concepts related to modelling biological and ecological systems using ordinary and partial differential equations.

Synopsis

Continuous and discrete population models for a single species, including Ludwig's 1978 insect outbreak models, hysterisis and harvesting. Introduction to delay differential equation models.

Modelling interacting populations, including predator-prey and the principle of competitive exclusion. Discrete models for several species.

Epidemic models.

Michaelis–Menten model for enzyme-substrate kinetics.

Travelling wave propagation with biological examples.

Biological pattern formation, including Turing's model for animal coat markings, and chemotaxis models.

Excitable systems. Threshold phenomena (nerve pulses) and nerve signal propagation.

Reading

J.D. Murray, *Mathematical Biology, Volume I: An Introduction (2002); Volume II: Spatial Models and Biomedical Applications (2003)* (3rd edition, Springer–Verlag).

1. Volume I: 1.1, 1.2, 1.3, 1.6, 2.1–2.4, 3.1, 3.3–3.6, 3.8, 6.1–6.3, 6.5, 6.6, 8.1, 8.2, 8.4, 8.5, 10.1, 10.2, 11.1–11.5, 13.1–13.5, Appendix A.

2. Volume II: 1.6, 2, 3.1, 3.2, 5.1, 5.2, 13.1–13.4.

Dr R.E. Baker, online lecture notes.

Further Reading

1. J. Keener and J. Sneyd, *Mathematical Physiology* (First Edition Springer, Berlin, 1998) 1.1, 1.2, 9.1, 9.2.

2. N. F . Britton, Essential Mathematical Biology (Springer, London, 2003). 1.1, 1.2, 1.3, 1.5, 2.1, 2.3, 2.4, 2.5, 2.7, 3.1, 3.2, 3.3, 5.1, 5.2, 5.3, 5.6, 7.1, 7.2, 7.3, 7.4, 7.5.

2.17 B5.6: Nonlinear Systems — Prof. Irene Moroz — 16 HT

Level: H-level **Method of Assessment:** Written examination.

Weight: Unit

Recommended Prerequisites: Part A Differential Equations 2.

Overview

This course aims to provide an introduction to the tools of dynamical systems theory which are essential for the realistic modelling and study of many disciplines, including mathematical ecology and biology, fluid dynamics, granular media, mechanics, and more.

The course will include the study of both nonlinear ordinary differential equations and maps. It will draw examples from appropriate model systems and various application areas. The problem sheets will require numerical computation (using programs such as Matlab).

Learning Outcomes

Students will have developed a sound knowledge and appreciation of some of the tools, concepts, and computations used in the study of nonlinear dynamical systems. They will also get some exposure to some modern research topics in the field.

Synopsis

1. Bifurcations

   Bifurcation theory: standard codimension one (saddle-node, pitchfork, trans-critical, Hopf) normal forms; Conservative and Non-Conservative systems.

2. Nonlinear Oscillations

   Van der Pol and Duffing's equations. Poincaré-Lindstedt method; Method of Multiple Scales; Krylov-Bogoliubov Method of Averaging; Relaxation and Forced oscillations; Synchronization of coupled oscillators.

3. Maps

   Poincaré sections and first-return maps. Stability and periodic orbits; bifurcations of one-dimensional maps. Two-dimensional maps: Hénon map, Chirikov (standard) map.

4. Chaos in Maps and Differential Equations

   Maps: Logistic map, Bernoulli shift map, symbolic dynamics, Skinny Baker's map, Smale's Horseshoe Map.
   Differential equations: Lorenz equations, Rossler equations.

Reading

Students are by no means expected to read all these sources. These are suggestions intended to be helpful.

1. S. H. Strogatz, *Nonlinear Dynamics and Chaos with Applications to Physics, Biology, Chemistry and Engineering* (Westview Press, 2000).

2. E. Ott, *Chaos in Dynamical Systems* (Second edition, Cambridge University Press, Cambridge, 2002).

3. R. H. Rand, *Lecture Notes on Nonlinear Vibrations.* [Available for free online at http://audiophile.tam.cornell.edu/randdocs/nlvibe52.pdf]

4. P. G. Drazin, *Nonlinear Systems* (Cambridge University Press, Cambridge, 1992).

5. S. Wiggins, *Introduction to Applied Nonlinear Dynamical Systems and Chaos* (Second edition, Springer, 2003).

6. S. H. Strogatz, 'From Kuramoto to Crawford: exploring the inset of synchronization in populations of coupled oscillators', *Physica D* 143 (2000) 1-20.

2.18 B6.1 Numerical Solution of Differential Equations I — Prof. Ian Sobey — 16 MT

Level: H-level **Method of Assessment:** Written examination.

Weight: Unit

Recommended Prerequisites: None.

Overview

To introduce and give an understanding of numerical methods for the solution of ordinary differential equations and parabolic partial differential equations; including their derivation, analysis and applicability.

Learning Outcomes

At the end of the course the student will be able to:

1. construct one-step and linear multistep methods for the numerical solution of initial-value problems for ordinary differential equations and systems of such equations, and to analyse their stability, accuracy, and preserved geometric properties;
2. construct numerical methods for the numerical solution of initial-boundary-value problems for parabolic partial differential equations, and to analyse their stability and accuracy properties of these methods.

Synopsis

The course is devoted to the development and analysis of methods for numerical solution of initial value problems for ordinary differential equations and initial-boundary-value problems for second-order parabolic partial differential equations. The course begins by considering classical techniques for the numerical solution of initial value ordinary differential equations. The problem of stiffness is discussed in tandem with the associated questions of step-size control and adaptivity. Topics include: Euler, multistep, and Runge–Kutta methods; stability; stiffness; error control; symplectic and adaptive algorithms.

The remaining lectures focus on the numerical solution of initial-boundary-value problems for parabolic partial differential equations, Topics include: explicit and implicit methods; accuracy, stability and convergence, use of Fourier methods for analysis.

Reading List

The course will be based on the following textbooks:

1. A. Iserles, *A First Course in the Numerical Analysis of Differential Equations* (Cambridge University Press, second edition, 2009). ISBN 978-0-521-73490-5 [Chapters 1–6, 16].

2. R. LeVeque, *Finite difference methods for ordinary and partial differential equations* (SIAM, 2007). ISBN 978-0-898716-29-0 [Chapters 5-9].

3. E. Süli and D. Mayers, *An Introduction to Numerical Analysis* (Cambridge University Press, 2006). ISBN 0-521-00794-1 [Chapter 12].

2.19 B6.2 Numerical Solution of Differential Equations II — Prof. Jared Tanner — 16 HT

Level: H-level **Method of Assessment:** Written examination.

Weight: Unit

Recommended Prerequisites: Part A Differential Equations 1. B5.2 Applied Partial Differential Equations is desirable but not essential.

Overview

To introduce and give an understanding of numerical methods for the solution of hyperbolic and elliptic partial differential equations, including their derivation, analysis and applicability.

