Handbook for the Undergraduate Mathematics Courses
Supplement to the Handbook
Honour School of Mathematics
Syllabus and Synopses for Part C 2010–11
for examination in 2011

Contents

1 Foreword

The synopses for Part C will be available on the website at:

http://www.maths.ox.ac.uk/current-students/undergraduates/handbooks-synopses/

before the start of Michaelmas Term 2010.

See the current edition of the Examination Regulations for the full regulations governing these examinations.

Notice of misprints or errors of any kind, and suggestions for improvements in this booklet should be addressed to the Academic Assistant in the Mathematical Institute.

In the unlikely event that any course receives a very low registration we may offer this course as a reading course (this would include with some lectures but fewer classes).

Honour School of Mathematics

"Units" and "Half Units"

Students staying on to take Part C will take the equivalent of three units. The equivalent of two units must be taken from the schedule of "Mathematics Department units and half units" and may include a dissertation on a mathematical topic. Up to one unit may be taken from the schedule of "Other Units"

Most Mathematics Department lecture courses are available as half units, the exceptions being:

1. C7.4 Theoretical Physics - this is available as a whole-unit only.
2. C11.1b Probabilistic Combinatorics, where C11.1a Graph Theory is an essential pre-requisite.

All the units and half units described in this booklet are "M–Level".

Language Classes

Mathematics students may apply to take classes in a foreign language. In 2010-11 classes will be offered in French. Students' performances in these classes will not contribute to the degree classification awarded. However, successful completion of the course may be recorded on students' transcripts. See section 5 for more details.

Extract from Examination Conventions

The Examination Papers in Part C

Each half unit paper in Part C will be of one and a half hours duration and consist of three questions, each marked out of 25. Candidates may submit answers to as many questions as they wish: the best two answers will count towards the total mark for the paper.

In all papers the questions set should give a reasonable coverage of the syllabus.

Marking of Papers in Part C

For the Mathematics Department papers, mark schemes for question out of 25 will aim to ensure that the following qualitative criteria hold:

- 20-25 marks. A completely, or almost completely, correct answer, showing excellent understanding of the concepts and skill in carrying through the arguments and/or calculations.
- 13-19 marks. A good though not complete answer, showing understanding of the concepts and competence in handling the arguments and/or calculations. Such an answer might consist of an excellent answer to a substantial part of the question, or a good answer to the whole question which nevertheless shows some flaws in calculation or in understanding or in both.

This should be regarded only as a guide, conveying the intention of examiners.

Marking of Dissertations

Marks for other mathematical and non-mathematical papers will be reported to the Mathematics Examiners by the relevant assessing panel in University Standardised Mark (USM) form.

All dissertations are independently marked by at least two assessors. The examiner responsible for dissertations will oversee the reconciliation of marks. If agreement is not possible an additional assessor will be appointed.

Analysis of Marks in Part C

The Board of Examiners in Part C will assign USMs for papers taken in Part C and may recalibrate the raw marks to arrive at USMs reported to candidates. Examiners will take into account the relative difficulty of papers when assigning USMs. In order to achieve this, Examiners may use information on candidates' performances on the earlier parts of the examination when recalibrating the raw marks. They may also use other statistics to check that the USMs assigned fairly reflect the students' performances on a paper.

For the MMath. in Mathematics one of the classifications will be based on Part C alone.

Classification in the Honour School of Mathematics

Each candidate will receive a numerical mark on each paper in each Part of the examination in the University standardised range 0-100, such that

- a First Class performance (on that paper) is indicated by a mark of 70 to 100;

- an Upper Second Class performance (on that paper) is indicated by a mark of 60 to 69;
- a Lower Second Class performance (on that paper) is indicated by a mark of 50 to 59
- a Third Class performance (on that paper) is indicated by a mark of 40 to 49;
- a Pass performance (on that paper) is indicated by a mark of 30 to 39;
- a Fail performance (on that paper) is indicated by a mark of 0 to 29.

The USMs awarded to a candidate for papers in Part C will be used to arrive at a classification for Part C of the MMath. Let $Av\ USM$ = Average USM in Part C (rounded up to whole number);

- First Class:
  $Av\ USM \geq 70$
- Upper Second Class:
  $70 > Av\ USM \geq 60$
- Lower Second Class:
  $60 > Av\ USM \geq 50$
- Third Class:
  $50 > Av\ USM \geq 40$

A 'Pass' will not be awarded for Year 4. Candidates achieving:

$$Av\ USM < 40,$$

may supplicate for a BA.

[Note: Half-unit papers count as half a paper when determining the average USM.]

Candidates satisfying the Examiners for Parts A, B, and C may supplicate for an MMath. in Mathematics, with two associated classifications; for example: MMath. in Mathematics: Parts A and B - First Class; Part C - First Class

Note that successful candidates may only supplicate for one degree - either a BA or an MMath. The MMath. has two classifications associated with it but a candidate will not be awarded a BA degree and an MMath. degree.

Class Descriptors

The average USM ranges used in the classifications reflect the following descriptions:

- First Class: the candidate shows excellent skills in reasoning, deductive logic and problem-solving. He/she demonstrates an excellent knowledge of the material, and is able to use that in unfamiliar contexts.

- Upper Second Class: the candidate shows good or very good skills in reasoning, deductive logic and problem-solving. He/she demonstrates a good or very good knowledge of much of the material.
- Lower Second Class: the candidate shows adequate basic skills in reasoning, deductive logic and problem-solving. He/she demonstrates a sound knowledge of much of the material.
- Third Class: the candidate shows reasonable understanding of at least part of the basic material and some skills in reasoning, deductive logic and problem-solving.
- Pass: the candidate shows some limited grasp of at least part of the basic material.

  [Note that the aggregation rules in some circumstances allow a stronger performance on some papers to compensate for a weaker performance on others.]
- Fail: little evidence of competence in the topics examined; the work is likely to show major misunderstanding and confusion, coupled with inaccurate calculations; the answers to questions attempted are likely to be fragmentary only.

Registration

Classes

Students will have to register in advance for the classes they wish to take. Students will have to register by Friday of Week 9 of Trinity Term 2009 using the online system which can be accessed at http://www.maths.ox.ac.uk. Further guidance on how to use the online system can be found at:
http://www.maths.ox.ac.uk/help/faqs/undergrads/course-registration.

Note on Intercollegiate Classes

Where undergraduate registrations for lecture courses fall below 5, classes will not run as part of the intercollegiate scheme but will be arranged informally by the lecturer.

Lectures

Some combinations of subjects are not advised and lectures may clash. Details are given below. We will use the information on your registration forms to aim to keep clashes to a minimum. However, because of the large number of options available in Part C some clashes are inevitable, and we must aim to accommodate the maximum number of student preferences.

Lecture Timetabling in Part C, 2010–11

The Teaching Committee has agreed that the following clashes be allowed.

C1.1a Model Theory & C1.1b Godel's Incompleteness Theorems	may clash with	C6.1a Solid Mechanics
C1.2a Analytic Topology & C1.2b Axiomatic Set Theory		C6.1b Elasticity and Plasticity
C2.1a Lie Algebras & C2.1b Representation Theory of Symmetric Groups		C6.3a Perturbation Methods & C6.3b Applied Complex Variables
C2.2a Finite Group Theory & C2.2b Building Infinite Groups		C6.4a Topics in Fluid Mechanics
C3.1a Algebraic Geometry & C3.1b Algebraic Topology		C8.1a Mathematics and the Environment & C8.1b Mathematical Physiology
C9.1a Analytic Number Theory & C9.1b Elliptic Curves		All Statistics options
		C12.1a Numerical Linear Algebra & C12.1b Continuous Optimization
		C12.2a Approximation Theory & C12.2b Finite Element Methods

Mathematics vs. Computing		
C6.1a Solid Mechanics	may clash with	CCS1a Categories, Proofs and Processes
C6.1b Elasticity and Plasticity		
C6.3a Perturbation Methods & C6.3b Applied Complex Variables		
C3.1a Algebraic Geometry &C3.1b Algebraic Topology		

Other and Non-mathematical subjects		
All Statistics options	may clash with	All Philosophy papers

2 Mathematics Department Units

C1.1a: Gödel's Incompleteness Theorems — Dr Isaacson — 16MT

Level: M-level **Method of Assessment:** Written examination.

Weight: Half-unit (OSS paper code 2A60)

Recommended Prerequisites

This course presupposes basic knowledge of first-order predicate logic up to and including soundness and completeness theorems for a formal system of first-order predicate logic (B1 Logic).

Overview

The starting point is Gödel's mathematical sharpening of Hilbert's insight that manipulating symbols and expressions of a formal language has the same formal character as arithmetical operations on natural numbers. This allows the construction for any consistent formal system containing basic arithmetic of a 'diagonal' sentence in the language of that system which is true but not provable in the system. By further study we are able to establish the intrinsic meaning of such a sentence. These techniques lead to a mathematical theory of formal provability which generalizes the earlier results. We end with results that further sharpen understanding of formal provability.

Learning Outcomes

Understanding of arithmetization of formal syntax and its use to establish incompleteness of formal systems; the meaning of undecidable diagonal sentences; a mathematical theory of formal provability; precise limits to formal provability and ways of knowing that an unprovable sentence is true.

Synopsis

Gödel numbering of a formal language; the diagonal lemma. Expressibility in a formal language. The arithmetical undefinability of truth in arithmetic. Formal systems of arithmetic; arithmetical proof predicates. Σ_0-completeness and Σ_1-completeness. The arithmetical hierarchy. ω-consistency and 1-consistency; the first Gödel incompleteness theorem. Separability; the Rosser incompleteness theorem. Adequacy conditions for a provability predicate. The second Gödel incompleteness theorem; Löb's theorem. Provable Σ_1-completeness. Provability logic; the fixed point theorem. The ω-rule. The Bernays arithmetized completeness theorem; formally undecidable Δ^0_2-sentences of arithmetic.

Reading

1. Raymond M. Smullyan, *Gödel's Incompleteness Theorems* (oup, 1992).

Further Reading

1. George S. Boolos and Richard C. Jeffrey, *Computability and Logic* (3rd edition, Cambridge University Press, 1989), Chs 15, 16, 27 (pp 170–190, 268-284).
2. George Boolos, *The Logic of Provability* (Cambridge University Press, 1993), pp. 44–49.

C1.1b: Model Theory — Dr Koenigsmann — 16HT

Level: M-level **Method of Assessment:** Written examination.

Weight: Half-unit (OSS paper code 2B60).

Recommended Prerequisites

This course presupposes basic knowledge of First Order Predicate Calculus up to and including the Soundness and Completeness Theorems. Also a familiarity with (at least the statement of) the Compactness Theorem would also be desirable.

Overview

The course deepens a student's understanding of the notion of a mathematical structure and of the logical formalism that underlies every mathematical theory, taking B1 Logic as a starting point. Various examples emphasise the connection between logical notions and practical mathematics.

The concepts of completeness and categoricity will be studied and some more advanced technical notions, up to elements of modern stability theory, will be introduced.

Learning Outcomes

Students will have developed an in depth knowledge of the notion of an algebraic mathematical structure and of its logical theory, taking B1 Logic as a starting point. They will have an understanding of the concepts of completeness and categoricity and more advanced technical notions.

Synopsis

Structures. The first-order language for structures. The Compactness Theorem for first-order logic. Elementary embeddings. Löwenheim–Skolem theorems. Preservation theorems for substructures.

Categoricity for first-order theories. Types and saturation. Omitting types. The Ryll Nardzewski theorem characterizing aleph-zero categorical theories. Theories with few types.

Reading

1. D. Marker, *Model Theory: An Introduction* (Springer, 2002).
2. W. Hodges, *Shorter Model Theory* (Cambridge University Press, 1997).
3. J. Bridge, *Beginning Model Theory* (Oxford University Press, 1977). (Out of print but can be found in libraries.)

Further reading

1. All topics discussed (and much more) can also be found in W. Hodges, *Model Theory* (Cambridge University Press, 1993).