Learning Outcomes

At the end of the course the student will be able to:

1. construct practical methods for the numerical solution of boundary-value problems arising from ordinary differential equations and elliptic partial differential equations; analysis of the stability, accuracy, and uniqueness properties of these methods,

2. construct methods for the numerical solution of initial-boundary-value problems for first- and second-order hyperbolic partial differential equations, and to analyse their stability and accuracy properties.

Synopsis

The course is devoted to the development and analysis of numerical solutions of boundary value problems for second-order ordinary differential equations, boundary-value problems for second-order elliptic partial differential equations, and initial-boundary-value problems for first- and second-order hyperbolic partial differential equations. The course begins by considering classical techniques for the numerical solution of boundary-value problems for second-order ordinary differential equations and elliptic boundary-value equations, in particular the Poisson equation in two dimensions. Topics include: discretisations (e.g., finite difference, finite element, and spectral methods), error and convergence analysis, and the use of maximum principles. The remaining lectures focus on the numerical solution of initial-boundary-value problems for hyperbolic partial differential equations with topics such as: discretisations (e.g., finite difference and finite volume), method of lines, accuracy, stability (including CFL condition) and convergence, limiters, total variation, WENO schemes and energy methods.

Reading List

The course will be based on the following textbooks:

1. A. Iserles, *A First Course in the Numerical Analysis of Differential Equations* (Cambridge University Press, second edition, 2009), Chapters 8-10, 17.
2. R. LeVeque, *Finite difference methods for ordinary and partial differential equations* (SIAM, 2007). ISBN 978-0-898716-29-0 [Chapter 10].
3. R. LeVeque, *Numerical methods for conservation laws* (Birkhaüser 1992), ISBN 0-8176-2723-5 [Chapters 10–16].

2.20 B6.3 Integer Programming — Prof. Coralia Cartis — 16 MT

Level: H-level **Method of Assessment:** Written examination.

Weight: Unit

Recommended Prerequisites: Prelims Optimisation is recommended.

Overview

In many areas of practical importance linear optimisation problems occur with integrality constraints imposed on some of the variables. In optimal crew scheduling for example, a pilot cannot be fractionally assigned to two different flights at the same time. Likewise, in combinatorial optimisation an element of a given set either belongs to a chosen subset or it does not. Integer programming is the mathematical theory of such problems and of algorithms for their solution. The aim of this course is to provide an introduction to some of the general ideas on which attacks to integer programming problems are based: generating bounds through relaxations by problems that are easier to solve, and branch-and-bound.

Learning Outcomes

Students will understand some of the theoretical underpinnings that render certain classes of integer programming problems tractable ("easy" to solve), and they will learn how to solve them algorithmically. Furthermore, they will understand some general mechanisms by which intractable problems can be broken down into tractable subproblems, and how these mechanisms are used to design good heuristics for solving the intractable problems. Understanding these general principles will render the students able to guide the modelling phase of a real-world problem towards a mathematical formulation that has a reasonable chance of being solved in practice.

Synopsis

1. Course outline. What is integer programming (IP)? Some classical examples.
2. Further examples, hard and easy problems.
3. Alternative formulations of IPs, linear programming (LP) and the simplex method.
4. LP duality, sensitivity analysis.
5. Optimality conditions for IP, relaxation and duality.
6. Total unimodularity, network flow problems.
7. Optimal trees, submodularity, matroids and the greedy algorithm.
8. Augmenting paths and bipartite matching.
9. The assignment problem.
10. Dynamic programming.
11. Integer knapsack problems.
12. Branch-and-bound.
13. More on branch-and-bound.
14. Lagrangian relaxation and the symmetric travelling salesman problem.
15. Solving the Lagrangian dual.
16. Branch-and-cut.

Reading

1. L. A. Wolsey, *Integer Programming* (John Wiley & Sons, 1998), parts of chapters 1–5 and 7.

Time Requirements

The course consists of 16 lectures and 6 problem classes. There are no practicals. It is estimated that 8–10 hours of private study are needed per week for studying the lecture notes and relevant chapters in the textbook, and for solving the problem sheets, so that the total time requirement is circa 12 hours per week.

2.21 B7.1: Classical Mechanics — Prof. James Sparks — 16 MT

Level: H-level **Method of Assessment:** Written examination.

Weight: Unit

Recommended Prerequisites: Calculus of Variations.

Overview

This course builds on the Prelims Dynamics course, recasting Newtonian mechanics in the Lagrangian and Hamiltonian formalisms. As well as being elegant and computationally useful, these formulations of classical mechanics give important insights into symmetries and conservation laws, and are the language used to describe all modern theories of physics.

Learning Outcomes

Students will be able to demonstrate knowledge and understanding of the Lagrangian and Hamiltonian formalisms. They will understand how symmetries and conserved quantities are described in this language, and be able to apply the ideas developed to small oscillations around equilibria, rigid body motion and some other elementary systems.

Synopsis

Review of Newtonian mechanics. Generalized coordinates. The principle of least action. Constraints. Symmetries and Noether's Theorem. Examples with simple systems.

Equilibria. Small oscillations about a stable equilibrium and normal modes, with examples.

Rigid bodies. Angular velocity, angular momentum and the inertia tensor. Euler's equations and tops. Euler angles and $SO(3)$.

Legendre transformations and the Hamiltonian. Phase space and its geometry. Poisson brackets. Canonical transformations. Liouville's theorem. The Hamilton- Jacobi equation.

Reading

1. H. Goldstein, C, Poole, J. Safko, *Classical Mechanics* Third Edition (Addison-Wesley, 2002).

2. L. D. Landau, E. M. Lifshitz, *Mechanics (Course of Theoretical Physics, Vol. 1)*, Third Edition, (Butterworth-Heinemann, 1976).

3. N. M. J. Woodhouse, *Introduction to Analytical Mechanics* (OUP, 1987)

2.22 B7.2: Electromagnetism — Prof. Fernando Alday — 16MT

Level: M-level **Method of Assessment:** Written examination.

Weight: Unit

Recommended Prerequisites: None

Overview

The idea is to have a classical course on Electromagnetism, similar to the one in a theoretical Physics degree. We will follow closely the book by Jackson, first 8 chapters.

Learning Outcomes

Students will have a clear understanding of what electromagnetism is, they will dominate many techniques and will be able to solve most classic problems of electromagnetism. This course should also enable them to continue learning by themselves, or take more advanced courses.

Synopsis

Basics of electrostatics;
Boundary value problems in electrostatics;
Multipoles, electrostatics of macroscopic media, dielectrics;
Magnetostatics;
Time-varying fields, Maxwell equations, conservation laws;
Plane electromagnetic waves;
Wave guides and resonant cavities [if time allows]

Reading

The lectures will follow:

1. J.D. Jackson, *Classical Electrodynamics* (John Wiley, 1962), chapters 1 to 8.

Further Reading

1. R. Feynman, *Lectures in Physics*, Vol.2. Electromagnetism, Addison Wesley.
2. L.D. Landau and E.M. Lifshitz, *A classical theory of fields* Volume 2.