C1.2a: Analytic Topology — Dr Suabedissen — 16MT

Level: M-level **Method of Assessment:** Written examination.

Weight: Half-unit (OSS paper code 2A61)

We find it necessary to run Analytic Topology in MT. Classes for Analytic Topology will be run both in MT and HT, so that students who find themselves overburdened in MT will have the opportunity to attend classes on Analytic Topology in HT.

Recommended Prerequisites

Part A Topology; a basic knowledge of Set Theory, including cardinal arithmetic, ordinals and the Axiom of Choice, will also be useful.

Overview

The aim of the course is to present a range of major theory and theorems, both important and elegant in themselves and with important applications within topology and to mathematics as a whole. Central to the course is the general theory of compactness and Tychonoff's theorem, one of the most important in all mathematics (with applications across mathematics and in mathematical logic) and computer science.

Synopsis

Bases and initial topologies (including pointwise convergence and the Tychonoff product topology). Separation axioms, continuous and semicontinuous functions, Urysohn's lemma. Separable, Lindelöf and second countable spaces. Urysohn's metrization theorem. Filters and ultrafilters. Tychonoff's theorem. Compactifications, in particular the Alexandroff One-Point Compactification and the Stone–Čech Compactification. Connectedness and local connectedness. Components and quasi-components. Totally disconnected compact spaces, Boolean algebras and Stone spaces. Paracompactness (brief treatment).

Reading

1. S. Willard, *General Topology* (Addison–Wesley, 1970), Chs. 1–8.
2. N. Bourbaki, *General Topology* (Springer-Verlag, 1989), Ch. 1.

C1.2b: Axiomatic Set Theory — Prof. Zilber — 16HT

Level: M-level **Method of Assessment:** Written examination.

Weight: Half-unit (OSS paper code 2B61).

Recommended Prerequisites

This course presupposes basic knowledge of First Order Predicate Calculus up to and including the Soundness and Completeness Theorems, together with a course on basic set theory, including cardinals and ordinals, the Axiom of Choice and the Well Ordering Principle.

Overview

Inner models and consistency proofs lie at the heart of modern Set Theory, historically as well as in terms of importance. In this course we shall introduce the first and most important of inner models, Gödel's constructible universe, and use it to derive some fundamental consistency results.

Synopsis

A review of the axioms of ZF set theory. The recursion theorem for the set of natural numbers and for the class of ordinals. The Cumulative Hierarchy of sets and the consistency of the Axiom of Foundation as an example of the method of inner models. Levy's Reflection Principle. Gödel's inner model of constructible sets and the consistency of the Axiom of Constructibility ($V = L$). The fact that $V = L$ implies the Axiom of Choice. Some advanced cardinal arithmetic. The fact that $V = L$ implies the Generalized Continuum Hypothesis.

Reading

For the review of ZF set theory:

1. D. Goldrei, *Classic Set Theory* (Chapman and Hall, 1996).

For course topics (and much more):

1. K. Kunen, *Set Theory: An Introduction to Independence Proofs* (North Holland, 1983) (now in paperback). Review: Chapter 1. Course topics: Chapters 3, 4, 5, 6 (excluding section 5).

Further Reading

1. K. Hrbacek and T. Jech, *Introduction to Set Theory* (3rd edition, M Dekker, 1999).

C2.1a: Lie Algebras — Dr Williamson — 16MT

Level: M-level **Method of Assessment:** Written examination.

Weight: Half-unit (OSS paper code 2A62)

Recommended Prerequisites

Part B course B2a. A thorough knowledge of linear algebra and the second year algebra courses; in particular familiarity with group actions, quotient rings and vector spaces, isomorphism theorems and inner product spaces will be assumed. Some familiarity with the Jordan–Hölder theorem and the general ideas of representation theory will be an advantage.

Overview

Lie Algebras are mathematical objects which, besides being of interest in their own right, elucidate problems in several areas in mathematics. The classification of the finite-dimensional complex Lie algebras is a beautiful piece of applied linear algebra. The aims of this course are to introduce Lie algebras, develop some of the techniques for studying them, and describe parts of the classification mentioned above, especially the parts concerning root systems and Dynkin diagrams.

Learning Outcomes

Students will learn how to utilise various techniques for working with Lie algebras, and they will gain an understanding of parts of a major classification result.

Synopsis

Definition of Lie algebras, small-dimensional examples, some classical groups and their Lie algebras (treated informally). Ideals, subalgebras, homomorphisms, modules.

Nilpotent algebras, Engel's theorem; soluble algebras, Lie's theorem. Semisimple algebras and Killing form, Cartan's criteria for solubility and semisimplicity.

The root space decomposition of a Lie algebra; root systems, Cartan matrices and Dynkin diagrams. Classification of irreducible root systems. Description (with few proofs) of the classification of complex simple Lie algebras; examples.

Reading

1. J. E. Humphreys, *Introduction to Lie Algebras and Representation Theory,* Graduate Texts in Mathematics 9 (Springer-Verlag, 1972, reprinted 1997). Chapters 1–3 are relevant and part of the course will follow Chapter 3 closely.

2. B. Hall, *Lie Groups, Lie Algebras, and Representations. An Elementary Introduction,* Graduate Texts in Mathematics 222 (Springer-Verlag, 2003).

3. K. Erdmann, M. J. Wildon, *Introduction to Lie Algebras* (Springer-Verlag, 2006), ISBN: 1846280400.

Additional Reading

1. J.-P. Serre, *Complex Semisimple Lie Algebras* (Springer, 1987). Rather condensed, assumes the basic results. Very elegant proofs.

2. N. Bourbaki, *Lie Algebras and Lie Groups* (Masson, 1982). Chapters 1 and 4–6 are relevant; this text fills in some of the gaps in Serre's text.

C2.2a: Finite Group Theory — Dr Craven — 16MT

Level: M-level **Method of Assesment:** Written examination.

Weight: Half-unit. OSS paper code to follow.

Recommended Prerequisites

Part B course B2b. A thorough knowledge of the third-year group theory will be assumed. Familiarity with B2a will be an advantage, but is not necessary.

Overview

Finite group theory has changed considerably over the course of the last hundred years, with miniature revolutions every thirty years or so. This course will chart this progress by describing some important concepts in finite group theory that have emerged over this period, dealing with nilpotent and soluble groups, before considering permutation groups (with an eye towards the finite simple groups), then transfer and fusion, ending with the modern view towards fusion, that of fusion systems, which is one of main research areas of finite group theory today and has interactions with representation theory and algebraic topology.

Synopsis

Nilpotent and soluble groups: Hall's theorem. Schur–Zassenhaus theorem.

Permutation groups, Iwasawa's lemma. Primitive permutation groups.

Transfer and fusion. Alperin's fusion theorem, the focal subgroup theorem.

Fusion systems.

Reading List

1. Michael Aschbacher, *Finite Group Theory* (Cambridge, 2000). Various sections deal with topics in this course.

2. Daniel Gorenstein, *Finite Groups* (Chelsea, 1980).

3. Derek Robinson, *A Course in the Theory of Groups* (Springer-Verlag, 1996).

4. John S. Rose, *A Course on Group Theory* (Dover, 1994). Chapters 10 and 11 are relevant; all previous chapters should be known.

5. Joseph Rotman, *An Introduction to the Theory of Groups* (Springer-Verlag, 1995)

Further Reading:

All but the fourth book above have extra material not taught in this course, that would be valuable to learn.

C2.2b: Building Infinite Groups — Prof. Wilson — 16HT

Level: M-level **Method of Assesment:** Written examination.

Weight: Half-unit. OSS paper code to follow.

Recommended Prerequisites

A thorough knowledge of the second-year algebra courses; in particular, familiarity with group actions, with quotient rings and quotient groups, and with isomorphism theorems, will be assumed. Some familiarity with the general ideas of representation theory will be an advantage.

Overview

By describing old and very new methods for constructing groups from given building blocks, this course will introduce many important families of groups. These will include the following: soluble and nilpotent groups, free groups, wreath products, Golod–Shafarevich groups and branch groups. A unifying theme throughout the course will be the notion of the growth of a group, a topic of great current interest.

Synopsis

Nilpotent, polycyclic and soluble groups; examples and basic properties.

Growth of finitely generated groups. Examples: nilpotent groups and free groups.

Proof that a soluble groups has polynomial growth if and only if it is virtually nilpotent.

Golod–Shafarevich groups; torsion examples. Proof that finitely presented soluble groups require many relations.

The Gupta–Sidki group. Branch groups. Structure theory and examples with various types of growth.

Reading

D. J. S. Robinson, *A course in group theory*, Graduate Texts in Mathematics (Springer-Verlag)1995.

C3.1a: Algebraic Geometry — Dr Berczi — 16MT

Level: M-level. **Method of Assessment:** Written examination.

Weight: Half-unit (OSS paper codes to follow)

Recommended Prerequisites

Part A Group Theory and Introduction to Fields (B3 Algebraic Curves useful but no essential.

Overview

Algebraic geometry is the study of algebraic varieties: an algebraic variety is roughly speaking, a locus defined by polynomial equations. One of the advantages of algebraic geometry is that it is purely algebraically defined and applied to any field, including fields of finite characteristic. It is geometry based on algebra rather than calculus, but over the real or complex numbers it provides a rich source of examples and inspiration to other areas of geometry.

Synopsis

Affine algebraic varieties, the Zariski topology, morphisms of affine varieties. Irreducible varieties.

Projective space and general position points. Projective varieties, affine cones over projective varieties. The Zariski topology on projective varieties. The projective closure of affine variety. Morphisms of projective varieties. Projective equivalence.

Veronese morphism: definition, examples. Veronese morphisms are isomorphisms onto their image; statement, and proof in simple cases. Subvarieties of Veronese varieties. Segre maps and products of varieties, Categorical products: the image of Segre map gives the categorical product.

Coordinate rings. Hilbert's Nullstellensatz. Correspondence between affine varieties (and morphisms between them) and finitely generate reduced k-algebras (and morphisms between them). Graded rings and homogeneous ideals. Homogeneous coordinate rings.

Categorical quotients of affine varieties by certain group actions. The maximal spectrum.

Discrete invariants projective varieties: degree dimension, Hilbert function. Statement of theorem defining Hilbert polynomial.

Quasi-projective varieties, and morphisms of them. The Zariski topology has a basis of affine open subsets. Rings of regular functions on open subsets and points of quasi-projective varieties. The ring of regular functions on an affine variety in the coordinate ring. Localisation and relationship with rings of regular functions.

Tangent space and smooth points. The singular locus is a closed subvariety. Algebraic re-formulation of the tangent space. Differentiable maps between tangent spaces.

Function fields of irreducible quasi-projective varieties. Rational maps between irreducible varieties, and composition of rational maps. Birational equivalence. Correspondence between dominant rational maps and homomorphisms of function fields. Blow-ups: of affine space at appoint, of subvarieties of affine space, and general quasi-projective varieties along general subvarieties. Statement of Hironaka's Desingularisation Theorem. Every irreducible variety is birational to hypersurface. Re-formulation of dimension. Smooth points are a dense open subset.

Reading

KE Smith et al, *An Invitation to Algebraic Geometry*, (Springer 2000), Chapters 1–8.

Further Reading

1. M Reid, *Undergraduate Algebraic Geometry*, LMS Student Texts 12, (Cambridge 1988).
2. K Hulek, *Elementary Algebraic Geometry*, Student Mathematical Library 20. (American Mathematical Society, 2003).

C3.1b: Algebraic Topology — Prof. Lackenby — 16HT

Level: M-level. **Method of Assessment:** Written examination.

Weight: Half-unit (OSS paper codes to follow).

Overview

Homology theory is a subject that pervades much of modern mathematics. Its basic ideas are used in nearly every branch, pure and applied. In this course, the homology groups of topological spaces are studied. These powerful invariants have many attractive applications. For example we will prove that the dimension of a vector space is a topological invariant and the fact that 'a hairy ball cannot be combed'.