2.23 C7.3: Further Quantum Theory — Prof. Lionel Mason — 16 HT

It is not possible to take C7.3 to examination in Part C in 2016 if it was taken as part of the Part B examination in 2015, or if C7.1b was taken to examination prior to this.

Level: M-level **Method of Assessment:** Written examination.

Weight: Unit

Recommended Prerequisites: Part A Quantum Theory.

Overview

This course builds directly on the first course in quantum mechanics and covers a series of important topics, particularly features of systems containing several particles. The behaviour of identical particles in quantum theory is more subtle than in classical mechanics, and an understanding of these features allows one to understand the periodic table of elements and the rigidity of matter.

There are rarely neat solutions to problems involving several particles, so usually one needs some approximation methods. These are developed so as to study both energy levels of interacting Hamiltonians, and scattering.

Learning Outcomes

Students will be able to demonstrate knowledge and understanding of the mathematics of quantum mechanics of many particle systems, and atomic structure and scattering.

Synopsis

Symmetries in quantum mechanics. Rotations, spin and angular momentum and their addition.

Identical particles, symmetric and anti-symmetric states, Fermi-Dirac and Bose-Einstein statistics and atomic structure.

Approximation methods, Rayleigh-Schrödinger time-independent perturbation theory and variation principles. The virial theorem. Helium. WKB approimation.

Heisenberg representation, interaction representation, time dependent perturbation theory and FeynmanDyson expansion.

Scattering theory: propagators, Coulomb scattering.

Reading

The lectures will partly follow:

S. Weinberg, *Lectures on quantum mechanics*, CUP, (2013). Sections 4.3-5, 5.1-7, 6.1-3, 7.1-4.

together with:

K. C. Hannabuss, *Introduction to quantum mechanics*, OUP (1997). Chapter 8.1-4, 8.8, 9, 16.1-4, 11.1-5, 12.1-4, 14.1-4, 15.1-3, 13.5.

But the following are also recommended:

J. Binney and D. Skinner, *The physics of quantum mechanics*, PUP, 2011.

Landau and Lifschitz, *Quantum Mechanics, non-relativistic theory*, Vol 3 of a course in theoretical physics, Pergamon press.

Gordon Screaton, *Further Quantum Theory*, Mathematical Institute Notes (1991). Also designed for an Oxford course, though only covering some material: This can be found online at `https://www.maths.ox.ac.uk/system/files/legacy/3171/Further_Quantum_Mechanics.pdf`

L. I. Schiff, *Quantum Mechanics* (3rd edition, Mc Graw Hill, 1968).

B. J. Bransden and C. J. Joachain, *Introduction to Quantum Mechanics* (Longman, 1995).

A. I. M. Rae, *Quantum Mechanics* (4th edition, Institute of Physics, 1993).

A popular non-technical account of the subject: A. Hey and P. Walters, *The New Quantum Universe* (Cambridge, 2003).

2.24 B8.1: Martingales Through Measure Theory — Prof Zhongmin Qian — 16 MT

Level: H-level **Method of Assessment:** Written examination.

Weight: Unit

Recommended Prerequisites: Part A Integration is a prerequisite, so that the corresponding material will be assumed to be known. Part A Probability is a prerequisite.

Overview

Probability theory arises in the modelling of a variety of systems where the understanding of the "unknown" plays a key role, such as population genetics in biology, market evolution in financial mathematics, and learning features in game theory. It is also very useful in various areas of mathematics, including number theory and partial differential equations. The course introduces the basic mathematical framework underlying its rigorous analysis, and is therefore meant to provide some of the tools which will be used in more advanced courses in probability.

The first part of the course provides a review of measure theory from Integration Part A, and develops a deeper framework for its study. Then we proceed to develop notions of conditional expectation, martingales, and to show limit results for the behaviour of these martingales which apply in a variety of contexts.

Learning Outcomes

The students will learn about measure theory, random variables, independence, expectation and conditional expectation, product measures and discrete-parameter martingales.

Synopsis

A branching-process example. Review of σ-algebras, measure spaces. Uniqueness of extension of π-systems and Carathéodory's Extension Theorem [both without proof], monotone-convergence properties of measures, $\limsup$ and $\liminf$ of a sequence of events, Fatou's Lemma, reverse Fatou Lemma, first Borel–Cantelli Lemma.

Random variables and their distribution functions, σ-algebras generated by a collection of random variables. Product spaces. Independence of events, random variables and σ-algebras, π-systems criterion for independence, second Borel–Cantelli Lemma. The tail σ-algebra, Kolomogorov's 0–1 Law. Convergence in measure and convergence almost everywhere.

Integration and expectation, review of elementary properties of the integral and L^p spaces [from Part A Integration for the Lebesgue measure on $\mathbb{R}$]. Scheffé's Lemma, Jensen's inequality. The Radon–Nikodym Theorem [without proof]. Existence and uniqueness of conditional expectation, elementary properties. Relationship to orthogonal projection in L^2.

Filtrations, martingales, stopping times, discrete stochastic integrals, Doob's Optional-Stopping Theorem, Doob's Upcrossing Lemma and "Forward" Convergence Theorem, martingales bounded in L^2, Doob decomposition, Doob's submartingale inequalities.

Uniform integrability and L^1 convergence, backwards martingales and Kolmogorov's Strong Law of Large Numbers.

Examples and applications, including branching processes.

Reading

1. D. Williams, *Probability with Martingales*, Cambridge University Press, 1995.

2. Lecture Notes for the course.

Further Reading

1. Z. Brzeźniak and T. Zastawniak, Basic stochastic processes. A course through exercises. Springer Undergraduate Mathematics Series. (Springer-Verlag London, Ltd., 1999) [more elementary than D. Williams' book, but can provide with a complementary first reading].

2. M. Capinski and E. Kopp, *Measure, integral and probability*, Springer Undergraduate Mathematics Series. (Springer-Verlag London, Ltd., second edition, 2004).

3. R. Durrett, *Probability: Theory and Examples.* (Second Edition Duxbury Press, Wadsworth Publishing Company, 1996).

4. A. Etheridge, *A Course in Financial Calculus*, (Cambridge University Press, 2002).

5. J. Neveu, *Discrete-parameter Martingales.* (North-Holland, Amsterdam, 1975).

6. S. I. Resnick, *A Probability Path*, (Birkhäuser, 1999).

2.25 B8.2: Continuous Martingales and Stochastic Calculus — Prof. Jan Obłój — 16 HT

Level: H-level **Method of Assessment:** Written examination.

Weight: Unit

Recommended Prerequisites: B8.1 Martingales through Measure Theory is a prerequisite. Consequently, Part A Integration and Part A Probability are also prerequisites.

Overview

Stochastic processes - random phenomena evolving in time - are encountered in many disciplines from biology, through geology to finance. This course focuses on mathematics needed to describe stochastic processes evolving continuously in time and introduces the basic tools of stochastic calculus which are at the cornerstone of modern probability theory. The motivating example of a stochastic process is Brownian motion, also called the Wiener process - a mathematical object initially proposed by Bachelier and Einstein, which originally modelled displacement of a pollen particle in a fluid. The paths of Brownian motion, or of any continuous martingale, are of infinite variation (they are in fact nowhere differentiable and have non-zero quadratic variation) and one of the aims of the course is to define a theory of integration along such paths equipped with a suitable integration by parts formula (Itô formula).

Learning Outcomes

The students will develop an understanding of Brownian motion and continuous martingales in continuous time. They will became familiar with stochastic calculus and in particular be able to use Itô's formula.

Synopsis

Stochastic process: collection of random variables vs a random variable on the path space. Brownian motion - definition, construction and basic properties, regularity of paths. Filtrations and stopping times, first hitting times. Brownian motion - martingale and strong Markov properties, reflection principle. Martingales - definitions, regularisation and convergence theorems, optional sampling theorem, maximal and Doob's L^p inequalities. Quadratic variation, local martingales, semimartingales. Recall of Stieltjes integral. Stochastic integration and Itô's formula with applications.

Reading

There are a large number of textbooks which cover the course material with a varying degree of detail/rigour. Precise references for reading from two excellent reference books will be given. These are:

1. D. Revuz and M. Yor, "Continuous martingales and Brownian motion", Springer (Revised 3^{rd} ed.), 2001. Selected pages from Chapters 0–4: *exact pages covering each lecture will be indicated in the course materials.*
2. I. Karatzas and S. Shreve, "Brownian motion and stochastic calculus", Springer (2^{nd} ed.), 1991. Selected pages from Chapters 1–3: *exact pages covering each lecture will be indicated in the course materials.*

Further Reading

Further helpful references include:

1. R. Durrett, "Stochastic Calculus: A practical introduction", CRC Press, 1996. Sections 1.1 – 2.10.
2. F. Klebaner, "Introduction to Stochastic Calculus with Applications", 3rd edition, Imperial College Press, 2012. Chapters 1, 2, 3.1–3.11, 4.1–4.5, 7.1–7.8, 8.1–8.7.
3. J. M. Steele, "Stochastic Calculus and Financial Applications", Springer, 2010. Chapters 3 – 8.
4. B. Oksendal, "Stochastic Differential Equations: An introduction with applications", 6th edition, Springer (Universitext), 2007. Chapters 1 – 3.
5. S. Shreve, "Stochastic calculus for finance", Vol 2: Continuous-time models, Springer Finance, Springer-Verlag, New York, 2004. Chapters 3 – 4.

2.26 B8.3: Mathematical Models of Financial Derivatives — Dr Jeff Dewynne — 16 HT

Level: H-level **Method of Assessment:** Written examination.

Weight: Unit

Recommended Prerequisites: B8.1 (Martingales Through Measure Theory) would be good background. Part A Probability is a prerequisite. Part A Integration is also good background, though not a prerequisite.

Overview

The course aims to introduce students to derivative security valuation in financial markets. At the end of the course the student should be able to formulate a model for an asset price and then determine the prices of a range of derivatives based on the underlying asset using arbitrage free pricing ideas.

Learning Outcomes

Students will have a familiarity with the mathematics behind the models and analytical tools used in Mathematical Finance. This includes being able to formulate a model for an asset price and then determining the prices of a range of derivatives based on the underlying asset using arbitrage free pricing ideas.

Synopsis

Introduction to markets, assets, interest rates and present value; arbitrage and the law of one price: European call and put options, payoff diagrams. Probability spaces, random variables, conditional expectation, discrete-time martingales. The binomial model; European and American claim pricing.

Introduction to Brownian motion and its quadratic variation , continuous-time martingales, informal treatment of Itô's formula and stochastic differential equations. Discussion of the connection with PDEs through the Feynman–Kac formula.

The Black–Scholes analysis via delta hedging and replication, leading to the Black–Scholes partial differential equation for a derivative price. General solution via Feynman–Kac and risk neutral pricing, explicit solution for call and put options.

American options, formulation as a free-boundary problem. Simple exotic options. Weakly path-dependent options including barriers, lookbacks and Asians. Implied volatility. Introduction to stochastic volatility. Robustness of Black-Scholes formula.

Reading

1. S.E Shreve, *Stochastic Calculus for Finance*, vols I and II, (Springer 2004).
2. T. Bjork, *Arbitrage Theory in Continuous Time* (Oxford University Press, 1998).
3. P. Wilmott, S. D. Howison and J. Dewynne, *Mathematics of Financial Derivatives* (Cambridge university Press, 1995).
4. A. Etheridge, *A Course in Financial Calculus* (Cambridge University Press, 2002).

Further Reading

Background on Financial Derivatives

1. J. Hull, *Options Futures and Other Financial Derivative Products*, 4th edition (Prentice Hall, 2001).

2.27 B8.4: Communication Theory — Dr Noah Forman — 16 MT

Level: H-level **Method of Assessment:** Written examination.

Weight: Unit

Recommended Prerequisites: Part A Probability would be helpful, but not essential.

Overview

The aim of the course is to investigate methods for the communication of information from a sender, along a channel of some kind, to a receiver. If errors are not a concern we are interested in codes that yield fast communication; if the channel is noisy we are interested in achieving both speed and reliability. A key concept is that of information as reduction in uncertainty. The highlight of the course is Shannon's Noisy Coding Theorem.

Learning Outcomes

(i) Know what the various forms of entropy are, and be able to manipulate them.

(ii) Know what data compression and source coding are, and be able to do it.

(iii) Know what channel coding and channel capacity are, and be able to use that.

Synopsis

Uncertainty (entropy); conditional uncertainty; information. Chain rules; relative entropy; Gibbs' inequality; asymptotic equipartition and typical sequences. Instantaneous and uniquely decipherable codes; the noiseless coding theorem for discrete memoryless sources; constructing compact codes.

The discrete memoryless channel; decoding rules; the capacity of a channel. The noisy coding theorem for discrete memoryless sources and binary symmetric channels.

Extensions to more general sources and channels.

Reading

1. D. J. A. Welsh, *Codes and Cryptography* (Oxford University Press, 1988), Chapters 1–3, 5.

2. T. Cover and J. Thomas, *Elements of Information Theory* (Wiley, 1991), Chapters 1–5, 8.

Further Reading

1. R. B. Ash, *Information Theory* (Dover, 1990).

2. D. MacKay, *Information Theory, Inference, and Learning Algorithms* (Cambridge, 2003). [Can be seen at: http://www.inference.phy.cam.ac.uk/mackay/itila. Do not infringe the copyright!]