Learning Outcomes

At the end of the course, students are expected to understand the basic algebraic and geometric ideas that underpin homology and cohomology theory. These include the CUP product and Poincaré Duality for manifolds. They should be able to choose between the different homology theories and to use calculational tools such as the Mayer-Vietoris sequence to compute the homology and cohomology of easy examples, including projective spaces, surfaces, certain simplicial spaces and cell complexes. At the end of the course, students should also have developed a sense of how the ideas of homology and cohomology may be applied to problems from other branches of mathematics.

Synopsis

Chain complexes of Abelian groups and their homology. Short exact sequences. Delta (and simplicial) complexes and their homology. Euler characteristic.

Singular homology of topological spaces. Relation of the first homology group to the fundamental group. Relative homology and the Five Lemma. Homotopy invariance and excision (details of proofs not examinable). Mayer-Vietoris Sequence. Equivalence of simplicial and singular homology. Axioms of homology.

Degree of a self-map of a sphere. Cell complexes and cellular homology. Application: the hairy ball theorem.

Cohomology of spaces and the Universal Coefficient Theorem (proof not examinable). CUP products. Künneth Theorem (without proof). Topological manifolds and orientability. The fundamental class of an orientable, closed manifold and the degree of a map between manifolds of the same dimension. Poincaré Duality (without proof).

Reading

1. A. Hatcher, *Algebraic Topology* (Cambridge University Press, 2001). Chapter 3 and 4.
2. G. Bredon, *Topology and Geometry* (Springer, 1997). Chapter 4 and 5.
3. J. Vick, *Homology Theory,* Graduate Texts in Mathematics 145 (Springer, 1973).

C4.1a: Functional Analysis — Prof. Haydon — 16MT

Level: M-level **Method of Assessment:** Written examination.

Weight: Half-unit (OSS paper code 2A64)

Recommended Prerequisites

Part A Topology, B4 Analysis

Overview

This course builds on B4, by extending the theory of Banach spaces and operators. As well as developing general methods that are useful in Operator Theory, we shall look in more detail at the structure and special properties of "classical" sequence-spaces and function-spaces.

Synopsis

Normed spaces and Banach spaces; dual spaces, subspaces, direct sums and completions; quotient spaces and quotient operators.

Baire's Category Theorem and its consequences (review).

Classical Banach spaces and their duals; smoothness and uniform convexity of norms.

Compact sets and compact operators. Ascoli's theorem.

Hahn–Banach extension and separation theorems; the bidual space and reflexivity.

Weak and weak* topologies. The Banach–Alaoglu theorem and Goldstine's theorem. Weak compactness.

Schauder bases; examples in classical spaces. Gliding-hump arguments.

Fredholm operators.

Reading

1. M. Fabian et al., *Functional Analysis and Infinite-Dimensional Geometry* (Canadian Math. Soc, Springer 2001), Chapters 1,2,3,6,7.

Alternative Reading

1. N. L. Carothers, *A Short Course on Banach Space Theory*, (LMS Student Text, Cambridge University Press 2004).

C4.1b: Banach and C*- algebras — Dr Edwards — 16HT

Level: M-level **Method of Assessment:** Written examination.

Weight: Half-unit (OSS paper code 2B64).

Recommended Prerequisites

B4 Analysis.

Overview

The suggestion that the observables of quantum mechanics should be represented by linear operators on Hilbert spaces led Murray and von Neumann into a detailed study of algebras of bounded linear operators on Hilbert space. The abstract version of such algebras are known as C*-algebras and investigations into their structure continue today. Their importance now extends beyond functional analysis and physics into geometry and number theory.

Learning Outcome

The outcome of this course should be the appreciation and understanding of the main problems in Banach algebras and C*-algebras. Details are included in the synopsis below.

Synopsis

The course begins with an introduction to the general theory of Banach algebras and C*-algebras culminating in the Gelfand Representation Theorem for commutative C*-algebras. The second part of the course concentrates on the relationship that exists between the algebraic and geometric properties of non-commutative C*-algebras. The final part concerns itself with representations of C*-algebras on Hilbert spaces culminating with the Gelfand–Naimark Theorem.

Reading

1. G.I. Murphy, *C*-algebras and Operator Theory* (Academic, 1990), Chs. 2, 3 and parts of Ch. 5.

Alternative (more advanced) sources:

1. G.K. Pedersen, *C*-algebras and their Automorphism Groups* (Academic, 1979).
2. S. Sakai, *C*-algebras and W*-algebra* (Springer, 1971).

C5.1a Methods of Functional Analysis for PDEs — Dr Kristensen — 16MT

Only available to students who have not offered C5.1a Methods of Functional Analysis for PDEs at Part B.

Level: M-level **Method of assessment:** Written examination.

Weight: Half-unit (OSS paper code 2A65)

Recommended Prerequisites

Part A Integration. There will be a 'Users' Guide to Integration' on the subject website and anyone who has not done Part A Integration can read it up over the summer vacation. In addition some knowledge of functional analysis, in particular Banach spaces (as in B4) and compactness (as in Part A Topology), is useful. We will however recall the relevant definitions as we go along so these prerequisites are not strictly needed.

Overview

The course will introduce some of the modern techniques in partial differential equations that are central to the theoretical and numerical treatments of linear and nonlinear partial differential equations arising in science, geometry and other fields.

It provides valuable background for the Part C courses on Calculus of Variations, Fixed Point Methods for Nonlinear PDEs, and Finite Element Methods.

Learning Outcomes

Students will learn techniques and results, such as Sobolev spaces, weak convergence, weak solutions, the direct method of calculus of variations, embedding theorems, the Lax–Milgram theorem and the Fredholm Alternative and how to apply these to obtain existence and uniqueness results for linear and nonlinear elliptic partial differential equations.

Synopsis

Part I Function Spaces:

Why are function spaces important for partial differential equations?

Review of relevant definitions from functional analysis, including Banach and Hilbert spaces, separability and dual spaces. The Lebesgue spaces $L^p(\Omega)$, $1 \leq p \leq \infty$, where $\Omega \subset \mathbb{R}^n$ is open. Minkowski and Hölder inequalities. Elementary properties of Lebesgue spaces: completeness and separability. The dual space of $L^p(\Omega)$ is $L^{p'}(\Omega)$ for $\frac{1}{p} + \frac{1}{p'} = 1$.

Weak and weak* convergence in L^p spaces: oscillation and concentration. Equi–integrability and Vitali's Convergence Theorem. Examples. A bounded sequence in the dual of a separable Banach space has a weak* convergent subsequence. Statement of Mazur's Lemma.

Mollifiers and the density of smooth functions in L^p for $1 \leq p < \infty$.

Definition of weak derivatives and their uniqueness. Comparison with classical derivatives and absolutely continuous functions of one variable. Statement of the Fundamental Theorem of Calculus for absolutely continuous functions. Definition of Sobolev space $W^{m,p}(\Omega)$, $1 \leq p \leq \infty$. $H^m(\Omega) = W^{m,2}(\Omega)$. Definitions of $W_0^{1,p}(\Omega)$ and $W_g^{1,p}(\Omega)$, $1 \leq p < \infty$ and $g \in W^{1,p}(\Omega)$. Lipschitz functions and other examples. Local spaces L^p_{loc}, $W^{1,p}_{\text{loc}}$. Brief introduction to distributions. Difference—quotient characterization of $W^{1,2}_{\text{loc}}$.

Part II Elliptic Problems:

The direct method of calculus of variations: The Poincaré inequality. Proof of the existence and uniqueness of a weak solution to Poisson's equation $-\Delta u = f$, with Dirichlet boundary conditions of Sobolev class and $f \in L^2(\Omega)$, with Ω bounded. Elementary L^2 regularity theory, including difference-quotient method. Brief discussion of other types of regularity.

The Lax–Milgram theorem and Gårding's inequality. Existence and uniqueness of weak solutions to general linear uniformly elliptic equations.

Embedding theorems (proofs omitted except $W^{1,1}(a,b) \hookrightarrow C[a,b]$).

Brief discussion of compact operators, self–adjoint operators and the Fredholm Alternative.

A nonlinear elliptic problem treated by the direct method.

Reading

Lawrence C. Evans, *Partial differential equations*, (Graduate Studies in Mathematics 2004), American Mathematical Society

Elliott H. Lieb and Michael Loss, *Analysis*, 2nd Edition, (Graduate Studies in Mathematics 2001), American Mathematical Society

Additional Reading

E. Kreyszig, *Introductory Functional Analysis with Applications*, Wiley (revised edition, 1989)

P.D. Lax *Functional analysis* (Wiley-Interscience, New York, 2002).

J. Rauch, *Partial differential equations*, (Springer–Verlag, New York, 1992).

C5.1b Fixed Point Methods for Nonlinear PDEs — Prof. Niethammer — 16HT

Level: M-level **Method of assessment:** Written examination.

Weight: Half-unit (OSS paper code 2B75)

Recommended Prerequisites

C5.1a: Methods of Functional Analysis for PDEs. Some knowledge of functional analysis, in particular Banach spaces (as in B4) and compactness (as in Part A Topology), is useful.

Overview

This course gives an introduction to the techniques of nonlinear functional analysis with emphasis on the major fixed point theorems and their applications to nonlinear differential equations and variational inequalities, which abound in applications such as fluid and solid mechanics, population dynamics and geometry.

Learning Outcomes

Besides becoming acquainted with the fixed point theorems of Banach, Brouwer and Schauder, students will see the abstract principles in a concrete context. Hereby they also reinforce techniques from elementary topology, functional analysis, Banach spaces, compactness methods, calculus of variations and Sobolev spaces.

Synopsis

Examples of nonlinear differential equations and variational inequalities. Banach's fixed point theorem and applications. Brouwer's fixed point theorem, proof via Calculus of Variations and Null-Lagrangians. Compact operators and Schauder's fixed point theorem. Applications of Schauder's fixed point theorem to nonlinear elliptic equations. Weak convergence in Lebesgue- and Sobolev spaces; Variational inequalities and monotone operators. Applications of monotone operator theory to nonlinear elliptic equations (p-Laplace, stationary Navier-Stokes)

Further Reading

1. E. Zeidler, *Nonlinear Functional Analysis I & II* (Springer-Verlag, 1986/89).
2. M. S. Berger, *Nonlinearity and Functional Analysis* (Academic Press, 1977).
3. K. Deimling, *Nonlinear Functional Analysis* (Springer-Verlag, 1985).
4. Lawrence C. Evans, *Partial Differential Equations,* Graduate Studies in Mathematics (American Mathematical Society, 2004).
5. L. Nirenberg, *Topics in Nonlinear Functional Analysis,* Courant Institute Lecture Notes (American Mathematical Society, 2001).
6. R.E. Showalter, *Monotone Operators in Banach Spaces and Nonlinear Partial Differential Equations,* Mathematical Surveys and Monographs, vol.49 (American Mathematical Society, 1997).

C5.2b: Calculus of Variations — Prof. Seregin — 16HT

Weight: Half-unit, OSS paper code 2B65

Recommended Prerequisites

C5.1a: Methods of Functional Analysis for PDEs. Some familiarity with the Lebesgue integral is essential, and some knowledge of elementary functional analysis (e.g. Banach spaces and their duals, weak convergence) an advantage.

Overview

The aim of the course is to give a modern treatment of the calculus of variations from a rigorous perspective, blending classical and modern approaches and applications.

Learning Outcomes

Students will learn rigorous results in the classical and modern one-dimensional calculus of variations and see possible behaviour and application of these results in examples. They will see some examples of multi-dimensional problems.

Synopsis

Classical and modern examples of variational problems (e.g. brachistochrone, models of phase transformations).

One-dimensional problems, function spaces and definitions of weak and strong relative minimizers. Necessary conditions; the Euler–Lagrange and Du Bois–Reymond equations, theory of the second variation, the Weierstrass condition. Sufficient conditions; field theory and sufficiency theorems for weak and strong relative minimizers. The direct method of the calculus of variations and Tonelli's existence theorem. Regularity of minimizers. Examples of singular minimizers and the Lavrentiev phenomenon. Problems whose infimum is not attained. Relaxation and generalized solutions. Isoperimetric problems and Lagrange multipliers.