3. G. Jones and J. M. Jones, *Information and Coding Theory* (Springer, 2000), Chapters 1–5.

4. Y. Suhov & M. Kelbert, *Information Theory and Coding by Example* (Cambridge University Press, not yet published - available at the end of 2013), Relevant examples.

2.28 B8.5: Graph Theory — Prof. Alex Scott — 16 HT

Level: H-level **Method of Assessment:** Written examination.

Weight: Unit

Recommended Prerequisites: Part A Graph Theory is recommended.

Overview

Graphs (abstract networks) are among the simplest mathematical structures, but nevertheless have a very rich and well-developed structural theory. Since graphs arise naturally in many contexts within and outside mathematics, Graph Theory is an important area of mathematics, and also has many applications in other fields such as computer science.

The main aim of the course is to introduce the fundamental ideas of Graph Theory, and some of the basic techniques of combinatorics.

Learning Outcomes

The student will have developed a basic understanding of the properties of graphs, and an appreciation of the combinatorial methods used to analyze discrete structures.

Synopsis

Introduction: basic definitions and examples. Trees and their characterization. Euler circuits; long paths and cycles. Vertex colourings: Brooks' theorem, chromatic polynomial. Edge colourings: Vizing's theorem. Planar graphs, including Euler's formula, dual graphs. Maximum flow - minimum cut theorem: applications including Menger's theorem and Hall's theorem. Tutte's theorem on matchings. Extremal Problems: Turan's theorem, Zarankiewicz problem, Erdős-Stone theorem.

Reading

1. B. Bollobas, *Modern Graph Theory*, Graduate Texts in Mathematics 184 (Springer-Verlag, 1998)

Further Reading

1. J. A. Bondy and U. S. R. Murty, *Graph Theory: An Advanced Course*, Graduate Texts in Mathematics 244 (SpringerVerlag, 2007).

2. R. Diestel, *Graph Theory*, Graduate Texts in Mathematics 173 (third edition, Springer-Verlag, 2005).

3. D. West, *Introduction to Graph Theory*, Second edition, (PrenticeHall, 2001).

2.29 SB3a: Applied Probability — Dr Matthias Winkel — 16 MT

[Teaching responsibility of the Department of Statistics. Please note, this course is offered from the schedule of Mathematics Department Units]

Level: H-Level

Method of Assessment: Written examination.

Weight: Unit

The double-unit (SB3a and SB3b) has been designed so that a student obtaining at least an upper second class mark on the double unit can expect to gain exemption from the Institute of Actuaries' paper CT4, which is a compulsory paper in their cycle of professional actuarial examinations. The first unit, clearly, and also the second unit, apply much more widely than just to insurance models.

Recommended Prerequisites: Part A Probability.

Overview

This course is intended to show the power and range of probability by considering real examples in which probabilistic modelling is inescapable and useful. Theory will be developed as required to deal with the examples.

Synopsis

Poisson processes and birth processes. Continuous-time Markov chains. Transition rates, jump chains and holding times. Forward and backward equations. Class structure, hitting times and absorption probabilities. Recurrence and transience. Invariant distributions and limiting behaviour. Time reversal. Renewal theory. Limit theorems: strong law of large numbers, strong law and central limit theorem of renewal theory, elementary renewal theorem, renewal theorem, key renewal theorem. Excess life, inspection paradox.

Applications in areas such as: queues and queueing networks - M/M/s queue, Erlang's formula, queues in tandem and networks of queues, M/G/1 and G/M/1 queues; insurance ruin models; applications in applied sciences.

Reading

1. J. R. Norris, *Markov Chains* (Cambridge University Press, 1997).

2. G. R. Grimmett and D. R. Stirzaker, *Probability and Random Processes* (3rd edition, Oxford University Press, 2001).

3. G. R. Grimmett and D. R. Stirzaker, *One Thousand Exercises in Probability* (Oxford University Press, 2001).

4. S. M. Ross, *Introduction to Probability Models* (4th edition, Academic Press, 1989).

5. D. R. Stirzaker: *Elementary Probability* (2nd edition, Cambridge University Press, 2003).

2.30 BEE “Mathematical” Extended Essay

Level: H-level **Method of Assessment:** Written extended essay.

Weight: Double unit (7,500 words).

An essay on a mathematical topic may be offered for examination at Part B as a double unit. It is equivalent to a 32-hour lecture course. Generally, students will have 8 hours of supervision distributed over Michaelmas and Hilary terms. In addition there are lectures on writing mathematics and using LaTeX in Michaelmas and Hilary terms. See the lecture list for details.

Students considering offering an essay should read the *Guidance Notes on Extended Essays and Dissertations in Mathematics* available at:

https://www.maths.ox.ac.uk/members/students/undergraduate-courses/teaching-and-learning/projects

Application

Students must apply to the Mathematics Projects Committee for approval of their proposed topic in advance of beginning work on their essay. Proposals should be addressed to the Chairman of the Projects Committee, c/o Mrs Helen Lowe, Room S0.20, Mathematical Institute and are accepted from the end of Trinity Term. All proposals must be received before 12noon on Friday of Week 0 of Michaelmas Full Term. Note that a BEE essay must have a substantial mathematical content. The application form is available at:
https://www.maths.ox.ac.uk/members/students/undergraduate-courses/teaching-and-learning/projects

Once a title has been approved, it may only be changed by approval of the Chairman of the Projects Committee.

Assessment

Each project is blind double marked. The marks are reconciled through discussion between the two assessors, overseen by the examiners. Please see the *Guidance Notes on Extended Essays and Dissertations in Mathematics* for detailed marking criteria and class descriptors.

Submission

THREE copies of your essay, identified by your candidate number only, should be sent to the Chairman of Examiners, FHS of Mathematics Part B, Examination Schools, Oxford, to arrive no later than **12noon on Monday of week 10, Hilary Term 2016**. An electronic copy of your dissertation should also be submitted via the Mathematical Institute website. Further details may be found in the *Guidance Notes on Extended Essays and Dissertations in Mathematics.*

2.31 BSP, Structured projects, MT and HT

Level: H-level **Assessment:** Written work, oral presentation, and peer review.

Weight: Double unit.

Recommended prerequisites: None.

Quota: Students will be able to choose a project from a menu of three or four possibilities. There is likely to be a cap of between four to eight on the numbers allowed to take any one project.

Learning outcomes

This option is designed to help students understand applications of mathematics to live research problems and to learn some of the necessary techniques. For those who plan to stay on for the MMath or beyond, the course will provide invaluable preliminary training. For those who plan to leave after the BA, it will offer insights into what mathematical research can involve, and training in key skills that will be of long term benefit in any career.

Students will gain experience of:

- Applications of numerical computation to current research problems.
- Reading and understanding research papers.
- Working with new people in new environments.
- Meeting the expectations of different disciplines.
- Presenting a well structured written report, using LaTeX.
- Making an oral presentation to a non-specialist audience.
- Reading and assessing the work of other students.
- Independent study and time management.

Students will be expected to:

a. Learn about a current research problem by reading one or more relevant research papers together with appropriate material from textbooks.

b. Carry out the required calculations using Maple, MuPAD or Matlab. Students are not expected to engage in original research but there will be scope for able students to envisage new directions.

c. Write up the problem and their findings in a report that is properly supported with detail, discussion, and good referencing.

d. Give an oral presentation to a non-specialist audience.

e. Undertake peer review.

In past years projects have included applications to biology, finance, and earth sciences. It is expected that a similar menu of topics, from which students will select one, will be available for 2015-2016.

Teaching

At the beginning of the course students will be given written instructions for their chosen project.

Michaelmas Term

There will be a group meeting with the organiser (Cath Wilkins) at the beginning of MT to set out expectations and deal with queries. The organisers will meet again with students individually at the end of MT. Between those meetings students will read around their chosen topic and take preparatory courses in LaTeX and Matlab, both of which are available from the department and are well documented online. Individual contact with the organisers by email, or if necessary in person, will be encouraged.

Hilary Term

Week 1
Lecture on expectations for the term, and advice on writing up.

Weeks 2 to 8
Students will meet regularly with their specialist supervisors. In addition, each student will meet at least once with one of the organisers, who will together maintain an overview of the student's progress.

Week 10
Submission of written paper.

Easter vacation

Peer review

Trinity Term

Week 1
Oral presentation

Assessment

Students (and tutors) have sometimes expressed doubts about the predictability or reliability of project assessment. We are therefore concerned:

i. to make the assessment scheme as transparent as possible both to students and to assessors;

ii. that students who produce good project work should be able to achieve equivalent grades to students who write good exam papers.

The mark breakdown will be as follows:

a. Written work 75%, of which:

    50% of available marks will be for general explanation and discussion of the problem

    50% of available marks will be for mathematical calculations and commentary

b. Oral presentation 15%

c. Peer review 10%

Note on (c):
This is a new kind assessment in Oxford mathematics, though other universities have used it with great success. As with journal peer review, the anonymity of both writer and reviewer will be strictly maintained. Each student will be expected to read one other student's project write-up and to make a careful and well explained judgement on it. Credit for this will go to the reviewer, not to the writer, whose work will already have been assessed by examiners in the usual way.

3 Other units

3.1 Statistics Options

Students in Part B may take units drawn from Part B of the Honour School of Mathematics and Statistics. For full details of these units see the syllabus and synopses for Part B of the Honour School Mathematics and Statistics, which are available on the web at http://www.stats.ox.ac.uk/current_students/bammath/course_handbooks/

The Statistics units available are as follows:

- SB1 Applied Statistics (double unit)
- SB2a Foundations of Statistical Inference (unit)
- SB3b Statistical Lifetime Models (unit; can only be taken as a double unit with SB3a)
- SB4a Actuarial Science I (unit; please note that if taken as a unit this course will not qualify candidates for an exemption from actuarial science exams)
- SB4b Actuarial Science II (unit; can only be taken as a double unit with SBS4a)

3.2 Computer Science Options

Students in Part B may take units drawn from Part B of the Honour School of Mathematics and Computing. For full details of these units see the Department of Computer Science's website (http://www.cs.ox.ac.uk/teaching/courses/)

The Computer Science units available are as follows:

- OCS1 Lambda Calculus and Types (unit)
- OCS2 Computational Complexity (unit)

3.3 Other Options

3.3.1 BO1.1: History of Mathematics — Dr Chris Hollings — 16 lectures in MT and reading course of 8 seminars in HT

Level: H-level

Assessment: 2-hour written examination paper for the MT lectures and 3000-word essay for the reading course.

Weight: Double unit.

Recommended prerequisites: None.

Quota: The maximum number of students that can be accepted will be 20.

Learning outcomes

This course is designed to provide the historical background to some of the mathematics familiar to students from A-level and the first four terms of undergraduate study, and looks at a period from approximately the mid-sixteenth century to the end of the nineteenth century. The course will be delivered through 16 lectures in Michaelmas Term, and a reading course consisting of 8 seminars (equivalent to a further 16 lectures) in Hilary Term. Guidance will be given throughout on reading, note-taking, and essay-writing.

Students will gain:

- an understanding of university mathematics in its historical context;
- an enriched understanding of the mathematical content of the topics covered by the course

  together with skills in:

- reading and analysing historical mathematical sources;
- reading and analysing secondary sources;
- efficient note-taking;
- essay-writing (from 1000 to 3000 words);
- construction of references and bibliographies;
- oral discussion and presentation.

Lectures

The Michaelmas Term lectures will cover the following material:

- Introduction.
- Seventeenth century: analytic geometry; the development of calculus; Newton's *Principia*.
- Eighteenth century: from calculus to analysis; functions, limits, continuity; equations and solvability.

- Nineteenth century: group theory and abstract algebra; the beginnings of modern analysis; sequences and series; integration; complex analysis; linear algebra.

Classes to accompany the lectures will be held in Weeks 3, 5, 6, 7. For each class students will be expected to prepare one piece of written work (1000 words) and one discussion topic.

Reading course

The Hilary Term part of the course is run as a reading course during which we will study two or three primary texts in some detail, using original sources and secondary literature. Details of the books to be read in HT 2016 will be decided and discussed towards the end of MT 2015. Students will be expected to write two essays (2000 words each) during the first six weeks of term. The course will then be examined by an essay of 3000 words to be completed during Weeks 7 to 9.

Recommended reading

Jacqueline Stedall, *Mathematics emerging: a sourcebook 1540–1900*, (Oxford University Press, 2008).

Victor Katz, *A history of mathematics* (brief edition), (Pearson Addison Wesley, 2004), or:

Victor Katz, *A history of mathematics: an introduction* (third edition), (Pearson Addison Wesley, 2009).

Benjamin Wardhaugh, *How to read historical mathematics*, (Princeton, 2010).

Supplementary reading

John Fauvel and Jeremy Gray (eds), *The history of mathematics: a reader*, (Macmillan, 1987).

Assessment

The Michaelmas Term material will be examined in a two-hour written paper at the end of Trinity Term. Candidates will be expected to answer two half-hour questions (commenting on extracts) and one one-hour question (essay). The paper will account for 50% of the marks for the course. The Reading Course will be examined by a 3000-word essay at the end of Hilary Term. The title will be set at the beginning of Week 7 and two copies of the project must be submitted to the Examination Schools by midday on Monday of Week 10. This essay will account for 50% of the marks for the course.

3.3.2 BOE "Other Mathematical" Extended Essay

Level: H-level **Method of Assessment:** Written essay.

Weight: Double unit (7,500 words).