Multi-dimensional problems, done via some examples.

Reading

1. G. Buttazzo, M. Giaquinta, S. Hildebrandt, *One-dimensional Variational Problems,* Oxford Lecture Series in Mathematics, Vol. 15 (Oxford University Press, 1998). Ch 1, Sections 1.1, 1.2 (treated differently in course), 1.3, Ch 2 (background), Ch 3, Sections 3.1, 3.2, Ch 4, Sections 4.1, 4.3.

Additional Reading

1. U. Brechtken-Manderscheid, *Introduction to the Calculus of Variations* (Chapman & Hall, 1991).
2. H. Sagan, *Introduction to the Calculus of Variations* (Dover, 1992).
3. J. Troutman, *Variational Calculus and Optimal Control* (Springer-Verlag, 1995).

C6.1a: Solid Mechanics — Dr Ortner — 16 MT

Level: M-level **Method of Assessment:** Written examination.

Weight: Half-unit. OSS paper code 2466.

Recommended Prerequisites
There are no formal prerequisites. In particular it is not necessary to have taken any courses in fluid mechanics, though having done so provides some background in the use of similar concepts. Use is made of some elementary linear algebra in $\mathbb{R}^3$ (for example, eigenvalues, eigenvectors and diagonalization of symmetric matrices), and revision of this material, for example from the Mods Linear Algebra course, is useful preparation. The necessary material is summarized in the course.

The course is useful preparation for C6.1b Elasticity and Plasticity. Taken together the two courses will provide a broad overview of modern solid mechanics, with a variety of approaches.

Overview

Solid mechanics is a vital ingredient of materials science and engineering, and is playing an increasing role in biology. It has a rich mathematical structure. The aim of the course is to derive the basic equations of elasticity theory, the central model of solid mechanics, and give some interesting applications to the behaviour of materials.

Learning Outcomes

Students will learn basic techniques of modern continuum mechanics, such as kinematics of deformation, stress, constitutive equations and the relation between nonlinear and linearized models. They will also gain an insight into some recent developments in applications of mathematics to a variety of different materials.

Synopsis

Kinematics: Lagrangian and Eulerian descriptions of motion, deformation gradient, invertibility

Analysis of strain: polar decomposition, stretch tensors, Cauchy–Green tensors Stress Principle: forces in continuum mechanics, balance of forces, Cauchy stress tensor, the Piola–Kirchhoff stress

Constitutive Models: stress-strain relations, hyperelasticity and stored energy function, boundary value problems, the variational problem, frame indifference, material symmetry, isotropic materials

Further topics: incompressible elasticity, linearized elasticity, brittle fracture.

Reading

1. O. Gonzales and A. Stuart, *A first course in continuum mechanics*, (Cambridge University Press, 2008).
2. M. E. Gurtin, *A introduction to continuum mechanics*, (Academic Press, 1981).

Further Reading

1. P. G. Ciarlet, *Mathematical Elasticity.* Vol. I Three-dimensional Elasticity, (North-Holland, 1988)
2. S. S. Antman, Nonlinear *Problems of Elasticity*, (Springer, 1995)
3. J. E. Marsden and T.J.R. Hughes, *Mathematical Foundations of Elasticity*, Prentice–Hall, 1983

C6.1b: Elasticity and Plasticity — Prof. Goriely — 16HT

Level: M-level **Method of Assessment:** Written examination.

Weight: Half-unit. OSS paper code 2467.

Recommended Prerequisites

Familiarity with classical and fluid mechanics (Part A) and simple perturbation theory (Part C course) will be useful. The complementary Part C course, Solid Mechanics, will be especially useful.

Overview

The course gives a rapid review of mathematical models for basic solid mechanics. Benchmark solutions are reviewed for static problems and wave propagation in linear-elastic materials. It is then shown how these results can be used as a basis for practically useful problems involving rods, plates, and shells. Also simple geometrically nonlinear models will be introduced to explain buckling, contact and fracture at the most basic level. Yield and plasticity will be discussed at a similar level, both microscopically and macroscopically and there will be a brief introduction to composite fields (composite materials, thermo- and visco-elasticity).

Synopsis

Review of tensors, conservation laws, Navier equations. Antiplane strain, plain strain, torsion. Elastic wave propagation, Rayleigh waves. Ad hoc approximations for thin materials; simple bifurcation theory and buckling; simple mixed boundary value problems, brittle fracture and smooth contact; simple ideas about homogenization, composite materials, thermo- and visco-elasticity.

Reading

1. R.M. Hill, *Mathematical Theory of Plasticity* (Oxford Clarendon Press, 1998).
2. A.E.H. Love, *Treatise on the Mathematical Theory of Elasticity* (Dover, 1944).
3. L.D. Landau and E.M. Lifshitz, *Theory of Elasticity* (Pergamon Press, 1986).

C6.3a: Perturbation Methods — Prof. Maini — 16MT

Level: M-level **Method of Assessment:** Written examination.

Weight: Half-unit (OSS paper code 2A68)

Recommended Prerequisites

Part A Differential Equations and Core Analysis (Complex Analysis). B5, B6 and B8 are helpful but not officially required.

Overview

Perturbation methods underlie numerous applications of physical applied mathematics: including boundary layers in viscous flow, celestial mechanics, optics, shock waves, reaction-diffusion equations, and nonlinear oscillations. The aims of the course are to give a clear and systematic account of modern perturbation theory and to show how it can be applied to differential equations.

Synopsis

Asymptotic expansions. Asymptotic evaluation of integrals (including Laplace's method, method of stationary phase, method of steepest descent). Regular and singular perturbation theory. Multiple-scale perturbation theory. WKB theory and semiclassics. Boundary layers and related topics. Applications to nonlinear oscillators. Applications to partial differential equations and nonlinear waves.

Reading

1. E.J. Hinch, *Perturbation Methods* (Cambridge University Press, 1991), Chs. 1–3, 5–7.
2. C.M. Bender and S.A. Orszag, *Advanced Mathematical Methods for Scientists and Engineers* (Springer, 1999), Chs. 6, 7, 9–11.
3. J. Kevorkian and J.D. Cole, *Perturbation Methods in Applied Mathematics* (Springer-Verlag, 1981), Chs. 1, 2.1–2.5, 3.1, 3.2, 3.6, 4.1, 5.2.

C6.3b: Applied Complex Variables — Dr Oliver — 16HT

Level: M-level **Method of Assessment:** Written examination.

Weight: Half-unit in C6.3b (OSS paper code 2B68).

Recommended Prerequisites

The course requires second year core analysis (complex analysis). It continues the study of complex variables in the directions suggested by contour integration and conformal mapping. Part A Fluid Dynamics and Waves is desirable as it provides motivation for some of the topics studied.

Overview

The course begins where core second-year complex analysis leaves off, and is devoted to extensions and applications of that material. It is assumed that students will be familiar with inviscid two-dimensional hydrodynamics (Part A Fluid Dynamics and Waves) to the extent of the existence of a harmonic stream function and velocity potential in irrotational icompressible flow, and Bernoulli's equation.

Synopsis

1–2 Review of core real and complex analysis, especially contour integration, Fourier transforms.

2–4 Conformal mapping. Riemann mapping theorem (statement only). Schwarz–Christoffel formula. Solution of Laplace's equation by conformal mapping onto a canonical domain.

5–6 Applications to inviscid hydrodynamics: flow past an aerofoil and other obstacles by conformed mapping; free streamline flows, hodograph plane.

7–8 Flow with free boundaries in porous media. Construction of solutions using conformal mapping. The Schwarz function.

9–16 Transform methods, complex Fourier transform. Contour integral solutions of ODEs. Applications of Cauchy integrals and Plemelj formulae. Solution of mixed boundary value problems motivated by thin aerofoil theory and the theory of cracks in elastic solids. Riemann Hilbert problems. Wiener–Hopf method.

Reading

1. M. J. Ablowitz and A. S. Fokas, *Complex Variables: Introduction and Applications* (2nd edition, Cambridge University Press., Cambridge, 2003). ISBN 0521534291.

2. J. Ockendon, Howison, Lacey and Movichan, *Applied Partial Differential Equations* (Oxford, 1999) Pages 195–212.

3. G. F. Carrier, M. Krook and C. E. Pearson, *Functions of a Complex Variable* (McGraw–Hill, New York, 1966). (Reprinted by Hod Books, 1983.) ISBN 0962197300 (Out of print).

C6.4a: Topics in Fluid Mechanics — Dr Gaffney & Dr Peppin — 16MT

Level: M-level **Method of Assessment:** Written examination,

Weight: Half-unit. OSS paper code 2A74.

Recommended Prerequisites

B6 fluid mechanics.

Overview

The course will expand and illuminate the 'classical' fluid mechanics taught in the third year course B6, and illustrate its modern application in a number of different areas in industry and geoscience.

Synopsis

Convection and solidification: Earth's mantle and core; magma chambers; sea ice. Stability, boundary layers, parameterised convection.

Rotating flows: atmosphere and oceans. Waves, geostrophy, quasi-geostrophy, baroclinic instability.

Thin film flows: coatings and foams. Lubrication theory: gravity flows, Marangoni effects. Droplet dynamics, contact lines, menisci. Drying and wetting.

Reading

1. J. S. Turner, *Buoyancy Effects in Fluids* (Cambridge University Press, Cambridge, 1973).
2. A. E. Gill, *Atmosphere-Ocean Dynamics* (Academic Press, San Diego, 1982).
3. J. Pedlosky, *Geophysical Fluid Dynamics* (Springer-Verlag, Berlin, 1979).
4. D. A. Drew and S. L. Passman,*Theory of Multicomponent Fluids* (Springer-Verlag, Berlin, 1999).
5. P. B. Whalley, *Boiling, Condensation and Gas-Liquid Flow* (Oxford University Press, Oxford, 1987).
6. D. Weaire and S. Hutzler, *The Physics of Foams* (Oxford University Press, Oxford, 1999).

Further Reading

1. G. K. Batchelor, H. K. Moffatt and M. G. Worster (eds.), *Perspectives in Fluid Dynamics* (Cambridge University Press, Cambridge, 2000).

C7.1b: Quantum Theory and Quantum Computers — Dr Hannabuss — 16HT

Level: M-level **Method of Assessment:** Written examination.

Weight: Half-unit, OSS paper code 2A78.

Prerequisites

B7.1a Quantum Mechanics.

Overview

This course builds directly on the first course in quantum mechanics and covers a series of important topics, particularly features of systems containing several particles. The behaviour of identical particles in quantum theory is more subtle than in classical mechanics, and an understanding of these features allows one to understand the periodic table of elements and the rigidity of matter. It also introduces a new property of entanglement linking particles which can be quite widely dispersed.

There are rarely neat solutions to problems involving several particles, so usually one needs some approximation methods. In very complicated systems, such as the molecules of gas in a container, quantum mechanical uncertainty is compounded by ignorance about other details of the system and requires tools of quantum statistical mechanics.

Two state quantum systems enable one to encode binary information in a new way which permits superpositions. This leads to a quantum theory of information processing, and by exploiting entanglement to other ideas such as quantum teleportation.

Learning Outcomes

Students will be able to demonstrate knowledge and understanding of quantum mechanics of many particle systems, statistics, entanglement, and applications to quantum computing.

Synopsis

Identical particles, symmetric and anti-symmetric states, Fermi-Dirac and Bose-Einstein statistics and atomic structure.

Heisenberg representation, interaction representation, time dependent perturbation theory and Feynman–Dyson expansion. Approximation methods, Rayleigh-Schrö)dinger time-independent perturbation theory and variation principles. The virial theorem. Helium.

Mixed states, density operators. The example of spin systems. Purification. Gibbs states and the KMS condition.

Entanglement. The EPR paradox, Bell's inequalities, Aspect's experiment.

Quantum information processing, qubits and quantum computing. The no-cloning theorem, quantum teleportation. Quantum logic gates. Quantum operations. The quantum Fourier transform.

Reading

K. Hannabuss, *Introduction to Quantum Mechanics* (oup, 1997). Chapters 10–12 and 14, 16, supplemented by lecture notes on quantum computers on the web.

Further Reading

A popular non-technical account of the subject:

A. Hey and P. Walters, *The New Quantum Universe* (Cambridge, 2003).

Also designed for an Oxford course, though only covering some material:

I. .P Grant, *Classical and Quantum Mechanics*, Mathematical Institute Notes (1991).

A concise account of quantum information theory:

S. Stenholm and K.-A. Suominen, *Quantum Approach to Informatics* (Wiley, 2005).

An encyclopaedic account of quantum computing:

M. A. Nielsen and I. L. Chuang, *Quantum Computation* (Cambridge University Press, 2000).

Even more paradoxes can be found in:

Y. Aharanov and D. Rohrlich, *Quantum Paradoxes* (Wiley–VCH, 2005).

Those who read German can find further material on entanglement in:

J. Audretsch, *Verschränkte Systeme* (Wiley–VCH, 2005).

Other accounts of the first part of the course:

L. I. Schiff, *Quantum Mechanics* (3rd edition, Mc Graw Hill, 1968).

B. J. Bransden and C. J. Joachain, *Introduction to Quantum Mechanics* (Longman, 1995).

A. I. M. Rae, *Quantum Mechanics* (4th edition, Institute of Physics, 1993).

John Preskill's on-line lecture notes (http://www.theory.caltech.edu/ preskill/ph219/index.html).

C7.2a: General Relativity I — Dr Reid-Edwards — 16MT

Level: M-level **Method of Assessment:** Written examination.

Weight: Half-unit. OSS paper code 2B78.

Recommended Prerequisites

B7.2a Relativity and Electromagnetism.

Overview

The course is intended as an elementary introduction to general relativity, the basic physical concepts of its observational implications, and the new insights that it provides into the nature of space time, and the structure of the universe. Familiarity with special relativity and electromagnetism as covered in the B7 course will be assumed. The lectures will review Newtonian gravitation, tensor calculus and continuum physics in special relativity, physics in curved space time and the Einstein field equations. This will suffice for an account of simple applications to planetary motion, the bending of light and the existence of black holes.

Learning Outcomes

This course starts by asking how the theory of gravitation can be made consistent with the special-relativistic framework. Physical considerations (the principle of equivalence, general covariance) are used to motivate and illustrate the mathematical machinery of tensor calculus. The technical development is kept as elementary as possible, emphasising the use of local inertial frames. A similar elementary motivation is given for Einstein's equations and the Schwarzschild solution. Orbits in the Schwarzschild solution are given a unified treatment which allows a simple account of the three classical tests of Einstein's theory. Finally, the analysis of extensions of the Schwarzschild solution show how the theory of black holes emerges and exposes the radical consequences of Einstein's theory for space-time structure. Cosmological solutions are not discussed.

The learning outcomes are an understanding and appreciation of the ideas and concepts described above.

Synopsis

Review of Newtonian gravitation theory and problems of constructing a relativistic generalisation. Review of Special Relativity. The equivalence principle. Tensor formulation of special relativity (including general particle motion, tensor form of Maxwell's equations and the energy momentum-tensor of dust). Curved space time. Local inertial coordinates. General coordinate transformations, elements of Riemannian geometry (including connections, curvature and geodesic deviation). Mathematical formulation of General Relativity, Einstein's equations (properties of the energy-momentum tensor will be needed in the case of dust only). The Schwarzschild solution; planetary motion, the bending of light, and black holes.

Reading

1. L.P. Hughston and K.P. Tod, *An Introduction to General Relativity,* LMS Student Text 5 (London Mathematical Society, Cambridge University Press, 1990), Chs 1–18.

2. N.M.J. Woodhouse, *Notes on Special Relativity*, Mathematical Institute Notes. Revised edition; published in a revised form as *Special Relativity, Lecture notes in Physics m6* (Springer-Verlag, 1992), Chs 1–7

Further Reading

1. B. Schutz, *A First Course in General Relativity* (Cambridge University Press, 1990).

2. R.M. Wald, *General Relativity* (Chicago, 1984).

3. W. Rindler, *Essential Relativity* (Springer-Verlag, 2nd edition, 1990).

C7.4 — Theoretical Physics

Note: This unit is offered by the Physics Department.

Level: M-level **Method of Assessment:** Written examination.

Weight: Whole-unit only. OSS paper code 2756.

Recommended Prerequisites

Part A Electromagnetism, Part A Classical Mechanics, B7.1a Quantum Mechanics, C7.1b Quantum Theory and Quantum Computers, B7.2a Special Relativity and Electromagnetism.

Overview

This course is intended to give an introduction to some aspects of field theory and related ideas. These are important in particular for treating systems with an infinite number of degrees of freedom. An aim is to present some core ideas and important applications in a unified way. These applications include the classical mechanics of continuum systems, the quantum mechanics and statistical mechanics of many-particle systems, and some basic aspects of relativistic quantum field theory.

C7.4a Theoretical Physics I — Prof. Chalker, Prof. Essler & Prof. Lucas — 24MT

Synopsis

1. The mathematical description of systems with an infinite number of degrees of freedom: functionals, functional differentiation, and functional integrals. Multi-dimensional Gaussian integrals. Random fields: properties of a Gaussian field. Perturbation theory for non-Gaussian functional integrals. Path integrals and quantum mechanics. Treatment of free particle and of harmonic oscillator. [5 lectures]

2. Classical field theory: fields, Lagrangians and Hamiltonians. The least action principle and field equations. Space-time and internal symmetries: $U(1)$ example, Noether current. The idea of an irreducible representation of a group. Irreducible representations of $SU(2)$ and application to global internal symmetry. Simple representations of the Lorentz group via $SU(2) \times SU(2)$ without proof. $U(1)$ gauge symmetry, action of scalar QED and derivation of Maxwell's equations in covariant form. [5 lectures]

3. Landau theory and phase transitions: phase diagrams, first-order and continuous phase transitions. Landau–Ginsburg–Wilson free energy functionals. Examples including liquid crystals. Critical phenomena and scaling theory. [5 lectures]

4. The link between quantum mechanics and the statistical mechanics of one-dimensional systems via Wick rotation. Transfer matrices for one-dimensional systems in statistical mechanics. [4 lectures]

5. Stochastic processes and path integrals: the Langevin and Fokker–Planck equation. Brownian motion of single particle. Rouse model of polymer dynamics. [4 lectures]

C7.4b — Theoretical Physics II — Prof. Chalker, Prof. Essler & Prof. Lucas — 16HT

Synopsis

1. Canonical quantisation and connection to many body theory: quantised elastic waves; quantisation of free scalar field theory; many-particle quantum systems. [4 lectures]
2. Path integrals and quantum field theory: generating functional and free particle propagator for scalar and $U(1)$ gauge fields (in Lorentz gauge). [5 lectures]
3. Perturbation theory at tree level for decay and scattering processes. Examples from pure scalar theories and scalar QED. Goldstone theorem. [4 lectures]
4. Canonical transformations in quantum field theory: Bogoliubov transformations applied to bose condensates, magnons in antiferromagnets, and to BCS theory. [4 lectures]

[Total: 40 lectures]

Reading

The lecturers are aware of no book that presents all parts of this course in a unified way and at an appropriate level. For this reason, detailed lecture notes will be made available.

Some books that cover parts of the course are:

1. D. Bailin and A. Love, *Introduction to Gauge Field Theory* Graduate Student Series in Physics, mainly chapters 1 – 6 for sections 1, 2, 6, 7 and 8.
2. R. P. Feynman, *Statistical Mechanics* A Set of Lectures, mainly chapters 3, 4 and 6 for sections 1, 6 and 9.
3. F. Reif, *Statistical and Thermal Physics*,(Fundamentals of Physics) chapter 15 for section 5.
4. J. M. Yeomans, *Statistical Mechanics of Phase Transitions*, (Oxford Science Publications) chapters 1 – 5 for sections 3 and 4.

C8.1a: Mathematics and the Environment — Prof. Sander & Dr Fowler — 16MT

Level: M-level **Method of Assessment:** Written examination.

Weight: Half-unit (OSS paper code 2A81)

Recommended Prerequisites

B6 highly recommended.

Overview

The aim of the course is to illustrate the techniques of mathematical modelling in their particular application to environmental problems. The mathematical techniques used are drawn from the theory of ordinary differential equations and partial differential equations. However, the course does require the willingness to become familiar with a range of different scientific disciplines. In particular, familiarity with the concepts of fluid mechanics will be useful.

Synopsis

Applications of mathematics to environmental or geophysical problems involving the use of models with ordinary and partial differential equations. Examples to be considered are: Climate dynamics. River flow and sediment transport. Glacier dynamics.

Reading

1. A. C. Fowler, *Mathematics and the Environment*, Mathematical Institute Notes (Revised edition, September 2004.)
2. K. Richards, *Rivers* (Methuen, 1982).
3. G. B. Whitham, *Linear and Nonlinear Waves* (Wiley, New York, 1974).
4. W. S. B. Paterson, *The Physics of Glaciers* (3rd edition, Pergamon Press, 1994).
5. J. T. Houghton, *The Physics of Atmospheres* (3rd ed., Cambridge University Press., Cambridge, 2002).

C8.1b: Mathematical Physiology — Prof. Maini — 16HT

Level: M-level **Method of Assessment:** Written examination.

Weight: Half-unit (OSS paper code 2B81).

Recommended Prerequisites

B8a highly recommended.

Overview

The course aims to provide an introduction which can bring students within reach of current research topics in physiology, by synthesising a coherent description of the physiological background with realistic mathematical models and their analysis. The concepts and treatment of oscillations, waves and stability are central to the course, which develops ideas introduced in the more elementary B8a course. In addition, the lecture sequence aims to build understanding of the workings of the human body by treating in sequence problems at the intracellular, intercellular, whole organ and systemic levels.

Synopsis

Review of enzyme reactions and Michaelis–Menten theory. Trans-membrane ion transport: Hodgkin–Huxley and Fitzhugh–Nagumo models.
Excitable media; wave propagation in neurons
Calcium dynamics; calcium-induced calcium release. Intracellular oscillations and wave propagation.
The electrochemical action of the heart. Cardiac fibres and propagation. Bidomian model and homogenization. Pacemakers, phase oscillators and the sinoatrial node. Spiral waves, tachycardia and fibrillation
Discrete delays in physiological systems. The Glass–Mackey model of respiration. Regulation of stem cell and blood cell production. Dynamical diseases.

Reading

The principal text is:

1. J. Keener and J. Sneyd, *Mathematical Physiology* (Springer-Verlag, 1998). Chs. 1, 4, 5, 9–12, 14–17. [Or: Second edition Vol I: Chs. 1, 2, 4, 5, 6, 7. Vol II: Chs. 11, 13, 14. (Springer-Verlag, 2009)]

Subsidiary mathematical texts are:

1. J. D. Murray, *Mathematical Biology* (Springer-Verlag, 2nd ed., 1993). [Third edition, Vols I and II, (Springer-Verlag, 2003).]
2. L. A. Segel, *Modeling Dynamic Phenomena in Molecular and Cellular Biology* (Cambridge University Press, 1984).
3. L. Glass and M. C. Mackey, *From Clocks to Chaos* (Princeton University Press, 1988).
4. P. Grindrod, *Patterns and Waves* (oup, 1991).

General physiology texts are:

1. R. M. Berne and M. N. Levy, *Principles of Physiology* (2nd ed., Mosby, St. Louis, 1996).
2. J. R. Levick, *An Introduction to Cardiovascular Physiology* (3rd ed. Butterworth–Heinemann, Oxford, 2000).
3. A. C. Guyton and J. E. Hall, *Textbook of Medical Physiology* (10th ed., W. B. Saunders Co., Philadelphia, 2000).