An essay on a topic related to mathematics may be offered for examination at Part B as a double unit. It is equivalent to a 32-hour lecture course. Generally, students will have 8 hours of supervision distributed over Michaelmas and Hilary terms. In addition there are lectures on writing mathematics and using LaTeX in Michaelmas and Hilary terms. See the lecture list for details.

Students considering offering an essay should read the *Guidance Notes on Extended Essays and Dissertations in Mathematics* available at:

https://www.maths.ox.ac.uk/members/students/undergraduate-courses/teaching-and-learning/projects.

Application

Students must apply to the Mathematics Projects Committee for approval of their proposed topic in advance of beginning work on their essay. Proposals should be addressed to the Chairman of the Projects Committee, c/o Mrs Helen Lowe, Room S0.20, Mathematical Institute and are accepted from the end of Trinity Term. All proposals must be received before 12noon on Friday of Week 0 of Michaelmas Full Term. The application form is available at https://www.maths.ox.ac.uk/members/students/undergraduate-courses/teaching-and-learning/projects.

Once a title has been approved, it may only be changed by approval of the Chairman of the Projects Committee.

Assessment

Each project is blind double marked. The marks are reconciled through discussion between the two assessors, overseen by the examiners. Please see the *Guidance Notes on Extended Essays and Dissertations in Mathematics* for detailed marking criteria and class descriptors.

Submission

THREE copies of your essay, identified by your candidate number only, should be sent to the Chairman of Examiners, FHS of Mathematics Part B, Examination Schools, Oxford, to arrive no later than **12noon on Monday of week 9, Hilary Term 2016**. An electronic copy of your dissertation should also be submitted via the Mathematical Institute website. Further details may be found in the *Guidance Notes on Extended Essays and Dissertations in Mathematics*.

3.3.3 BN1: Mathematics Education and Undergraduate Ambassadors Scheme

Level: H-level **Method of Assessment:** See individual synopses for each unit

Weight: Double-unit, or BN1.1 may be taken as a unit.

BN1.1 Mathematics Education — Dr Jenni Ingram and Mr Nick Andrews — MT

Level: H-level

Method of Assessment: Two examined written assignments and a short presentation.
1500 word written assignment (not examined) due: end of week 2
2500 word written assignment (examined) due: end of week 5
Presentation (examined) week 8
3000 word written assignment (examined) due: week 1 Hilary Term

Weight: Unit.

Quota: There will be a quota of approximately 20 students for this course.

Recommended Prerequisites: None

Overview

The Mathematics Education option will be a unit, run in Michaelmas Term. The course is appropriate for all students of the appropriate degree courses, whether or not they are interested in teaching subsequently. Final credit will be based on two examined written assignments (35 % each), one of which will be submitted at the start of Hilary Term, and a presentation (30%). Teaching will be 22 hours of contact time which will include lecture, seminar, class and tutorial formats as follows:

- A two-hour lecture/class per week. These will be interactive and involve discussion and other tasks as well as input from the lecturer.
- Four hours of tutorial workshops in small groups to review ideas from the course and preparation for written assignments.

Learning Outcomes

1. Understanding:
    - the psychology of learning mathematics;
    - the nature of mathematics and the curriculum;
    - relations between teaching and learning at primary, secondary and tertiary level;
    - the role of mathematics education in society;

- issues associated with communicating mathematics.

2. Understanding connections between mathematics, education issues and the mathematical experience of learners.

3. The ability to express ideas about the study and learning of mathematics in writing, verbally, and in other forms of communication.

Synopsis

1. Introduction to mathematics education as a field of study.
    - Issues and problems; relation to learning mathematics. Introduction to course, the education library, and the expected forms of study including pair work. Setting assignment.
2. Relations between teaching and learning
    - Theories about teaching and learning, e.g. variation theory; deep/surface approaches; observable characteristics in comparative studies; task design.
3. Mathematics education in society.
    - Achievement according to class, gender, ethnicity in UK and elsewhere
4. Communicating mathematics.
    - Verbal; symbolic; diagram; gestural; dynamic representation.

Reading

Main Texts:

1. Tall, D. (1991) *Advanced Mathematical Thinking.* (Mathematics Education Library, 11). Dordrecht: Kluwer
2. Gates, P. (ed.) (2001) *Issues in Mathematics Teaching.* London: RoutledgeFalmer
3. Mason, J., Burton, L. & Stacey, K. (2010) *Thinking Mathematically.* Any edition by any publisher will do.
4. Polya, G. (1957) *How to Solve It.* Any edition by any publisher will do.
5. Davis, P. And Hersh, R. (1981) *The Mathematical Experience.* Any edition by any publisher will do.
6. Mason, J. & Johnston-Wilder, S. (2004) *Fundamental Constructs in Mathematics Education*, London: RoutledgeFalmer.
7. Carpenter, T., Dossey, J. & Koehler, J. (2004) *Classics in Mathematics Education Research.* Reston, VA: National Council of Teachers of Mathematics.

Important websites:

1. ncetm.org.uk

BN1.2 Undergraduate Ambassadors' Scheme — Mr Nick Andrews — mainly HT

[This course is not available as a unit; it must be taken alongside BN1.1 Mathematics Education]

Weight: Unit

Method of Assessment: Journal of activities, Oral presentation, Course report and project, Teacher report.

Quota: There will be a quota of approximately 10 students for this course.

Recommended Prerequisites: BN1.1 is an essential prerequisite.

Option available to Mathematics, Mathematics & Statistics, Mathematics & Philosophy students.

Learning Outcomes

The Undergraduate Ambassadors' Scheme (UAS) was begun by Simon Singh in 2002 to give university undergraduates a chance to experience assisting and, to some extent, teaching in schools, and to be credited for this. The option focuses on improving students' communication, presentation, cooperation and organizational skills and sensitivity to others' learning difficulties.

Course Description and Timing:

The Oxford UAS option, BN1.2, is a unit, mainly run in Hilary Term. A quota will be in place, of approximately 10 students, and so applicants for the UAS option will be asked to name a second alternative unit. The course is appropriate for all students, whether or not they are interested in teaching subsequently.

A student on the course will be assigned to a mathematics teacher in a local secondary school (in the Oxford, Kidlington, Wheatley area) for half a day per week during Hilary Term. Students will be expected to keep a journal of their activities, which will begin by assisting in the class, but may widen to include teaching the whole class for a part of a period, or working separately with a smaller group of the class. Students will be required at one point to give a presentation to one of their school classes relating to a topic from university mathematics, and will also run a small project based on some aspect of mathematics education with advice from the course co-ordinator and teacher/s. Final credit will be based on the journal (20%), the presentation (30%), an end of course report (approximately 3000 words) including details of the project (35%), together with a report from the teacher (15%).