C9.1a: Analytic Number Theory — Prof. Heath-Brown — 16MT

Level: M-Level. **Method of Assessment:** Written examination.

Weight: Half-unit (OSS paper code 2A82)

Prerequisites

Complex analysis (holomorphic and meromorphic functions, Cauchy's Residue Theorem, Evaluation of integrals by contour integration, Uniformly convergent sums of holomorphic functions). Elementary number theory (Unique Factorization Theorem).

Overview

The course aims to introduce students to the theory of prime numbers, showing how the irregularities in this elusive sequence can be tamed by the power of complex analysis. The course builds up to the Prime Number Theorem which is the corner-stone of prime number theory, and culminates in a description of the Riemann Hypothesis, which is arguably the most important unsolved problem in modern mathematics.

Learning Outcomes

Students will learn to handle multiplicative functions, to deal with Dirichlet series as functions of a complex variable, and to prove the Prime Number Theorem and simple variants.

Synopsis

Introductory material on primes.

Arithmetic functions — Möbius function, Euler function, Divisor function, Sigma function — multiplicativity.

Dirichlet series — Euler products — von Mangoldt function.

Riemann Zeta-function — analytic continuation to $\mathrm{Re}(s) > 0$.

Non-vanishing of $\zeta(s)$ on $\mathrm{Re}(s) = 1$.

Proof of the prime number theorem.

The Riemann hypothesis and its significance.

The Gamma function, the functional equation for $\zeta(s)$, the value of $\zeta(s)$ at negative integers.

Reading

1. T.M. Apostol, *Introduction to Analytic Number Theory,* Undergraduate Texts in Mathematics (Springer-Verlag, 1976). Chapters 2,3,11,12 and 13.
2. M. Ram Murty, *Problems in Analytic Number Theory* (Springer, 2001). Chapters 1 – 5.
3. G.H. Hardy and E.M. Wright, *An Introduction to the Theory of Numbers* (Fifth edition, Oxford University Press, 1979). Chapters 16 ,17 and 18.
4. G.J.O. Jameson, *The Prime Number Theorem,* LMS Student Texts 53 (Cambridge University Press, 2003).

C9.1b Elliptic Curves — Prof. Heath-Brown — 16HT

Level: M-Level. **Method of Assessment:** Written examination.

Weight: Half-unit (OSS paper code 2B82).

Recommended Prerequisites

It is helpful, but not essential, if students have already taken a standard introduction to algebraic curves and algebraic number theory. For those students who may have gaps in their background, I have placed the file "Preliminary Reading" permanently on the Elliptic Curves webpage, which gives in detail (about 30 pages) the main prerequisite knowledge for the course. Go first to: http://www.maths.ox.ac.uk/courses/material then click on "C9.1b Elliptic Curves" and then click on the pdf file "Preliminary Reading".

Overview

Elliptic curves give the simplest examples of many of the most interesting phenomena which can occur in algebraic curves; they have an incredibly rich structure and have been the testing ground for many developments in algebraic geometry whilst the theory is still full of deep unsolved conjectures, some of which are amongst the oldest unsolved problems in mathematics. The course will concentrate on arithmetic aspects of elliptic curves defined over the rationals, with the study of the group of rational points, and explicit determination of the rank, being the primary focus. Using elliptic curves over the rationals as an example, we will be able to introduce many of the basic tools for studying arithmetic properties of algebraic varieties.

Learning Outcomes

On completing the course, students should be able to understand and use properties of elliptic curves, such as the group law, the torsion group of rational points, and 2-isogenies between elliptic curves. They should be able to understand and apply the theory of fields with valuations, emphasising the p-adic numbers, and be able to prove and apply Hensel's Lemma in problem solving. They should be able to understand the proof of the Mordell–Weil Theorem for the case when an elliptic curve has a rational point of order 2, and compute ranks in such cases, for examples where all homogeneous spaces for descent-via-2-isogeny satisfy the Hasse principle. They should also be able to apply the elliptic curve method for the factorisation of integers.

Synopsis

Non-singular cubics and the group law; Weierstrass equations.
Elliptic curves over finite fields; Hasse estimate (stated without proof).
p-adic fields (basic definitions and properties).
1-dimensional formal groups (basic definitions and properties).
Curves over p-adic fields and reduction mod p.
Computation of torsion groups over $\mathbb{Q}$; the Nagell–Lutz theorem.
2-isogenies on elliptic curves defined over $\mathbb{Q}$, with a $\mathbb{Q}$-rational point of order 2.
Weak Mordell–Weil Theorem for elliptic curves defined over $\mathbb{Q}$, with a $\mathbb{Q}$-rational point of order 2.
Height functions on Abelian groups and basic properties.
Heights of points on elliptic curves defined over $\mathbb{Q}$; statement (without proof) that this gives a height function on the Mordell–Weil group.
Mordell–Weil Theorem for elliptic curves defined over $\mathbb{Q}$, with a $\mathbb{Q}$-rational point of order 2.
Explicit computation of rank using descent via 2-isogeny.
Public keys in cryptography; Pollard's $(p-1)$ method and the elliptic curve method of factorisation.

Reading

1. J.W.S. Cassels, *Lectures on Elliptic Curves,* LMS Student Texts 24 (Cambridge University Press, 1991).
2. N. Koblitz, *A Course in Number Theory and Cryptography,* Graduate Texts in Mathematics 114 (Springer, 1987).
3. J.H. Silverman and J. Tate, *Rational Points on Elliptic Curves,* Undergraduate Texts in Mathematics (Springer, 1992).
4. J.H. Silverman, *The Arithmetic of Elliptic Curves,* Graduate Texts in Mathematics 106 (Springer, 1986).

Further Reading

1. A. Knapp, *Elliptic Curves, Mathematical Notes 40* (Princeton University Press, 1992).
2. G, Cornell, J.H. Silverman and G. Stevans (editors), *Modular Forms and Fermat's Last Theorem* (Springer, 1997).
3. J.H. Silverman, *Advanced Topics in the Arithmetic of Elliptic Curves,* Graduate Texts in Mathematics 151 (Springer, 1994).

C10.1a: Stochastic Differential Equations —Prof. Lyons—16MT

Level: M-level **Method of Assessment:** Written examination.

Weight: Half-unit (OSS paper code 2A83)

Recommended Prerequisites

A course on integration (in future years Part A integration will be required) and an introduction to discrete martingales, B10a Martingales Through Measure Theory, is expected.

Overview

Stochastic differential equations have been used extensively in many areas of application, including finance and social science as well as chemistry. This course develops the basic theory of Itô's calculus and stochastic differential equations, and gives a few applications.

Learning Outcomes

The student will have developed an appreciation of stochastic calculus as a tool that can be used for defining and understanding diffusive systems.

Synopsis

Itô's calculus: stochastic integrals with respect to martingales, Itô's lemma, Levy's theorem on characteristic of Brownian motion, exponential martingales, exponential inequality, Girsanov's theorem, The Martingale Representation Theorem. Stochastic differential equations: strong solutions, questions of existence and uniqueness, diffusion processes, Cameron–Martin formula, weak solution and martingale problem. Some selected applications chosen from option pricing, stochastic filtering etc.

Reading — Main Texts

1. Dr Qian's online notes:
   www.maths.ox.ac.uk/courses/2007/part-c/c101a-stochastic-differential-equations/material
2. B. Oksendal, *Stochastic Differential Equations: An introduction with applications* (Universitext, Springer, 6th edition). Chapters II, III, IV, V, part of VI, Chapter VIII (F).
3. F. C. Klebaner, *Introduction to Stochastic Calculus with Applications* (Imperial College Press, 1998, second edition 2005). Sections 3.1 – 3.5, 3.9, 3.12. Chapters 4, 5, 11.

Alternative Reading

1. H. P. McKean, *Stochastic Integrals* (Academic Press, New York and London, 1969).

Further Reading

1. N. Ikeda & S. Watanabe, *Stochastic Differential Equations and Diffusion Processes* (North–Holland Publishing Company, 1989).
2. I. Karatzas and S. E. Shreve, *Brownian Motion and Stochastic Calculus,* Graduate Texts in Mathematics 113 (Springer-Verlag, 1988).
3. L. C. G. Rogers & D. Williams, *Diffusions, Markov Processes and Martingales Vol 1 (Foundations) and Vol 2 (Ito Calculus)* (Cambridge University Press, 1987 and 1994).

C10.1b: Brownian Motion in Complex Analysis — Dr Cass — 16HT

Level: M-level **Method of Assessment:** Written examination.

Weight: Half-unit (OSS paper code 2B83).

Recommended Prerequisites

Part A Analysis (and Part A Integration for future years), (B10a) Martingales through Measure and (C10.1) Stochastic Differential Equations.

Overview

Randomness plays a key role in determining the behaviour of many high dimensional systems and so is intimately connected with applications. However, it also plays a key role in our understanding of many aspects of pure mathematics. This course will look at the deep interaction between 2 dimensional Brownian motion and complex analysis. At the core of these interactions is the conformal invariance of Brownian motion observed by Levy and the relationship with harmonic functions (based on martingales) first observed by Kakutani and Doob.

Since that time there have been many developments and connections. The Hardy spaces of Fefferman and Stein, Value Distribution Theory, and most recently the stochastic Lowener equation (a topic of current and very exciting research).

We will use the conformal properties of Brownian motion to examine and prove some deep theorems about value distributions for complex functions.

Learning Outcomes

The student will have developed an appreciation of the role that Brownian motion and martingales can play in pure mathematics.

Synopsis

Brownian motion. Continuous martingales and Levy's characterisation in terms of Brownian motion. Conformal invariance of Brownian motion. Brownian motion tangles about two points and a proof of Picard's theorems. Harmonic functions on the disk and the solution of the Dirichlet problem. Burkholder's Inequalities in Hardy spaces. Harmonic and superharmonic functions — via probability. [Nevanlinna's Theorems].

Reading

There isn't a perfect book for this course and we will refer to research papers to a limited extent.

1. The Notes of Prof. Lyons

2. D. Burkholder, *Distribution Function Inequalities for Martingales*, Ann. Probability, 1 (1973) 19–42.

Further Reading

1. McKean, *Stochastic Integrals* (1969). Hard, short, with much relevant material and some mistakes! Excellent for the able!
2. K. E. Petersen, *Brownian Motion, Hardy Spaces and Bounded Mean Oscillation,* LMS Lecture Note Series, 28 (Cambridge University Press, 1977).
3. T. K. Carne, 'Brownian Motion and Nevanlinna Theory'. *Proc. London. Math. Soc.* (52) (1986), 349–68.
4. Richard F. Bass, *Probabilistic Techniques in Analysis* (Springer-Verlag, New York Inc, 1995) ISBN: 0387943870.
5. Lars Ahlfors, *Complex Analysis* (McGraw–Hill, 1979) ISBN: 0070006571.
6. Jean-Claude Gruet, 'Nevanlinna Theory, Fuchsian Functions and Brownian Motion Windings', Source: *Rev. Mat. Iberoamericana* 18 (2002), no. 2, 301–324.

C11.1a: Graph Theory — Prof. McDiarmid — 16MT

Level: M-level **Method of Assessment:** Written examination.

Weight: Half-unit (OSS paper code 2A85).

Recommended Prerequisites

None beyond elementary probability theory.

Overview

Graphs are among the simplest mathematical structures, but nevertheless have a very rich and well-developed structural theory. Graph Theory is an important area of mathematics, and also has many applications in other fields such as computer science.

The main aim of the course are to introduce the analysis of discrete structures, and particularly the use of extremal methods.

Learning Outcomes

The student will have developed an appreciation of extremal methods in the analysis and understanding of graphical structures.