Short interviews will take place on Thursday or Friday of 0th week in Michaelmas term to select students for this course. The interview (of roughly 15 minutes) will include a presentation by the student on an aspect of mathematics of their choosing. Students will be chosen on the basis of their ability to communicate mathematics, and two references will be sought from college tutors on these qualities. Applicants will be quickly notified of the decision.

Those on the course will also need to fill in a CRB form, or to have done so already. By the end of Michaelmas term students will have been assigned to a teacher and have made a first, introductory, visit to their school. The course will begin properly in Hilary term with students helping in schools for half a day each week. Funds are available to cover travel expenses. Support classes will be provided throughout Hilary for feedback and to discuss issues such as the planning of the project. The deadline for the journal and report will be noon on Monday of 1st week of Trinity term.

Any further questions on the UAS option should be passed on to the option's co-ordinator, via (director-ugrad-studies@maths.ox.ac.uk).

Reading List

Clare Tickly, Anne Watson, Candia Morgan, *Teaching School Subjects: Mathematics* (Routledge Falmer, 2004).

3.3.4 Philosophy: Double Units

Students in Part B may take units drawn from courses provided by the Faculty of Philosophy. For full details of these units see the Philosophy Faculty website http://www.philosophy.ox.ac.uk/.

Students interested in taking a Philosophy double unit are encouraged to contact their college tutors well in advance of term, to ensure that teaching arrangements can be made.

The Philosophy units available are as follows:

- 101 Early Modern Philosophy (double unit).
- 102 Knowledge and Reality (double unit).
- 122 Philosophy of Mathematics (double unit).
- 127 Philosophical Logic (double unit).

[Paper 101, 102, 122 and 127 in all Honour Schools including Philosophy. Teaching responsibility of the Philosophy Faculty.]

4 Language Classes: French and Spanish

Language courses in French and German or Spanish (in alternate years) are offered by the University Language Centre.

Students in the FHS Mathematics may apply to take language classes. In 2015-2016, French and Spanish language classes will be run in MT and HT. We have a limited number of places but if we have spare places we will offer these to joint school students , Mathematics and Computer Science, Mathematics and Philosophy and Mathematics and Statistics.

Two levels of French courses are offered, a lower level for those with a good pass at GCSE, and a higher level course for those with A/S or A level. Acceptance on either course will depend on satisfactory performance in the Preliminary Qualifying Test held in Week 1 of Michaelmas Term (usually on Monday, 17.00-19.00 at the Language Centre). Classes at both levels will take place on Mondays, 17.00-19.00. A single class in German or Spanish at a lower or higher level will be offered on the basis of the performances in the Preliminary Qualifying Test, held at the same time as the French test. Classes will also be held on Mondays, 17-00-19.00.

Performance on the course will not contribute to the class of degree awarded. However, upon successful completion of the course, students will be issued with a certificate of achievement which will be sent to their college.

Places at these classes are limited, so students are advised that they must indicate their interest at this stage. If you are interested please contact Nia Roderick (roderick@maths.ox.ac.uk or tel. 01865 615205), Academic Assistant in the Mathematical Institute, as soon as possible for an application form.

Aims and rationale

The general aim of the language courses is to develop the student's ability to communicate (in both speech and writing) in French, German and Spanish to the point where he or she can function in an academic or working environment in a French-speaking, German-speaking or Spanish-speaking country.

The courses have been designed with general linguistic competence in the four skills of reading, writing, speaking and listening in mind. There will be opportunities for participants to develop their own particular interests through presentations and assignments.

Form and Content

Each course will consist of a thirty-two contact hour course, together with associated work. Classes will be held in the Language Centre at two hours per week in Michaelmas and Hilary Terms.

The content of the courses is based on coursebooks together with a substantial amount of supplementary material prepared by the language tutors. Participants should expect to spend an additional two hours per week on preparation and follow-up work.

Each course aims to cover similar ground in terms of grammar, spoken and written language and topics. Areas covered will include:

Grammar:

- all major tenses will be presented and/or revised, including the subjunctive
- passive voice
- pronouns
- formation of adjectives, adverbs, comparatives
- use of prepositions
- time expressions

Speaking

- guided spoken expression for academic, work and leisure contact (e.g. giving presentations, informal interviews, applying for jobs)
- expressing opinions, tastes and preferences
- expressing cause, consequence and purpose

Writing

- Guided letter writing for academic and work contact
- Summaries and short essays

Listening

- Listening practice of recorded materials (e.g. news broadcasts, telephone messages, interviews)
- developing listening comprehension strategies

Topics (with related readings and vocabulary, from among the following)

- life, work and culture
- the media in each country
- social and political systems
- film, theatre and music
- research and innovation
- sports and related topics
- student-selected topics

Teaching staff

The courses are taught by Language Centre tutors or Modern Languages Faculty instructors.

Teaching and learning approaches

Each course uses a communicative methodology which demands active participation from students. In addition, there will be some formal grammar instruction. Students will be expected to prepare work for classes and homework will be set. Students will also be given training in strategies for independent language study, including computer-based language learning, which will enable them to maintain and develop their language skills after the course.

Entry

Two classes in French and one in German or Spanish (probably at Basic and Threshold levels) will be formed according to level of French/German/Spanish at entry. The minimum entry standard is a good GCSE Grade or equivalent. Registration for each option will take place at the start of Michaelmas Term. A preliminary qualifying test is then taken which candidates must pass in order to be allowed to join the course.

Learning Outcomes

The learning outcomes will be based on internationally agreed criteria for specific levels (known as the ALTE levels), which are as follows:

Basic Level (corresponds to ALTE Level 2 "Can-do" statements)

- Can express opinions on professional, cultural or abstract matters in a limited way, offer advice and understand instructions.
- Can give a short presentation on a limited range of topics.
- Can understand routine information and articles, including factual articles in newspapers, routine letters from hotels and letters expressing personal opinions.
- Can write letters or make notes on familiar or predictable matters.

Threshold Level (corresponds to ALTE Level 3 "Can-do" statements)

- Can follow or give a talk on a familiar topic or keep up a conversation on a fairly wide range of topics, such as personal and professional experiences, events currently in the news.
- Can give a clear presentation on a familiar topic, and answer predictable or factual questions.
- Can read texts for relevant information, and understand detailed instructions or advice.
- Can make notes that will be of reasonable use for essay or revision purposes.
- Can make notes while someone is talking or write a letter including non- standard requests.

Assessment

There will a preliminary qualifying test in Michaelmas Term. There are three parts to this test: Reading Comprehension, Listening Comprehension, and Grammar. Candidates who have not studied or had contact with French, German or Spanish for some time are advised to revise thoroughly, making use of the Language Centre's French, German or Spanish resources.

Students' achievement will be assessed through a variety of means, including continuous assessment of oral performance, a written final examination, and a project or assignment chosen by individual students in consultation with their language tutor.

Reading comprehension, writing, listening comprehension and speaking are all examined. The oral component may consist of an interview, a presentation or a candidate's performance in a formal debate or discussion.