Synopsis

Introduction. Trees. Euler circuits. Planar graphs.
Matchings and Hall's Theorem. Connectivity and Menger's Theorem.
Extremal problems. Long paths and cycles. Turán's Theorem. Erdős–Stone Theorem.
Graph colouring. The Theorem of Brooks. The chromatic polynomial.
Ramsey's Theorem.
Szemerédi's Regularity Lemma.

Reading

1. B. Bollobás, *Modern Graph Theory,* Graduate Texts in Mathematics 184 (Springer-Verlag, 1998)

Further Reading

1. J. A. Bondy and U. S. R. Murty, *Graph Theory: An Advanced Course*, Graduate Texts in Mathematics 244 (Springer–Verlag, 2007).
2. R. Diestel, *Graph Theory,* Graduate Texts in Mathematics 173 (third edition, Springer-Verlag, 2005).
3. D. West, *Introduction to Graph Theory* (second edition, Prentice–Hall, 2001).

C11.1b: Probabilistic Combinatorics — Prof. Riordan — 16HT

Level: M-level **Method of Assessment:** Written examination.

Weight: Half Unit (OSS code to be confirmed)

Recommended Prerequisites

C11.1a Graph Theory and Part A Probability.

Overview

Probabilistic combinatorics is a very active field of mathematics, with connections to other areas such as computer science and statistical physics. Probabilistic methods are essential for the study of random discrete structures and for the analysis of algorithms, but they can also provide a powerful and beautiful approach for answering deterministic questions. The aim of this course is to introduce some fundamental probabilistic tools and present a few applications.

Learning Outcomes

The student will have developed an appreciation of probabilistic methods in discrete mathematics.

Synopsis

Spaces of random graphs. Threshold functions.
First and second moment methods. Chernoff bounds. Applications to Ramsey numbers and random graphs.
Lovasz Local Lemma. Two-colourings of hypergraphs (property B).
Poisson approximation, and application to the distribution of small subgraphs. Janson's inequality.
Concentration of measure. Martingales and the Azuma–Hoeffding inequality.
Chromatic number of random graphs.
Talagrand's inequality.

Reading

1. N. Alon and J.H. Spencer, *The Probabilistic Method* (second edition, Wiley, 2000).

Further Reading

1. B. Bollobás, *Random Graphs* (second edition, Cambridge University Press, 2001).
2. M. Habib, C. McDiarmid, J. Ramirez-Alfonsin, B. Reed, ed., *Probabilistic Methods for Algorithmic Discrete Mathematics* (Springer, 1998).
3. S.Janson, T. Luczak and A.Rucinski, *Random Graphs* (John Wiley and Sons, 2000).
4. M. Mitzenmacher and E. Upfal, *Probability and Computing : Randomized Algorithms and Probabilistic Analysis* (Cambridge University Press, New York (NY), 2005).
5. M. Molloy and B. Reed, *Graph Colouring and the Probabilistic Method* (Springer, 2002).
6. R. Motwani and P. Raghavan, *Randomized Algorithms* (Cambridge University Press, 1995).

Numerical Linear Algebra and Analysis

C12.1a Numerical Linear Algebra — Prof. Wendland — 16MT

Level: M-level **Method of Assessment:** Written examination.

Weight: Half-unit (OSS paper code 2A88)

Recommended Prerequisites

Only elementary linear algebra is assumed in this course. The part A Numerical Analysis course would be helpful, indeed some swift review and extensions of some of the material of that course is included here.

Overview

Linear Algebra is a central and widely applicable part of mathematics. It is estimated that many (if not most) computers in the world are computing with matrix algorithms at any moment in time whether these be embedded in visualization software in a computer game or calculating prices for some financial option. This course builds on elementary linear algebra and in it we derive, describe and analyse a number of widely used constructive methods (algorithms) for various problems involving matrices.

Numerical Methods for solving linear systems of equations, computing eigenvalues and singular values and various related problems involving matrices are the main focus of this course.

Synopsis

Common problems in linear algebra. Matrix structure, singular value decomposition. QR factorization, the QR algorithm for eigenvalues. Direct solution methods for linear systems, Gaussian elimination and its variants. Iterative solution methods for linear systems.

Chebyshev polynomials and Chebyshev semi-iterative methods, conjugate gradients, convergence analysis, preconditioning.

Reading List

L. N. Trefethen and D. Bau III, *Numerical Linear Algebra* (SIAM, 1997).

J. W. Demmel, *Applied Numerical Linear Algebra* (SIAM, 1997).

A. Greenbaum, *Iterative Methods for Solving Linear Systems* (SIAM, 1997).

G. H. Golub and C. F. van Loan, *Matrix Computations* (John Hopkins University Press, 3rd edition, 1996).

H. C. Elman, D. J. Silvester and A. J. Wathen, *Finite Elements and Fast Iterative Solvers* (Oxford University Press, 1995), only chapter 2.

C12.1b Continuous Optimization — Dr Hauser — 16HT

Level: M-level **Method of Assessment:** Written examination.

Weight: Half-Unit (OSS paper code 2B88).

Overview

Optimization deals with the problem of minimising or maximising a mathematical model of an objective function such as cost, fuel consumption etc. under a set of side constraints on the domain of definition of this function. Optimization theory is the study of the mathematical properties of optimization problems and the analysis of algorithms for their solution. The aim of this course is to provide an introduction to nonlinear continuous optimization specifically tailored to the background of mathematics students.

Synopsis

Part 1: Unconstrained Optimization
Optimality conditions, Newton's method for nonlinear systems, Convergence rates, Steepest descent method, General line search methods (alternative search directions, e.g. Newton, CG, BFG, ...), Trust region methods, Inexact evaluation of linear systems, iterative methods and the role of preconditioners.

Part 2: Constrained Optimization
Optimality/KKT conditions, Lagrange Multipliers, Penalty methods and SQP for equality constrained optimization, Interior penalty / barrier methods for inequality constrained optimization.

Reading List

Lecture notes will be made available for downloading from the course webpage.
To complement the notes, reading assignments will be given from the book of J.Nocedal and S.J.Wright, *Numerical Optimisation*, (Springer, 1999).

C12.2a Approximation of Functions — Dr Sobey — 16MT

Level: M-Level. **Method of Assessment:** Written examination.

Weight: Half-unit (OSS paper code 2A89)

Recommended Prerequisites

None

Overview

The central idea in approximation of functions can be illustrated by the question: Given a set of functions A and an element $u \in A$, if we select a subset $B \subset A$, can we choose an element $U \in B$ so that U approximates u in some way? The course focuses on this question in the context of functions when the way we measure 'goodness' of approximation is either with an integral least square norm or with an infinity norm of the difference $u - U$.

The choice of measure leads to further questions: is there a best approximation; if a best approximation exists, is it unique, how accurate is a best approximation and can we develop algorithms to generate good approximations? This course aims to give a grounding in the advanced theory of such ideas, the analytic methods used and important theorems for real functions. As well as being a beautiful subject in its own right, approximation theory is the foundation for many of the algorithms of computational mathematics and numerical analysis.

Synopsis

Introduction to approximation. Approximation in L^2. Approximation in L^∞: Oscillation Theorem, Exchange Algorithm. Approximation with splines. Rational approximation. Approximation of periodic functions.

Reading

1. M. J. D. Powell, *Approximation Theory and Methods* Cambridge University Press.
2. P. J. Davis, *Interpolation & Approximation* (Dover 1981).

C12.2b Finite Element Methods for Partial Differential Equations — Dr Wathen — 16HT

Level: M-Level. **Method of Assessment:** Written examination.

Weight: Half-unit (OSS paper code 2B89).

Recommended Prerequisites

No formal prerequisites are assumed. The course builds on elementary calculus, analysis and linear algebra and, of course, requires some acquaintance with partial differential equations such as the material covered in the Maths Mods Waves and Diffusion course, in particular the Divergence Theorem. Part A Numerical Analysis would be helpful but is certainly not essential. Function Space material will be introduced in the course as needed.

Overview

Computational algorithms are now widely used to predict and describe physical and other systems. Underlying such applications as weather forecasting, civil engineering (design of structures) and medical scanning are numerical methods which approximately solve partial differential equation problems. This course gives a mathematical introduction to one of the more widely used methods: the finite element method.

Synopsis

Finite element methods represent a powerful and general class of techniques for the approximate solution of partial differential equations. The aim of this course is to introduce these methods for boundary value problems for the Poisson and related elliptic partial differential equations. Attention will be paid to formulation, analysis, implementation and applicability of these methods.

Reading List

The main text will be

Howard Elman, David Silvester & Andy Wathen, *Finite Elements and Fast Iterative Solvers* (Oxford University Press, 2005) [mainly Chapters 1 and 5]

and some of the introductory material is usefully covered in

Endre Suli and David Mayers, *An Introduction to Numerical Analysis* (Cambridge University Press, 2003) [Chapter 11 and in particular Chapter 14]

and

David Silvester, *A Finite Element Primer*, notes which can be found at
http://www.maths.manchester.ac.uk/ djs/primer.pdf

Another book on finite elements which may be useful for different parts of the course is

Claes Johnson, *Numerical Solution of Partial Differential Equations by the Finite Element Method* (Cambridge University Press, 1990). (Unfortunately out of print) [Chapter 1-4]

and the Computing Lab lecture notes

Endre Suli, *Finite Element Methods for Partial Differential Equations*, Oxford University Computing Laboratory, 2001.

are also useful.

CD : Dissertations on a Mathematical Topic

Level : M-level

Weight : Half unit (5,000 words) or whole unit (10,000).

You may offer either a whole unit or a half unit Dissertation in a Mathematical topic. A whole-unit is equivalent to a 32-hour lecture course and a half-unit is equivalent to a 16-hour lecture course. Students will have approximately 8 hours of supervision for a whole-unit dissertation distributed over the two terms. Similarly, for the half-unit approximately 4 hours of supervision. There are special lectures given on projects, in Trinity Term and Michaelmas Term on projects in general and also lectures on Latex. See the lecture list for details.

Candidates offering a dissertation should read the *Guidance Notes on Extended Essays and Dissertations in Mathematics* by the Projects Committee available at:

http://www.maths.ox.ac.uk/current-students/undergraduates/projects/.

Application You must apply to the Mathematics Project Committee in advance for approval. Proposals should be addressed to The Chairman of the Projects Committee, c/o Mrs Helen Lowe, Room DH61, Dartingtion House, and must be received before 12 noon on Friday of Week 0 of Michaelmas Full Term. For CD dissertations candidates should take particular care to remember that the project must have substantial mathematical content. The application form is available at http://www.maths.ox.ac.uk/current-students/undergraduates/projects/. Once your title has been approved, it may only be changed by approval of the Chairman of the Projects Committee.

Assessment Each project is blind double marked. The marks are reconciled through discussion between the two assessors which is overseen by the examiners. Please see the *Guidance Notes on Extended Essays and Dissertations in Mathematics* for detailed marking criteria and class descriptors.

Submission THREE copies of your dissertation,identified by your candidate number only, should be sent to the Chairman of Examiners, FHS of Mathematics Part C, Examination Schools, Oxford, to arrive no later than **12noon on Friday of week 9, Hilary Term 2011**. An electronic copy of your dissertation should also be submitted via the Mathematical Institute website. Further details may be found in the *Guidance Notes on Extended Essays and Dissertations in Mathematics.*

Extra Units- application required

If you wish to offer the following course for examination in 2011, you should apply in writing to the Academic Administrator, Mathematical Institute, no later that Friday of the seventh week of Michaelmas Term 2010. You must apply formally in this way. This is to ensure the questions on these subjects will be set as part of your examination in 2011.

C2.1b — Representation Theory of Symmetric Groups — Dr Danz —16HT

Level: M-level **Method of Assessment:** Written examination.

Weight: Half-unit (OSS paper code 2B62).

Recommended Prerequisites

A thorough knowledge of linear algebra and the second year algebra courses; in particular familiarity with the symmetric groups, (symmetric) group actions, quotient vector spaces, isomorphism theorems and inner product spaces will be assumed. Some familiarity with basic representation theory from B2 (group algebras, simple modules, reducibility, Maschke's theorem, Wedderburn's theorem, characters) will be an advantage.

Overview

The representation theory of symmetric groups is a special case of the representation theory of finite groups. Whilst the theory over characteristic zero is well understood, this is not so over fields of prime characteristic. The course will be algebraic and combinatorial in flavour, and it will follow the approach taken by G. James. One main aim is to construct and parametrise the simple modules of the symmetric groups over an arbitrary field. Combinatorial highlights include combinatorial algorithms such as the Robinson–Schensted–Knuth correspondence. The final part of the course will discuss some finite-dimensional representations of the general linear group $\mathrm{GL}_n(\mathbb{C})$, and connections with representations of symmetric groups. In particular we introduce tensor products, and symmetric and exterior powers.

Synopsis

Counting standard tableaux of fixed shape: Young diagrams and tableaux, standard-tableaux, Young–Frobenius formula, hook formula. Robinson–Schensted-Knuth algorithm and correspondence.

Construction of fundamental modules for symmetric groups: Action of symmetric groups on tableaux, tabloids and polytabloids; permutation modules on cosets of Young subgroups. Specht modules, and their standard bases. Examples and applications.

Simplicity of Specht modules in characteristic zero and classification of simple S_n-module over characteristic zero. Characters of symmetric groups, Murnaghan–Nakayama rule.

Submodule Theorem, construction of simple S_n-modules over a field of prime characteristic. Decomposition matrices. Examples and applications.

Some finite-dimensional $\mathrm{GL}_n(\mathbb{C})$-modules, in particular the natural module, its tensor powers, and its symmetric and exterior powers. Connections with representations of S_n over $\mathbb{C}$.

Reading

1. W. Fulton, *Young Tableaux,* London Mathematical Society Student Texts 35 (Cambridge University Press, 1997). From Part I and II.

2. D. Knuth, *The Art of Computer Programming, Volume 3* (Addison–Wesley, 1998). From Chapter 5.

3. B. E. Sagan, *The Symmetric Group: Representations, Combinatorial Algorithms, and Symmetric Functions,* Graduate Texts in Mathematics 203 (Springer–Verlag, 2000). Chapters 1 – 2.

Additional Reading

1. W. Fulton, J. Harris, *Representation Theory: A first course,* Graduate Texts in Mathematics, Readings in Mathematics 129 (Springer–Verlag, 1991). From Part I.

2. G. James, *The Representation Theory of the Symmetric Groups,* Lecture Notes in Mathematics 682 (Springer–Verlag, 1978).

3. G. James, A. Kerber, *The Representation Theory of the Symmetric Groups*, Encyclopaedia of Mathematics and its Applications 16, (Addison–Wesley, 1981). From Chapter 7.

4. R. Stanley, *Enumerative Combinatorics. Volume 2*, Cambridge Studies in Advanced Mathematics 62 (Cambridge University Press, 1999).

MS: Statistics Half-units

The other half units that students in Part C may take are drawn from Part C of the Honour School of Mathematics and Statistics. For full details of these half units see the syllabus and synopses for Part C of the Honour School Mathematics and Statistics, which are available on the Web at http://www.stats.ox.ac.uk/current_students/bammath/course_handbooks/

The Statistics half units available are as follows:

- MS1a Graphical Models and Inference
- MS1b Statistical Data Mining
- MS2a Bioinformatics and Computational Biology
- MS2b Stochastic Models in Mathematical Genetics

Computer Science: Half Units

The other half units that students in Part C may take are drawn from Part C of the Honour School of Mathematics and Computing. For full details of these half units see the syllabus and synopses for Part C of the Honour School Mathematics and Computing, which are available on the Web at http://www.comlab.ox.ac.uk/teaching/mcs/PartC/

Please note that these three courses will be examined by mini-project (as for MSc students). Mini-projects will be handed out to candidates on the last Friday of the term in which the subject is being taught, and you will have to hand it in to the Exam Schools by noon on Monday of Week 1 of the following term. The mini-project will be designed to be completed in about three days. It will include some questions that are more open-ended than those on a standard sit-down exam. The work you submit should be your own work, and include suitable references.

Please note that the Computer Science courses in Part C are 50% bigger than those in earlier years, i.e. for each Computer Science course in the 3rd year undergraduates are expected to undertake about 10 hours of study per week, but 4th year courses will each require about 15 hours a week of study. Lecturers are providing this extra work in a variety of ways, e.g. some will give 16 lectures with extra reading, classes and/or practicals, whereas others will be giving 24 lectures, and others still will be doing something in between. Students will need to look at each synopsis for details on this.

The Computing half units available are as follows:

- CCS1a Categories, Proofs and Processes
- CCS3b Quantum Computer Science
- CCS4b Automata, Logics and Games

Philosophy

The other half units that students in Part C may take are drawn from Part C of the Honour School of Mathematics and Philosophy. For full details of these half units see the syllabus and synopses for Part C of the Honour School Mathematics and Philosophy, which are available on the Web at http://www.philosophy.ox.ac.uk/undergraduate/course_descriptions

The Philosophy half units available are as follows:

- Rise of Modern Logic

OD : Dissertations on a Mathematically related Topic

Level : M-level

Weight : Half unit (5,000 words) or whole unit (10,000 words).

You may offer either a whole-unit or a half-unit Dissertation in a Mathematically related topic. For example, applications of mathematics to another field (eg Maths in Music), historical topics, topics concentrating on the analysis of statistical data, or topics concentrating on the production of computer-generated data will be acceptable as an Other mathematical option. (Topics in mathematical education are not allowed).

A whole-unit is equivalent to a 32-hour lecture course and a half-unit is equivalent to a 16-hour lecture course. Students will have approximately 8 hours of supervision for a whole-unit dissertation or 4 hours for a half unit distributed over the two terms. There are special lectures given on projects, in Trinity Term and Michaelmas Term on projects in general and also lectures on Latex. See the lecture list for details.

Candidates offering a dissertation should read the *Guidance Notes on Extended Essays and Dissertations in Mathematics* by the Projects Committee available at:

http://www.maths.ox.ac.uk/current-students/undergraduates/projects/.

Application You must apply to the Mathematics Project Committee in advance for approval. Proposals should be addressed to The Chairman of the Projects Committee, c/o Mrs Helen Lowe, Room DH61, Dartingtion House, and must be received before 12 noon on Friday of Week 0 of Michaelmas Full Term. The application form is available at http://www.maths.ox.ac.uk/current-students/undergraduates/projects/. Once your title has been approved, it may only be changed by approval of Chairman of the Project's Committee.

Assessment Each project is blind double marked. The marks are reconciled through discussion between the two assessors which is overseen by the examiners. Please see the *Guidance Notes on Extended Essays and Dissertations in Mathematics* for detailed marking criteria and class descriptors.

Submission THREE copies of your dissertation,identified by your candidate number only, should be sent to the Chairman of Examiners, FHS of Mathematics Part C, Examination Schools, Oxford, to arrive no later than **12noon on Friday of week 9, Hilary Term 2011**. An electronic copy of your dissertation should also be submitted via the Mathematical Institute website. Further details may be found in the *Guidance Notes on Extended Essays and Dissertations in Mathematics*.

3 Language Classes: French

Language courses in French may be offered by the University Language Centre.

Students in the FHS Mathematics may apply to take language classes. In 2010-2011, French language classes will be run in MT and HT. We have a limited number of places but if we have spare places we will offer these to joint school students, Mathematics and Computer Science, Mathematics and Philosophy and Mathematics and Statistics.

Two levels of French courses are offered, a lower level for those with a good pass at GCSE, and a higher level course for those with A/S or A level. Acceptance on either course will depend on satisfactory performance in the Preliminary Qualifying Test held in Week 1 of Michaelmas Term (Monday, 17.00-19.00 at the Language Centre). Classes at both levels will take place on Mondays, 17.00-19.00.

Performance on the course will not contribute to the class of degree awarded. However, upon successful completion of the course, students will be issued with a certificate of achievement which will be sent to their college.

Places at these classes are limited, so students are advised that they must indicate their interest at this stage. If you are interested but are unable to attend this presentation for some reason please contact the Academic Administrator in the Mathematical Institute (academic.administrator@maths.ox.ac.uk; (6)15203) as soon as possible.

Aims and rationale

The general aim of the language course is to develop the student's ability to communicate (in both speech and writing) in French to the point where he or she can function in an academic or working environment in a French-speaking country.

The course has been designed with general linguistic competence in the four skills of reading, writing, speaking and listening in mind. There will be opportunities for participants to develop their own particular interests through presentations and assignments.

Form and Content

The course will consist of a thirty-two contact hour course, together with associated work. Classes will be held in the Language Centre at two hours per week in Michaelmas and Hilary Terms.

The content of the course is based on coursebooks together with a substantial amount of supplementary material prepared by the language tutors. Participants should expect to spend an additional two hours per week on preparation and follow-up work.

The course aims to cover similar ground in terms of grammar, spoken and written language and topics. Areas covered will include:

Grammar:

- all major tenses will be presented and/or revised, including the subjunctive
- passive voice
- pronouns
- formation of adjectives, adverbs, comparatives
- use of prepositions
- time expressions

Speaking

- guided spoken expression for academic, work and leisure contact (e.g. giving presentations, informal interviews, applying for jobs)
- expressing opinions, tastes and preferences
- expressing cause, consequence and purpose

Writing

- Guided letter writing for academic and work contact
- Summaries and short essays

Listening

- Listening practice of recorded materials (e.g. news broadcasts, telephone messages, interviews)

- developing listening comprehension strategies

Topics (with related readings and vocabulary, from among the following)

- life, work and culture
- the media in each country
- social and political systems
- film, theatre and music
- research and innovation
- sports and related topics
- student-selected topics

Teaching staff

The course is taught by Language Centre tutors or Modern Languages Faculty instructors.

Teaching and learning approaches

The course uses a communicative methodology which demands active participation from students. In addition, there will be some formal grammar instruction. Students will be expected to prepare work for classes and homework will be set. Students will also be given training in strategies for independent language study, including computer-based language learning, which will enable them to maintain and develop their language skills after the course.

Entry

Two classes in French at (probably at Basic and Threshold levels) will be formed according to level of French at entry. The minimum entry standard is a good GCSE Grade or equivalent. Registration for each option will take place at the start of Michaelmas Term. A preliminary qualifying test is then taken which candidates must pass in order to be allowed to join the course.

Learning Outcomes

The learning outcomes will be based on internationally agreed criteria for specific levels (known as the ALTE levels), which are as follows:

Basic Level (corresponds to ALTE Level 2 “Can-do” statements)

- Can express opinions on professional, cultural or abstract matters in a limited way, offer advice and understand instructions.
- Can give a short presentation on a limited range of topics.
- Can understand routine information and articles, including factual articles in newspapers, routine letters from hotels and letters expressing personal opinions.
- Can write letters or make notes on familiar or predictable matters.

Threshold Level (corresponds to ALTE Level 3 “Can-do” statements)

- Can follow or give a talk on a familiar topic or keep up a conversation on a fairly wide range of topics, such as personal and Prof.essional experiences, events currently in the news.
- Can give a clear presentation on a familiar topic, and answer predictable or factual questions.
- Can read texts for relevant information, and understand detailed instructions or advice.
- Can make notes that will be of reasonable use for essay or revision purposes.
- Can make notes while someone is talking or write a letter including non- standard requests.

Assessment

There will a preliminary qualifying test in Michaelmas Term. There are three parts to this test: Reading Comprehension, Listening Comprehension, and Grammar. Candidates who have not studied or had contact with French for some time are advised to revise thoroughly, making use of the Language Centre’s French resources.

Students’ achievement will be assessed through a variety of means, including continuous assessment of oral performance, a written final examination, and a project or assignment chosen by individual students in consultation with their language tutor.

Reading comprehension, writing, listening comprehension and speaking are all examined. The oral component may consist of an interview, a presentation or a candidate’s performance in a formal debate or discussion.